# The potential rate of bed-load transport

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**Averages of measurements made in three rivers characterized by a high availability of sediment in relation to runoff, and comparable data that represent the high end of the range of transport rates observed during three sets of laboratory experiments, confirm there is an upper, particle-size-dependent limit to bed-load transport efficiency. Incorporating the regression relation derived from these six diverse and unrelated data sets into R. A. Bagnold's classic formulation yields $i_b = \omega\ [0.0115 \cdot D_{50}^{-0.51}]/0.63$. This straightforward scale correlation can be used to estimate the potential rate of bed-load transport (for average conditions) in gravel-bed rivers when sediment transport is constrained neither by the supply of sediment to nor by the amount of sediment available in the channel. The independent application of this empirical limit formula to two rivers with applicable bed-load transport regimes reveals a good (±10%) correspondence between average observed and predicted transport rates.**

bed-load transport efficiency | gravel-bed rivers | sediment transport

Bed-load transport provides the major process linkage between the hydraulic and material conditions that govern river-channel morphology, and knowledge of bed-load movement is required not only to elucidate the causes and consequences of changes in fluvial form but also to make informed management decisions that affect a river's function. Unfortunately the collection of high-quality bed-load transport data is an expensive and time-consuming task, and for many practical purposes recourse is made to a bed-load transport formula (1). The ability of any formula to predict the bed-load transport rate under given flow conditions is predicated on the assumption that it is possible to describe the rate at which bed load is transported in terms of measurable hydraulic and sedimentological quantities (1). Even assuming there are no limitations on sediment supply, the task is complicated by the realization that, at all but the highest flows, the movement of heterogeneous sediment is governed by absolute and relative size effects, so that the local transport rate depends on the population of particles immediately available at the bed surface (2). However, little is known about how the composition of the bed surface changes over time, and despite more than a century of effort it is not yet possible to make reliable predictions of bed-load transport rates. One way to address this impasse is to remove the restrictions imposed by sediment supply and availability from the problem and determine how much bed load a river is capable of transporting, rather than how much it actually transports. That is, to articulate a limit formula for potential transport. I accomplish this by incorporating an empirical, particle-size-dependent term for bed-load transport efficiency into an existing formulation that has been shown to be relatively successful in predicting bed-load transport in gravel-bed rivers (1, 2), where the dominant bed material size ranges from 0.002 to ~0.2 m.

G. K. Gilbert and R. A. Bagnold made the analogy between a river and a machine (3, 4), and the latter related the rate at which bed load is transported to the rate of energy expenditure in the channel, such that

$$i_b = \omega\, e_b/\tan\alpha, \quad [1]$$

where (in mass units) $i_b$ is the transport rate of bed load measured as immersed mass per unit width, $\omega$ is the stream power per unit width, $e_b$ is the bed-load transport efficiency, and $\tan\alpha$ is a friction coefficient for the bed material (5). Notwithstanding that the status of individual gravel particles is known to vary and to depend on clast shape and sorting (6), for the general case, $\alpha$ is assumed constant and to be approximated by the angle of internal dynamic friction of sand, which is a degree or so less than the static angle of repose (5); thus, $\tan\alpha = 0.63$. Stream power is $\rho uYS$ (where $\rho$ is the mass density of water, $u$ is mean velocity, $Y$ is mean depth, and $S$ is water-surface slope) and is readily quantified. The overall efficiency of the transport process ($i_b \tan\alpha/\omega$) is much more difficult to define (7). Actual efficiencies are less than unity because $\omega$ is not a measure of the power directly available to transport bed load and there are always inefficiencies involved in energy conversion. The issue has been addressed empirically (7, 8), but no straightforward expression relating the bed-load transport rate to stream power and some function of a third parameter, such as particle size ($D$), has emerged. This situation arises in part because there are uncertainties about the fundamental physics (7, 8), but most of the available data also apply either to conditions that are below the threshold for general bed motion or to situations where the supply of incoming sediment to a reach is less than the capacity of the flow. In such circumstances, the transport rate is not directly proportional to the available power, and efficiency necessarily is moderated by sediment supply and availability (9). Sometimes, however, in gravel-bed rivers and during experiments made in laboratory flumes the bed is fully mobile, and neither the composition of the bed surface nor sediment supply regulates transport (1). Using the few data relating to conditions in gravel-bed rivers under which the constraints on sediment supply and availability are relaxed, I reexamine the assertion that there is an upper, particle-size-dependent limit to bed-load transport efficiency (5).

