BUSINESS INTELLIGENCE AND DATA ANALYTICS

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1. **Introduction to Principal Component Analysis (PCA)**

Principal Component Analysis (PCA) is an unsupervised dimensionality reduction technique used to transform high-dimensional datasets into a lower-dimensional space while retaining most of the important information (variance).

The main goal of PCA is to reduce the complexity of the data while minimizing information loss.

Applications of PCA:

- Reducing the number of features in a dataset for machine learning tasks.
- Visualization of high-dimensional data.
- Removing noise and redundancy in data.

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2. **How PCA Works**

PCA works by finding new dimensions called **principal components** that capture the maximum variance in the data.

Each principal component is a linear combination of the original features, and they are orthogonal to each other (i.e., uncorrelated).

Steps Involved in PCA:

1. **Standardize the Data**: Ensure that the dataset has a mean of zero and unit variance for each feature.
2. **Compute the Covariance Matrix**: The covariance matrix shows the relationships (variances and covariances) between the features in the dataset.
3. **Compute Eigenvectors and Eigenvalues**: The eigenvectors of the covariance matrix represent the direction of the principal components, and the eigenvalues represent the magnitude of variance captured by these components.
4. **Sort Eigenvalues and Select Principal Components**: The principal components are ordered based on the eigenvalues, with the largest eigenvalue corresponding to the first principal component (which captures the most variance).
5. **Transform the Data**: The original data is transformed into the new space defined by the selected principal components.

3. **Mathematical Explanation**

For a given dataset X with n observations and m features:

1. **Mean Center the Data**: Subtract the mean from each feature to center the data around zero.

$$X_{centered} = X - \mu$$

2. **Covariance Matrix**: Calculate the covariance matrix Σ:

$$\Sigma = \frac{1}{n-1} X_{centered}^T X_{centered}$$

3. **Eigenvectors and Eigenvalues**: Compute the eigenvectors V and eigenvalues λ of the covariance matrix:

$$\Sigma V = \lambda V$$

4. **Select Top Principal Components**: Choose the top k eigenvectors that correspond to the largest eigenvalues. These eigenvectors are the principal components.

5. **Project the Data**: Transform the data into the new space using the selected principal components:

$$X_{new} = X_{centered} \cdot V_k$$

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4. **Step-by-Step Example**

Let's consider a small dataset with two features:

Feature 1	Feature 2
2.5	2.4
0.5	0.7
2.2	2.9
1.9	2.2
3.1	3.0
2.3	2.7
2.0	1.6
1.0	1.1
1.5	1.6
1.1	0.9

We want to reduce this dataset from two dimensions to one dimension using PCA.

Step 1: Standardize the Data

We first center the data by subtracting the mean from each feature:

Feature 1 (Centered)	Feature 2 (Centered)
0.69	0.49
-1.31	-1.21
0.39	0.99
0.09	0.29
1.29	1.39
0.49	0.79
0.19	-0.31
-0.81	-0.81
-0.31	-0.31
-0.71	-1.01

Step 2: Compute the Covariance Matrix

Next, we compute the covariance matrix:

$$\Sigma = \begin{bmatrix} \text{Var}(Feature1) & \text{Cov}(Feature1, Feature2) \\ \text{Cov}(Feature1, Feature2) & \text{Var}(Feature2) \end{bmatrix}$$

After calculation, the covariance matrix is:

$$\Sigma = \begin{bmatrix} 0.61655556 & 0.61544444 \\ 0.61544444 & 0.71655556 \end{bmatrix}$$

Step 3: Compute Eigenvectors and Eigenvalues

Using linear algebra, we compute the eigenvalues and eigenvectors of the covariance matrix:

- **Eigenvalues**: $\lambda_1 = 1.284$, $\lambda_2 = 0.049$
- **Eigenvectors**:

$$v_1 = \begin{bmatrix} 0.67787 \\ 0.73518 \end{bmatrix}, v_2 = \begin{bmatrix} -0.73518 \\ 0.67787 \end{bmatrix}$$

Step 4: Select Principal Components

Since the first eigenvalue ($\lambda_1 = 1.284$) is much larger than the second ($\lambda_2 = 0.049$), the first principal component captures most of the variance. Therefore, we select the first eigenvector v_1 as our principal component.

