# Introduction To Multiple Linear Regression

## Topics and Outcomes

- Introduce Multiple Regression.

## What is multiple Linear Regression?

Multiple Linear Regression is an extension of simple linear regression that allows for predicting a dependent variable based on multiple independent variables. The general form of the model is expressed as:

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_n X_n$$

Where:

- $y \in Y$ is the dependent variable (the value we aim to predict).
- $\beta_0$ is the intercept (constant term).
- $\beta_1, \beta_2, \ldots, \beta_n$ are the coefficients for the independent variables.
- $X_1, X_2, \ldots, X_n$ are the independent variables (features that influence $y$).

The model gets the best regression fit line by finding the best values for $\beta_0$, $\beta_1$ and so on.

## Steps of Multiple Linear Regression Model

1. **Data Pre Processing**

- Importing the Libraries.
- Importing the Data Set.
- Splitting the Data set into Training Set and Test Set.

2. **Define the Model**:

   - The model is defined as a linear combination of the independent variables. The coefficients ($\beta_1$, $\beta_2$, . . ., $\beta_n$) represent how much each independent variable contributes to predicting ( y ).

3. **Fit the Model**:

   - Using a method such as **Ordinary Least Squares (OLS)**, estimate the coefficients by minimizing the sum of the squared differences between the actual values of $y$ and the predicted values $\hat{y}$.

4. **Make Predictions**:
   - Once the model is trained, use the estimated coefficients to make predictions for new data points.
5. **Evaluate the Model**:
   - After fitting the model, evaluate its performance using metrics such as **R-squared** (which tells how well the model explains the variance in the data) and **Mean Squared Error (MSE)** to measure prediction accuracy.

# Practical Example

# Predicting Apartment Price based on Appartment Features

In this example, we'll predict **price** of an apartment based on its area size, number of rooms, age of the building, floor number.
**We will use the city factor in the next lesson.**

### Step 1: Import Libraries

In [44]:
```
import numpy as np
import pandas as pd
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LinearRegression
from sklearn.metrics import mean_squared_error, mean_absolute_error, r2_scor
import matplotlib.pyplot as plt
```

### Step 2: Open Dataset

We will use the house/apartment prices dataset to demonstrate multiple linear regression

In [45]:
```
df = pd.read_csv("../datasets/apartment_prices.csv")
df
```

Out[45]:

| | Square_Area | Num_Rooms | Age_of_Building | Floor_Level | City | Price |
|---|---|---|---|---|---|---|
| **0** | 162 | 1 | 15 | 12 | Amman | 74900.0 |
| **1** | 152 | 5 | 8 | 8 | Aqaba | 79720.0 |
| **2** | 74 | 3 | 2 | 8 | Irbid | 43200.0 |
| **3** | 166 | 1 | 3 | 18 | Irbid | 69800.0 |
| **4** | 131 | 3 | 14 | 15 | Aqaba | 63160.0 |
| **...** | ... | ... | ... | ... | ... | ... |
| **495** | 177 | 1 | 6 | 12 | Irbid | 64100.0 |
| **496** | 79 | 5 | 9 | 13 | Irbid | 52700.0 |
| **497** | 106 | 3 | 7 | 14 | Aqaba | 60160.0 |
| **498** | 108 | 3 | 9 | 18 | Amman | 72600.0 |
| **499** | 73 | 1 | 18 | 6 | Aqaba | 19280.0 |

500 rows × 6 columns

### Step 3: Define Features and Target

We use **Square_Area**, **Num_Rooms**, **Age_of_Building** and **Floor_Level** as features and **Price** as the target.

