

GenAI and mathematical education

What should we do?

Just for this meeting:

AI = GenAI = LLM

Think of “ChatGPT or some equivalent technology”

People have too inflated sense of what it means to "ask an AI" about something. The AI are language models trained basically by imitation on data from human labelers. Instead of the mysticism of "asking an AI", think of it more as "asking the average data labeler" on the internet. [...]

Post triggered by someone suggesting we ask an AI how to run the government etc. TLDR you're not asking an AI, you're asking some mashup spirit of its average data labeler.

— Andrej Karpathy (Nov 2024)

Poll on your AI experiences

Please stand up for “yes”

- Have you used AI recently (for work, curiosity, ...)?
- Have you used AI productively in your workflow?
- Do you know RUG policy on the use of AI (for us)?
- Do you know RUG policy on the use of AI (for our students)?
- Do you provide your own extra guidelines on AI to your students?
- Have you experienced the use of AI from your students?

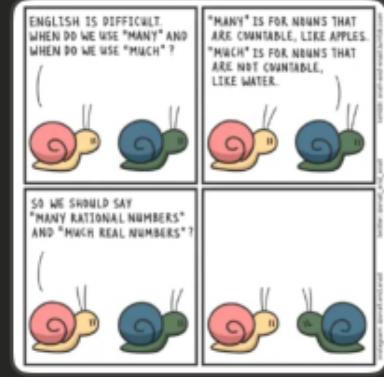
*Ethical*** and environmental** concerns* (which are *big* and *important*) aside, I think these can be useful tools and I am skeptical about forbidding them. I will try to argue why.

But we need to educate our students (and ourselves) on *if*, *when* and *how* to use them.

How I use AI #1

Accessibility: generate ALT text for posts on social networks

Marcello



Alt text for the image

A four-panel comic with two snails discussing grammar.

Panel 1: "ENGLISH IS DIFFICULT. WHEN DO WE USE 'MANY' AND WHEN DO WE USE 'MUCH'?" Panel 2: "'MANY' IS FOR NOUNS THAT ARE COUNTABLE, LIKE APPLES. 'MUCH' IS FOR NOUNS THAT ARE NOT COUNTABLE, LIKE WATER." Panel 3: "SO WE SHOULD SAY 'MANY RATIONAL NUMBERS' AND 'MUCH REAL NUMBERS'?" Panel 4: Shows the snails looking confused



Claude can make mistakes.
Please double check responses.

Reply to Claude



How I use AI #2

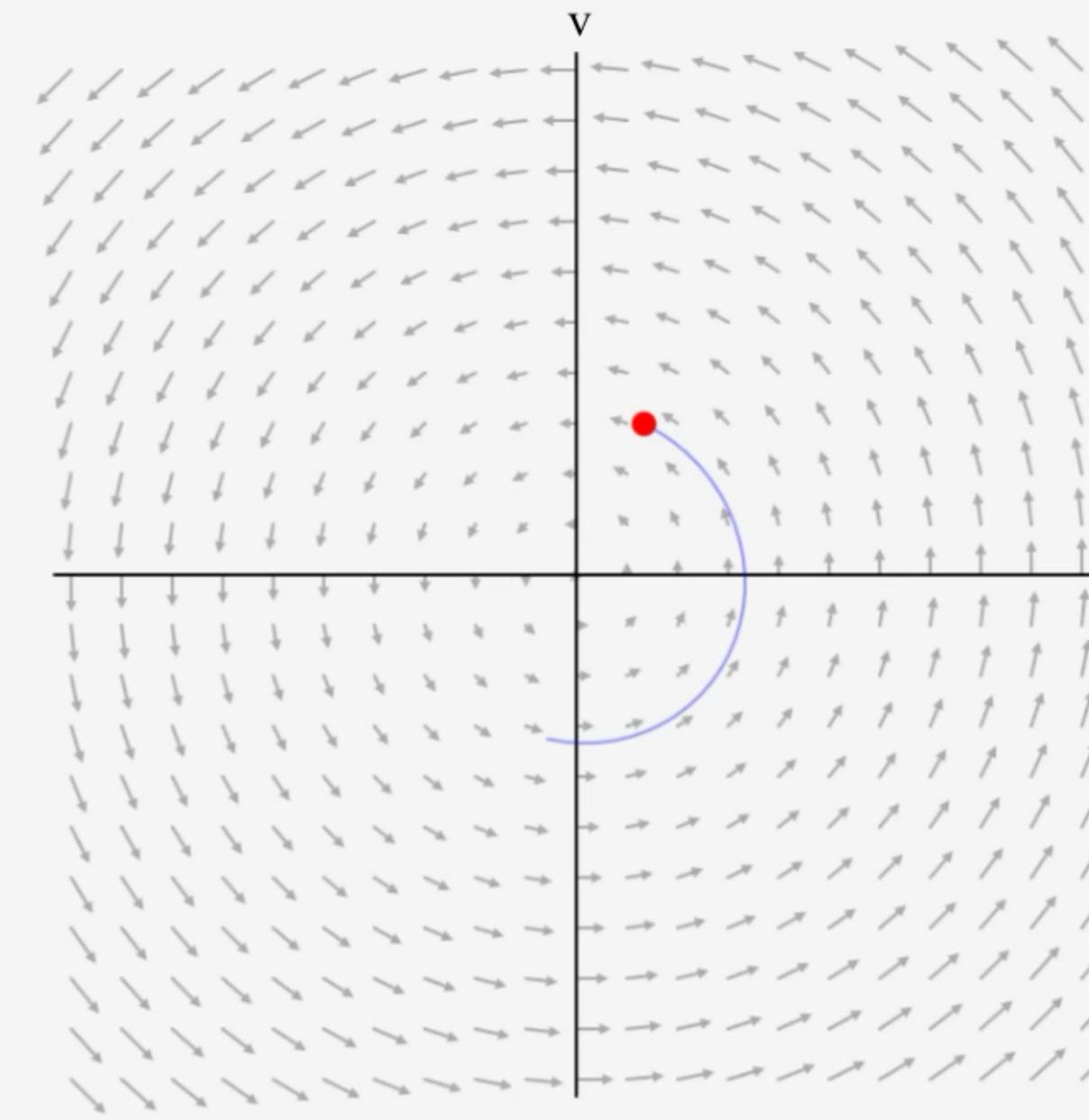
Generate interactive browser demonstrations (translating my python code)

