Design Method for Differentially-Driven Capacitive Wireless Power Transfer Systems

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***Abstract*—A method of matching a capacitive power transfer system (CPT) is shown. A six-plate design driven by a balanced (differential) source is analyzed. The balanced design neutralizes several plate capacitances by imposing a virtual ground. The capacitance network linking the transmitter and receiver is simplified to an equivalent half circuit. A matching method is shown that reduces the network capacitances to two resonators coupled through a lumped element transmission line. The theory is described. Using this method, a CPT system was built for 13.56 MHz that delivers 230 W of load power at 80% efficiency and 950 W at 75% efficiency. Experimental results demonstrate how the differential source and load configuration neutralize a floating ground plate at the load.**

***Index Terms*—capacitive power transfer (CPT), wireless power transfer (WPT), six-plate CPT**

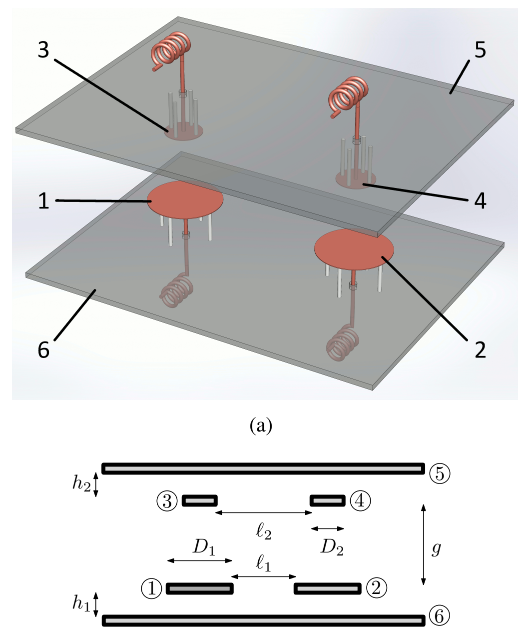
I. Introduction

Capacitive power transfer (CPT) systems deliver power across a dielectric gap to a load. Utilizing a network of capacitive electrodes, energy is transferred by the electric field. A full CPT system consists of three parts: an RF source, a wireless "link", and a load.

To achieve effective RF power transfer across the gap, the CPT link includes inductances which resonate with the electrode capacitances at the operating frequency. Additionally, 1

In our design method, an analysis is shown to determine the required values of matching inductors for a six conductor CPT. An example of how the six conductors (electrodes) are physically arranged is shown in Fig. 1(a). Various analytic methods have been used to design matching circuit for this type of electrode configuration and most employ circuit analysis methods to determine plate voltages and branch currents using Kirchoff's circuit laws [1], [2]. Other matching methods include the synthesis of networks based on filter theory [3].

We present an elegant method of analyzing the circuit by exploiting the symmetry that comes from using differential (balanced) source and load impedances. Using symmetry, the capacitance matrix is reduced to an equivalent half circuit. A transformation is used that includes the effect of cross-coupling capacitance between the transmit and receive electrodes and all plate self-capacitance terms. An equivalent circuit model of capacitances is transformed into two resonators coupled by a lumped element transmission line. The model includes matching inductors and the values of the inductors are easily obtained. The theory and experimental verification of the design methodology are described next.



(b)

Fig. 1. (a) Physical layout of the six-plate CPT system. For clarity, the wire diameter and vertical spacing between plates have been exaggerated, and the two large plates (5 and 6) are shown with partial transparency. Not pictured are the differential source and the loads, which connect to the inductors on the bottom (TX side) and top (RX side), respectively as shown in Fig. 2. (b) Physical dimensions associated with the six-plate CPT system.

II. Circuit Model

The physical structure of a six-plate CPT link is shown in Fig. 1(a). The electrodes consist of four circular disks sandwiched between two large rectangular plates. The two rectangular plates shield the fields between the circular disks and provide local ground plane references for the transmit and receive systems. Disks labelled 1 and 2 are associated with the transmitter, and disks 3 and 4 are associated with the receiver. The transmit electrodes have a larger diameter than the receive electrodes to accommodate physical misalignment error between the transmitter and receiver.

All six conductors are electrically isolated from one another, with Teflon supports holding the smaller electrodes parallel to the large plates. Small clearance holes are drilled in the large rectangular plates (5 and 6) to allow a wire to connect each electrode to a matching inductor located outside the plates.