In [46]:
```
# Features and Target
X = df[['Square_Area', 'Num_Rooms', 'Age_of_Building','Floor_Level']]  # Inc
y = df['Price']  # Dependent variable (Sales)

# Split the data into training and testing sets (80% train, 20% test)
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2, rar
```

### Step 4: Train the Model

In [47]:
```
# Initialize the Linear Regression model
model = LinearRegression()

# Train the model on the training data
model.fit(X_train, y_train)

# Coefficients and Intercept
print("Coefficients:", model.coef_)
print("Intercept:", model.intercept_)
```

```
Coefficients: [ 364.36926791 5064.2938487  -925.54218137  951.56065025]
Intercept: 461.27020385613287
```

### Step 5: Make Predictions

In [48]:
```
# Predict the target variable for the test set
y_pred = model.predict(X_test)

# Display the predictions alongside the actual values
df = pd.DataFrame({'Actual': np.round(y_test,2), 'Predicted': np.round(y_pre
df
```

Out[48]:

| | Actual | Predicted | Residual |
|---|---|---|---|
| **361** | 102550.0 | 89530.89 | 13019.11 |
| **73** | 54200.0 | 62947.21 | 8747.21 |
| **374** | 44000.0 | 44989.04 | 989.04 |
| **155** | 67000.0 | 77635.12 | 10635.12 |
| **104** | 63700.0 | 57098.21 | 6601.79 |
| **...** | ... | ... | ... |
| **347** | 73360.0 | 75233.49 | 1873.49 |
| **86** | 36800.0 | 32110.44 | 4689.56 |
| **75** | 85500.0 | 75366.09 | 10133.91 |
| **438** | 89200.0 | 74516.43 | 14683.57 |
| **15** | 52960.0 | 53594.27 | 634.27 |

100 rows × 3 columns

### Step 6: Evaluate the Model

We evaluate the model using **MSE**, **MAE**, and **R-squared**.

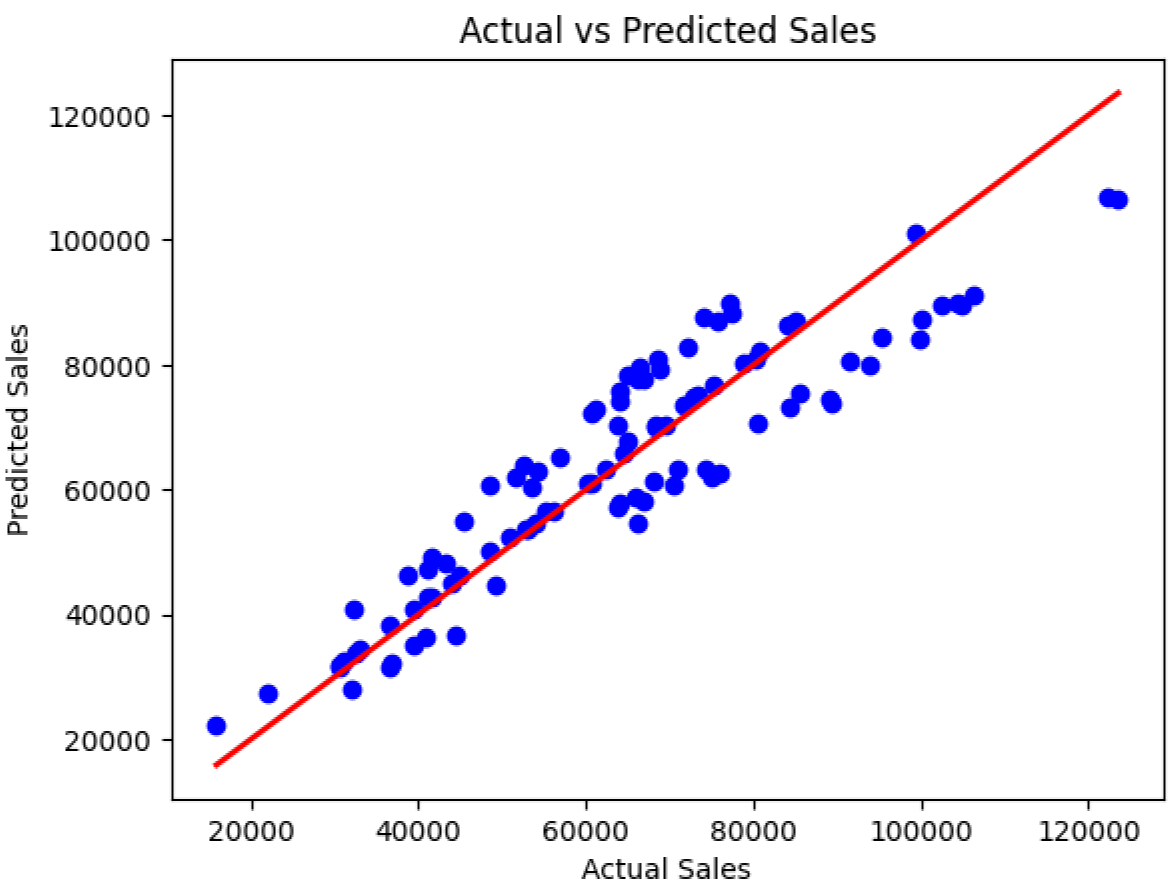
In [49]:
```
# Mean Squared Error (MSE)
mse = mean_squared_error(y_test, y_pred)
print("Mean Squared Error (MSE):", round(mse,2))

# Mean Absolute Error (MAE)
mae = mean_absolute_error(y_test, y_pred)
print("Mean Absolute Error (MAE):", round(mae,2))

# R-squared (R²)
r2 = r2_score(y_test, y_pred)
print("R-squared (R²):", round(r2,2))
```

```
Mean Squared Error (MSE): 73900149.47
Mean Absolute Error (MAE): 6997.8
R-squared (R²): 0.84
```

