Exploring Dark Energy Models

Michael Sekatchev
UBC Department of Physics and Astronomy

Introduction

The **Friedmann equation** relates the evolution of the scale factor $a(t)$ to different components making up our universe via

$$\left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \sum_i \Omega_{i,0}\, a^{-3(1+w_i)},$$

where w_i is related to the **equation of state**,

$$P_i = w_i \epsilon_i, \quad w_i = \begin{cases} 0 & \text{matter,} \\ 1/3 & \text{radiation,} \\ -1/3 & \text{curvature.} \end{cases}$$

As seen from the **acceleration equation**, which describes the acceleration of the scalar factor,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\epsilon + 3P),$$

where $\epsilon = \sum_i \epsilon_i$, $P = \sum_i P_i$, the observed positive acceleration of our Universe, $\ddot{a} > 0$, can **only** be achieved by introducing a new component with $w_\chi < -1/3$. Writing the Friedmann equation today with this included gives

$$\begin{aligned} 1 = \; & \Omega_{r,0} + \Omega_{m,0} + \Omega_{\chi,0} \\ \approx \; & 10^{-4} + 0.3 + 0.7, \end{aligned}$$

with r for radiation, m for matter, k for curvature, χ for this new term.
This poster summarizes a few candidates for this "dark energy" term which drives the expansion of the Universe and makes up $\sim$70% of it.

Cosmological Constant

First introduced by Einstein in 1917, this is the simplest and most popular dark energy model, which proposes that Ω_χ is a **constant** by setting its equation of state parameter to $w_\Lambda = -1$,

$$\Omega_\chi = \Omega_\Lambda = \Omega_{\Lambda,0} a^{-3(1+w_\Lambda)} = \Omega_{\Lambda,0} = \frac{\Lambda}{3H_0^2},$$

where Λ is the cosmological constant. Physically, it can be thought of as the energy density of empty space / **vacuum energy**.

Quintessence

An alternative model is **quintessence** which suggests that dark energy is a dynamical **scalar field** $Q(z)$. The w_Q parameter varies with time (redshift z) via

$$-1 < w_Q(z) = \frac{P_Q}{\varepsilon_Q} = \frac{\frac{1}{2}\dot{Q} - V(Q)}{\frac{1}{2}\dot{Q} + V(Q)} < -1/3,$$

where $V(Q)$ is the potential of the scalar field, and $\frac{1}{2}\dot{Q}$ is the kinetic term. There is not enough experimental evidence to put any constraint on the form of $Q(z)$ or $w_Q(z)$. It is only known that $\dot{Q}(z)$ is small to ensure $P_Q < 0$. It is common to use two parameters for the equation of state, one for $w_Q(z=0)$ and one for $w_Q'(z=0)$, for example:

$$w(z) = w_0 + \frac{w_a z}{1+z},$$

$$\text{or } w(z) = \begin{cases} w_0 + w_1 z & \text{if } z < 1, \\ w_0 + w_1 & \text{if } z \geq 1. \end{cases}$$

Phantom Energy

A variation of quintessence, **phantom energy** also proposes a scalar field but with equation of state parameter $w_Q < -1$, **more negative** than the cosmological constant.

In this model, as soon as phantom energy density becomes dominant in the Universe, the scale factor will accelerate at an ever-increasing rate until every particle in the Universe is infinitely far away from every other one. This is the **Big Rip** scenario.

Problems

There are two problems with the dark energy models presented:

1. **Cosmological constant problem**: The energy density of dark energy is $\sim 10^{-47}$ GeV^4 (see figure). **Quantum Field Theory** predicts it should be many orders of magnitude **higher**. It is also $\sim$14 orders of magnitude **lower** than the **electroweak scale** — a new mass scale is required.
2. **Cosmic coincidence problem**: We live in a time where $\mathcal{O}(\Omega_m) = \mathcal{O}(\Omega_\Lambda)$. For this to happen, the ratio Ω_m/Ω_Λ must have been "set" exactly right with **infinitesimal precision** from early on for this coincidence $\sim$13 billion years later.

