

# MSD 2019 Final Project

A replication and extension of “Wage disparity and team productivity: evidence from Major League Baseball” by Craig A. Depken II, 1999

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## Contents

<b>Introduction</b>	<b>1</b>
Problem and Paper Description . . . . .	1
Problem Motivation . . . . .	1
Data Source . . . . .	1
<b>Reproduction</b>	<b>2</b>
Reproduction Code and Analysis . . . . .	2
Reproduction Notes . . . . .	4
<b>Extension</b>	<b>4</b>
Extension Motivation . . . . .	4
Extension Code and Analysis . . . . .	4
Extension Notes . . . . .	14
<b>Postface</b>	<b>15</b>

## Introduction

### Problem and Paper Description

Our goal was to replicate and extend the results seen in “Wage disparity and team productivity: evidence from major league baseball” by Craig A. Depken II. Depken’s paper explores whether or not wage disparity impacts a team’s performance. He examines this in the context of baseball where there are ready-made measures of both performance (team win percentage) and publicly available information about wages (player salaries). He evaluates his results with respect to two hypotheses: the danger-potential hypothesis (Ramaswamy and Rowthorn, 1991), which suggests that wage disparity indicates complementary skills of team-members and thus leads to better performance, and the team-cohesiveness theory (Levine, 1991), which suggests that it creates disfunction and thus negatively impacts performance.

### Problem Motivation

### Data Source

The data is taken from Sean Lahman’s baseball database, up to date as of 2019. The database is the most commonly used archive for baseball statistics. The latest version of the data can be accessed at <http://www.seanlahman.com/baseball-archive/statistics>.

# Reproduction

## Reproduction Code and Analysis

Read in the data.

```
teams <- read_csv(here('data/teams.csv'))
salaries <- read_csv(here('data/salaries.csv'))
```

Clean the data by calculating the win percentage and removing an incomplete data point. The Lahman database is clearly missing some data for the 1987 Texas Rangers. For full data, see <https://www.baseball-reference.com/teams/TEX/1987.shtml>.

```
teams$WSWin <- as.logical(teams$WSWin == 'Y')
teams <- teams %>%
  filter(1985 <= yearID & yearID <= 2016) %>%
  mutate(winPercentage = W / (W + L) * 1000) %>%
  filter(yearID != 1987 & teamID != 'TEX')

salaries <- salaries %>%
  filter(1985 <= yearID & yearID <= 2016) %>%
  mutate(salaryMil = salary / 1000000) %>%
  filter(yearID != 1987 & teamID != 'TEX')
```

Compute total team salaries, measured in millions of dollars.

```
teams <- teams %>%
  inner_join(salaries) %>%
  group_by(yearID, teamID, G, W, L, WSWin, winPercentage) %>%
  summarize(totalSalaryMil = sum(salaryMil))
```

Compute the salary share of each player on their respective team for each year.

```
salaries <- salaries %>%
  inner_join(teams) %>%
  mutate(salaryShare = salaryMil / totalSalaryMil * 100) %>%
  mutate(salaryShareSquared = salaryShare ^ 2) %>%
  select(yearID, teamID, playerID, salary, salaryShare, salaryShareSquared)
```

Compute the Herfindahl-Hirschman Index for each team's salary. For more information, see [https://en.wikipedia.org/wiki/Herfindahl\\_index](https://en.wikipedia.org/wiki/Herfindahl_index). The index ranges from 0 to 10,000 where smaller values represent more equality in team salary distribution and larger values represent more inequality.

```
teams <- teams %>%
  inner_join(salaries) %>%
  group_by(yearID, teamID, G, W, L, winPercentage, WSWin, totalSalaryMil) %>%
  summarize(HHI = sum(salaryShareSquared))
```

Take the subset of the data from the years 1985 to 1998, inclusive, to match Depken's analysis.

```
teams_old <- teams %>%
  filter(1985 <= yearID & yearID <= 1998) %>%
  mutate(normalizedYear = yearID - 1985)
```

View the summary statistics of win percentage, total team salary, and team HHI for the subset of data.

```
summary(teams_old$winPercentage)
```

```
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
```

```
##    327.2    456.8    496.9    500.1    543.2    703.7
```

```
sd(teams_old$winPercentage)
```

```
## [1] 67.34053
```

```
summary(teams_old$totalSalaryMil)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    4.613 14.217  23.655  26.220  37.022  72.356
```

```
sd(teams_old$totalSalaryMil)
```

```
## [1] 14.00647
```

```
summary(teams_old$HHI)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    427.5  666.8   754.7   801.5   876.3  2158.3
```

```
sd(teams_old$HHI)
```

```
## [1] 220.2525
```

Our reproduction of summary statistics shows results that are nearly identical to the original paper. In particular, we see the same values for win percentage, with slight differences only due to rounding. Our total salary replication tends to show slightly higher values in terms of minimum, maximum, and mean. Our hypothesis for this can be seen in the notes section below. We also see slightly different values for HHI in terms of minimum and mean. This is likely due to slight differences between our dataset and the one used in the original paper. As noted in our initial step, there are occasionally incomplete data points, or teams where not every player was accounted for, which would explain the difference in mean. We are confident in our calculation of the metric given that our max HHI is identical to that in the original paper.