### Step 7: Plotting the Performance

We visualize the predicted vs actual sales values to assess model performance.

In [50]:
```
# Plot actual vs predicted values
plt.scatter(y_test, y_pred, color='blue')
plt.plot([min(y_test), max(y_test)], [min(y_test), max(y_test)], color='red'
plt.xlabel('Actual Sales')
plt.ylabel('Predicted Sales')
plt.title('Actual vs Predicted Sales')
plt.show()
```




## Comments on the Performance Plot

This plot is a scatter plot of **actual sales** values versus **predicted sales** values, with a red line representing the line of perfect fit (where actual equals predicted). Here are some thoughts based on the shape and distribution of the points:

1. **General Alignment with Perfect Fit**:
   - The data points generally align along the red line, indicating that the model is making reasonably accurate predictions.
   - Points closely clustered around the line suggest good model performance.
2. **Variance and Spread**:
   - The spread of points widens slightly as sales values increase, indicating that the model may be slightly less accurate for higher sales values. This pattern can be common in regression models if there's more variability in the data at higher

ranges, potentially due to heteroscedasticity (non-constant variance of residuals).**We will use the city factor in the next lesson.**

3. **Outliers**:
   - A few points are noticeably distant from the line, suggesting possible outliers where the model predictions differ significantly from actual values.
   - Outliers might indicate unusual cases that the model didn't capture well, or they could highlight instances where the data is noisy.
4. **Linear Model Appropriateness**:
   - If this plot generally follows a linear pattern (as it does here), it suggests that a linear regression model is appropriate. If you noticed a nonlinear pattern, it might suggest trying a nonlinear regression approach.

### Suggestions

- **Investigate Outliers**: Look into the data points that are far from the line to understand why the model struggled with those predictions.
- **Check for Heteroscedasticity**: You could perform a residual plot (residuals vs. predicted values) to check if there's increasing variance, which may indicate heteroscedasticity.
- **Consider Model Complexity**: If the model consistently struggles at higher values, a more complex model (e.g., polynomial regression) or transformations on the target variable might improve performance.

Overall, this plot suggests that the model is performing reasonably well, but further analysis (especially of residuals) could help refine it.

## Discussion of Results

The accuracy, as represented by the (R^2) score of **0.87**, suggests that the model is capturing about 87% of the variance in the data, which is lower than desired for many business applications.

Here are a few potential steps you can take to improve the model's accuracy:

### 1. **Check Feature Importance**

- The current model may not be capturing all the important relationships in the data. Try using **feature importance** techniques (like looking at the coefficients of the linear model) to determine if any of the variables have a disproportionately low or high impact.
- You could try adding more relevant features if available or removing features that don't seem to contribute significantly.