Screenshot of Example 1.8 at <https://www.mseri.me/hammech20/chapter-1.html#autosec-46>

Example 1.8. Note that Theorem 1.6 applies to Example 1.1.

- In the case of the spring, also called the *harmonic oscillator*, $U(x) = \frac{1}{2}\omega^2x^2$ and the integral curve $y^2 + \omega^2x^2 = C$. Recalling (1.6), $C = \omega^2R^2 \geq 0$ and the curves are ellipses parametrized by C . This example throughout the course.

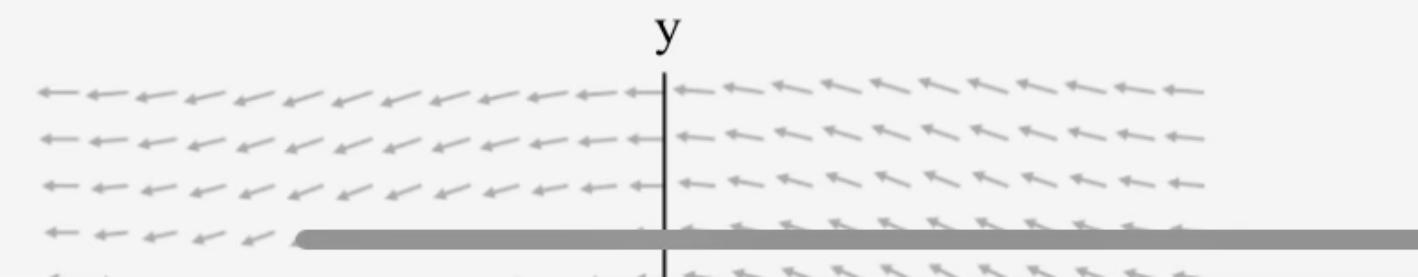
p5* Spring - phase portrait by mseri



Use the sliders to see how the phase plane is changing. What happens if you vary

- In the case of the pendulum, $U(x) = -\omega^2 \cos(x)$ and the integral curves are solutions of $\frac{1}{2}y^2 - \omega^2 \cos(x) = C$. Even though we cannot easily give a time parametrization of the motion itself, this formalism allows us to describe the evolution of the system. We will come back to this fact in more generality in the next chapter.

p5* Pendulum - phase portrait by mseri



How I use AI #3

Generate *interactive browser demonstrations* (from scratch)

Screenshot of <https://www.mseri.me/riemann-rearrangement-theorem>

Rearrangements generator for conditionally convergent series

Enter a number:

0.378

Number of terms:

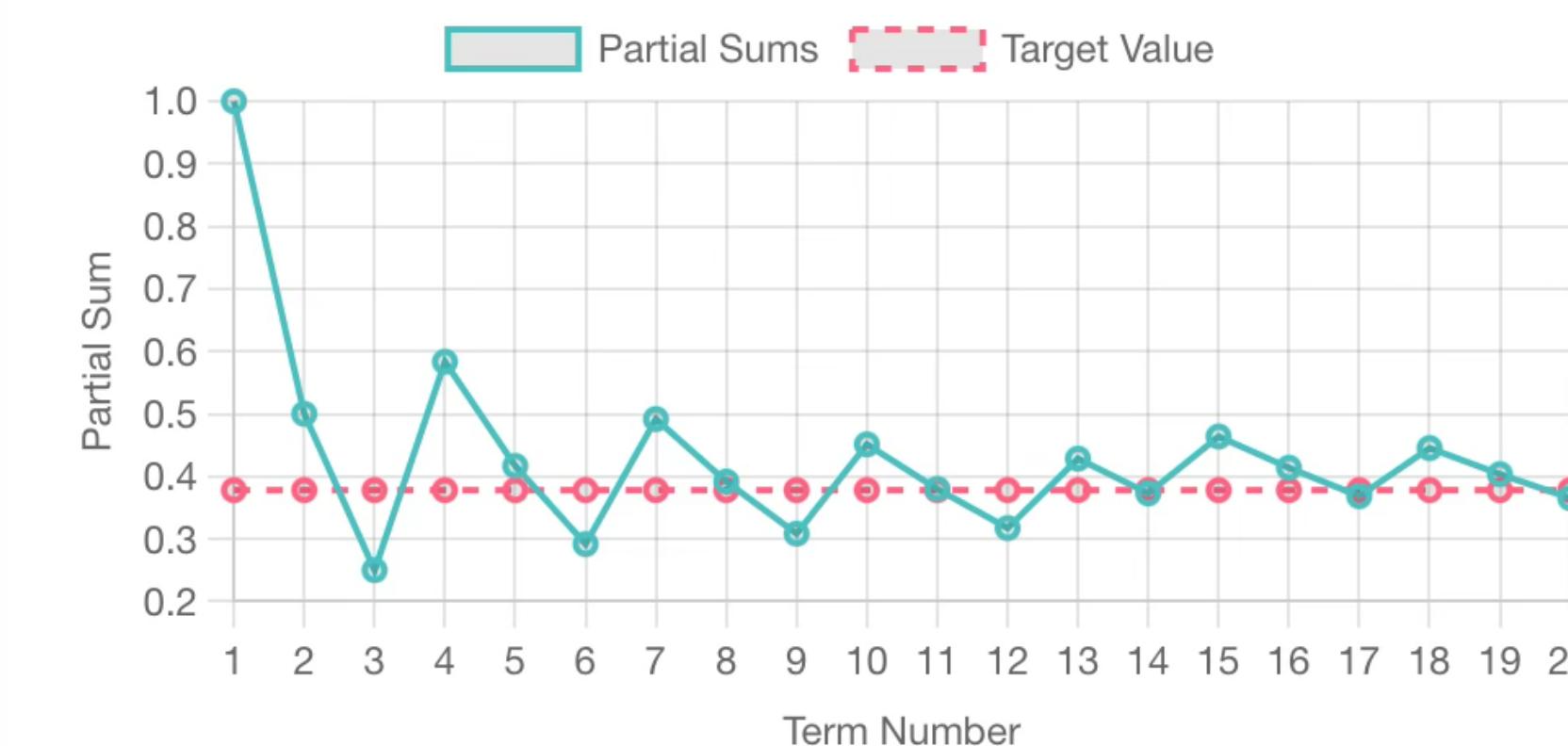
20

Select sequence type:

$(-1)^{n+1}/n$

Generate Series

Reset



The first 20 terms of the rearranged alternating harmonic series that converges to 0.378 are:

Reveal Terms

How I use AI #4

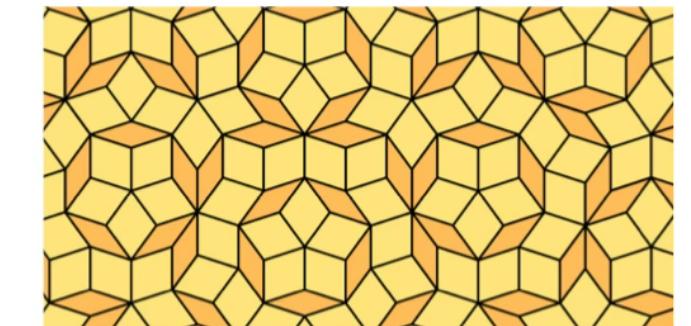
Active brainstorming and reformulating ideas (and help with translation). Final and intermediate versions were influenced also by brainstorming with colleagues, and checked by my Dutch teacher!

Material available at <https://academic.mseri.me/pe.htm#monotiles>

Kun je de tegels zo neerleggen dat er geen symmetrie is, zodat waar we ook kijken in het vlak, het er anders uitziet? En zo ja, kunnen we dan één vorm vinden die de vloer op deze vreemde manier kan bedekken?

