First Test

NAME: _____

You will have one hour and fifteen minutes to complete this exam. **YOU MUST SHOW ALL YOUR WORK FOR FULL CREDIT.**. The majority of the credit you receive will be based on the completeness and the clarity of your responses. Unless Stated otherwise, answers should be exact values rather than approximations. Scientific calculators are allowed, along with standard graphing calculators (i.e., TI-84), but not calculators that can do symbolic differentiation/integration (e.g., IT-89, TI-Nspire). No other electronic device is permitted. This test is closed book and closed notes.

Question	Points	Score
1	15	
2	11	
3	8	
4	8	
5	8	
6	16	
7	18	
8	16	
9	0	
Total:	100	

(15 points) 1. Let $\vec{v} = \hat{\imath} + \hat{\jmath}$ and $\vec{w} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$. Find each of the following. Unless otherwise indicated, find the exact value.

(a) $|\vec{w}|$

(b) $\vec{v} \cdot \vec{w}$

(c) The angle between $\vec{v}$ and $\vec{w}$ to the nearest whole degree

(d) The vector projection of $\vec{v}$ onto $\vec{w}$

(11 points) 2. Find where the two lines below intersect, or show that they do not intersect.

$$\mathcal{L}_1: \quad \begin{aligned} x &= 3 + 2t \\ y &= -1 + 4t \\ z &= 2 - t \end{aligned} \qquad \mathcal{L}_2: \quad \begin{aligned} x &= 1 + 4s \\ y &= 1 + 2s \\ z &= -3 + 4s \end{aligned}$$

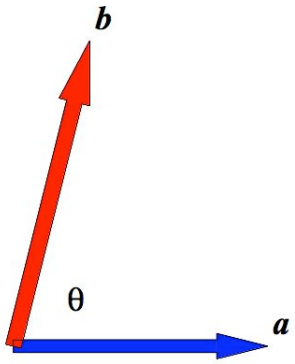
(8 points) 3. Find the length of the curve $\langle \sqrt{1+t^3},\ \sqrt{1+t^3} \rangle$ between $t = 0$ and $t = 2$.

(8 points) 4. Given the surface $-3x^2 - y^2 - z^2 + 4y = -22$.

(a) Rewrite in standard form.

(b) Identify the quadric surface

(8 points) 5. Let $\vec{a}$ and $\vec{b}$ be two vectors in $\mathbb{R}^2$ as shown in the figure below. Sketch the vectors $2\vec{b}$, $-3\vec{a}$ and $2\vec{b} - 3\vec{a}$. It is recommended to draw the vectors over the ones in the figure 1.



(16 points) 6. (a) Find the equation of a plane determined by the points (1,-1,2), (3,0,0) and (4,2,1).

(b) Is this plane parallel to the plane $x + y - z = 5$? Show your work, or otherwise justify your answer.

7. Let a space curve be given by $\vec{r}(t) = (e^t + e^{-t})\,\hat{\imath} + (e^t - e^{-t})\,\hat{\jmath} + 2t\hat{k}$.

(6 points) (a) Find the unit tangent vector at the point corresponding to $t = 1$.

(6 points) (b) Find the parametric equations of the tangent line to the space curve when $t = 1$.

(6 points) (c) Find $\displaystyle\int_0^1 \vec{r}(t)\,dt$

8. A particle moves in a path defined by the function $\vec{r}(t) = t^2\ \vec{\imath} + (2t-3)\ \vec{\jmath} + (3t^2-3t)\ \vec{k}$.

(4 points) (a) Find the velocity of the particle.

(4 points) (b) Find the speed of the particle.

(4 points) (c) Find the acceleration of the particle.

(4 points) (d) Find the curvature $\kappa = \dfrac{|\vec{r}\,' \times \vec{r}\,''|}{|\vec{r}\,'|^3}$

(6 (bonus)) 9.

1. Let $\vec{u} = \langle 2, 4, -5 \rangle$, and $\vec{w} = \langle -4, -8, k \rangle$. For what values of k will $\vec{u}$ and $\vec{w}$ be parallel?
2. Let $\vec{u} = \langle 2, 4, -5 \rangle$, and $\vec{w} = \langle -4, -8, k \rangle$. For what values of k will $\vec{u}$ and $\vec{w}$ be orthogonal?