### 2. **Polynomial Features**

- Linear regression assumes a linear relationship between the features and the target. However, if the true relationship is nonlinear, linear regression might not be a good fit.
- Try adding **polynomial features** (interaction terms) using `PolynomialFeatures` from `scikit-learn`.

```
from sklearn.preprocessing import PolynomialFeatures

poly = PolynomialFeatures(degree=2, include_bias=False)
X_poly = poly.fit_transform(X)

X_train_poly, X_test_poly, y_train, y_test =
train_test_split(X_poly, y, test_size=0.2, random_state=42)

model = LinearRegression()
model.fit(X_train_poly, y_train)

y_pred = model.predict(X_test_poly)
```

## 3. Regularization

- Try using **regularization techniques** like **Ridge Regression** or **Lasso Regression** to penalize large coefficients and reduce overfitting.

```
from sklearn.linear_model import Ridge

model = Ridge(alpha=1.0)
model.fit(X_train, y_train)
```

## 4. Cross-Validation

- Use **cross-validation** to ensure your model isn't overfitting on the training set.

```
from sklearn.model_selection import cross_val_score

scores = cross_val_score(model, X, y, cv=5, scoring='r2')
print("Average R^2 from cross-validation:", np.mean(scores))
```

## 5. Scale the Data

- Scaling the numerical features can often improve the performance of linear models, especially if the features have different magnitudes.

```
from sklearn.preprocessing import StandardScaler

scaler = StandardScaler()
X_scaled = scaler.fit_transform(X)
```

## 6. Outlier Detection

- Outliers in the dataset could be adversely affecting the performance of the model. Check if any observations have extreme values that might be skewing the results.

### 7. **Interaction Terms**

- Consider adding interaction terms between variables. For example, TV and Radio spend might have an interaction effect that influences sales.

### 8. **Add More Data**

- If possible, adding more data (e.g., more records, more features) can help improve the accuracy by giving the model more examples to learn from.

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## Complete Model Code

In [51]:
```
df = pd.read_csv("../datasets/apartment_prices.csv")

# Features and Target
X = df[['Square_Area', 'Num_Rooms', 'Age_of_Building','Floor_Level']]  # Inc
y = df['Price']  # Dependent variable (Sales)

# Split the data into training and testing sets (80% train, 20% test)
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# Initialize the Linear Regression model
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# Train the model on the training data
model.fit(X_train, y_train)

# Coefficients and Intercept
print("Coefficients:", model.coef_)
print("Intercept:", model.intercept_)
```

```
Coefficients: [ 364.36926791 5064.2938487  -925.54218137  951.56065025]
Intercept: 461.27020385613287
```