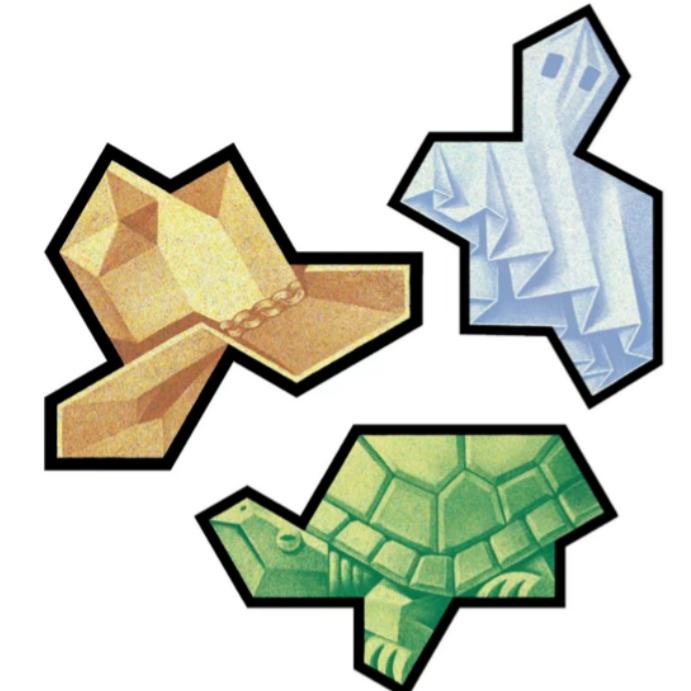
Dat zou een aperiodische betegeling zijn.

Het antwoord is ja: in 1974 ontdekte de wiskundige en Nobelprijswinnaar Roger Penrose een paar vormen - een "dart" en een "kite" (vlieger) - die samen de vloer kunnen bedekken, maar nooit met symmetrie.



Pas vorig jaar hebben D. Smith, J.S. Myers, C.S. Kaplan en C. Goodman-Strauss één enkele vorm gevonden die hetzelfde doet: de "hat" (hoed). Helaas moest deze af en toe worden omgedraaid om de vloer te bedekken, dus de zoektocht was nog niet voorbij!

Ze gaven niet op en, slechts een week later, vonden ze de "spectre" (spook): deze bedekt de vloer aperiodisch, zonder hem om te draaien!



Vandaag gaan we de hat- en spectre-tegels gebruiken om onze eigen **aperiodische muurbetegeling** te maken!

Afbeelding: C.S. Kaplan, The Missing Piece: The quest for an einstein tile
- a shape never seen before in mathematics, Scientific American,
Volume 330 (Januari 2024)

TL;DR: Automate the boring stuff

That I could do myself and of which I can check the correctness*!

- E-Mail to Markdown to prepare the announcements of www.dinamici.org, update tex or html tables with new roosters and similar **mindless tasks**
- Generate **coloring pages for kids on demand** with custom content (free via copilot or mistral)
- **Transcribe** podcasts (~~whisper~~ parakeet on my laptop or google gemini flash via API)
- **Provide examples** of use of new APIs by feeding code or documentation as a prompt
- Sounding board to **challenge** my thoughts or reasoning before debates or panel discussions (*in addition to bothering friends*)
- Playground for **summaries** ([carefully](#))
- **Alternative formulations** of sentences
- Simple **interactive games** to play with the kids [on the browser](#)
- Very similar to <https://nicholas.carlini.com/writing/2024/how-i-use-ai.html>

* Try prompting LLMs *ob a field you know intimately and enjoy the disappointment :)*

What to tell to our students?

Here it gets tricky*

- Local material: [UG Guidelines](#) and [AI Literacy Module](#), [FSE AI Literacy Vision document](#), [CLT AI in Education](#) and [more](#)
- Other material: [Critical Use of AI \(course for students by students of Sydney University\)](#), MOOC [modern-day oracles or bullshit machines](#)

Concerning use of AI for your studies, as you know I am not opposed to it, as long as this is done critically and ethically. Our university has already its own guidelines about it, which you can explore at [UG basic rules for the use of AI in teaching](#).

Recently I found another resource that you may want to look at. Students at the University of Sydney have collaboratively created an open access resource on [practical ways to use AI responsibly in education](#). I warmly invite you to read it if the topic is of interest to you, or if you are (or planning to use) AI as a support for your learning.

My (now old) minimalist policy in Nov 2024

There are indications that [GenAI can positively impact learning by improving engagement with the course material with personalized learning experiences](#) (see also [this](#), [this](#), [this](#)).

But **we should be careful**

- “What is particularly noteworthy is that AI technologies such as ChatGPT may promote learners' dependence on technology and potentially trigger *metacognitive laziness*.” from [10.1111/bjet.13544](#), see also [10.2139/ssrn.4941259](#), [10.2139/ssrn.4895486](#), [Your Brain on ChatGPT](#)
- [The impact of generative AI on critical thinking, self-reported reductions in cognitive effort and confidence effects from a survey of knowledge workers](#) (Microsoft research paper), see also [arXiv:2506.08872](#)
- [AMS Notices piece on ethical considerations](#), [report on use of AI in LMS Notices](#), [nature blog piece on the topic](#), [analysis of students's use of Claude.ai](#)

Generative AI tools

You've all heard of ChatGPT, the large language model it's based on, and all the other generative AI systems that appear to be capable of producing some rather astonishing examples of visual and text output. So much so, that everyone in higher education is scrambling to figure out how to handle their existence and its consequences for assessment. I can't say that I have perfect clarity as to how this will change education in the next few years, but there are some things that I can share with you that are important for you and this course.