## Results

Under field conditions, high bed-load transport rates oftentimes are recorded in circumstances when there is a high availability of sediment in relation to runoff. Such was the case in the North Fork Toutle River as sediment emplaced after the 1980 eruption of Mount St. Helens (WA) was reworked; in Nahal Yatir (Israel), an ephemeral dry-land stream subject to discontinuous flash floods, where the supply of alluvium stored in the channel was essentially unlimited; and in Hilda Creek (Alberta, Canada), a braided proglacial stream with unconfined and freely erodible banks (10–12). These data, which represent the high end of the reported range of bed-load transport rates measured in both relatively large and small rivers at discharges less than bankfull (1), are plotted in Fig. 1. Also plotted are analogous data from flume experiments with uniform or bimodal sediments, and no apparent limitations on supply (13–15), that fall outside the range of partial transport when the bed-load transport rate increases rapidly in response to a small increase in stream power (16). Together, the diverse, unrelated field and laboratory data

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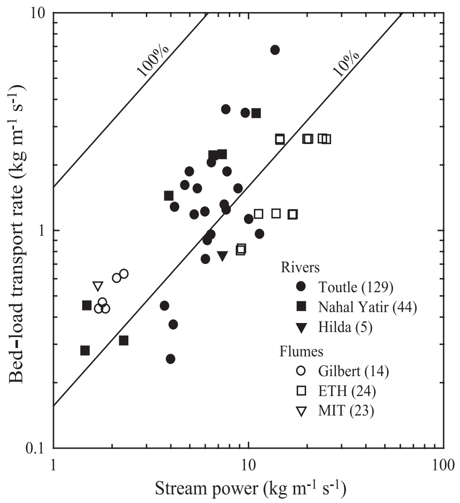


**Fig. 1.** Bed-load transport rate as a function of stream power under conditions in gravel-bed rivers that do not appear to be moderated by either sediment supply or availability (filled symbols) and flumes (open symbols) and selected levels of percent bed-load transport efficiency (solid lines). Values in parentheses are $\bar{Y}$:$\bar{D}_{50}$ in each river or flume.

encompass a 26-fold range of $\bar{Y}$:$\bar{D}_{50}$ (where $D_{50}$ is the bed load median particle size), and they extend across the spectrum of gravel-bed river typologies encountered in nature. They appear to delineate an upper limit on bed-load transport rates, as governed by the efficiency of energy expenditure (Fig. 2) and, although there are differences in the behavior of ephemeral and perennial rivers and flumes, to be part of the same overall trend that relates bed-load transport to the flow characteristics (2, 5).

The averaged data suggest that, at high transport rates, there is a straightforward relation between the bed-load transport efficiency attained in both rivers and flumes and the median particle size of the bed load (Fig. 2). Even accepting that internal variability has been abstracted by averaging, and given the diversity and autonomy of the data sources, the coherence ($R^2$ = 0.99) of the regression relation ($e_b = 0.0115{\cdot}D_{50}{}^{-0.51}$) is quite remarkable. As others have shown, efficiency declines with increasing particle size (9), as the overall rate of energy dissipation involved in the transfer of stress from fluid to solids increases (4, 5), and the proportion of the available stream power directly available for bed-load transport is an inverse function of $Y$:$D_{50}$ (5, 16). The suggested scaling is $Y^{-2/3}$ and $D_{50}{}^{-1/2}$ (7, 8), and the empirically derived power essentially is in agreement with the latter proposition.

