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Finite elements on accelerators


An experience using **FEniCSx** and **SYCL**

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What's Performance Portability?

And why do we care about it?

An application is performance portable if it:

- ✓ Achieves reasonable level of performance
- ✓ Requires minimal platform specific code



Programming Model



SYCL is a high-level single source parallel programming model, that can target a range of heterogeneous platforms:

- uses completely standard C++;
- both host CPU and device code can be written in the same C++ source file;
- open standard coordinated by the **Khronos** group.

SYCL implementations:

Intel
SYCL*

hipSYCL*

Compute
Cpp

triSYCL*

**open source*

```
cl::sycl::queue q{cl::sycl::gpu_selector()};

int N = 100;
auto a = cl::sycl::malloc_device<double>(N, q);
auto b = cl::sycl::malloc_shared<double>(N, q);

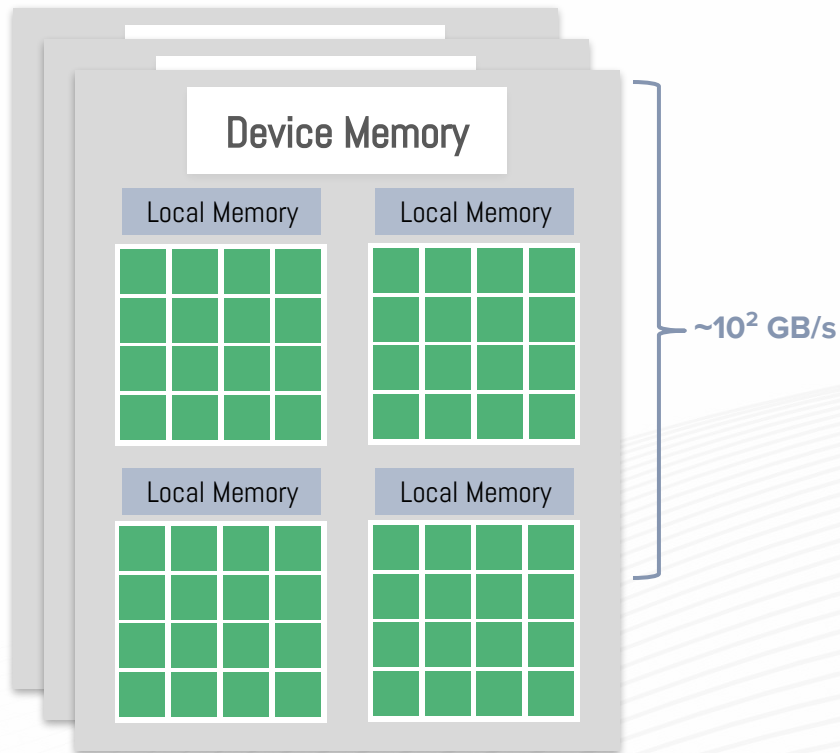
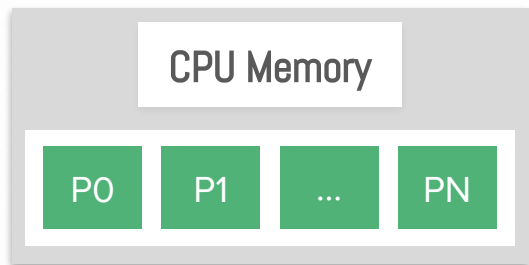
auto e = q.fill(a, 3.0, N);

q.parallel_for(cl::sycl::range<1>(N), e,
[=](cl::sycl::id<1> Id) {
    int i = Id.get(0);
    b[i] = 2 * a[i];
});

q.wait();

for (int i = 0; i < N; i++)
    assert(b[i] == 6.);
```