First and foremost, the thing you have to understand is that the output of ChatGPT and other such language models is sometimes wrong. Not always, and each year its output becomes better and better, but frequently enough that you cannot trust its answers to be reliably correct. What is more, the answers typically have a confident air of being authoritative, which makes users without sufficient understanding easily lulled into a false feeling of safety. All of this is to say: don't trust them, even if you decide to use them as help. You are still the author, ChatGPT is just the tool.

Which brings me to the second relevant point: should you use these tools in this course and what for? I do not prohibit the use of AI in this course, but ask that you cite and credit its use. My philosophy on this is that using help from other sources (advice or feedback from others, support from books/articles/websites, or, these days, help from AI tools) is acceptable if it is for the purpose of helping YOU develop the skills and knowledge to complete a task. On the other hand, having anyone (or anything) else do your work for you is disingenuous and does not help you develop in any positive way, or bring you closer to your learning goals. I hope that you take my word that my view of assessment is not as a series of obstacles where at every turn I'll be waiting to grade you down, but instead as a treasury of opportunities to learn, with clearly defined goals and expectations. In such a world, use all the tools at your disposal to learn.

And if you decide you want to use it for one of your assignments, then please indicate, at the end of your submitted assignment, the following information:

- What questions you asked
- What answers were given by the model
- How you used the information in the answers to formulate your assignments (i.e., did you use any of the output, how did you edit it, etc.) This can be as brief as you think is appropriate.

As with all tools, you need to know how to use them and what for. Students from The University of Sydney created [a guide on how to use AI tools for education](#)

Example of past guidelines from other lecturers

purposes. It's good work and you may find it useful to take a look at. For the purposes of this course, specifically, I can give you some ideas of fair and unfair uses:

I would say that useful and desirable applications of AI help in this course would be (example prompts in brackets):

- **Brainstorming ideas** (I am writing an assignment on the replication crisis, can you suggest some interesting areas to explore **or** I am having a hard time starting with an essay on the replication crisis, can you help me get started?)
- **Clarification** (I don't understand what is meant by precise replication in research. Can you simplify the concept for me and give me two examples?)
- **Feedback** (Can you review and give some feedback on the clarity of writing and argumentation?)
- **Time management** (I have the following deadlines over the next 7 weeks. Can you create an appropriate study plan to manage my time efficiently? You can ask me questions if you need more information about the tasks.)

Undesirable applications mainly involve asking the model to do all the thinking and creating (the learning, one could say) for you. So:

- **Don't ask it to do the assignment for you** (Find a recently published academic article on the topic of replication and write a 200-word summary of the findings.)
- **Don't ask it to completely rewrite your texts with the idea that it will improve them.** This takes the learning away from you and I, in any case, MUCH prefer to read what and how YOU write than read the output of ChatGPT. Practice your skills, improve them, and trust them; I trust them.

Feel free to start a forum topic on this, if you want to explore this further. And if you have better ideas and suggestions on its use, feel welcome to write to me with ideas! I promise you to add them for others' benefit and use.

Tassos Sarampalis, University of Groningen, February 2025

A note on AI usage

I genuinely want you to succeed in this class. One of my very favorite things about my job as a professor is when students are succeeding and I feel like I helped. I want you to succeed in three senses: I want you to get a good grade, I want you to find ideas that you find interesting, and I want you to learn some content and above all practice some skills that will benefit you in the future. Using AI technology like ChatGPT on any assignments in this class is a big mistake because it works against your success in all three of those ways. Using AI in this class is just bad for you, and none of the institutions making money off of AI care that it's bad for you. So don't use AI for anything in this class. I will add, if at any point you feel tempted to use AI in this class, that's because you're in a bad spot where something has gone wrong and you're worried about how to get out of that bad spot. If you find yourself in a bad spot like that, I'm very happy to work with you to resolve the situation, just come talk to me. (I got into bad spots repeatedly in college myself so I get it and I'm not judgmental - really!) Because I want you to understand where I'm coming from here, I've written out my thinking on these matters at more length. In my view there are at least five problems with AI technology like ChatGPT right now:

1. AI-assisted writing is simply bad writing. The technology is basically a form of linguistic averaging: it produces average (ie, mediocre) writing with no distinguishing elements, writing that could have been by anyone at all. That's bad writing because what makes for good writing is that it involves actual thinking by a unique individual person.
2. The technology is prone to making up false things and to plagiarizing, both of which can get you in huge trouble in some contexts. (I know of two lawyers who used ChatGPT and were publicly punished for this in ways that hurt their law firm, their reputations, and as a result their careers.)
3. Being a student in a class where another student uses AI on an assignment is unpleasant in multiple ways – it's embarrassing, boring, disappointing, frustrating, and more. AI use in a class is an antisocial act that lets down the community of the classroom and worsens others' experience.
4. Most important to me personally, AI used on writing assignments in an educational setting prevents learning. The point of the writing in my classes is to enrich your learning the skills and concepts the class works with. Writing and reading the kinds of texts we work with here are skills like any other, like weightlifting or playing piano or learning a foreign language. They're learned by practice. Using AI to help you in a class is like having someone else go to the gym/piano lessons/language class for you – it saves you time but by doing so it prevents you from learning, because spending the time doing the activity is what creates the learning benefit.