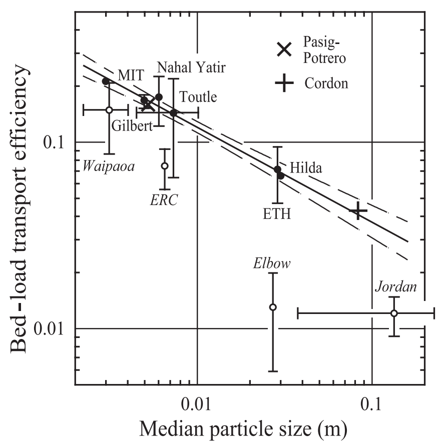


**Fig. 2.** Relation of average bed-load transport efficiency ($i_b \tan \alpha/\omega$) to bed load median particle size for conditions in gravel-bed rivers and flumes where there are no constraints on sediment supply or availability (filled circles) compared with other rivers and flumes (open circles, labels in italics) in which high bed-load transport rates, that nonetheless were reported to have been moderated by armoring or inadequate supply, have been observed. Error bars indicate the standard deviation of bed-load transport efficiency and the median particle size of the bed load (when reported). Solid line represents the regression relation used to derive Eq. **2**, and the dashed lines indicate the 95% confidence intervals. Crosses denote the Rio Cordon and Pasig-Potrero River ($\bar{Y}$:$\bar{D}_{50}$ = 10 and 18, respectively). Note that these data were not used to derive Eq. **2**.

Substituting for $e_b$ in Eq. **1** gives:

$$i_b = \omega\,[0.0115{\cdot}D_{50}{}^{-0.51}]/0.63, \qquad \textbf{[2]}$$

which is a simple, generalized expression for the relation between stream power and the bed-load transport rate under conditions where there are no constraints on the availability of sediment in the channel and there is an adequate supply of sediment for the duration of the flow. Uncertainties about the fundamental physics have confounded attempts to explain the variation of $i_b$ as $D^{-1/2}$ under the widely differing conditions encountered in rivers and flumes (7, 8, 16, 17). However, assuming that in steady, uniform flow the mean bed shear stress ($\rho gYS$) is approximately proportional to the square of the vertically averaged velocity, this empirical limit formula predicts that the potential bed-load transport rate is roughly proportional to velocity to the third power, or shear stress to the 1.5 power. In this respect, it has a similar form to many other empirical relations (17), including that proposed by Meyer-Peter and Müller (14), which frequently is applied to gravel-bed rivers.

Dimensional analysis has a long history of application to problems relating to the hydraulics of sediment transport, and recently it has been shown (2) that, if consideration is given to the bed surface median particle size ($D_{s50}$), it is possible to relate the bed-load transport rate to flow strength in both perennial and ephemeral gravel-bed rivers by recasting the relation between bed-load transport rate and stream power in terms of the dimensionless parameters $i_b{}^* = i_b/[\rho_s\ (s\,g\,D_{s50}{}^3)^{0.5}]$ and $\omega^* = \omega/[\rho_s\,(s\,g\,D_{s50}{}^3)^{0.5}]$, where $s$ (the submerged specific gravity of sediment) = $(\rho_s/\rho) - 1$; $g$ is acceleration due to gravity; $\rho$ and $\rho_s$ are the densities of water and sediment, respectively; and $e_b{}^* = i_b{}^* \tan \alpha/\omega^*$. For the data considered herein, $D_{50} = D_{s50}$, and introducing a nondimensional particle diameter $D_{50}{}^* = [(\rho_s - \rho)\,g\,D_{50}{}^3]/(\rho\,\nu^2)$ (18) (where $\nu$ is the kinematic viscosity of water) permits the regression relation between average bed-load transport efficiency and median particle size to be reconstituted, and the appropriate substitutions yield a dimensionless form of Eq. **2**:

$$i_b{}^* = \omega^*[2{\cdot}D_{50}{}^{*-0.34}]/0.63. \qquad \textbf{[3]}$$

Few independent data are available to conduct an authoritative test of its applicability, but, on the basis of the measurements made during the extraordinary September 14, 1994, event in the Rio Cordon basin (Italy) (19), I compare the average bed-load transport rate computed with Eq. **2** to the rate calculated from the volume of sediment amassed in a retention basin by using the reported hydraulic and sedimentological data for the two measurements made on the falling limb of the flood hydrograph. I also compare computed and observed rates by using data collected with a Helley-Smith sampler in the Pasig-Potrero River after the 1991 eruption of Mount Pinatubo (Philippines) (20). In geomorphological terms, these are very different fluvial systems, and the median particle size of the bed load in the Rio Cordon lies outside the range used to derive Eq. **2**. However, there was a high availability of sediment in relation to runoff in each river during the monitoring period when, albeit for a limited time, their respective bed-load transport regimes were in accord with that described by the field and laboratory data used to derive Eq.

