Statistical Uncertainty and the Estimation of Log Law Parameters

Clinton L. Dancey[1] and Panayiotis Diplas, M.ASCE[2]

Abstract: It is experimentally demonstrated that fitting the log law to the mean velocity profile in turbulent open-channel flow to determine the friction velocity, roughness length, and zero-plane displacement, can lead to relatively high standard errors in these parameters. It is also demonstrated that this approach, where the log law alone is used to estimate these parameters, may yield values of these parameters that are significantly different from those obtained when the friction velocity is determined independently of the log law. Although the statistical estimation of the three log law parameters from mean velocity data alone is frequently necessary, this procedure can lead to inaccurate and imprecise estimates of these quantities.

DOI: 10.1061/(ASCE)0733-9429(2008)134:9(1353)

CE Database subject headings: Open channel flow; Velocity; Flow measurement; Turbulent flow; Statistics; Parameters.

Introduction

In many experimental investigations of turbulent channel flow, whether laboratory or field studies, it is common to assume that the mean, streamwise velocity profile is well represented by the "log law." One common expression of this "law" is given by

$$\frac{\langle U\rangle}{U_*} = \frac{1}{\kappa}\ln\left(\frac{z}{Z_0}\right) \tag{1}$$

(e.g., Raupach et al. 1991) where

$$z = Z + d \tag{2}$$

In this form of the log law, that is appropriate to flow above a rough surface, U_*=friction velocity ($\tau=\rho U_*^2$, where τ=mean wall shear stress and ρ=fluid density); κ=von Kármán constant; z=position perpendicular to the bed; and Z_0=roughness length. Z=position above the bed measured relative to a conveniently located origin, such as the top of the bed roughness; and d=zero plane displacement. $\langle U\rangle$=local mean streamwise velocity. In this technical note angle brackets, $\langle\rangle$, are used to indicate time averaged quantities.

It is not uncommon, particularly in the case of field measurements, for the parameters that appear in Eq. (1), including U_*, Z_0, and d, to be found through least squares analysis using available experimental data on the measured streamwise velocity alone. Here this is termed the "one-equation" approach. (For a general treatment of linear regression analysis see Brownlee 1965.) Usually κ is treated as a universal constant between 0.4 and 0.41 (Nezu and Rodi 1986) and it is not included as a parameter to be determined in the analysis. On the other hand, in controlled laboratory investigations it is frequently possible to determine U_* independently of Eq. (1). For example, in the case of fully developed, two-dimensional, turbulent channel flow (Tennekes and Lumley 1992)

$$\frac{\nu\dfrac{\partial\langle U\rangle}{\partial z} - \langle uw\rangle}{U_*^2} = 1 - \frac{Z}{H} \tag{3}$$

is valid. In Eq. (3) lowercase quantities (e.g., u and w) represent fluctuations about the mean. u=component in the streamwise direction; v=spanwise component; w=component perpendicular to the sediment bed, the normal component; ν=kinematic viscosity; and H=flow depth. In flows where Eq. (3) is valid, additional measurements of the Reynolds shear stress, $-\langle uw\rangle(Z)$, can be used to estimate U_* by fitting Eq. (3) to the measured stress profile (Nezu and Nakagawa 1993). A subsequent least-squares analysis on Eq. (1) is applied to obtain Z_0 and d. Here this is termed the "two-equation" approach. Unfortunately, this method is not always practical and it may be necessary to determine all three parameters, U_*, Z_0, and d, from Eq. (1) alone, with no independent measurement of U_*. Although this "one-equation" approach is convenient and relatively easy to apply, the uncertainty in the computed parameters may be higher than desired since several adjustable parameters are available to fit a single equation to the data. In this technical note both approaches are used to analyze the same high quality set of data obtained in a fully developed, fully rough, two-dimensional, turbulent open-channel flow; and the estimated values of the parameters and their respective standard errors are compared.