An equivalent circuit model for the capacitances between the conductors is shown in Fig. 2. The nodes labelled 1–6

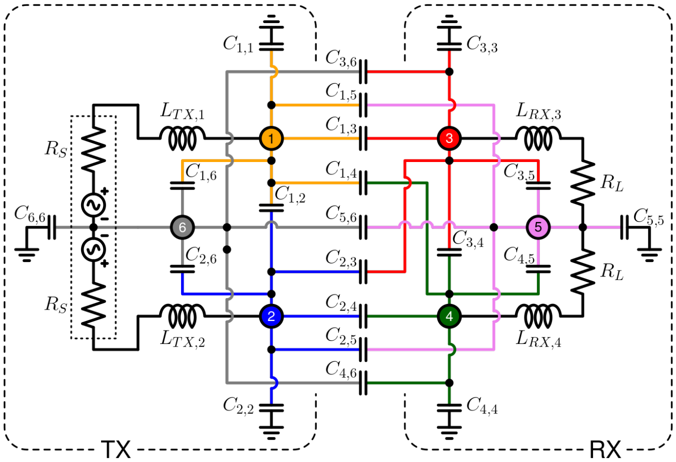


Fig. 2. Equivalent circuit diagram of the CPT system showing mutual capacitances between all six plates. The transmit (TX) side is on the left and receive (RX) is on the right. A circuit node representing each plate is shown as a numbered circle. Each plate is identified by a different color.

identify the corresponding physical conductors in Fig. 1(a). The circuit includes a differential source, matching inductors, and a differential load. The circuit also has a 'ground' which models earth ground and all six electrodes are in general floating with respect to earth.

Since all six conductors are identified uniquely and separate from ground, electrode capacitances are modelled by a six by six mutual capacitance matrix. The capacitance between a pair of conductors i and j is identified as $C_{i,j}$, and the capacitance between a conductor and earth ground is identified as $C_{i,i}$.The values of these capacitances are determined from 3D electrostatic simulation. For this work, modeling was done using the finite element analysis software COMSOL. The electrodes are simulated inside a grounded sphere with infinite element domain boundaries

Assuming the transmit and receive plates are aligned, the physical symmetry in the system leads to:

$$\begin{aligned} C_{1,1} &= C_{2,2} & C_{3,3} &= C_{4,4} \\ C_{1,3} &= C_{2,4} & C_{1,4} &= C_{2,3} \\ C_{1,6} &= C_{2,6} & C_{1,5} &= C_{2,5} \\ C_{3,5} &= C_{4,5} & C_{3,6} &= C_{4,6} \end{aligned} \tag{1}$$

Since our goal is to decompose the circuit in Fig. 2 into two symmetrical half-circuits, the matching inductors are also chosen to be symmetrical. This means that $L_{TX,1} = L_{TX,2} = L_{TX}$ and $L_{RX,3} = L_{RX,4} = L_{RX}$.

If we now use a differential source for the transmitter, the two outputs have the same amplitude with a phase difference of 180°. A differential load with a total resistance of $2R_L$ completes the circuit. Combining the physical symmetry of the circuit and the differential source and load, we conclude that the node voltages have the following symmetries:

$$\begin{aligned} V_1 &= -V_2 \\ V_3 &= -V_4 \\ V_5 = V_6 &= 0\,\mathrm{V} \end{aligned} \tag{2}$$

The implication of the circuit symmetry and differential excitation are significant and create virtual grounds on the two large rectangular plates, nodes 5 and 6. When these nodes are at virtual ground, this neutralizes the self-capacitance of the rectangular plates ($C_{5,5}$ and $C_{6,6}$), as well as the inter-electrode capacitance $C_{5,6}$. The effect of neutralizing these capacitances is to effectively isolate the system from earth ground by providing a local ground reference for all the system capacitances. This makes the system less susceptible to the capacitance of the surrounding environment.

III. Analysis

We now proceed with an analysis of the circuit in Fig. 2 by reducing the network to two identical half-circuits. These half-circuits are each comprised of a Π-network of capacitances. A key step is to absorb the cross-plate capacitances, $C_{1,4}$ and $C_{2,3}$, into an equivalent network without cross-plate capacitances. The direct capacitances, $C_{1,3}$ and $C_{2,4}$, along with cross-plate capacitances, $C_{1,4}$ and $C_{2,3}$, form a lattice network. We use the well-known lattice-to-ladder transform (see Fig. 3) to remove the cross-coupled branches [4]. From the physical symmetry of the coupler, $C_{1,4} = C_{2,3}$ and $C_{1,3} = C_{2,4}$, and the admittances of the direct and cross-plate capacitances are denoted by Y_a and Y_b, respectively.