Run the fixed effects regression on data between 1985 and 1998, inclusive.

```
hhi_fixed_old <- lm(formula = winPercentage ~ totalSalaryMil + HHI + normalizedYear +
                    teamID + 0,
                    data = teams_old)
summary(hhi_fixed_old)$coefficients[1:3,]
```

```
##              Estimate Std. Error  t value    Pr(>|t|)
## totalSalaryMil  1.96115149 0.48091620  4.077948 5.803436e-05
## HHI             -0.04611478 0.02028827 -2.272977 2.372293e-02
## normalizedYear -4.08667575 1.76297250 -2.318060 2.110738e-02
```

Our regression results match the paper's regression results in terms of coefficients being of the same order of magnitude and sign. Notably, both show that wage disparity negatively impacts team performance, which supports the Levine's team-cohesiveness theory. The difference in coefficients is that our regression claims that every additional \$1M spent improves win percentage by almost 0.2%, while the paper claims this amount to be 0.17%. In addition, each additional year in time corresponds to a 0.4% drop in win percentage, compared to 0.26% in the original paper. Note that the interpretation for the effect of time on win percentage should not be taken literally. Instead, the negative relationship here suggests that equal amounts of expenditure are worth less in terms of win percentage as time goes on. It is encouraging that the HHI index coefficients are quite similar. We obtain a coefficient of -0.047, while the original paper obtains a coefficient of -0.064.

Run the random effects regression on data between 1985 and 1998, inclusive.

```
hhi_random_old <- lm(formula = winPercentage ~ totalSalaryMil + HHI + normalizedYear,
                    data = teams_old)
summary(hhi_random_old)$coefficients[1:4,]
```

##	Estimate	Std. Error	t value	Pr(> t )
## (Intercept)	515.9993052	15.18582553	33.979009	1.416422e-110
## totalSalaryMil	2.1067707	0.41153306	5.119323	5.180395e-07
## HHI	-0.0469241	0.01853488	-2.531665	1.180894e-02
## normalizedYear	-4.7782865	1.57044613	-3.042630	2.530509e-03

We see similar results when replicating the random effects model, which doesn't have team specific intercepts. It is notable that the relative changes in coefficients between our fixed and random effects regressions are very similar to that of the original paper. Our coefficients for salary and year increase in magnitude, while our coefficient for HHI remains almost identical. One odd note is that both our intercept and the original intercept suggest a default win percentage of greater than .500 or 50%.

## Reproduction Notes

We faced a couple of challenges when considering the context of the data.

First, the original paper did not describe how time fixed effects are accounted for. For example, one may choose to control for different expansion periods or different years. When experimenting with different regression formulas, we found that controlling for different years yielded results most similar to the original paper.

Additionally, the original paper did not include a discussion on how real world practices may affect the data. For example, baseball seasons have different periods in which a 25-man roster or 40-man roster period is allowed. Moreover, players are often designated to the minor leagues, cut, or traded to a different team in the middle of a season. We believe that this could have impacted our summary statistics results. We opted to take the data from the Lahman database verbatim, save for the removal of the incomplete data point mentioned above.

## Extension

### Extension Motivation

Given the paper's lack visualizations of the underlying data, we thought it was important to create several graphs to help readers understand the data, its distribution, and the various trends over time. Our other extensions were centered around better understanding the efficacy of the regression in the original paper and the parameters in it. As a result, we first wanted to see how the regression would change when it was re-run on more recent data. Next, we compared how the regressions would change with different inequality metrics, as HHI is a relatively uncommon metric. Lastly, we wanted to see how well the regression actually predicted the results and did so with different inequality metrics to see how the results changed.

### Extension Code and Analysis

Subset the data to the years 1999 to 2016, inclusive. This is because the Lahman dataset doesn't provide salary information past 2016.

```
teams_new <- teams %>%
  filter(1999 <= yearID & yearID <= 2016) %>%
  mutate(normalizedYear = yearID - 1999)
```

Run the fixed effects regression on data between 1999 and 2016, inclusive.

```
hhi_fixed_new <- lm(formula = winPercentage ~ totalSalaryMil + HHI + normalizedYear +
                    teamID + 0,
                    data = teams_new)
summary(hhi_fixed_new)$coefficients[1:3,]
```

```
##              Estimate Std. Error   t value    Pr(>|t|)
## totalSalaryMil  0.49949048 0.13266465  3.765061 0.0001868427
## HHI            -0.05358433 0.01442365 -3.715033 0.0002267173
## normalizedYear -2.00506563 0.73422847 -2.730847 0.0065462996
```

Run the random effects regression on data between 1999 and 2016, inclusive.

```
hhi_random_new <- lm(formula = winPercentage ~ totalSalaryMil + HHI + normalizedYear,
                     data = teams_new)
summary(hhi_random_new)$coefficients[1:4,]
```

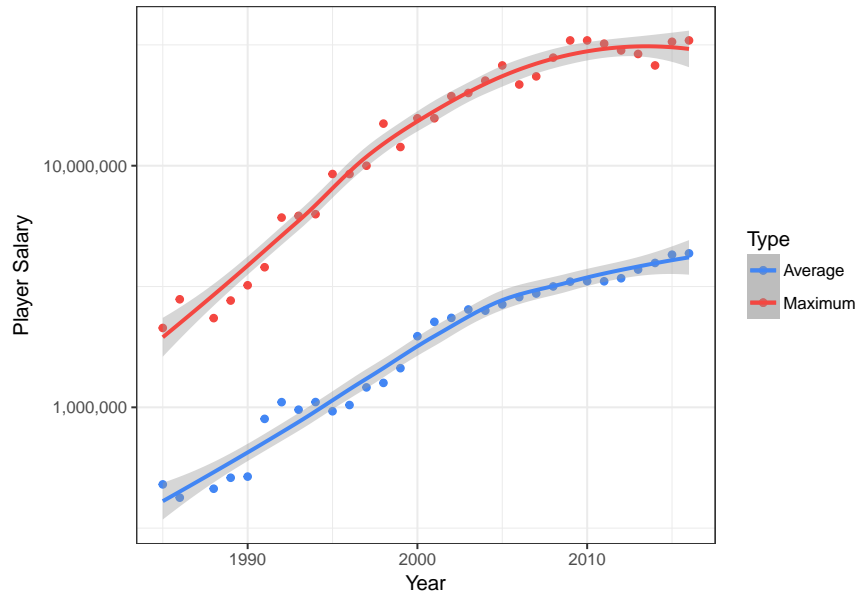
```
##              Estimate Std. Error   t value    Pr(>|t|)
## (Intercept)  503.61973192 15.73186679 32.012713 7.433408e-125
## totalSalaryMil  0.70690261 0.08558449  8.259705 1.226558e-15
## HHI            -0.04403327 0.01403935 -3.136419 1.807431e-03
## normalizedYear -2.75511624 0.63636357 -4.329469 1.794894e-05
```