Facilities, Instrumentation, and Flow Conditions

The experiments were performed in the Kelso S. Baker Environmental Hydraulics Laboratory at the Virginia Polytechnic Institute and State University in Blacksburg, Va. A tilting laboratory flume

[1]Associate Professor, Dept. of Mechanical Engineering, Virginia Tech Univ., Blacksburg, VA 24061-0238 (corresponding author). E-mail: cld@vt.edu

[2]Professor, Baker Environmental Hydraulic Lab, Dept. of Civil & Environmental Engineering, Virginia Tech. Univ., Blacksburg, VA 24061. E-mail: pdiplas@vt.edu

Note. Discussion open until February 1, 2009. Separate discussions must be submitted for individual papers. The manuscript for this technical note was submitted for review and possible publication on July 13, 2004; approved on July 23, 2007. This technical note is part of the ***Journal of Hydraulic Engineering***, Vol. 134, No. 9, September 1, 2008. ©ASCE, ISSN 0733-9429/2008/9-1353–1356/$25.00.

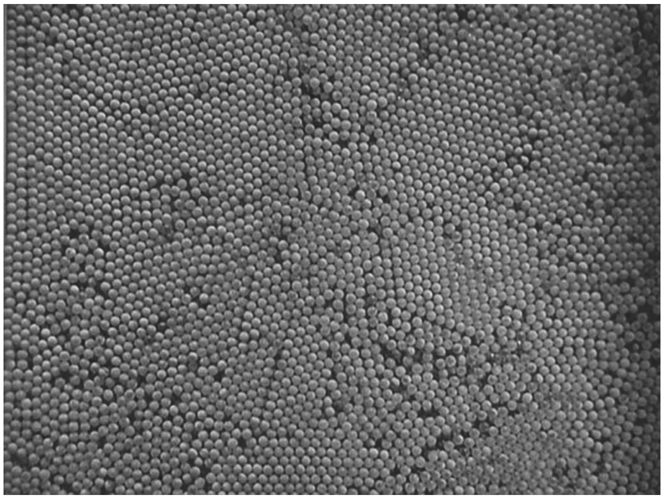

Fig. 1. Densely packed bed particles, but randomly oriented relative to channel flow (top view); flow is from top to bottom

and a three-component laser Doppler velocimeter (LDV) system were used for the experiments. The tilting flume is of rectangular cross section and is 14.6 m long, 0.6 m wide, and 0.3 m deep. The velocity profile data were measured at a location within the "test section" near the downstream end of the flume where fully developed turbulent flow conditions were achieved. The "test section" is 3 m long and composed of four layers of densely packed, 8 mm diameter glass balls (SG=2.54). Natural gravel of approximately the same median diameter was used upstream of the test section, extending to the flume entrance. A photograph of the bed in the measurement area is shown in Fig. 1. The glass balls were tightly packed in a general hexagonal pattern with adjacent regions oriented randomly, as depicted in the photo of Fig. 1.

LDV was used to measure the velocity profiles due to its high accuracy, its high spatial resolution (the measurement volume is estimated to be approximately 100 μm in diameter and 300 μm in length), and because of its nonintrusive nature. The entire laser system was mounted on a three-axis traverse table that can position the measurement volume with high accuracy and precision within the flume. For the measurements reported here the LDV system was mounted alongside of the flume with the laser beams probing through the side wall of the flume. The optical table on which the LDV system was mounted was rotated 4.8° to the horizontal to allow measurements very close to the bed. Conversion from the LDV coordinate system to flume coordinates (x,y,z) where x is streamwise along the flume axis, y is spanwise across the flume, and z is perpendicular to the flume bottom was performed during postprocessing of the data. Software developed at the Kelso Baker Laboratory was used for data analysis.