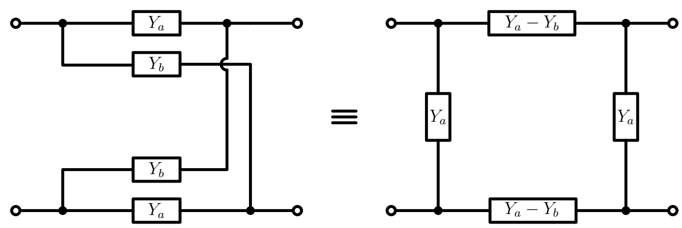


Fig. 3. Lattice-to-ladder transformation.

The transformation steps are summarized in Fig. 4(a) and Fig. 4(b). After removing the cross-plate capacitances, we define the following equivalent capacitances:

$$\begin{aligned} C_c &= C_{1,3} - C_{1,4} \\ C'_{1,2} &= C_{1,2} + C_{1,4} \\ C'_{3,4} &= C_{3,4} + C_{1,4} \end{aligned} \tag{3}$$

The corresponding circuit using equation (3) is shown in Fig. 4(c). The differential voltage conditions between nodes 1 and 2, and nodes 3 and 4, create virtual ground nodes that are shown in Fig. 4(d).

We can now consolidate the remaining capacitances connected to electrodes 1–4, as they are all connected in parallel between one electrode and ground. On the transmit side, define C_{TX} as the parallel combination of all capacitances connected between node 1 and ground (or equivalently node 2 and ground). Likewise, C_{RX} is defined on the receive side as the parallel combination of the capacitances between node 3 and ground. Therefore,

$$\begin{aligned} C_{TX} &= C_{1,1} + C_{1,6} + 2C'_{1,2} \\ C_{RX} &= C_{3,3} + C_{3,5} + 2C'_{3,4} \end{aligned} \tag{4}$$

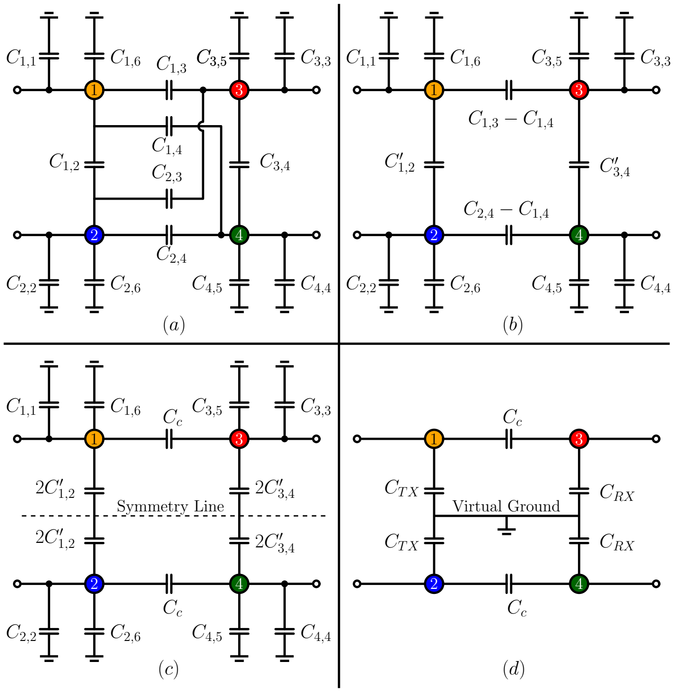


Fig. 4. Transformation of network capacitances: (a) equivalent circuit of the differential six-plate CPT; (b) transformed equivalent circuit (lattice to ladder network transformation); (c) circuit configured with a line of symmetry; (d) final equivalent network with two identical Π half-circuits.

Using these capacitances, the equivalent circuit model for the full system simplifies to the circuit shown in Fig. 5.

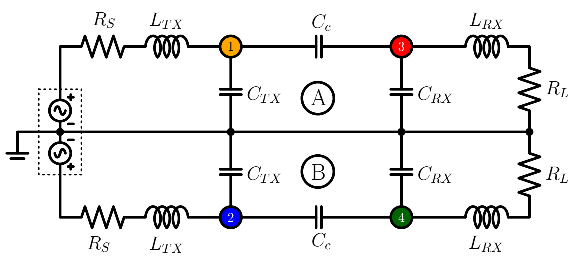


Fig. 5. Two equivalent half-circuits in a Π configuration assuming physical symmetry and differential excitation.

The circuit in Fig. 5 is composed of two half-circuits connected by a virtual ground plane. The remaining steps of the analysis use the half-circuit A shown in Fig. 5.

