



UPPSALA  
UNIVERSITET

# Coupling of thermomechanics with electromagnetism in FEniCS

**Bilen Emek Abali**

Associate Professor in Solid Mechanics  
Department of Materials Science and Engineering  
Uppsala University

March 25, 2021, Cambridge



## Thermomechanics and electromagnetism

Challenges in theory and implementation

- ▶ Coupling of electromagnetism and thermomechanics,  
ABRAHAM–MINKOWSKI debate
- ▶ Thermodynamically sound derivation of all  
constitutive equations using MINKOWSKI momentum
- ▶ Balances of mass, momentum, energy, electric charge,  
and FARADAY law, jump conditions

## Thermomechanics and electromagnetism

### Challenges in theory and implementation

- ▶ Coupling of electromagnetism and thermomechanics,  
**ABRAHAM–MINKOWSKI** debate
- ▶ Thermodynamically sound derivation of all  
constitutive equations using **MINKOWSKI** momentum
- ▶ Balances of mass, momentum, energy, electric charge,  
and **FARADAY** law, jump conditions
- ▶ Numerical method depends on the chosen gauge  
conditions
- ▶ Jump conditions to be implemented as terms in the  
variational formulation rather than element  
formulation
- ▶ Monolithic computation by using **LORENZ** gauge and  
jump conditions

## Implementation

Solving the weak form by using open-source packages:

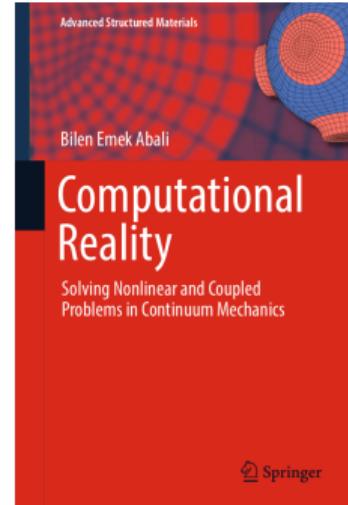
- ▶ CAD in Salome
- ▶ Mesh via NetGen in Salome
- ▶ Code in Python
- ▶ Assembly, linearization, solving via FEM in space and FDM in time by FEniCS
- ▶ Visualization in ParaView



## Implementation

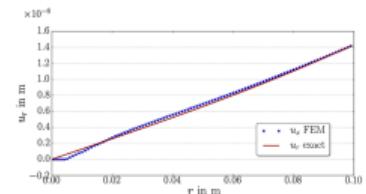
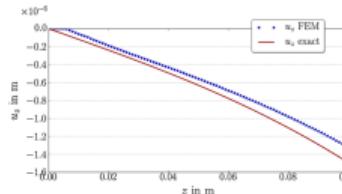
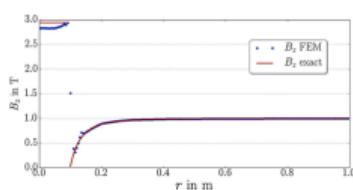
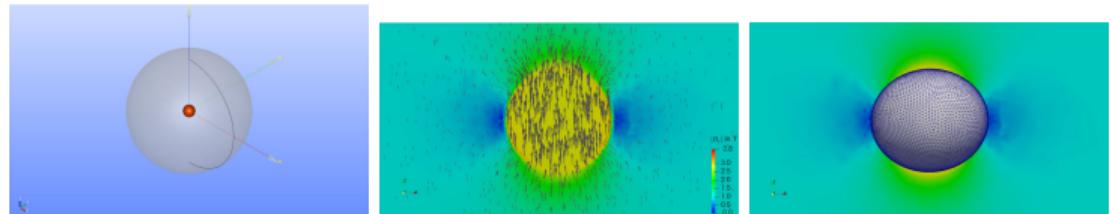
Simulation of multiphysics applications, FEM in space, FDM in time