For the measurements reported here the channel flow was fully developed with a Reynolds number (based upon the cross-sectional area average velocity and flow depth) of 21,500 and a Froude number of 0.26. The flow depth was 9.7 cm and $U_*=0.0166$ m/s [determined by fitting Eq. (3) to the stress measurements]. Thus $\mathsf{R}_*=U_* d_p/\nu=116$ and the flow is regarded as fully rough. d_p is the diameter of a roughness element (8 mm). The velocity profile was measured along a line perpendicular to and above the roughness bed, at a location 2.5 cm to one side of the centerline of the channel (toward the LDV system and such that the side-wall effect is negligible, i.e., $|y/H|\langle(B/H-5)/2$, where B is the flume width), but otherwise arbitrarily located over the roughness pattern. The width to depth ratio for the flow was 6.2, which exceeds the critical value required for two-dimensional (2D) flow in the central portion of the channel (Nezu and Rodi 1985; Neza and Nakagawa 1993). Furthermore, no secondary flows were measured by the three-component LDV at the measurement location. Three thousand seventy two individual velocity triplets (U,V,W) were measured at each of 49 positions above the bed where the average data rate ranged between 12 and 25 measurements (triplets) per second. Near the roughness the distance between measurement positions is small, $\Delta z=0.127$ mm, further from the bed this increases to 0.254 mm, and well above the roughness surface the distance increases to 0.508 mm. It is noted that for smooth walls deviation from the log law has been reported for $Z/H>0.2$. For rough wall flow it is not uncommon to apply the log law beyond this limit, and some researchers have reported using the log law over the entire depth (see for example, Chen and Chiew 2004, 2003). In the present case, a limited number of measurements were included beyond the $Z/H=0.2$ limit with seven of the 49 measurement locations in the range $0.2<Z/H<0.275$. No measurements were taken above $Z/H=0.275$. The kinematic Reynolds stresses, in particular $-\langle uw\rangle$, were obtained from the measured velocity triplets during postprocessing of the data. The estimated uncertainty in the mean streamwise velocity, $\langle U\rangle$, is 1.5%, the uncertainty in the streamwise RMS velocity, u', is 5%, and the uncertainty in the kinematic Reynolds stress, $-\langle uw\rangle$, is estimated to be 10%.

It is emphasized that in this investigation the exact same set of measurements was used for both the "one-equation" and "two-equation" approaches. Any change in the resulting values of the parameters and their standard error estimates is due to the method of analysis not the data. It is also noted that the authors have taken several sets of measurements under fully developed, 2D channel conditions. This particular set of data represents the highest quality data among these sets (as judged by the low standard error in the estimates). All of these sets reveal similar behavior with even larger standard errors, when using the "one-equation" method.

Results

The nondimensional total stress, $T/U_*^2=(\nu\,\partial\langle U\rangle/\partial z-\langle uw\rangle)/U_*^2$, as a function of nondimensional height above the bed surface, Z/H, is shown in Fig. 2. T, the total stress, is the sum of the mean viscous and Reynolds shear stresses. The value of U_* (0.0166 m/s) used in the nondimensionalization of T in Fig. 2 was obtained by regressing T on Z/H using Eq. (3). The result is shown in the figure. The regression yields a standard error of the parameter U_* of 0.55% and a coefficient of determination, $R^2=0.618$.

In Fig. 3 the respective turbulence intensities (u', v', and w' nondimensionalized by $U_*=0.0166$ m/s) are shown as functions of Z/H. These nondimensional distributions compare favorably to other reported measurements under similar (fully rough) conditions. As an example, the data of Nezu (1977) taken with hot-film anemometry is also shown in Fig. 3 for comparison to the present measurements.

Finally, the measured mean streamwise velocity profile, $\langle U\rangle(Z)$, was used together with the previously determined value of $U_*=0.0166$ m/s to obtain d and Z_0 by regression utilizing Eq. (1). This is the "two-equation" approach. The result ($d=2.0$ mm, $Z_0=0.194$ mm) is shown in Fig. 4, assuming $\kappa=0.41$. The stan-