The Π network of capacitances (C_{TX}, C_c, and C_{RX}) in the half-circuit are transformed to an equivalent T-network using a Δ–Y transform. The equivalent T-network circuit is shown in Fig. 6(a) where:

$$
\begin{aligned}
C_0 &= C_{TX} + C_{RX} + \frac{C_{TX}C_{RX}}{C_c} \\
C_1 &= C_{TX} + C_c + \frac{C_{TX}C_c}{C_{RX}} \\
C_2 &= C_{RX} + C_c + \frac{C_{RX}C_c}{C_{TX}}
\end{aligned}
\tag{5}
$$

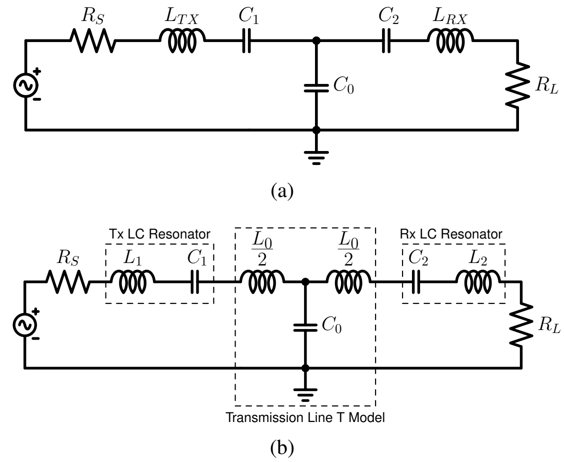


Fig. 6. Half-circuit after Π–T transform of capacitances is shown in (a). (b) depicts the half-circuit with single-section transmission line model at the center.

We now partition the inductors L_{TX} and L_{RX} into two series inductors to construct a circuit that consists of two series resonators coupled by a lumped element transmission line. The circuit is shown in Fig. 6(b).

The transmission line has a distributed inductance of L_o and a distributed capacitance of C_o where the capacitance and inductance are selected to implement a characteristic impedance $Z_o = \sqrt{L_o/C_o}$. If the source resistance R_S is equal to the load resistance R_L, then the characteristic impedance Z_o should be equal to the terminal impedances for maximum power. Since C_o is determined by the geometry of the physical conductors, for equal source and load resistances, L_o is given by

$$
L_o = {Z_o}^2\, C_o = {R_S}^2\, C_o. \tag{6}
$$

If source and load resistances were unequal, an L-match impedance network could be added to either the source or load to change the terminal impedance.

The resonator inductances L_1 and L_2 are selected to resonate with C_1 and C_2, respectively. For a power transfer frequency ω_o, the resonator inductances are:

$$
\begin{aligned}
L_1 &= \frac{1}{{\omega_o}^2 C_1} \\
L_2 &= \frac{1}{{\omega_o}^2 C_2}
\end{aligned}
\tag{7}
$$

In a circuit implementation, the resonator and transmission line inductors can be combined into single inductors as shown in Fig. 5 where

$$
\begin{aligned}
L_{TX} &= L_1 + \frac{L_o}{2} \\
L_{RX} &= L_2 + \frac{L_o}{2}
\end{aligned}
\tag{8}
$$

IV. Experimental Verification

An experimental CPT system was designed for a critically-coupled response with terminal resistances ($R_S = R_L$) of 50 Ω at 13.56 MHz. With reference to Fig. 1(b), the six-plate CPT system dimensions were: $D_1 = 20$ cm, $D_2 = 10$ cm, $g = 2$ cm, $\ell_1 = 20$ cm and $\ell_2 = 30$ cm. The rectangular shield plates, 5 and 6, were 84×61 cm^2.

COMSOL was used to find the capacitance matrix for a system configuration where plate 6 is grounded at the TX. Therefore, there are five floating conductors resulting in a 5x5 capacitance matrix. Matching inductances were found using the design equations in section III. Air core inductors were built using 14-gauge copper wire to handle the high power. The TX inductors had 12 turns wound on a 5 cm diameter coil frame, and the RX inductors had 9 turns wound on a 7 cm coil frame. Table I summarizes the equivalent circuit values for the CPT design.

TABLE I
CIRCUIT PARAMETERS; CAPACITANCES LESS THAN 5 fF ARE NEGLECTED.