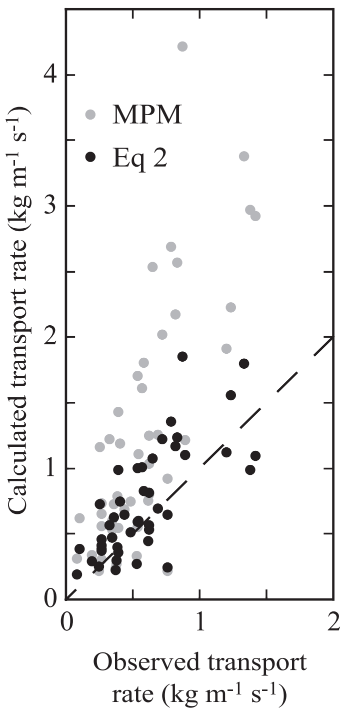


**Fig. 3.** Pasig-Potrero River observations compared with estimates calculated by using the Meyer-Peter–Müller (MPM) formula (12) and Eq. **2**. The dashed line denotes perfect agreement, but note that Eq. 2 applies to average conditions and is not expected to predict the instantaneous transport rate, which (assuming the total discharge is available and used for sediment transport) may be moderated by surface coarsening and variations in the rate at which sediment is supplied to a reach (1).

**2** (Fig. 2). Accordingly, there is a good correspondence between average observed and predicted transport rates as indicated by the discrepancy ratio ($\bar{i}_{b[\text{observed}]}:\bar{i}_{b[\text{calculated}]}$), which is 1.1 and 0.9 for the Rio Cordon and Pasig-Potrero River, respectively. Other familiar equations, which also assume a single effective particle diameter may be used to characterize the mixture, are not as reliable. For example, Meyer-Peter and Müller's formula (14) consistently overestimates ($\bar{i}_{b[\text{observed}]}:\bar{i}_{b[\text{calculated}]} = 0.6$) the transport rate in the Pasig-Potrero River (Fig. 3), and the discrepancy for the steeper Rio Cordon is even more pronounced. This finding is not surprising (1), as the outcome of previous tests of the applicability of this and other frequently used bed-load transport equations to steep gravel-bed rivers also indicate (15, 21).

## Discussion and Conclusion

I suggest that Eq. **2** (or its dimensionless equivalent) provides a straightforward scale correlation that can be used in conjunction with summary hydraulic data to estimate the potential rate of bed-load transport in gravel-bed rivers in situations when neither the supply of sediment to nor the amount of sediment available in the channel moderate bed-load transport. Under such circumstances, the bed-load transport efficiency varies between 0.026 and 0.27 for 0.2 m and 0.002 m gravel, respectively, with the latter figure being slightly below the average upper limit to bed-load transport efficiency (0.3) that it has been proposed applies to both ephemeral and perennial gravel-bed rivers (2). The general variation of the potential bed-load transport rate with stream power and median particle size applies to average conditions and attests to the robustness of a theory advanced to describe sand transport (4, 5), which was expected to apply to gravel (16). The resulting empirical relation accounts for major differences among gravel-bed rivers, in the absence of significant variations in the observed transport rate that are attributable to short-term adjustments in sediment supply and availability, to which all rivers are sensitive. I emphasize this point by plotting in Fig. 2 data from three other gravel-bed rivers and a flume experiment in which high bed-load transport rates also have been observed but where the transport rate either was reported to have been supply-limited or constrained by the characteristics of the bed surface. These data are averages of stream-wide measurements made in the Elbow River (Alberta, Canada), estimates made in the River Jordan (Israel), at-a-point measurements made in the Waipaoa River (New Zealand), and runs made in the 4-m flume at the Environmental Research Centre, University of Tsukuba (Tsukuba, Japan) (22–25). The bed-load transport regimes in these channels do not represent conditions to which Eq. **2** is applicable.