- ▶ Elastostatics
- ▶ Nonlinear elasticity
- ▶ Plasticity
- ▶ Linear and nonlinear fluid dynamics
- ▶ Fluid-structure interaction
- ▶ Thermomechanics
- ▶ Electromagnetism
- ▶ Thermoelectric coupling
- ▶ Piezoelectricity
- ▶ Magnetohydrodynamics



## Verification of the method

- ▶ Thermodynamically sound derivation of all constitutive equations in electromagnetism and thermomechanics
- ▶ Computation of displacement,  $\mathbf{u}$ , and magnetic potential,  $\mathbf{A}$ , such that magnetic flux,  $\mathbf{B}$
- ▶ Analytical solution for verifying the novel numerical implementation using LORENZ gauge and jump conditions

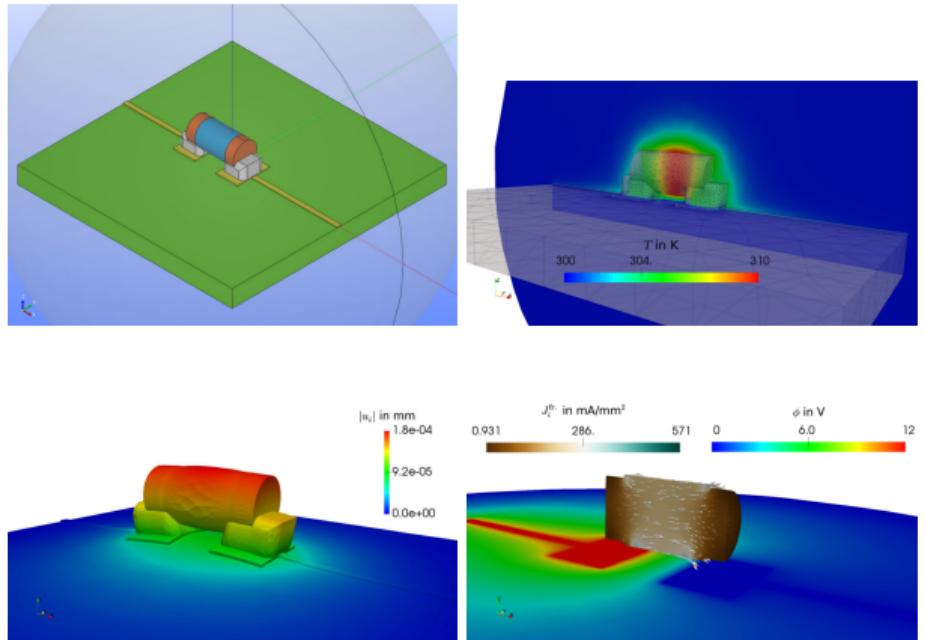


BEA and F. A. Reich. Continuum Mechanics and Thermodynamics 32.3 (2020), pp. 693-708.

BEA and F. A. Reich. Computer Methods in Applied Mechanics and Engineering 319 (2017), pp. 567-595.

## Multiphysics in electronics, transistor on a board

- ▶ Coupled constitutive equations in electromagnetism and thermomechanics
- ▶ Monolithic computation of displacement,  $\mathbf{u}$ , temperature,  $T$ , electric potential,  $\phi$ , magnetic potential,  $\mathbf{A}$
- ▶ Realistic Mini-MELF geometry and comparison to reduced order models



BEA and T. I. Zohdi. Journal of Computational Electronics 17.2 (2018), pp. 625–636.

## Thermal damage in lightning



BEA and T. I. Zohdi. Computational Mechanics 65.1 (2020), pp. 149–158.



