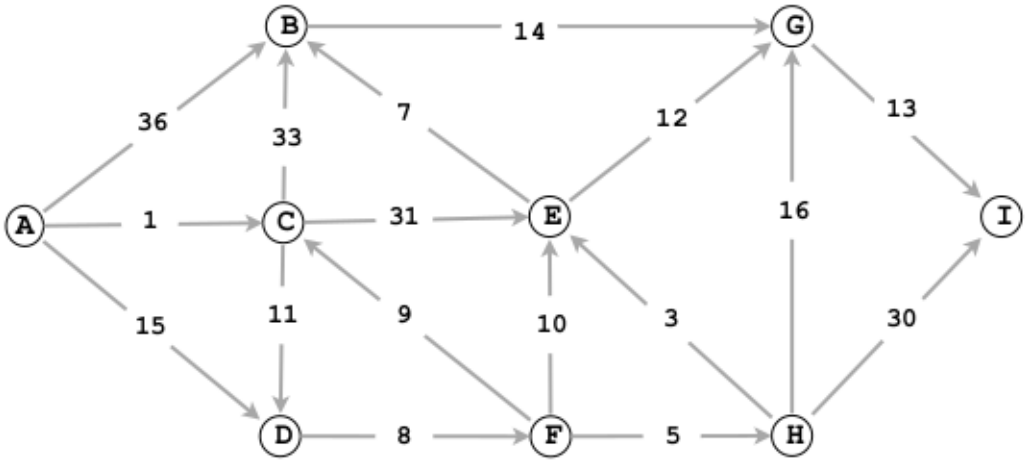
Homework 15

1. Spring 2008 Final Questions 2a and 2b

Run *Dijkstras algorithm* on the weighted digraph below, starting at vertex A.

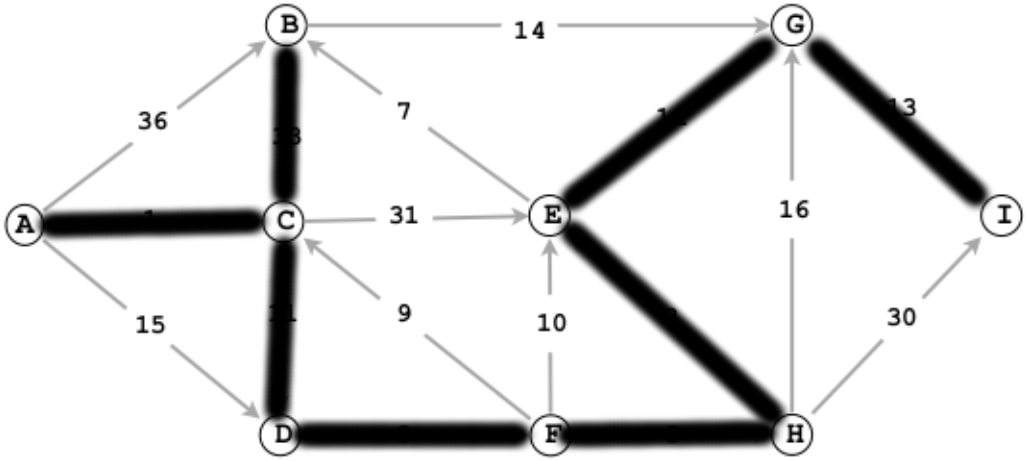


A. List the vertices in the order in which the vertices are dequeued (for the first time) from the priority queue and give the length of the shortest path from A.

vertex: A C D F H E B G I

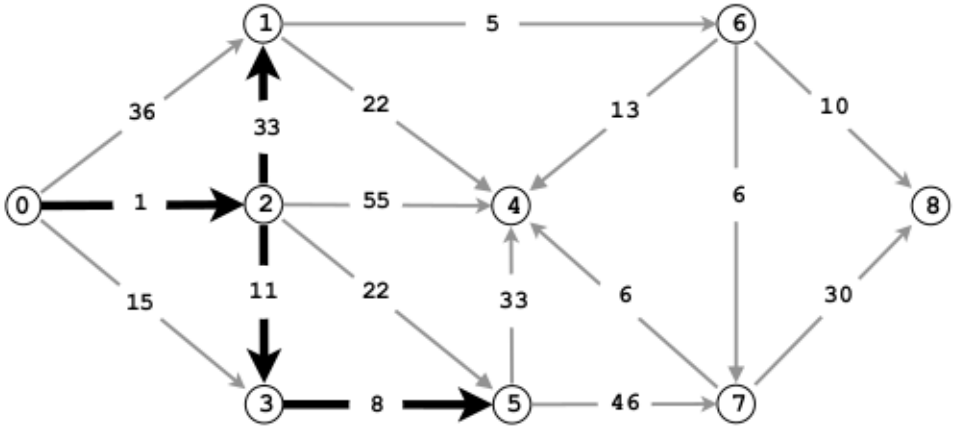
distance: 0 1 12 20 25 28 34 40 43

B. Draw the edges in the shortest path tree with thick lines in the figure above.



2. Fall 2010 Final Question 5

Run the eager version of Dijkstras algorithm on the following edge-weighted digraph, starting from vertex 0.



A. Complete the table of edgeTo[ ] and distTo[ ] values immediately after the first 5 vertices (0, 2, 3, 5, and 1) have been deleted from the priority queue and relaxed.

v	edgeTo[ ]	distTo[ ]
0	-	0.0
1	$2 \rightarrow 1$ 33.0	34.0
2	$0 \rightarrow 2$ 1.0	1.0
3	$2 \rightarrow 3$ 11.0	12
4	$5 \rightarrow 4$ 33.0	53.0
5	$3 \rightarrow 5$ 8.0	20.0
6	$1 \rightarrow 6$ 5.0	39.0
7	$5 \rightarrow 7$ 46.0	66.0
8	-	-

B. Complete the table of edgeTo[ ] and distTo[ ] values immediately after the 6 th vertexhas been deleted from the priority queue and relaxed. Circle those values that changed from A.

v	edgeTo[ ]	distTo[ ]
0	-	0.0
1	$2 \rightarrow 1$ 33.0	34.0
2	$0 \rightarrow 2$ 1.0	1.0
3	$2 \rightarrow 3$ 11.0	12
4	**$6 \rightarrow 4$ 13.0**	**52.0**
5	$3 \rightarrow 5$ 8.0	20.0
6	$1 \rightarrow 6$ 5.0	39.0
7	**$6 \rightarrow 7$ 6.0**	**45.0**
8	**$6 \rightarrow 8$ 10.0**	**49.0**

3. Question 4.3.1

Prove that you can rescale the weights by adding a positive constant to all of them or by multiplying them all by a positive constant without affecting the MST.

Edge weights increased by a common number maintain the same differences from each other and, thus, quite obviously maintain the same MST. While multiplication by a common number does not maintain the exact edge differences, the edge ratios are preserved. Thus, any edge E that was less than any other edge E' must continue to be less than E' when both are multiplied by a common factor.

4. Question 4.4.1

True of false: Adding a constant to every edge weight does not change the solution to the single-source shortest-paths problem.

False. In the case of negative edge weights, a constant can affect whether a path is the shortest.