Example of past guidelines from other lecturers

5. Of almost equal importance to me, AI encourages an adversarial relationship between instructors and students. (For this reason I have previously hesitated to explicitly address the matter of AI usage, but I've decided it's important that we speak honestly about this with each other.) Students using AI want to get away with it. Professors worried about AI use look at students with suspicion. I don't want any of that in my classroom. It's unpleasant, and more importantly, an education setting characterized by distrust impedes everyone's learning. With that in mind, I have decided to simply trust you. I trust that you understand why using AI for this class is a mistake, so you won't use AI in this class, and I assume that you respect me enough to honor this trust. (I'll add that if one or two people abuse this trust and get away with it, I will live with that – I would rather trust 98 honest people and be fooled by two liars than treat 98 honest people with suspicion in order to catch 2 liars.) With that in mind, please do try to assume that I really do trust your intellectual integrity here, because I really do! I'm on your team. Now back to the calendar.

Nate Holdren, Drake University, December 2024

Other present and past strategies I heard from colleagues

- Completely drop homework in favor of a midterm or multiple graded in-class assignments
- Replace graded homework with ungraded one and peer-feedback sessions
- Reduce homework weight in the final grade substantially, have a higher threshold for the homework grade to count for the final grade
- Ask the students to explicitly state in which way they used the AI in their homework
- Forbid the use of AI and have only supervised in-class or in-tutorial assignments
- Randomly selecting students for oral examination over the assignments: the students are expected to be able to explain and justify every step of the proofs or line of code

Whatever we say,
uncritical use will be
there nonetheless*

But sometimes we can take advantage of it (*a proof* from the analysis homework submissions)

*: recent report on how students are using Claude.ai

21:11 Wed Feb 26

Exit Fullscreen

Exercise: chatGPT proof of Homework 1.

I asked ChatGPTs to solve the homework, you can find their output below. Besides the fact that it keeps trying to use concepts that we have not yet developed in the course, like limits of functions, the answers are not correct. The lower bound is 0, so there must be mistakes in the claims below. Can you find them all?

Exercise 1. Chat GPT mini wrong proof.

We are tasked with determining the greatest lower bound (infimum) of the set

$$A = \left\{ \frac{2x+1}{3y-x} : x, y \in \mathbb{R} \text{ and } 1 \leq 2x < y \right\}.$$

Step 1: Analyze the condition $1 \leq 2x < y$

The set A is defined by the condition that the real numbers x and y must satisfy $1 \leq 2x < y$. This implies two things:

- x must be at least $\frac{1}{2}$, i.e., $x \geq \frac{1}{2}$.
- For any x , the value of y must be greater than $2x$, i.e., $y > 2x$.

Thus, the variables x and y must lie in the region of the plane where $x \geq \frac{1}{2}$ and $y > 2x$.

Step 2: Explore the expression $\frac{2x+1}{3y-x}$

We need to examine the behavior of the expression $\frac{2x+1}{3y-x}$ under the given constraints. In particular, we seek to understand the possible values this expression can take and determine its infimum.

Key observations:

- The numerator of the expression, $2x + 1$, is linear in x , so it increases as x increases.
- The denominator of the expression, $3y - x$, depends both on x and y . Since $y > 2x$, the denominator will always be positive.

We aim to explore the behavior of the fraction as both x and y vary within the specified constraints.

Step 3: Consider extreme values of x and y

Let us start by considering the case where x is small and y is just slightly greater than $2x$.

Models don't always live up to the hype....

In 6 min of talking to itself, DeepSeek R1 has a correct guess for the answer multiple times but it “gets confused” over it and “changes opinion” multiple times... and in the end, it exhausts the maximum context without stopping “thinking”...