Tracking Quintessence

Both problems can be solved by introducing **tracking**. The energy density $\Omega_{Q,T}$ of this proposed field closely tracks the energy density of radiation, with $\Omega_{Q,T} < \Omega_r$ until $z = 0.3$ when it starts to dominate.

For tracking quintessence, $Q(z)$ satisfies

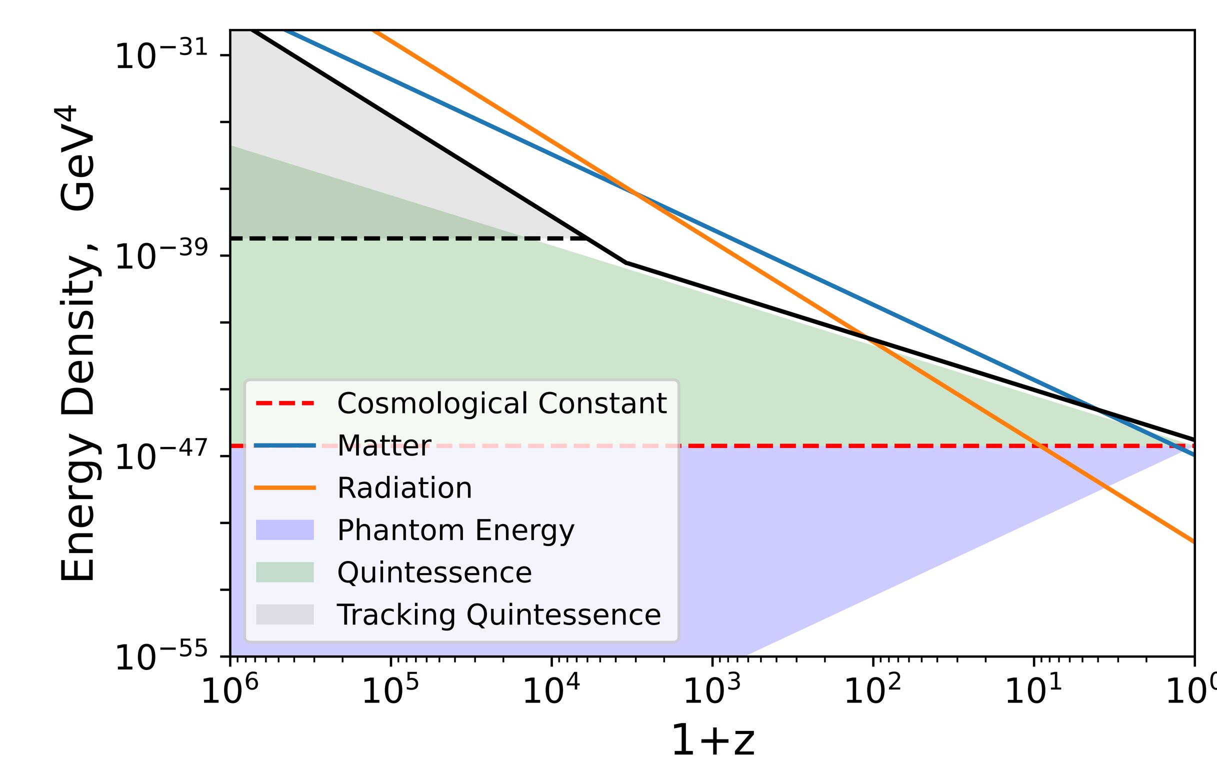
$$\ddot{Q} + 3\left(\frac{\dot{a}}{a}\right)\dot{Q} + V'(Q) = 0.$$

Several forms of potential are possible, for instance,

$$V(Q) = M^4\left(e^{\frac{m_p}{Q}} - 1\right),$$

where M is a parameter and m_p is the planck mass. This can be solved numerically to determine $w_{Q,T}$.

The plot below shows the evolution of all dark energy models, with matter and radiation components for comparison.



The shaded regions represent possible forms for each model. Phantom energy could occupy the purple region, with $w_Q \in \{-2, -1\}$, and regular quintessence could be in the green region with $w_Q \in \{-1, 1/3\}$. Tracking quintessence could have any initial energy density as long as it remains below the dominant component.