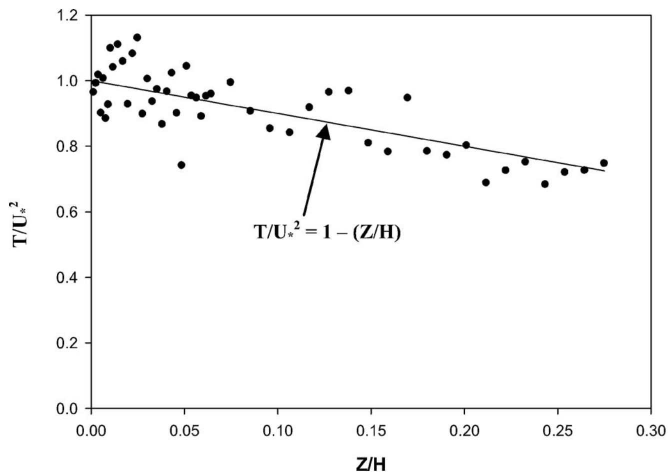

Fig. 2. Nondimensional total stress $T/U_*^2=[\nu(\partial\langle U\rangle/\partial z)-\langle uw\rangle]/U_*^2$ versus nondimensional distance from bed surface, Z/H: (•) experimental data; (—) $1-Z/H$; $U_*=0.0166$ m/s

dard error of the parameters d and Z_0 are 1.73 and 2.94%, respectively. The coefficient of determination, $R^2=0.993$. It is noted that the estimated values of d and Z_0 are comparable to values reported in the literature. For example, for sand grain roughness it is not uncommon to estimate $Z_0 \approx d_p/30$ (Raupach et al. 1991)$=0.266$ mm in the present case, and $d \approx 0.2d_p$ (Einstein and El-Samni 1949)$=1.6$ mm for this case.

The "two-equation" method described above uses the experimental data to the fullest to produce estimates for the friction velocity U_* independent of the log law and the parameters d and Z_0 from the log law. This is possible since the measurements included the distribution of $\langle uw\rangle$. In the event that the measured distribution of $\langle uw\rangle$ is not available, all three parameters (U_*, d, and Z_0) might be obtained by regression with Eq. (1) alone. In this case different estimated values of U_*, d, and Z_0 are expected. For the present set of measurements, this "one-equation" method yields $U_*=0.0187$ m/s, $d=2.73$ mm, and $Z_0=0.325$ mm (compared to 0.0166 m/s, 2.0 mm, and 0.194 mm, respectively, by the two-equation approach). The coefficient of determination, $R^2=0.994$ (compared to 0.993 for the two-equation method) and

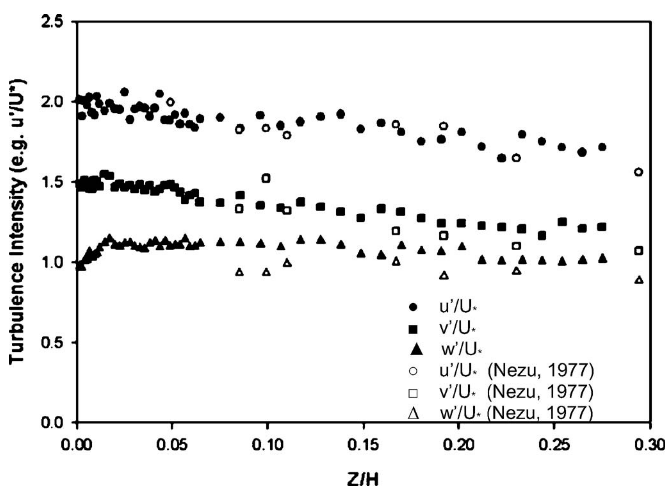

Fig. 3. Turbulence intensities versus Z/H: (•) u'/U_*, (■) v'/U_*, (▲) w'/U_*; (open symbols) Nezu (1977)