When both models were run on the data from 1999-2016, the coefficients matched the earlier years in terms of sign. The HHI coefficient is the same in magnitude as in the early year regressions, but the coefficients for both salary and time decrease. This makes sense, as the y-values the regression is fitting to remain between 40%-70% for win percentage but total salaries of teams are increasing over time, as is the year. As a result, a million dollar increase in total salary has a comparatively smaller effect on winning than in previous years.

Plot the relationship between annual player salary and time.

```
salary_vs_time <- salaries %>%
  group_by(yearID) %>%
  summarize(avg = mean(salary), max = max(salary))

ggplot(data = salary_vs_time) +
  geom_point(aes(x = yearID, y = avg, color = 'Average')) +
  geom_smooth(aes(x = yearID, y = avg, color = 'Average')) +
  geom_point(aes(x = yearID, y = max, color = 'Maximum')) +
  geom_smooth(aes(x = yearID, y = max, color = 'Maximum')) +
  scale_color_manual(values = c('#4286f4', '#f44741')) +
  scale_y_log10(labels = comma) +
  labs(color = 'Type') +
  xlab('Year') +
  ylab('Player Salary')
```

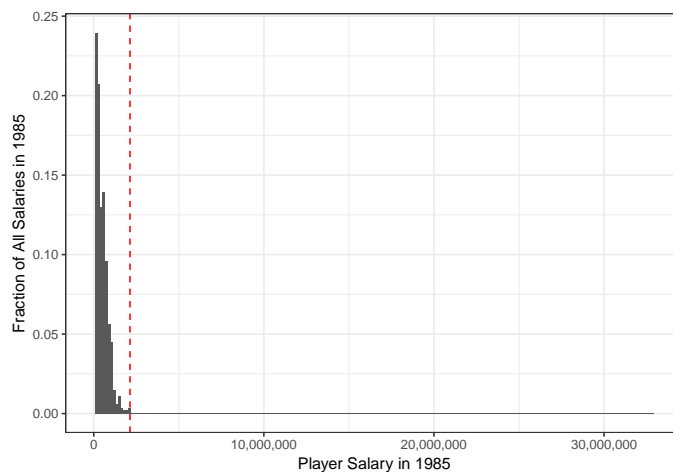


The plot suggests that both the average annual player salary and maximum annual player salary have increased over time. Note that the  $y$ -axis is displayed on a log scale. Therefore, the maximum annual player salary is growing at a much faster rate than the average annual player salary.

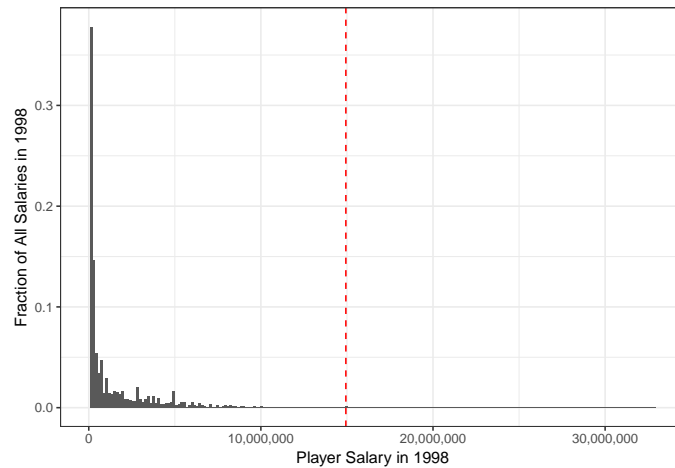
Plot the distributions of annual player salaries from three different years. First is 1985, the start year determined by the original paper. Second is 1998, the year of the last expansion of Major League Baseball in which the 29th and 30th teams were established. Third is 2016, the most recent year for which salary data is available. The red dashed lines denote the maximum annual player salary of each specific year.

```
salaries_1985 <- filter(salaries, yearID == 1985)
salaries_1998 <- filter(salaries, yearID == 1998)
salaries_2016 <- filter(salaries, yearID == 2016)

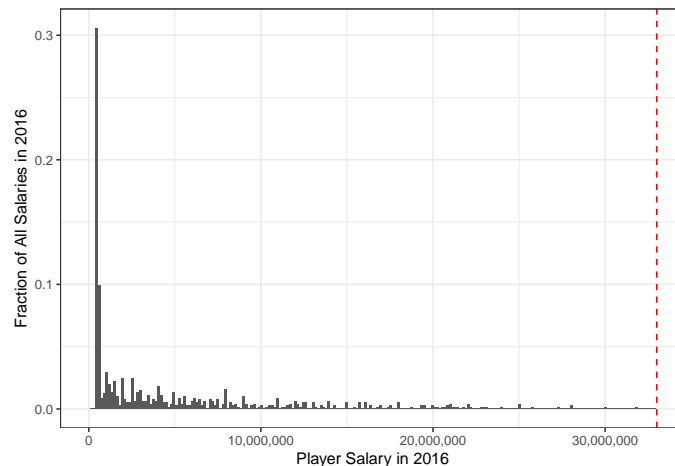
ggplot(data = salaries_1985) +
  geom_histogram(aes(x = salary, y = (..count..) / sum(..count..)), binwidth = 150000) +
  geom_vline(xintercept = max(salaries_1985$salary), color = 'red', linetype = 'dashed') +
  scale_x_continuous(limits = c(0, max(salaries$salary)), labels = comma) + xlab('Player Salary in 1985') +
  ylab('Fraction of All Salaries in 1985')
```



```
ggplot(data = salaries_1998) +
  geom_histogram(aes(x = salary, y = (..count..) / sum(..count..)), binwidth = 150000) +
  geom_vline(xintercept = max(salaries_1998$salary), color = 'red', linetype = 'dashed') +
  xlim(0, max(salaries$salary)) +
  scale_x_continuous(limits = c(0, max(salaries$salary)), labels = comma) +
  xlab('Player Salary in 1998') +
  ylab('Fraction of All Salaries in 1998')
```



```
ggplot(data = salaries_2016) +
  geom_histogram(aes(x = salary, y = (..count..) / sum(..count..)), binwidth = 150000) +
  geom_vline(xintercept = max(salaries_2016$salary), color = 'red', linetype = 'dashed') +
  scale_x_continuous(limits = c(0, max(salaries$salary)), labels = comma) +
  xlab('Player Salary in 2016') +
  ylab('Fraction of All Salaries in 2016')
```



