

MAC 2311 CALCULUS I CRN 85503

Exam 1

Term Fall, 2024

Full Name:

Instructions

1. Total time: **1 hour 15 minutes**.
2. Write the information requested above.
3. Switch off any electronic devices.
4. Calculators are not allowed.
5. Write the solution in the given space.
6. Show all your work for full credit.
7. Scratch papers are provided but will not be graded.
8. You are not allowed to use differentiation rules.
9. You are not allowed to use L'Hôpital's Rule in this exam.

Formulas

1. Slope of a tangent line at $x = a$ for the curve $f(x)$: $m = f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$
2. Derivative of a function using limit definition: $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

Q.N.	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
Bonus	10	
Total	70	

1. (10 points) Find the domain and the inverse of the below functions:

(a) $f(x) = \frac{x^5}{5} - 1$

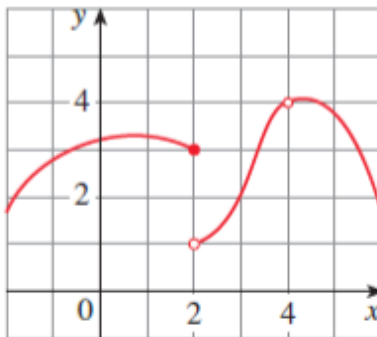
(b) $y = 3 - \sqrt{x}$

(c) $y = \ln(x + 2)$

2. (10 points)

(a) (4 pts) Find the equation of the vertical asymptotes ($x = a$) of the function: $f(x) = \frac{x^2 + 2}{x^2 - 7x + 10}$

(b) (6 pts) Using the graph of function $f(x)$ given below, answer the following questions.
(If a limit does not exist, write “**DNE**” and if function is undefined at a point, write “**undefined**”.)



(i) $\lim_{x \rightarrow 2^-} f(x) =$

(ii) $\lim_{x \rightarrow 2^+} f(x) =$

(iii) $\lim_{x \rightarrow 2} f(x) =$

(iv) $f(2) =$

(v) $\lim_{x \rightarrow 4} f(x) =$

(vi) $f(4) =$

3. (10 points)

(a) (5 pts) Determine the infinite limits (∞ or $-\infty$):

$$\lim_{x \rightarrow 2^+} \frac{x+1}{x-2} =$$

$$\lim_{x \rightarrow 4^-} \frac{x}{x-4} =$$

(b) (5 pts) Find $f \circ g \circ h$ for the functions: $f(x) = 2x + 1$, $g(x) = x^2$, $h(x) = \cos x$,

4. (10 points)

(a) (5 pts) Find the value of x in the below equation:

$$x = \log_2 8 + \log_2 \frac{1}{8} + \ln e$$

(b) (5 pts) Find the below limit (if exist) and the equation of the horizontal asymptotes ($y = L$):

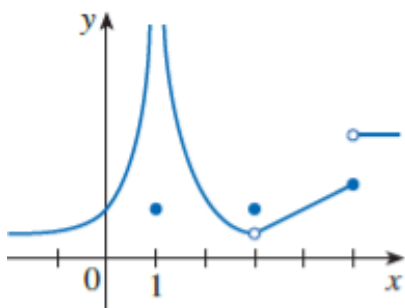
$$\lim_{x \rightarrow \infty} \frac{3x^2 - 5x + 2}{4 - 3x - x^2}$$

5. (10 points) Find the below limits algebraically, if exists (otherwise write DNE):

(a) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x^2 - 4x + 3}$

(b) $\lim_{x \rightarrow 0} \frac{(x - 1)^2 - 1}{x}$

6. (10 points) Find if the function f in the graph is continuous or discontinuous at the below points.
 If discontinuous then Why?
 (Choose one of the options: (i) f is undefined, or (ii) limit does not exist or (iii) $\lim_{x \rightarrow a} f(x) \neq f(a)$)



(a) At $x = 1$

(b) At $x = 2$

(c) At $x = 3$

(d) At $x = 4$

(e) At $x = 5$

7. (10 points) $f(x) = x^2 + 3$

(a) Find the derivative of the function $f'(x)$ using the limit definition of derivative.

(b) Use the derivative from previous part and find the slope of the tangent line to the curve $f(x)$ at $x = 1$.

Bonus Problem (Extra 10 points)

- (a) Find the value of constant c so that the function is continuous everywhere

$$G(x) = \begin{cases} 4 - \cos x, & \text{if } x < 0 \\ \sqrt{x + 3c}, & \text{if } x \geq 0 \end{cases}$$

- (b) Find the horizontal and vertical asymptotes of the curve

$$f(x) = \frac{2e^x}{e^x - 5}$$