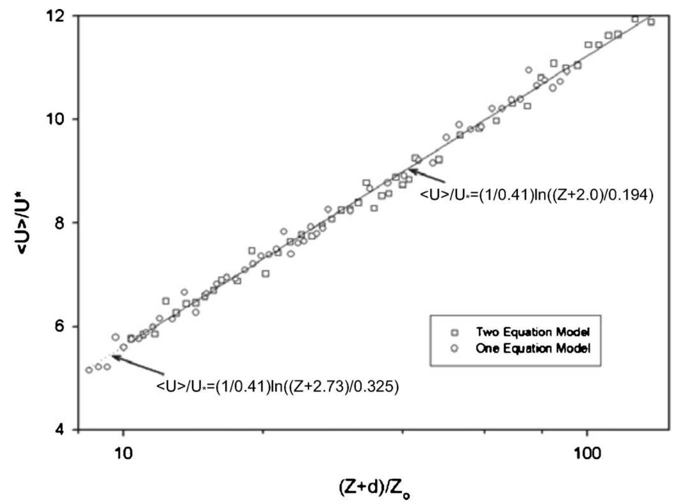

Fig. 4. Nondimensional mean streamwise velocity $\langle U\rangle/U_*$ versus nondimensional distance $(Z+d)/Z_0$ (□) data; (solid line) Eq. (1) $U_*=0.0166$ m/s; $d=2.0$ mm; $Z_0=0.194$ mm, using the "two-equation" method. (○) data; (...) Eq. (1): $U_*=0.0187$ m/s; $d=2.73$ mm; $Z_0=0.325$ mm, using "one-equation" method.

most significantly, the standard errors of the parameters U_*, d, and Z_0 are 3.88, 10.66, and 16.9%, respectively (compared to 0.55, 1.73, and 2.94%). The significant increase in the standard errors in the estimated parameters is noted. For comparison purposes Eq. (1) is plotted in Fig. 4 for both methods (using their respective values of U_*, d, and Z_0). It is noted that, when judged by eye, there appears to be no significant difference between the two results and yet the standard errors in U_*, d, and Z_0 are significantly different, as are their predicted values.

Finally, it is noted that the one-equation method (log law alone) yields estimates of the three parameters that are significantly larger than those estimated with the two-equation approach: i.e., the one-equation approach yields values for U_*, d, and Z_0 that are 12.7, 36.7, and 67.5% greater, respectively, than those predicted by the two-equation model. To further demonstrate the significance of the difference in the estimated value of U_* when using these two different approaches the total stress profile is plotted again, in Fig. 5, now using the value of $U_*=0.0187$ m/s obtained from the one-equation method to nondimensionalize the total stress. Fig. 5 should be compared with Fig. 2. It is apparent from Fig. 5 that the value of the friction velocity obtained from the log law alone is inconsistent with the stress measurements; while the two-equation method yields a value for U_* that is consistent with the measured stress profile as well as the root mean square (RMS) levels of the velocity component fluctuations (Fig. 3), and provides an excellent fit of the log law to the mean velocity data.

Conclusion

It has been experimentally demonstrated that estimating the three relevant parameters associated with the log law—the friction velocity, the roughness length, and the zero plane displacement—by statistical regression of the mean velocity data with Eq. (1) alone leads to relatively large standard errors in the parameters. This is not surprising, since with this method three parameters are available to fit the equation [Eq. (1)] to the data. That is, there is a

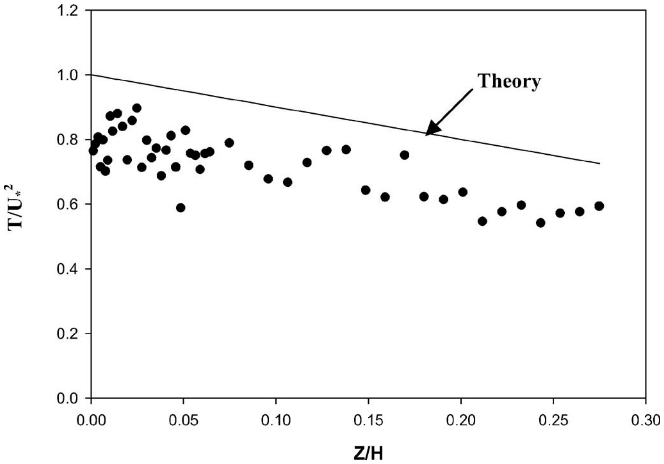

Fig. 5. Nondimensional total stress, $T/U_*^2=[\nu(\partial\langle U\rangle\partial z)-\langle uw\rangle]/U_*^2$ versus nondimensional distance from bed surface, Z/H: (•) experimental data; (—) $1-Z/H$, $U_*=0.0187$ m/s