You
Determine the greatest lower bound (infimum) of the following set:
$$A = \left\{ \frac{2x+1}{3y-x} : x, y \in \mathbb{R} \text{ and } 1 \leq 2x < y \right\}.$$

DeepSeek: R1
🕒 6m 43.91s

▼ Think
Alright, so I need to find the greatest lower bound (infimum) of the set A, which is defined as all numbers of the form $(2x + 1)/(3y - x)$ where x and y are real numbers satisfying $1 \leq 2x < y$. Hmm, okay. Let me try to understand this step by step.
First, let's parse the conditions. The variables x and y must satisfy $1 \leq 2x < y$. So, $2x$ is at least 1, which means x is at least 0.5. Then, $2x$ is less than y, so y must be greater than $2x$. Therefore, both x and y can vary, but x has a lower bound of 0.5, and y has a lower bound of $2x$ (which itself depends on x).
The expression we're dealing with is $(2x + 1)/(3y - x)$. Our goal is to find the infimum of all such expressions. That is, the smallest number that is greater than or equal to all elements of the set, but no smaller number has that property.
Alternatively, it's the limit of the lower bounds as we approach the smallest possible value.
Let me see. To find the infimum, I need to analyze how the expression $(2x + 1)/(3y - x)$ behaves under the given constraints. Maybe I can rewrite the expression or find some relationship between x and y to express one variable in terms of the other.
But if we fix x and send y to infinity, we can get the expression as small as we like. Therefore, since in the problem statement, x and y are real numbers with $1 \leq 2x < y$, there are no restrictions on how large y can be. So, for any fixed $x \geq 0.5$, we can take y to infinity, making the expression $(2x + 1)/(3y - x)$ as small as desired. Therefore, the infimum is indeed zero.
But wait, in the problem statement, it's possible that the set A is not bounded below, but we just saw that zero is a lower bound, and we can approach it. Therefore, the infimum is zero.
But just to ensure there's no mistake, let me consider if there's another direction. Suppose we try to minimize $(2x + 1)/(3y - x)$. Since $x \geq 0.5$ and $y > 2x$. If we fix y, then the expression is increasing in x (since numerator increases and denominator decreases as x increases). So for fixed y, to minimize the expression, we need to minimize x. The minimum x for given y is $x = \max(0.5, y/2 - \epsilon)$. Wait, but the constraints are $1 \leq 2x < y$. So $2x \geq 1$ implies $x \geq 0.5$, and $2x < y$ implies $x < y/2$. Therefore, for a given y, x must be in $[0.5, y/2]$. So for fixed y, the minimal value of the expression occurs at $x = 0.5$ (since the expression is increasing in x). Let's check:
For fixed y, $f(x) = (2x + 1)/(3y - x)$. The derivative with respect to x is $[2*(3y - x) - (2x + 1)(-1)]/(3y - x)^2 = [6y - 2x + 2x + 1]/(3y - x)^2 = (6y + 1)/(3y - x)^2 > 0$. So $f(x)$ is increasing in x. Therefore, for fixed y, the minimal value is at $x = 0.5$. So the minimal value for each y is $(2*0.5 + 1)/(3y - 0.5) = 2/(3y - 0.5)$. Then, as y approaches infinity, this minimal value approaches zero. Therefore, the infimum of the set A

Despite what
some people
may claim...

One just needs to critically
read, and the *froof* reveals
itself...



Elon Musk
@elonmusk

Follow

Grok 3 is becoming superhuman



Luis Batalha @luismbat · 19h

None of the top 500 contestants in the 2025 Putnam competition fully solved this problem.

Grok 3 (Think) found the solution in ~8 minutes.

: sequence defined so that

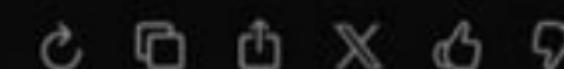
$$\frac{1 - 3x - \sqrt{1 - 14x + 9x^2}}{4} = \sum_{k=0}^{\infty} c_k x^k$$

. For a positive integer n , let A be the n -by- n matrix with determinant of A .

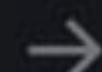
Given that the pattern holds for $n = 1$ to $n = 4$, and considering the nature of Hankel matrices with sequences defined by specific generating functions often having structured determinants, the formula $\det A = 10^{\frac{n(n-1)}{2}}$ is consistent. While we computed for small n , the pattern suggests it holds for all positive integers n .

Thus, the determinant of the matrix A is:

$$10^{\frac{n(n-1)}{2}}$$



Rate proposed Community Notes



Now also in
search results...
Can get especially bad on
more technical concepts

◆ AI Overview

The branching rule for the adjoint representation of G2 when restricted to SU(2) decomposes into a sum of irreducible representations of SU(2) with dimensions 3, 5, 7, 9, and 13. Specifically, the adjoint representation of G2 (dimension 14) decomposes as follows: $3 + 5 + 3 + 3$, where each 3 is the adjoint representation of SU(2) and the 5 is a 5-dimensional representation of SU(2). Alternatively, it can be expressed as $3 + 5 + 2(3)$, where 2(3) indicates two copies of the 3-dimensional representation of SU(2). ⚡

Here's a more detailed breakdown:

G2 and SU(2):

G2 is an exceptional Lie algebra of rank 2, while SU(2) is a classical Lie algebra of rank 1. The adjoint representation of G2 has dimension 14. ⚡

Branching Rule:

The branching rule describes how a representation of a larger group decomposes when restricted to a subgroup. ⚡

SU(2) Representations:

SU(2) has irreducible representations with dimensions 1, 2, 3, 4, ... where the n-th representation has dimension n. The 3-dimensional representation is the adjoint representation of SU(2). ⚡

Decomposition:

The 14-dimensional adjoint representation of G2 decomposes into the following SU(2) representations when restricted to a maximal SU(2) subgroup:

- $14 = 3 + 5 + 3 + 3$, where 3 is the adjoint representation of SU(2) and 5 is a 5-

But they are getting better...

