# Title

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March 30, 2023

**Abstract**

Hello blah bldsjlfdsjk sjdklfs

## 1 Introduction

More text

## 2 Main Results

We start with a definition:

**Definition 1.** *Define a (k,d,n)-generative net with layer widths* $k_0, k_1, \ldots, k_d$ *with* $k = k_0$ *and* $k_d = n$ *and coefficients* $\{l_i\}_{i\in[d]} \subseteq \mathbb{N}^d$ *to be a function* $G :^k \to^n$ *of the form where for each* $i \in [d]$ *,* $\sigma_i : \mathbb{R}^{k_i} \to \mathbb{R}^{k_i}$ *is a piecewise affine function with pieces generated by no more than* $l_i k_i$ *hyperplanes.*

Our main theorem is the following:

**Theorem 1.** *The self-difference of the range of a* **??** $G$ *is contained in the range of a positive-homogeneous piecewise linear function* $\tilde{G} :^{2k+1} \to^n$ *with a number of linear conical pieces* $M$ *such that*

$\log M \leq 2kd \log\left(\frac{es}{k}\right), where$

$s = (\prod$

## 3 Proofs

We now prove the main result:

*Proof.* Notice that $G$ is a piecewise affine function with $N$ pieces and **??** lemma::

**Lemma 2.** *A* **??** *will have at most*

$\log N \leq kd \log\left(\frac{es}{k}\right) affinepieceswhere$

$s := (\prod$

It follows that $(x, y) \to G(x) - G(y)$ is also piecewise affine and has no more than $2N$ pieces. Then from **??** lemma::

**Lemma 3.** *Let $f : \mathbb{R}^k \to \mathbb{R}^n$ be piecewise affine with $N$ pieces. Then there exists a function $\tilde{f} : \mathbb{R}^{k+1} \to \mathbb{R}^n$ which is piecewise linear with conical pieces such that $(\tilde{f}) \supseteq (f)$ . Furthermore, $\tilde{f}((x,1)) \equiv f(x)$ and in the case that $f$ has polyhedral pieces, $\tilde{f}$ will have pieces that are polyhedral cones.*

there exists a function $\tilde{G} : \mathbb{R}^{2k+1} \to \mathbb{R}^n$ , and one can check that it has all the properties in the statement.

□

We used two lemmas in the previous proof; **??**, **??**. These are based on [**vershyninHighDimensionalProbabilityIntroduction2018**].We now show **??**.

*Proof.* From the definition of a **??**, the *ith* layer has an activation that is piecewise affine with no more than $l_i k_i$ affine pieces. This *ith* layer will have a in its domain contained in a finite number of subspaces that are of dimension of no more than $\min(k, k_i)$ . Therefore consider the *ith* activation acting on a $\min(k, k_i)-$ dimensional subspace. By **??** lemma::

**Lemma 4.** *Let $U$ be a $k-$ dimensional hyperplane in general position in $^n$ . Let there be $P$ hyperplanes $\{H_i\}_{i \in [P]}$ in $^n$ . These hyperplanes induce connected components between the hyperplanes that we call "cells". The subspace $U$ may intersect at most*
$\sum_{i=0}^{k} \binom{P}{i} \leq \left(\frac{eP}{k}\right)^k$ *cells.*

, the layer will generate no more than $\left(\frac{el_i k_i}{\min(k,k_i)}\right)^{\min(k,k_i)}$ pieces for each subspace in its domain. Notice that this expression is monotonically increasing in $k_i$ . Therefore, by increase $k_i$ to $\max(k, k_i)$ we get the upper bound
$\left(\frac{el}{k}\right)^k .It follows that the number of affine pieces \mathrm{N} of the full network is such that$
$\mathrm{N} \leq \prod$ And we now give the proof of lemma **??**;

*Proof.* Letting $S$ be the finite partition of $dom(f)$ corresponding to the affine pieces of $f$ , we have that $f(x) = \sum_{s \in S} W_s x + b_s$ for some $R^{n \times k}$ matrices $\{W_s\}$ and some vectors $\{b_s\} \subseteq \mathbb{R}^k$ . Let $s_0 \in S$ . For any vector $\tilde{x} \in \mathbb{R}^{k+1}$ with $\tilde{x} = (\tilde{x}_{[k]}, \tilde{x}_{k+1})$ , let
$() :=$ $k \leq 0(W s$ *0x̃[k]+ bs0x̃k*$) +$ $k$ *¿ 0*$\sum$ Created: **??**