Simple Workflow



Copy input data from **Host** memory to **Device** memory



Launch kernels for execution on the **Device**

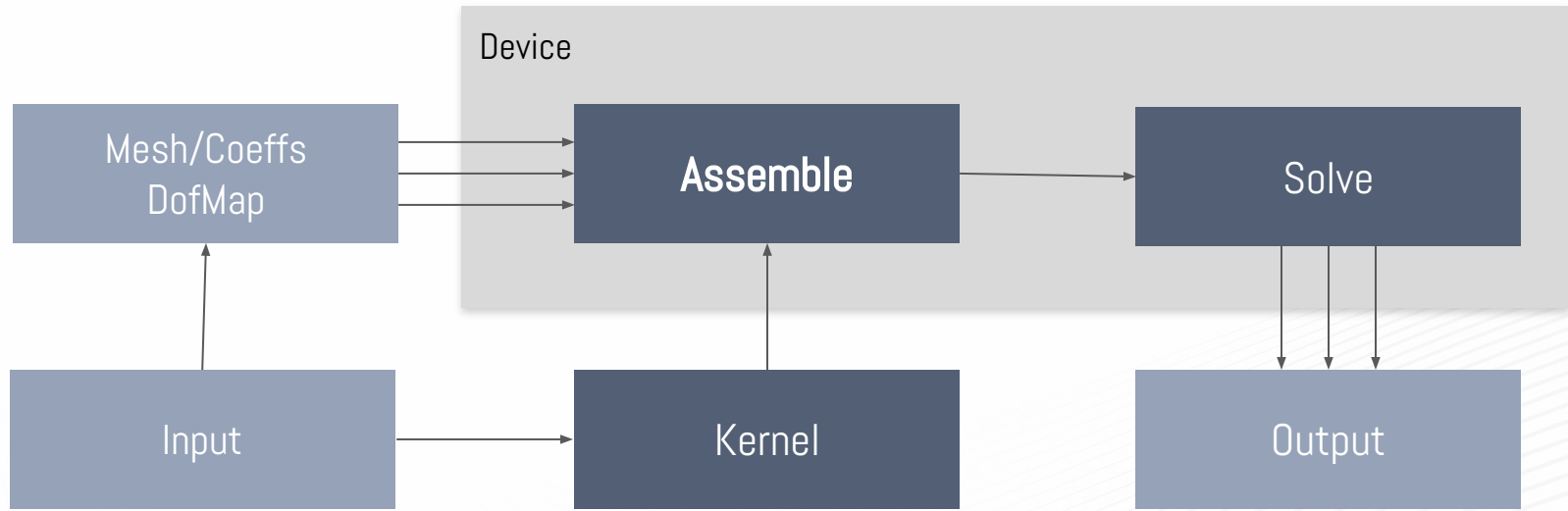


Wait for the execution queue to finish



Copy results back to **Host** from **Device**

An idealised modular Finite Element workflow

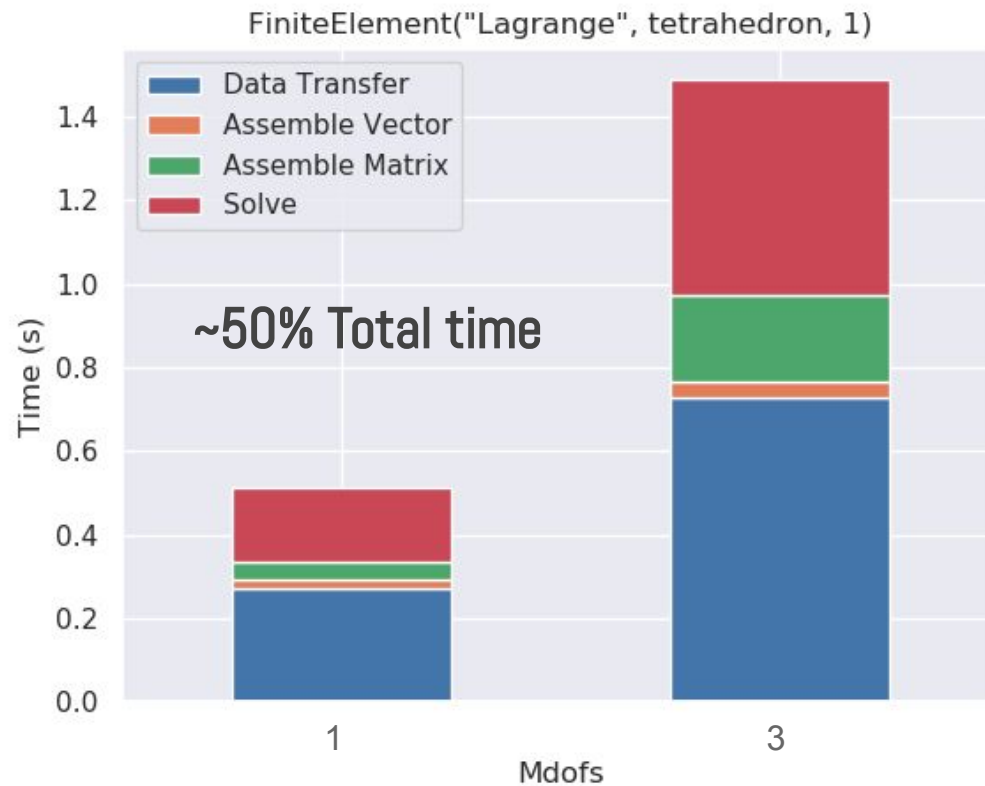


UFL
File

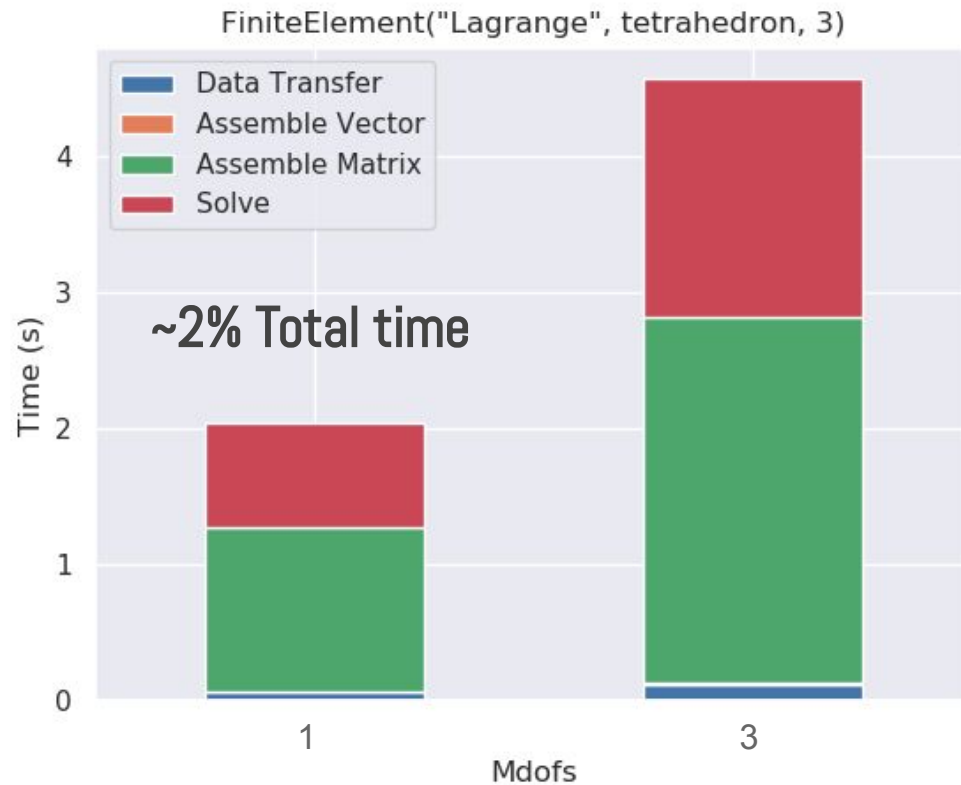
```
element = FiniteElement("Lagrange", tetrahedron, 3)
...
a = inner(grad(u), grad(v)) * dx + k*inner(u, v) * dx
L = inner(f, v) * dx
```

`ffcx --sycl_defines=True problem.ufl`

Data Transfer to Computation Ratio - P1



Data Transfer to Computation Ratio - P3



Matrix Assembly

For each cell:

- 01 Gather cell coordinates and coefficients
- 02 Compute element matrix
- 03 Update global CSR matrix

Global assembly strategies:

- Binary Search*
- Lookup Table*
- Two Stage

* *atomic operations*

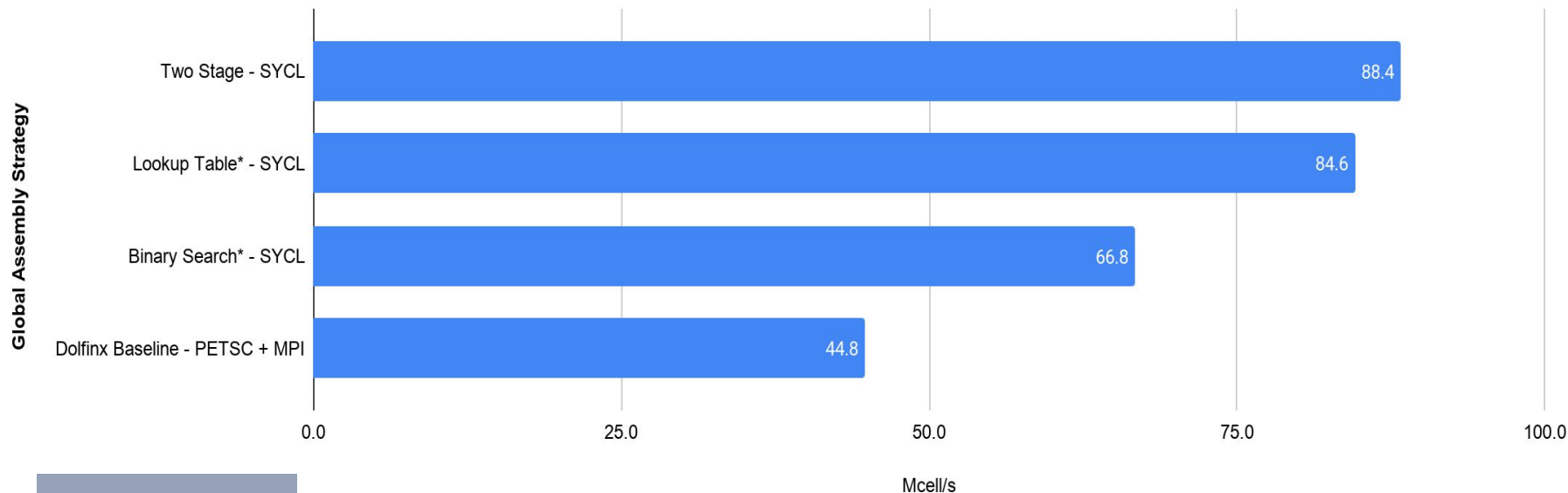
```
auto kernel = [=](cl::sycl::id<1> ID) {
    const int i = ID.get(0);
    ...
    double Ae [ndofs * nofs];

    // Gather cell coordinates and coefficients
    for (std::size_t j = 0; j < 4; ++j)
    {
        const std::size_t dmi = x_coor[i * 4 + j];
        for (int k = 0; k < gdim; ++k)
            cell_geom[j * gdim + k] = x[dmi * gdim + k];
    }
    ...
    // Compute element matrix
    tabulate_cell_a(Ae, coeffs, cell_geom);

    // Update global matrix - Binary Search
    for (int j = 0; j < ndofs; j++)
        for (int k = 0; k < ndofs; k++)
        {
            int ind = dofs[offset + k];
            int pos = find(indices, first, last, ind);
            atomic_ref atomic_A(data[pos]);
            atomic_A += Ae[j * ndofs + k];
        }
    };
```