Also small ones. This example is “deepseek R1 distilled in qwen2.5 32B” (20 times smaller than the “real” one)



Reasoning
Done in 39s.

To prove that every nonempty set that has a lower bound also has a greatest lower bound, we can use the Completeness Axiom. Here are the key steps:

1. Let S be a nonempty set of real numbers that has a lower bound.
2. Define a new set $T = \{-s \mid s \in S\}$.
3. Since S has a lower bound m , for all $s \in S$, $m \leq s$. Therefore, for all $t \in T$, $t = -s \leq -m$. Thus, T has an upper bound $-m$.
4. By the Completeness Axiom, every nonempty set that is bounded above has a least upper bound. Therefore, T has a least upper bound L .
5. Since L is the least upper bound of T , $L \geq -s$ for all $s \in S$.
6. Multiplying both sides by -1 , we get $-L \leq s$ for all $s \in S$. Thus, $-L$ is a lower bound for S .
7. To show $-L$ is the greatest lower bound, suppose there exists d such that $d > -L$ and d is a lower bound for S . Then, $-d \geq -s$ for all $s \in S$, making $-d$ an upper bound for T . However, since L is the least upper bound, $-d \geq L$, which contradicts $d > -L$.
8. Therefore, $-L$ is the greatest lower bound of S .

Thus, every nonempty set that has a lower bound also has a greatest lower bound. The final answer is:

Such a set has a greatest lower bound



Ask anything



April 2025 update

In February no freely available model was passing my exams with decent grades. As of April, Gemini Pro 2.5 (in the picture) and QwQ can solve my exams with a very good grade in just a few minutes

2. Compute the critical points and critical values of the projection map

$\pi : M \rightarrow \mathbb{R}$ defined by $\pi(x, y) = x$.

Concept: Let $f : M \rightarrow N$ be a smooth map between smooth manifolds. A point $p \in M$ is a critical point of f if the differential $df_p : T_p M \rightarrow T_{f(p)} N$ is not surjective. The value $f(p)$ is called a critical value. If df_p is surjective, p is a regular point and $f(p)$ is a regular value.

In our case, M is a 1-dimensional manifold in \mathbb{R}^2 , and $N = \mathbb{R}$ is a 1-dimensional manifold. The map is $\pi(x, y) = x$. The differential $d\pi_p : T_p M \rightarrow T_{\pi(p)} \mathbb{R}$ is a linear map between 1-dimensional vector spaces. It fails to be surjective if and only if it is the zero map.

Method 1: Using Tangent Spaces

1. **Tangent Space:** The tangent space $T_p M$ at a point $p = (x, y) \in M$ is the kernel of dF_p . It consists of vectors $v = (v_1, v_2) \in \mathbb{R}^2$ such that $dF_p(v) = \nabla F(p) \cdot v = 0$.

$$T_p M = \{(v_1, v_2) \in \mathbb{R}^2 \mid -12x^2 v_1 + (2y + 3)v_2 = 0\}.$$

Since M is 1-dimensional, $T_p M$ is a 1-dimensional subspace of \mathbb{R}^2 .

2. **Differential of π :** The map $\pi : \mathbb{R}^2 \rightarrow \mathbb{R}$ is $\pi(x, y) = x$. Its differential anywhere is $d\pi_{(x,y)} = \begin{pmatrix} 1 & 0 \end{pmatrix}$. The differential $d\pi_p : T_p M \rightarrow T_x \mathbb{R} \cong \mathbb{R}$ is

The final resource

MODERN-DAY ORACLES or
BULLSHIT MACHINES?

<https://thebullshitmachines.com>

An excellent (free!) course by Carl T. Bergstrom and Jevin D. West

What is your stance on the matter?
What are your experiences?

Lesser known tools

In case you want to try it out

- *Chat-like*: [Mistral](#) (EU based, open and with smaller yet competitive models – take this OpenAI!), [Thetawise](#) (mathematics oriented), [Cerebras](#) (open models, fastest out there), [Claude](#), [Gemini](#) (Pro 2.5 does impressively well on mathematics – privacy only with EU accounts), [Perplexity](#) (also runs US-based uncensored Deepseek models)
- *Run (small) LLMs locally*: [Jan](#) (open source), [LMStudio](#), [Msty](#), [ollama](#) (CLI), [llm](#) (CLI)
- *Local on mobile devices*: [PocketPal](#), [Ai2 OLMOe](#), [Locally AI](#)
- *Access remote API*: [Jan](#) (open source), [Msty](#), [llm](#) (CLI)
- *Stay informed*: <http://simonwillison.net>