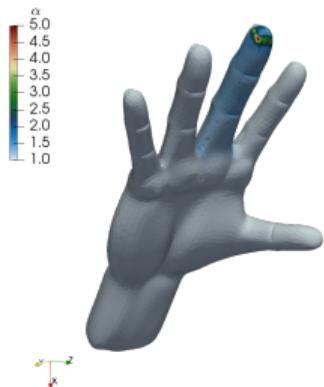
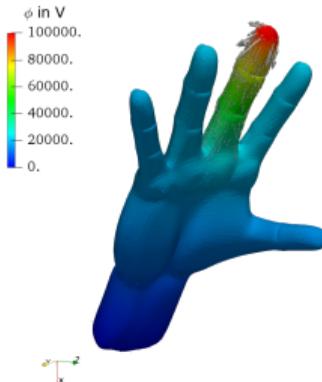
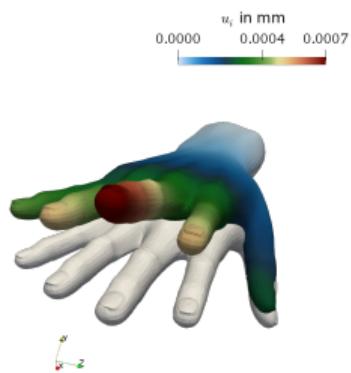
UPPSALA  
UNIVERSITET

Berkeley  
UNIVERSITY OF CALIFORNIA

DFG

Daimler und  
Benz Stiftung

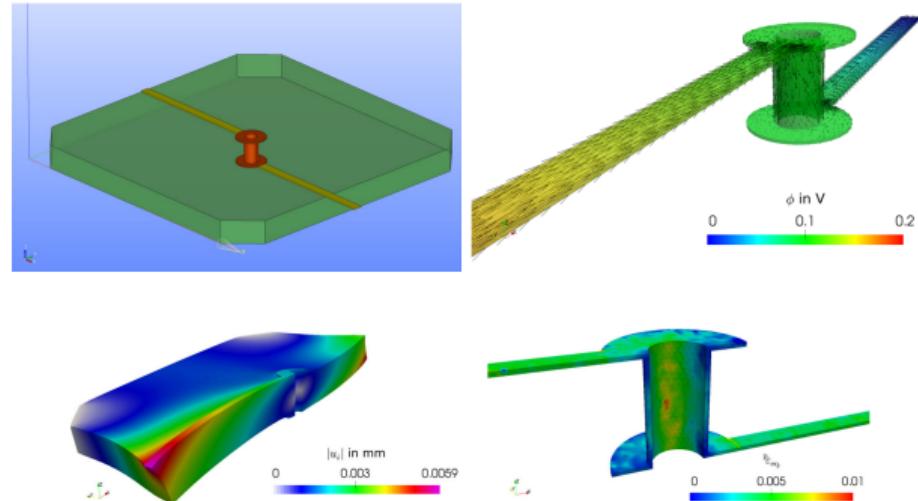
## Thermal damage in lightning



BEA and T. I. Zohdi. Computational Mechanics 65.1 (2020), pp. 149–158.

## Lifetime estimation in fatigue crack growth

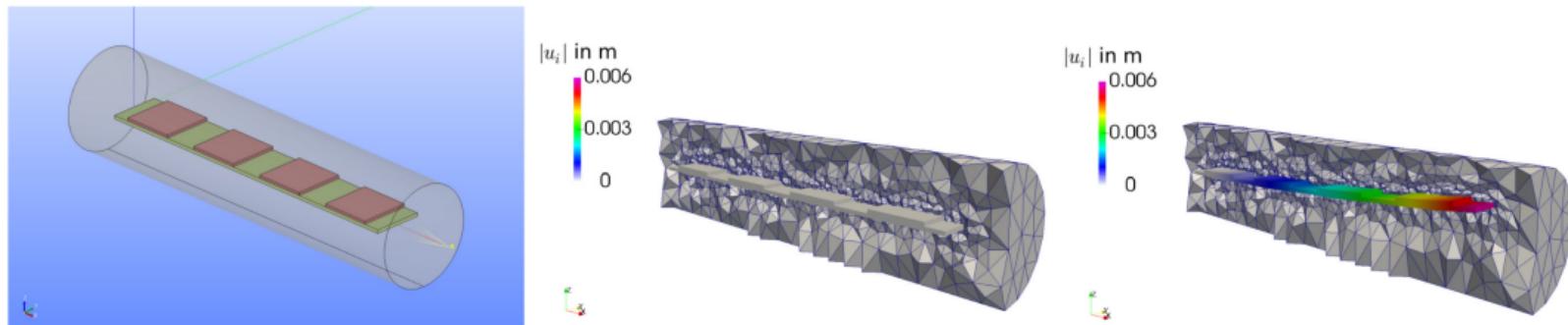
- ▶ Plasticity, thermodynamics, and electromagnetism by using experimentally determined material parameters
- ▶ Experimental validation of results
- ▶ COFFIN-MANSON type fatigue related damage by using accumulated plastic strain