range of combinations of parameter values that may satisfactorily fit the data. Furthermore, it is demonstrated that the estimated values of these parameters may differ significantly from those calculated using more complete methods (utilizing more extensive data and model equations, such as the two-equation approach demonstrated in this technical note).

It is remarked that this work does not imply that the single equation regression approach is invalid or should not be implemented when appropriate, but rather that the potential uncertainties associated with the approach should be considered whenever it is applied, and that they are not insignificant. These errors may contribute to the variability of the d and Z_0 values reported in the literature, as well as to the controversy about the discrepancy observed among the various methods used to calculate U_*.

Acknowledgments

The writers gratefully acknowledge the support of the National Science Foundation, NSF Grant No. EAR-0439663.

Notation

The following symbols are used in this technical note:

d = zero plane displacement;
d_p = sediment diameter;
H = flow depth (measured from top of sediment bed);
R^2 = coefficient of determination;
R_* = sediment Reynolds number, $U_* d_p/\nu$;
SG = specific gravity;
T = total mean shear stress, viscous+Reynolds shear stress;
U = instantaneous streamwise velocity in flume;
$\langle U\rangle$ = time mean streamwise velocity in flume;
U_* = friction velocity;
u' = RMS level of streamwise fluctuating velocity;
$\langle uw\rangle$ = kinematic Reynolds shear stress in x,z plane;
V = instantaneous spanwise velocity in flume;
v' = RMS level of spanwise fluctuating velocity;
W = instantaneous normal velocity component in flume (perpendicular to bottom);
w' = RMS level of normal fluctuating velocity;
x = streamwise coordinate in flume;
y = spanwise coordinate in flume;
Z = distance above top of roughness bed and normal to bed [Eq. (2)];
Z_0 = roughness length [Eq. (1)];
z = normal coordinate (perpendicular to bottom surface) in flume [Eqs. (1) and (2)];
κ = von Kármán's constant;
ν = kinematic viscosity;
ρ = fluid density; and
τ = mean bed shear stress.

References

Brownlee, K. A. (1965). *Statistical theory and methodology*, 2nd Ed., Wiley, New York.

Chen, X., and Chiew, Y.-M. (2003). "Response of velocity and turbulence to sudden change of bed roughness in open-channel flow." *J. Hydraul. Eng.*, 129(1), 35–43.

Chen, X., and Chiew, Y.-M. (2004). "Velocity distribution of turbulent open-channel flow with bed suction." *J. Hydraul. Eng.*, 130(2), 140–148.

Einstein, H. A., and El-Samni, E. A. (1949). "Hydrodynamic forces on a rough wall." *Rev. Mod. Phys.*, 21(3), 520–524.

Nezu, I. (1977). "Turbulent structure in open-channel flows." Doctoral dissertation, Kyoto Univ., Kyoto, Japan.

Nezu, I., and Nakagawa, H. (1993). *Turbulence in open channel flow*, IAHR, Balkema, Rotterdam.

Nezu, I., and Rodi, W. (1985). "Experimental study on secondary currents in open channel flow." *Proc., 21st IAHR Congress*, Vol. 2, Melbourne, Australia, 115–119.

Nezu, I., and Rodi, W. (1986). "Open-channel flow measurements with a laser Doppler anemometer." *J. Hydraul. Eng.*, 112(5), 335–355.

Raupach, M. R., Antonia, R. A., and Rajagopalan, S. (1991). "Rough-wall turbulent boundary layers." *Appl. Mater. Res.*, 44, 1–25.

Tennekes, H., and Lumley, J. L. (1992). *A first course in turbulence*, 14th printing, MIT Press, Boston.