The plots suggest that we see longer and longer tails as larger contracts are awarded over time. In 1998, we see that most annual player salaries are clumped at lower values. In 2016, we see that many annual player salaries are near the league minimum \$507,500 and few annual player salaries are above \$10,000,000.

Plot the relationship between average annual team salary and time. A few key teams are highlighted.

```
current_teamIDs <- c('ARI', 'ATL', 'BAL', 'BOS', 'CHA', 'CHN', 'CIN', 'CLE', 'COL', 'DET',
  'HOU', 'KCA', 'LAA', 'LAN', 'MIA', 'MIL', 'MIN', 'NYA', 'NYN', 'OAK',
  'PHI', 'PIT', 'SDN', 'SEA', 'SFN', 'SLN', 'TBA', 'TEX', 'TOR', 'WAS')
```

```

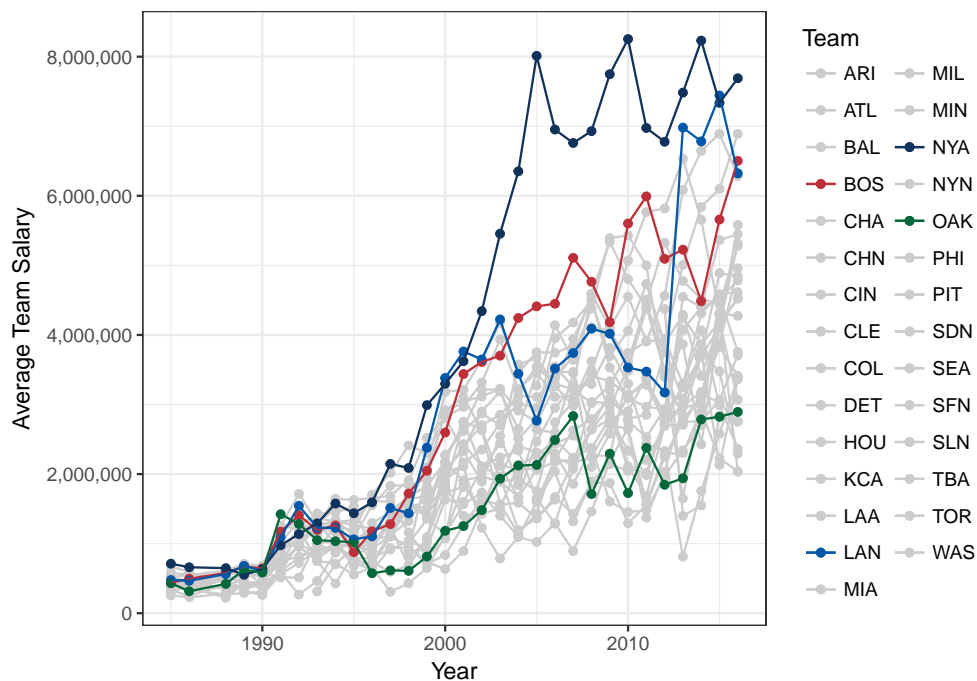
team_colors <- c('#cccccc', '#cccccc', '#cccccc', '#BD3039', '#cccccc',
                '#cccccc', '#cccccc', '#cccccc', '#cccccc', '#cccccc',
                '#cccccc', '#cccccc', '#cccccc', '#0157a8', '#cccccc',
                '#cccccc', '#cccccc', '#11325b', '#cccccc', '#04683b',
                '#cccccc', '#cccccc', '#cccccc', '#cccccc', '#cccccc',
                '#cccccc', '#cccccc', '#cccccc', '#cccccc', '#cccccc')
colored_teamIDs <- c('BOS', 'LAN', 'NYA', 'OAK')

team_salary_vs_time <- salaries %>%
  filter(teamID %in% current_teamIDs) %>%
  group_by(yearID, teamID) %>%
  summarize(avg = mean(salary)) %>%
  mutate(flag = teamID %in% colored_teamIDs)

underlay_data <- filter(team_salary_vs_time, !flag)
overlay_data <- filter(team_salary_vs_time, flag)

ggplot() +
  geom_point(data = underlay_data, aes(x = yearID, y = avg, color = teamID)) +
  geom_line(data = underlay_data, aes(x = yearID, y = avg, color = teamID)) +
  geom_point(data = overlay_data, aes(x = yearID, y = avg, color = teamID)) +
  geom_line(data = overlay_data, aes(x = yearID, y = avg, color = teamID)) +
  scale_y_continuous(labels = comma) +
  scale_color_manual(values = team_colors) +
  labs(color = 'Team') +
  xlab('Year') +
  ylab('Average Team Salary')