### Check the Variance Heteroscedasticity

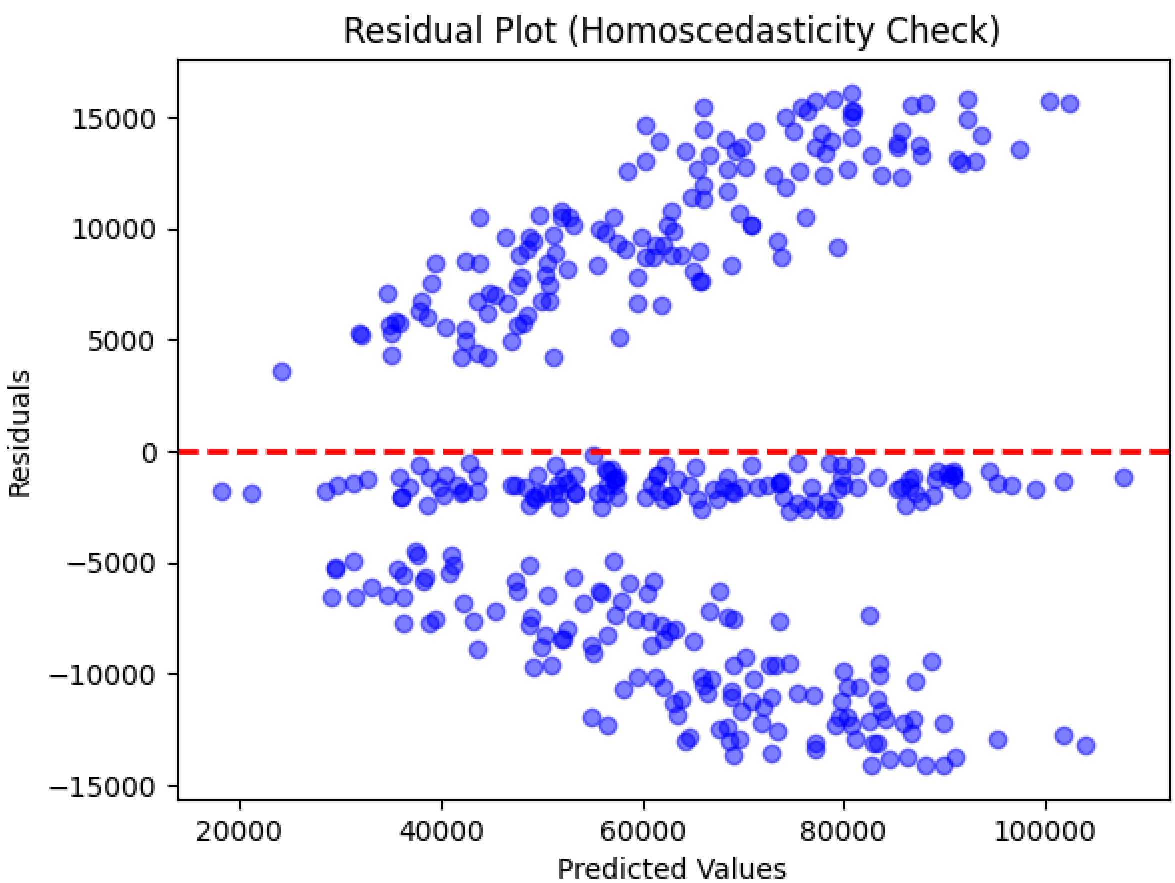
To check for heteroscedasticity (non-constant variance of residuals) in our linear regression model, we can create a residual plot. In this plot, we plot the residuals (differences between actual and predicted values) against the predicted values. If the residuals show a random pattern with constant spread, it suggests homoscedasticity. However, if the spread of the residuals increases or decreases with the predicted values, it suggests heteroscedasticity.

In [52]:
```
import matplotlib.pyplot as plt
import numpy as np
```

```
# Predict the target variable for the training set
y_train_pred = model.predict(X_train)

# Calculate residuals
residuals = y_train - y_train_pred

# Plot residuals vs. predicted values
plt.scatter(y_train_pred, residuals, color='blue', alpha=0.5)
plt.axhline(y=0, color='red', linestyle='--', linewidth=2)
plt.xlabel("Predicted Values")
plt.ylabel("Residuals")
plt.title("Residual Plot (Homoscedasticity Check)")
plt.show()
```




From the residual plot, it appears that there is a **fanning-out pattern** in the residuals, where the variance of the residuals increases as the predicted values increase. This pattern is indicative of **heteroscedasticity**.

## Interpretation of the Plot

1. **Fanning-Out Pattern**:

    - The residuals are not evenly distributed around the red horizontal line (at zero) with a constant spread.

- Instead, the spread of residuals grows wider as the predicted values increase, suggesting that the variance of errors is not constant.

2. **Implications**:

   - Heteroscedasticity means that the model's errors vary at different levels of the predicted values, which violates one of the core assumptions of linear regression.
   - This can lead to inefficient estimates of the coefficients and may make the model less reliable, especially in predictions for larger values.

## Potential Solutions

To address heteroscedasticity, consider the following:

1. **Transform the Dependent Variable**:

   - Applying a transformation to the target variable, such as the logarithmic ( `log(y)` ) or square root transformation ( `sqrt(y)` ), may help stabilize the variance.
   - After transformation, rerun the regression and check the residual plot again to see if heteroscedasticity is reduced.

2. **Weighted Least Squares Regression**:

   - Assign weights to observations based on the inverse of their variance (giving less weight to data points with higher variance).
   - This approach explicitly accounts for heteroscedasticity and can improve model reliability.

3. **Use Robust Standard Errors**:

   - If you’re interested in significance testing, you can use robust standard errors (e.g., with `statsmodels` in Python), which are less sensitive to heteroscedasticity.