**BEA**, W. H. Müller, H. Walter, O. Wittler, and M. Schneider-Ramelow. GMM-Facbericht, DVS 340 (2018), pp. 174–179.

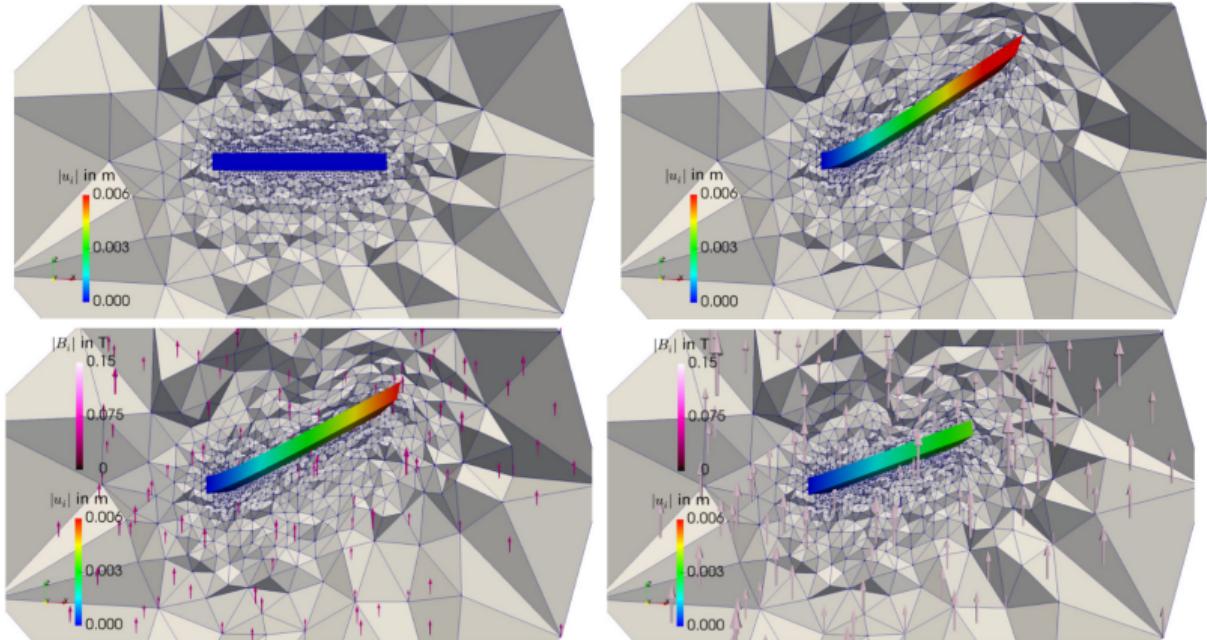
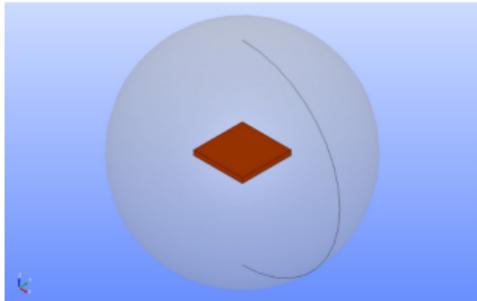
**BEA**. Mechanics of Advanced Materials and Modern Processes 3.1 (2017), pp. 1–11