```



The plot suggests, as is expected, that average annual team salary has increased with time for all teams. Such an increase in expenditure can be attributed to both general inflation and the rise of sports entertainment business. The New York Yankees, the most successful franchise by number of championships, have spent

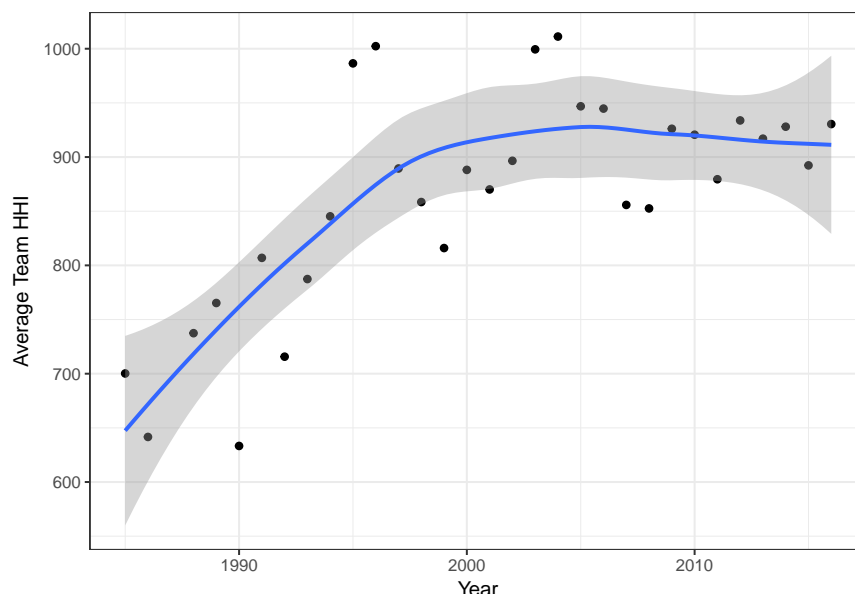


a lot throughout their history. Contrast this with the Oakland Athletics, a franchise who has historically traded for players based on value, who have spent relatively little throughout their history.

Plot the relationship between average team HHI and time.

```
hhi_vs_time <- teams %>%
  group_by(yearID) %>%
  summarize(avg = mean(HHI))

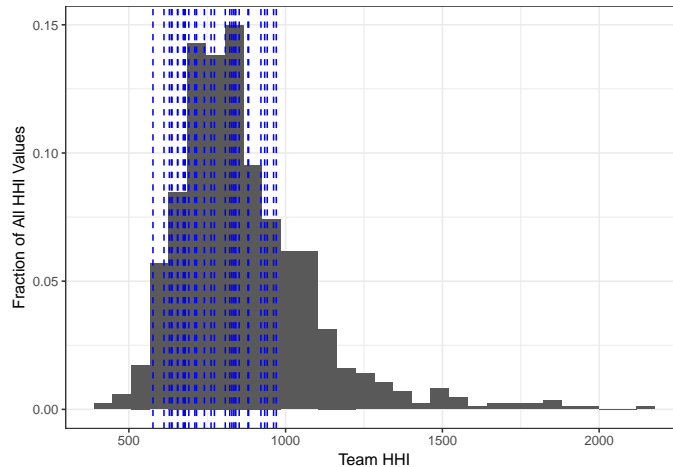
ggplot(data = hhi_vs_time) +
  geom_point(aes(x = yearID, y = avg)) +
  geom_smooth(aes(x = yearID, y = avg)) +
  xlab('Year') +
  ylab('Average Team HHI')
```



The plot suggests that average team HHI has experienced two trends through the lifetime of the data. From 1985 to 1999, we see an increase in intrateam wage disparity. From 2000 to 2016, we see that intrateam wage disparity remains relatively unchanged. This flattening of the curve may seem counterintuitive when considering that maximum annual player salaries have increased at a growing rate over time. We hypothesize that this phenomenon is due to the drastic increases in league minimum salary in the latter period. From 1985 to 1999, the league minimum rose from \$60,000 to \$109,000, or ~81%. From 2000 to 2016, the league minimum rose from \$200,000 to \$507,500, or ~153%. League minimum salary data is taken from [https://www.baseball-reference.com/bullpen/Minimum\\_salary](https://www.baseball-reference.com/bullpen/Minimum_salary).

Plot the distribution of team HHI using all teams from 1985 to 2016. The dashed vertical lines represent HHI values for teams that won the World Series.

```
ggplot(data = teams) +
  geom_histogram(aes(x = HHI, y = (..count..) / sum(..count..))) +
  geom_vline(data = filter(teams, WSWin), aes(xintercept = HHI), color = 'blue', linetype = 'dashed') +
  xlab('Team HHI') +
  ylab('Fraction of All HHI Values')
```