Eq. **2** serves to increase confidence that a formulation based on stream power has the capability to predict bed-load transport in gravel-bed rivers, although it affords no new insight into the mechanism of bed-load transport, and complexity necessarily remains to be reintroduced into the problem. Nevertheless, the empirical limit formula I have articulated with currently available measurements should have utility in situations where there is a high availability of sediment in relation to runoff and an estimate of the potential rate of bed-load transport is required, such as in the design of sediment retention basins and when assessing the impact of a natural or anthropogenic disturbance to the catchment environment. It also may help constrain the transport-limited models that are a component of many numerical investigations of drainage basin evolution and continental relief by placing an upper limit on the rate at which the dispersal system can remove coarse debris supplied by weathering or mass movements.

## Methods

It is accepted that a reliable estimate of the bed-load transport rate can be obtained if observations are made over a sufficiently long period (26), and both field and laboratory data that are integrated over lengthy periods of time and across the entire width of the channel often yield coherent relations (27–29). Averaging also removes much of the internal variance in a data set (8). The field data I employ to derive the regression relation (Figs. 1 and 2) are averages of measurements made in the North Fork Toutle River, near Kid Valley, between February 1985 and January 1987; in Nahal Yatir between January and March 1991; and in Hilda Creek on August 19, 1979 (10–12). In these rivers, the bed-load transport rate for the entire channel was determined from numerous direct measurements made by using a pressure-difference sampler of the Helley-Smith type, a continuously recording pit trap, and a basket-type sampler, respectively, and the gauging sites were located in straight reaches where concurrent observations of water discharge, flow width and depth, water surface slope, and the bed material particle size distribution also were made. The coincidence of the reported median particle size of the bed load and bulk samples of the bed material suggests that all sizes present on the bed were in motion in the same proportion as they were present on the bed and implies that transport was not inhibited by coarsening of the bed surface. The laboratory data include the flume-wide average transport rate for run H5 in the Massachusetts Institute of Technology (Cambridge, MA) flume (13); flume-wide average transport rates from the high end of the range encountered during G. K. Gilbert's experiments with 5-mm gravel (3), when only the bed slope was reported; and the Eidgenössiche Technische Hochschule (Zurich, Switzerland) runs with 26.7-mm gravel (14, 15). I apply a side-wall correction (30) to compensate for the narrow width of some flumes (3, 13).

The single data points for Hilda Creek and the Massachusetts Institute of Technology flume are the average of numerous channel-wide measurements made over protracted periods of time, under similar flow conditions (Fig. 1). Scatter in the remaining data sets reflects the sensitivity of bed-load transport to local factors that affect sediment availability in the short term, such as the interaction of particles on the bed, the passage of bed forms, and the limitations of the chosen measurement technique (1). Most scatter is encountered in the North Fork Toutle River data, which are averages of essentially instantaneous, not time-integrated measurements (10). Scatter in the Nahal Yatir data are reduced by computing average transport rates for measurement periods when the flow conditions remained relatively constant. Little residual scatter is abstracted by adjusting the transport rate to allow for variations of $Y{:}D_{50}$ with water discharge (7, 16). Curvature in plots of log $i_b$ versus log $\omega$, which arises because bed-load transport ceases at a finite stream power, often is removed by substituting $(\omega - \omega_o)$ for $\omega$, where $\omega_o$ is the threshold stream power (7, 16). Confident appraisals of $\omega_o$ are lacking (17), and the use of excess power confounds the notion of transport efficiency, which now depends on $\omega_o$, causing different curves to converge. However, the substitution is neither necessary nor is it used here. Like others, I assume that bulk averages characterize the bed-load transport and flow regimes in any given river or flume (8). Accordingly, the data used to derive the regression relation shown in Fig. 2 are the reported median particle size of the bed load in motion (averaged if information about individual samples was available) and the average bed-load transport efficiency for each river or flume, derived from individual values for the data points depicted in Fig. 1, computed by multiplying $\bar{i}_b$ by 0.63 and dividing by $\bar{\omega}$.

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