## Piezoceramic fan under large displacement



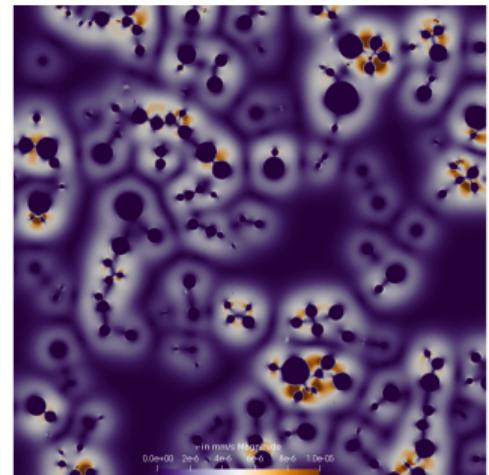
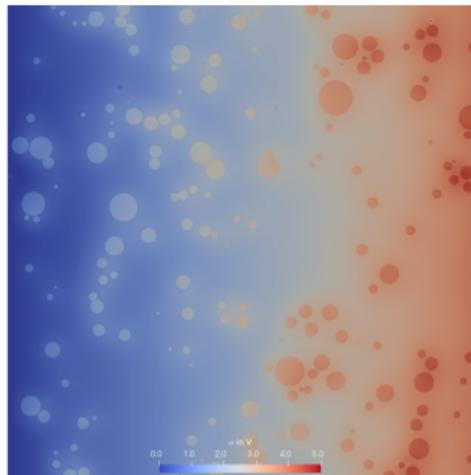
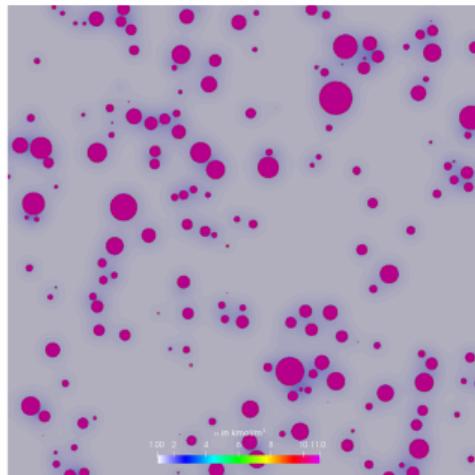
BEA and A. F. Queiruga. Journal of Computational Physics 394 (2019), pp. 200–231.

## Magnetorheological elastomer transducer



BEA and A. F. Queiruga. Journal of Computational Physics 394 (2019), pp. 200–231.

## Multiphysics in batteries, microscale computations



BEA. "Modeling mechanochemistry in Li-ion batteries". In: Scientific Computing in Electrical Engineering. Ed. by G. Nicosia and V. Romano. Vol. 32. Mathematics in Industry. Springer Nature, Cham, 2020. Chap. 8, pp. 79–91.

## What computations we can do?

- ▶ Solving coupled and nonlinear partial differential equations
- ▶ Thermomechanics and electromagnetism in solids and mixtures
- ▶ Reversible phenomena
  - ▶ Piezoelectricity
  - ▶ Pyroelectricity
  - ▶ Magnetothermal coupling
  - ▶ Electromagnetic coupling
- ▶ Irreversible phenomena
  - ▶ Thermoelectric coupling (Peltier elements)
  - ▶ Plasticity and damage



UPPSALA  
UNIVERSITET

*Thanks a lot!*

<http://bilenemek.abali.org>