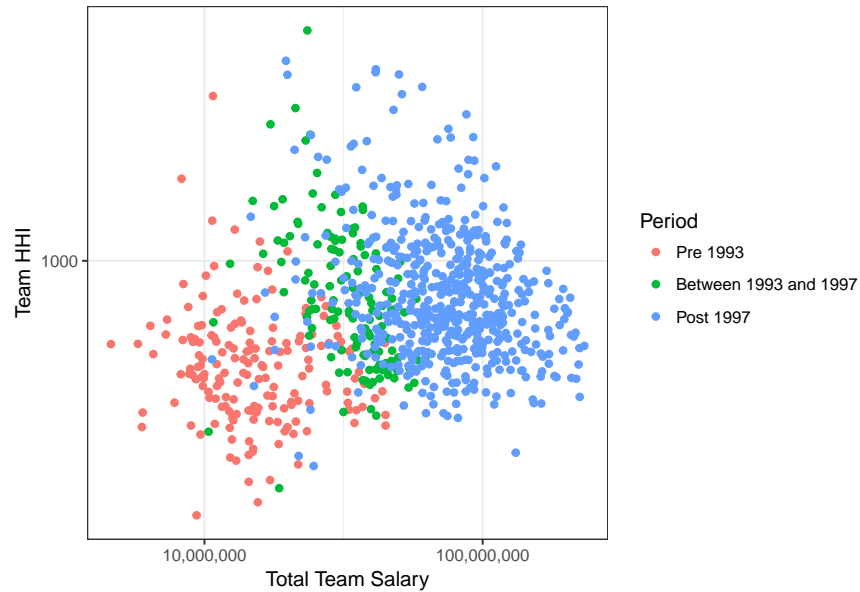
The plot suggests that too much intrateam wage disparity can negatively affect a team's chances of winning the World Series. Namely, all teams that won the World Series between 1985 and 2016 had an intrateam HHI of less than 1,000. For reference, an HHI of less than 1500 is considered to reflect a competitive marketplace, an HHI of 1,500 to 2,500 is considered to reflect a moderately concentrated marketplace, and an HHI of 2,500 or greater is considered to reflect a highly concentrated marketplace. We hypothesize that this finding is relevant to baseball itself and may not hold when considering other sports. Baseball is a game in which every player must take turns batting in a specified order, unlike basketball in which a single player can control the ball every possession. Hence, the affect of a star player on the outcome of a game is smaller in baseball relative to other sports. In other words, depth of a roster is far more important in baseball. Signing a star player to a huge contract may hurt a team's ability to offer decent contracts to other important role players, which may ultimately hurt a team's performance.

Plot the relationship between team HHI and total team salary. The data is split by time period based on the 1993 expansion and the 1998 expansion.

```
year_to_period <- function(year) {
  if (year <= 1992)
    return('Pre 1993')
  else if (1993 <= year & year <= 1997)
    return('Between 1993 and 1997')
  else
    return('Post 1997')
}

hhi_vs_total_salary <- mutate(teams, period = year_to_period(yearID))
hhi_vs_total_salary$period <- factor(hhi_vs_total_salary$period,
                                     levels = c('Pre 1993', 'Between 1993 and 1997', 'Post 1997'))

ggplot(data = hhi_vs_total_salary) +
  geom_point(aes(x = totalSalaryMil * 1000000, y = HHI, color = period)) +
  scale_x_log10(labels = comma) +
  scale_y_log10() +
  labs(color = 'Period') +
  xlab('Total Team Salary') +
  ylab('Team HHI')
```



The plot suggests some form of clustering between the three time periods. Since total team salary can be used as a proxy for time, we see that the average team HHI exhibits a similar growing and then flattening trend seen in a previous plot.

Compute the Gini coefficient for each team's salary. For more information, see [https://en.wikipedia.org/wiki/Gini\\_coefficient](https://en.wikipedia.org/wiki/Gini_coefficient). The index ranges from 0 to 1 where smaller values represent more equality and larger values represent more inequality.

```
gini <- salaries %>%
  group_by(yearID, teamID) %>%
  summarize(gini = Gini(salary))
teams <- inner_join(teams, gini)
```

Compute the Atkinson coefficient for each team's salary. For more information, see [https://en.wikipedia.org/wiki/Atkinson\\_index](https://en.wikipedia.org/wiki/Atkinson_index). The index ranges from 0 to 1 where smaller values represent more equality and larger values represent more inequality.

```
atkinson <- salaries %>%
  group_by(yearID, teamID) %>%
  summarize(atk = Atkinson(salary))
teams <- inner_join(teams, atkinson)
```

Subset the data after the Gini coefficient and Atkinson coefficient have been computed.

```
teams_old <- teams %>%
  filter(1985 <= yearID & yearID <= 1998) %>%
  mutate(normalizedYear = yearID - 1985)

teams_new <- teams %>%
  filter(1999 <= yearID & yearID <= 2016) %>%
  mutate(normalizedYear = yearID - 1999)
```

Run the fixed effects regression using the Gini coefficient on data between 1985 and 1998, inclusive.

```
gini_fixed_old <- lm(formula = winPercentage ~ totalSalaryMil + gini + normalizedYear +
  teamID + 0,
  data = teams_old)
```

```
summary(gini_fixed_old)$coefficients[1:3,]
```

```
##              Estimate Std. Error   t value    Pr(>|t|)
## totalSalaryMil    2.312489  0.4521101  5.114879 5.559085e-07
## gini              -129.827687 58.3962938 -2.223218 2.693371e-02
## normalizedYear    -3.793613  1.8442237 -2.057025 4.053398e-02
```

Run the random effects regression using the Gini coefficient on data between 1985 and 1998, inclusive.

```
gini_random_old <- lm(formula = winPercentage ~ totalSalaryMil + gini + normalizedYear,
                      data = teams_old)
summary(gini_random_old)$coefficients[1:4,]
```

```
##              Estimate Std. Error   t value    Pr(>|t|)
## (Intercept)    539.115522 24.3427437 22.146868 1.404711e-67
## totalSalaryMil    2.495248  0.3775998  6.608182 1.532621e-10
## gini            -133.160525 54.5170461 -2.442548 1.510025e-02
## normalizedYear   -4.559438  1.6394568 -2.781066 5.724596e-03
```