Parameter	Value	Parameter	Value
$C_{1,1}$	30.22 pF	$C_{1,2}$	0.00 pF
$C_{1,3}$	4.29 pF	$C_{1,4}$	0.00 pF
$C_{1,5}$	9.14 pF	$C_{3,3}$	0.00 pF
$C_{3,4}$	0.00 pF	$C_{3,5}$	15.55 pF
$C_{5,5}$	133.68 pF	C_{TX}	39.37 pF
C_{RX}	15.48 pF	C_c	4.3 pF
L_{TX}	2.84 uH	L_{RX}	6.77 uH

The first step in verifying the design was to evaluate the matching circuits. A two port vector network analyzer was used to measure the response of each half circuit and the results are shown in Fig. 8(a). At 13.56 MHz, the insertion loss of each half circuit was 0.3 dB. It is important to note that the RX shield plate must be grounded to accurately measure the S_{21} response, since the plate is at virtual ground when the source is differential. After verifying the CPT resonators, the second step was to measure power efficiency of the full system including inverters. The power efficiency versus load power is shown in Fig. 8(b). The system delivers a load power of 230 W with an efficiency of 80%, and 950 W with an efficiency of 75%. Load power is controlled by changing the dc drain voltage on the inverters and an overall system efficiency of 75% is maintained over a load power range of 100 W to 950 W.

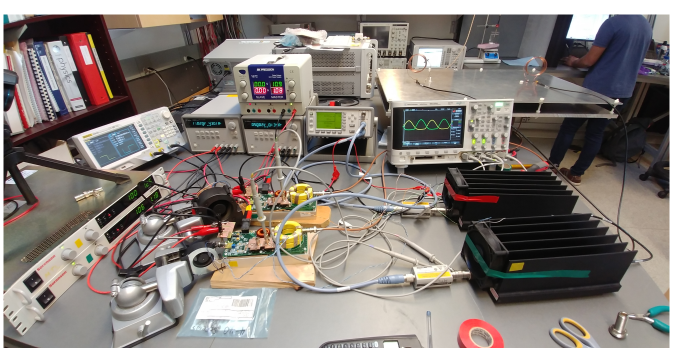


Fig. 7. Photo of the experimental test bed. Two class-E power inverters (model IXYS PRF-1150) were connected to the CPT. The output power of each inverter was measured using a 50 Ω 500 W attenuator and power meter.

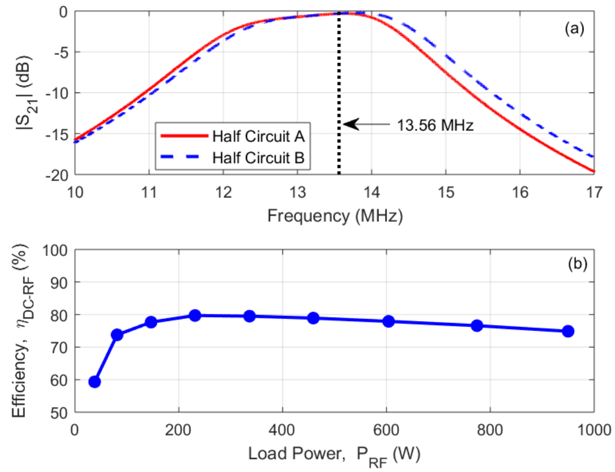


Fig. 8. (a) Measured frequency response of half circuits to confirm matching circuits. At 13.56 MHz the loss is 0.3 dB. (b) Efficiency versus load power. Efficiency is the total RF load power delivered by the CPT link over the total DC power supplied to the drain terminals of the inverters.

V. CONCLUSION

A design method for six-plate capacitive wireless power transfer systems has been described. The method can be applied to any six-plate design providing the electrodes satisfy the physical symmetry conditions that were described. The transmit and receive electrodes can have different sizes, but each pair of electrodes is assumed to be identical. When the six-plate design is excited with a differential source and load, the effect of changes to the capacitances of the shielding electrodes are neutralized. This Equations were derived for the matching inductors and they use a capacitance matrix obtained from a field simulation of the physical system. The method provides an easy way to design matching for CPT systems as demonstrated by the experimental results.

REFERENCES

[1] S. Sinha, A. Kumar, B. Regensburger, and K. K. Afridi, "A new design approach to mitigating the effect of parasitics in capacitive wireless power transfer systems for electric vehicle charging," *IEEE Transactions on Transportation Electrification*, vol. 5, no. 4, pp. 1040–1059, Dec 2019.

[2] F. Lu, H. Zhang, H. Hofmann, and C. Mi, "A double-sided lclc-compensated capacitive power transfer system for electric vehicle charging," *IEEE Transactions on Power Electronics*, vol. 30, no. 11, pp. 6011–6014, Nov 2015.

[3] M. Ahmadi, L. Markley, and T. Johnson, "A filter theory approach to the synthesis of capacitive power transfer systems," *IEEE Journal of Emerging and Selected Topics in Power Electronics*, pp. 1–13, 2020.

[4] E. A. Guillemin, *Synthesis of Passive Networks*. John Wiley and Sons. Inc., 1957.