Matrix Assembly - CPU Performance

Performance (MCells/s) P1 - 20 Mcells, 3 Mdofs

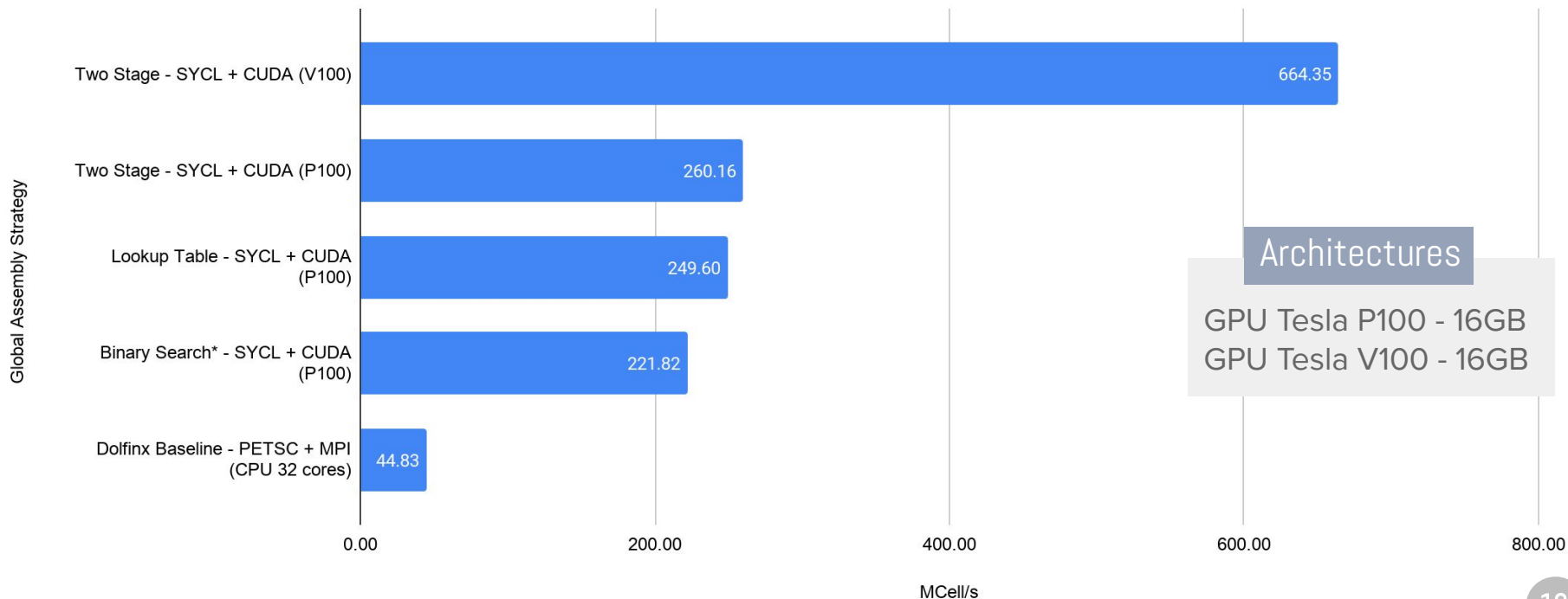


Architectures

2 x Intel Xeon Skylake 6142 processors, 2.6GHz 16-core
Theoretical peak performance: 2.7 TFlop/s.
192GB RAM

Matrix Assembly - GPU Performance

Performance (in Mcell/s) of assembling a CSR matrix for the Helmholtz problem on a GPU.
FiniteElement('Lagrange', tetrahedron, 1)

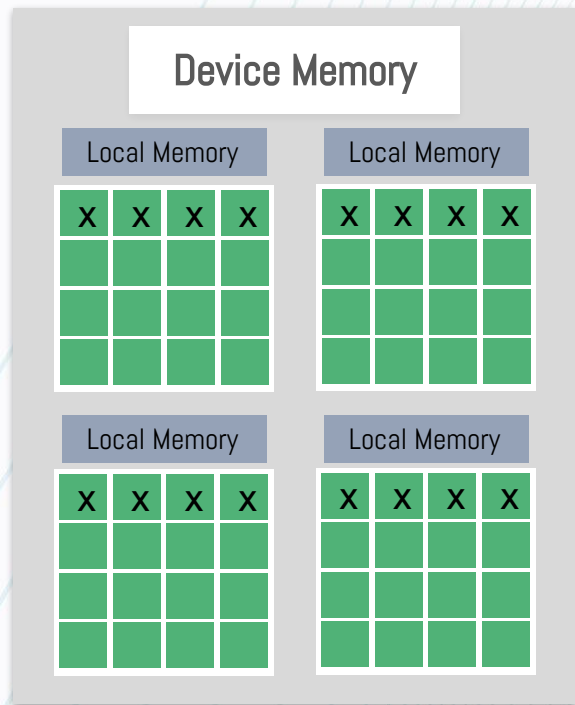


Matrix Assembly - GPU Performance

Performance (in Mcell/s) of assembling a CSR matrix for the Helmholtz problem on a GPU.
FiniteElement('Lagrange', tetrahedron, 1)



Low Achieved Occupancy



Achieved Occupancy: ~25%

The occupancy limited by register usage.

Solution:

Use shared memory for precomputed tables.

Each thread block (work-group) has shared memory visible to all threads (work-item) of the block.

	Occupancy	MCell/s
1st Version	25%	664 MCell/s
Shared Memory	63%	1660 MCell/s
Reference CUDA ¹	*	1627 MCells/s

[1] James Trotter - High-performance finite element computations - Performance modelling, optimisation, GPU acceleration & automated code generation - Phd Thesis 2021.

Thank you!

The code and reproducibility
instructions can be found at
<https://github.com/Excalibur-SLE/dolfinx.sycl>



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Future/Ongoing Work

Different problems,
and meshes

Linear Elasticity,
Maxwell's equations

Profiling in a wider
range of devices

AMD GPU, A64FX

Multi-GPU

MPI-based distributed
memory computations

Code transformation

Improve generated
code