Run the fixed effects regression using the Gini coefficient on data between 1999 and 2016, inclusive.

```
gini_fixed_new <- lm(formula = winPercentage ~ totalSalaryMil + gini + normalizedYear +
                    teamID + 0,
                    data = teams_new)
summary(gini_fixed_new)$coefficients[1:3,]
```

```
##              Estimate Std. Error   t value    Pr(>|t|)
## totalSalaryMil    0.6624573  0.1276041  5.191504 3.069387e-07
## gini              -218.5193970 58.0041403 -3.767307 1.852177e-04
## normalizedYear    -2.7481628  0.7167617 -3.834137 1.425403e-04
```

Run the random effects regression using the Gini coefficient on data between 1999 and 2016, inclusive.

```
gini_random_new <- lm(formula = winPercentage ~ totalSalaryMil + gini + normalizedYear,
                      data = teams_new)
summary(gini_random_new)$coefficients[1:4,]
```

```
##              Estimate Std. Error   t value    Pr(>|t|)
## (Intercept)    550.1960765 32.40207090 16.980275 9.795202e-52
## totalSalaryMil    0.8147846  0.08086385 10.076006 6.326317e-22
## gini            -158.9308612 55.31835762 -2.873022 4.232464e-03
## normalizedYear   -3.2653945  0.62553010 -5.220204 2.591533e-07
```

With both the Gini and Atkinson coefficient used in place of the HHI index, our regressions are very similar to that of our replications/extensions and the original work. Specifically, the signs remain the same, while the magnitude of the coefficient for both total salary and year stay roughly the same (total salary is slightly larger and year slightly smaller). The coefficients for the Gini index and the Atkinson index are much bigger because the value of this ranges between 0-1 as opposed to between 0-10,000 for HHI. This result is significant because it suggests that perhaps HHI metric used in the original paper is a solid one and consistent with most economist's understanding of income inequality.

Run the fixed effects regression using the Atkinson coefficient on data between 1985 and 1998, inclusive.

```
atk_fixed_old <- lm(formula = winPercentage ~ totalSalaryMil + atk + normalizedYear +
                    teamID + 0,
                    data = teams_old)
summary(atk_fixed_old)$coefficients[1:3,]
```

```
##              Estimate Std. Error   t value    Pr(>|t|)
```

```
## totalSalaryMil      2.376013  0.4504696  5.274524 2.527538e-07
## atk                 -179.252498 65.9775739 -2.716870 6.966615e-03
## normalizedYear      -3.413854  1.8123452 -1.883667 6.056128e-02
```

Run the random effects regression using the Atkinson coefficient on data between 1985 and 1998, inclusive.

```
atk_random_old <- lm(formula = winPercentage ~ totalSalaryMil + atk + normalizedYear,
                     data = teams_old)
summary(atk_random_old)$coefficients[1:4,]
```

```
##              Estimate Std. Error   t value    Pr(>|t|)
## (Intercept)   509.553006 11.7544621 43.349751 1.975329e-139
## totalSalaryMil  2.555104  0.3759984  6.795518 4.948012e-11
## atk           -181.545622 61.0995554 -2.971308 3.179705e-03
## normalizedYear  -4.188912  1.6071160 -2.606478 9.556548e-03
```

Run the fixed effects regression using the Atkinson coefficient on data between 1999 and 2016, inclusive.

```
atk_fixed_new <- lm(formula = winPercentage ~ totalSalaryMil + atk + normalizedYear +
                    teamID + 0,
                    data = teams_new)
summary(atk_fixed_new)$coefficients[1:3,]
```

```
##              Estimate Std. Error   t value    Pr(>|t|)
## totalSalaryMil  0.6943805  0.1274283  5.449185 8.049458e-08
## atk            -267.2160401 60.5183651 -4.415454 1.242698e-05
## normalizedYear  -2.8583544  0.7142924 -4.001659 7.268982e-05
```

Run the random effects regression using the Atkinson coefficient on data between 1999 and 2016, inclusive.

```
atk_random_new <- lm(formula = winPercentage ~ totalSalaryMil + atk + normalizedYear,
                     data = teams_new)
summary(atk_random_new)$coefficients[1:4,]
```

```
##              Estimate Std. Error   t value    Pr(>|t|)
## (Intercept)   512.1999193 17.31125767 29.587678 2.179218e-113
## totalSalaryMil  0.8479434  0.08182447 10.362956 5.387417e-23
## atk           -193.0102061 57.91250941 -3.332790 9.212586e-04
## normalizedYear  -3.3980916  0.62691299 -5.420356 9.131764e-08
```

With both the Gini and Atkinson coefficient used in place of the HHI index, our regressions are very similar to that of our replications/extensions and the original work. This is the case when we ran regressions using either index for all four cases: 1985-1998 fixed effects, 1985-1998 random effects, 1999-2016 fixed effects, and 1999-2016 random effects. Specifically, the signs remain the same, while the magnitude of the coefficient for both total salary and year stay roughly the same (total salary coefficient is slightly larger and year coefficient slightly smaller in comparison to when using HHI). The coefficients for the Gini index and the Atkinson index are much bigger because the value of this ranges between 0-1 as opposed to between 0-10,000 for HHI. This result is important because it suggests that the HHI metric used in the original paper is a solid one and consistent with most economist's understanding of income inequality.

