Distributed algorithms: a formulary

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1 Asynchronous systems

1.1 Broadcast

Model	Description	Messages	Time
Graph	Broadcast on a general graph	$\sum_n deg(v) - 1 = 2m - n + 1$	$n - 1$
Hypercube	Descending broadcast on an hypercube	$n - 1$	$n - 1$
Tree	Broadcast on a tree	$n + 1$	$n - 1$
Tree	Broadcast on a tree (k^* initiators)	$n + k^* - 2$	$n - 1$

1.2 Serialization

Model	Description	Messages	Time
Graph	Sequential token on generic graph	$\geq m$	$\geq n - 1$
Graph $[DF++]$	Asynchronous system with *received* and *ack* messages	$4m - n + 1$	$2n - 2$
Graph $[DF^*]$	Asynchronous system with *received* messages	$4m - 2n + 1$	$2n - 12$

1.3 Spanning tree formation

Model	Description	Messages	Time
Shout	Common knowledge eager tree formation	$4m - 2n + 2$	$n + 1$
Mega-Merger	Lazy mandate, eager-absorption city formation	$5n \log n + 2m - n + 1$	$2n - 2$
Yo-Yo	Asynchronous system with *received* messages	$2m + 2 \sum_n \log n + n + 1$	$\log(n)$

1.4 Minimum finding

Model	Description	Messages	Time
All the way	Wasteful minimum finding in ring	$\mathcal{O}(n^2)$	n
As far as it can	Message-efficient minimum finding in ring	$\mathcal{O}(n^2), \Theta(n), \Omega(.69 \log n)$	n
Controlled distance	Controlled As far	$7n \log(n)$	$\sum_{i=0}^{\log n} 2^i$
Staged distance	Limitless distance control	$2n \log n + n$	n
Stages with feedback	Stage with feedback on triples	$3n \log_3 n + n$ or $1.89n \log n$	$\log_3(n)$
Swinging pendulum	Limitless distance control	$1.44n \log n$	$\mathcal{O}(n)$

1.5 Saturation

Model	Description	Messages	Time
Saturation	Relax-contract saturation finding	$2n$	$2n$
Saturated eccentricity	Find eccentricity through saturation	$4n$	$4n$
Saturated center	Find center through saturation	$4n$	$4n$

2 Synchronous systems

2.1 As Far As It Can

Model	Description	Messages	Time
As Far As It Can	Exponential synchronous as far algorithm	1	$n2^{i+1}$

2.2 Bit communicators

Model	Description	Bits	Time
1-bit communicator	Wait $\frac{\mathcal{M}}{2}$, use bit for remainder	1	$\frac{\mathcal{M}}{2}$
2-bits communicator	Wait $\left\lceil\sqrt{\mathcal{M}}\right\rceil$, send remainder $\mathcal{M} - \left\lceil\sqrt{\mathcal{M}}\right\rceil$	2	$\mathcal{M} - \left\lceil\sqrt{\mathcal{M}}\right\rceil$

2.3 Waiting

Model	Description	Bits	Time
Round-Noise	Synchronous minimum in ring	1	$2min$
1 error guess	Guess minimum with one noise error	-	$min + 1$
2 errors guess	Guess minimum with two noise errors	-	$\leq 2\sqrt{\mathcal{M}}$
(q, k) model	Guess minimum with q questions and $k - 1$ overestimates	-	$\sum_{j=1}^{k} \binom{q}{k}$
Pipeline maximum	Compute maximum in pipeline with wait	$2n$	$(n-1) + max$

3 Sorting

3.1 Sorting

Model	Description	Messages	Time
Rankselect	Randomized rank selection	$\leq 4n^2$	$\leq 4n^2$
Selectsort	Mixed sorting through edge selection and internal sorting	$\leq \mathcal{NC}(edgeSelect) + n$	$\leq \mathcal{NC}(edgeSelect) + 1$

4 Routing

4.1 Routing

Model	Description	Messages	Time
Gossip map	Forward local map to every neighbour	$2mn + n \log n$	$n - 1$
Distance vector	Update local routing on demand	$\leq 2(n-1)nm$	$\leq (n-1)n$
Minimum Djikstra	Build MST on the total shortest outgoing road	$4m$	$2n^2 + 4n + 1$