Run a  $k$ -fold linear regression to generate a validation error and analyze predictive performance. Here we include all teams from 1985 to 2016 as potential training data points. We used the fixed effects model.

```
num_folds <- 5
num_rows <- nrow(teams)
shuffle_idx <- sample(1:num_rows, num_rows, replace = FALSE)

teams_k_fold <- teams[shuffle_idx,] %>%
  ungroup() %>%
```

```

mutate(fold = (row_number() %% num_folds) + 1) %>%
mutate(normalizedYear = yearID - 1985)

validate_err <- c()
train_err <- c()
for (f in 1:num_folds) {
  curr_train <- filter(teams_k_fold, fold != f)
  model <- lm(formula = winPercentage ~ totalSalaryMil + HHI + normalizedYear + teamID + 0,
              data = curr_train)
  train_err[f] <- sqrt(mean((predict(model, curr_train) - curr_train$winPercentage) ^ 2))

  curr_validate <- filter(teams_k_fold, fold == f)
  validate_err[f] <- sqrt(mean((predict(model, curr_validate) - curr_validate$winPercentage) ^ 2))
}

avg_validate_err <- mean(validate_err)
se_validate_err <- sd(validate_err) / sqrt(num_folds)

avg_train_err <- mean(train_err)
se_train_err <- sd(train_err) / sqrt(num_folds)

```

Use the linear regression trained on 1985 to 1998 data and predict on 1999 to 2016 data. We used the random effects model.

```

teams_pre_2011 <- teams %>%
  filter(yearID <= 2011) %>%
  mutate(normalizedYear = yearID - 1985)
teams_post_2012 <- teams %>%
  filter(yearID >= 2012) %>%
  mutate(normalizedYear = yearID - 1985)

time_model <- lm(formula = winPercentage ~ totalSalaryMil + HHI + normalizedYear,
                 data = teams_pre_2011)
time_train_err <- sqrt(mean((predict(model, teams_pre_2011) - teams_pre_2011$winPercentage) ^ 2))
time_validate_err <- sqrt(mean((predict(model, teams_post_2012) - teams_post_2012$winPercentage) ^ 2))

```

In this case we see an average error of 6.4 +/- 1.5 win% for the fixed effects and 6.6 +/- 1.5% for the random effects. While in an absolute sense these are relatively accurate, in a larger context they are less impressive. This is because the range of outcomes are reasonably between 40-70 win %, so 6.5% error represents roughly 20% of the range of possible outcomes. In a real world setting it means that the prediction could be off by almost 10 games. The fixed effects model was slightly more accurate than the random effects because it was trained on random selection of all data, as opposed to just the early years. Furthermore it has the additional parameter of team specific intercepts, as opposed to a general intercept, which likely allows for a better fitting of the data.

## Extension Notes

- Coefficients of different inequality indexes are the same sign and have the same predictive impact (magnitudes are different because HHI has a wider range than the other two which are between 0 and 1), so we just picked HHI for the predictive models.
- Used fixed effects model when split data independent of time, ie teams from 1985 and 2016 are in the training set. Meanwhile used random effects model when split data based on time, ie teams from 1985-2011 were in training set and 2012-2016 were test set. The use of the random effects model for the latter was to account for the fact that teams might be dominant in early years but not so much in more

recent years, ie dont want to worry about team being good in 80s/90s and bad in 2010s. Also handles the issue of teams not existing in training set but potentially in validation set (since teams come and go, move cities, rebrand, etc)

## Postface

The following is a list of all packages used to generate these results.

```
sessionInfo()
```

```
## R version 3.4.3 (2017-11-30)
## Platform: x86_64-apple-darwin15.6.0 (64-bit)
## Running under: macOS Sierra 10.12.6
##
## Matrix products: default
## BLAS: /Library/Frameworks/R.framework/Versions/3.4/Resources/lib/libRblas.0.dylib
## LAPACK: /Library/Frameworks/R.framework/Versions/3.4/Resources/lib/libRlapack.dylib
##
## locale:
## [1] en_US.UTF-8/en_US.UTF-8/en_US.UTF-8/C/en_US.UTF-8/en_US.UTF-8
##
## attached base packages:
## [1] stats      graphics  grDevices  utils      datasets  base
##
## other attached packages:
## [1] bindrcpp_0.2      forcats_0.3.0    stringr_1.3.0    dplyr_0.7.4
## [5] purrr_0.2.4      readr_1.1.1      tidyr_0.8.0      tibble_1.4.2
## [9] ggplot2_2.2.1    tidyverse_1.2.1  scales_0.5.0     ineq_0.2-13
## [13] here_0.1
##
## loaded via a namespace (and not attached):
## [1] reshape2_1.4.3   haven_1.1.1      lattice_0.20-35  colorspace_1.3-2
## [5] htmltools_0.3.6  yaml_2.1.17      rlang_0.2.0      pillar_1.2.1
## [9] foreign_0.8-69   glue_1.2.0       modelr_0.1.1     readxl_1.0.0
## [13] bindr_0.1        plyr_1.8.4       munsell_0.4.3    gtable_0.2.0
## [17] cellranger_1.1.0 rvest_0.3.2      psych_1.7.8      evaluate_0.10.1
## [21] labeling_0.3     knitr_1.20       parallel_3.4.3   broom_0.4.3
## [25] methods_3.4.3    Rcpp_0.12.15     backports_1.1.2  jsonlite_1.5
## [29] mnormt_1.5-5     hms_0.4.1        digest_0.6.15    stringi_1.1.6
## [33] grid_3.4.3       rprojroot_1.3-2  cli_1.0.0        tools_3.4.3
## [37] magrittr_1.5     lazyeval_0.2.1   crayon_1.3.4     pkgconfig_2.0.1
## [41] xml2_1.2.0       lubridate_1.7.3  assertthat_0.2.0 rmarkdown_1.9
## [45] httr_1.3.1       rstudioapi_0.7   R6_2.2.2         nlme_3.1-131
## [49] compiler_3.4.3
```