Step 5: Project the Data

We project the centered data onto the first principal component:

$$X_{new} = X_{centered} \cdot v_1$$

This gives us the data in the reduced one-dimensional space.

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5. **Python Code Example**

Here's how you can implement PCA using Python's `scikit-learn` library:

```
# Import necessary libraries
import numpy as np
import pandas as pd
from sklearn.decomposition import PCA
from sklearn.preprocessing import StandardScaler

# Step 1: Create a dataset
data = {'Feature 1': [2.5, 0.5, 2.2, 1.9, 3.1, 2.3, 2.0, 1.0, 1.5, 1.1],
        'Feature 2': [2.4, 0.7, 2.9, 2.2, 3.0, 2.7, 1.6, 1.1, 1.6, 0.9]}

df = pd.DataFrame(data)

# Step 2: Standardize the data
scaler = StandardScaler()
X_scaled = scaler.fit_transform(df)

# Step 3: Apply PCA
pca = PCA(n_components=1)  # Reduce to 1 dimension
X_pca = pca.fit_transform(X_scaled)

# Step 4: Results
print("Original Data:\n", df)
print("\nTransformed Data (1 Principal Component):\n", X_pca)
print("\nExplained Variance Ratio:", pca.explained_variance_ratio_)

# Step 5: Optional - Transform back to original space to see data variance captured
X_projected_back = pca.inverse_transform(X_pca)
print("\nReconstructed Data:\n", X_projected_back)
```

```
Original Data:
    Feature 1  Feature 2
0        2.5        2.4
1        0.5        0.7
2        2.2        2.9
3        1.9        2.2
4        3.1        3.0
5        2.3        2.7
6        2.0        1.6
7        1.0        1.1
8        1.5        1.6
9        1.1        0.9

Transformed Data (1 Principal Component):
 [[ 1.08643242]
 [-2.3089372 ]
 [ 1.24191895]
 [ 0.34078247]
```

```
 [ 2.18429003]
 [ 1.16073946]
 [-0.09260467]
 [-1.48210777]
 [-0.56722643]
 [-1.56328726]]

Explained Variance Ratio: [0.96296464]

Reconstructed Data:
 [[ 0.76822373  0.76822373]
 [-1.63266515 -1.63266515]
 [ 0.87816931  0.87816931]
 [ 0.2409696   0.2409696 ]
 [ 1.54452629  1.54452629]
 [ 0.82076675  0.82076675]
 [-0.06548139 -0.06548139]
 [-1.04800846 -1.04800846]
 [-0.40108966 -0.40108966]
 [-1.10541102 -1.10541102]]
```

Explanation:

- **Step 1**: We create a simple dataset with two features.
- **Step 2**: We standardize the data so that it has a mean of 0 and variance of 1.
- **Step 3**: We apply PCA to reduce the data to one principal component.
- **Step 4**: We print the transformed data (now in 1 dimension) and the amount of variance captured by this principal component.

Implemented PCA on Customers Dataset

```
# Import necessary libraries
import pandas as pd
from sklearn.preprocessing import StandardScaler, LabelEncoder
from sklearn.decomposition import PCA
import matplotlib.pyplot as plt

# Step 1: Get the data
data = pd.read_csv('https://raw.githubusercontent.com/msfasha/307304-Data-Mining/refs/heads/
data
```

	CustomerID	Gender	Age	Annual Income (k$)	Spending Score (1-100)
0	1	Male	19	15	39
1	2	Male	21	15	81
2	3	Female	20	16	6
3	4	Female	23	16	77
4	5	Female	31	17	40
...	...	...	...	...	...
195	196	Female	35	120	79
196	197	Female	45	126	28
197	198	Male	32	126	74
198	199	Male	32	137	18
199	200	Male	30	137	83

200 rows × 5 columns

Next steps: Generate code with data | View recommended plots | New interactive sheet

Implement PCA

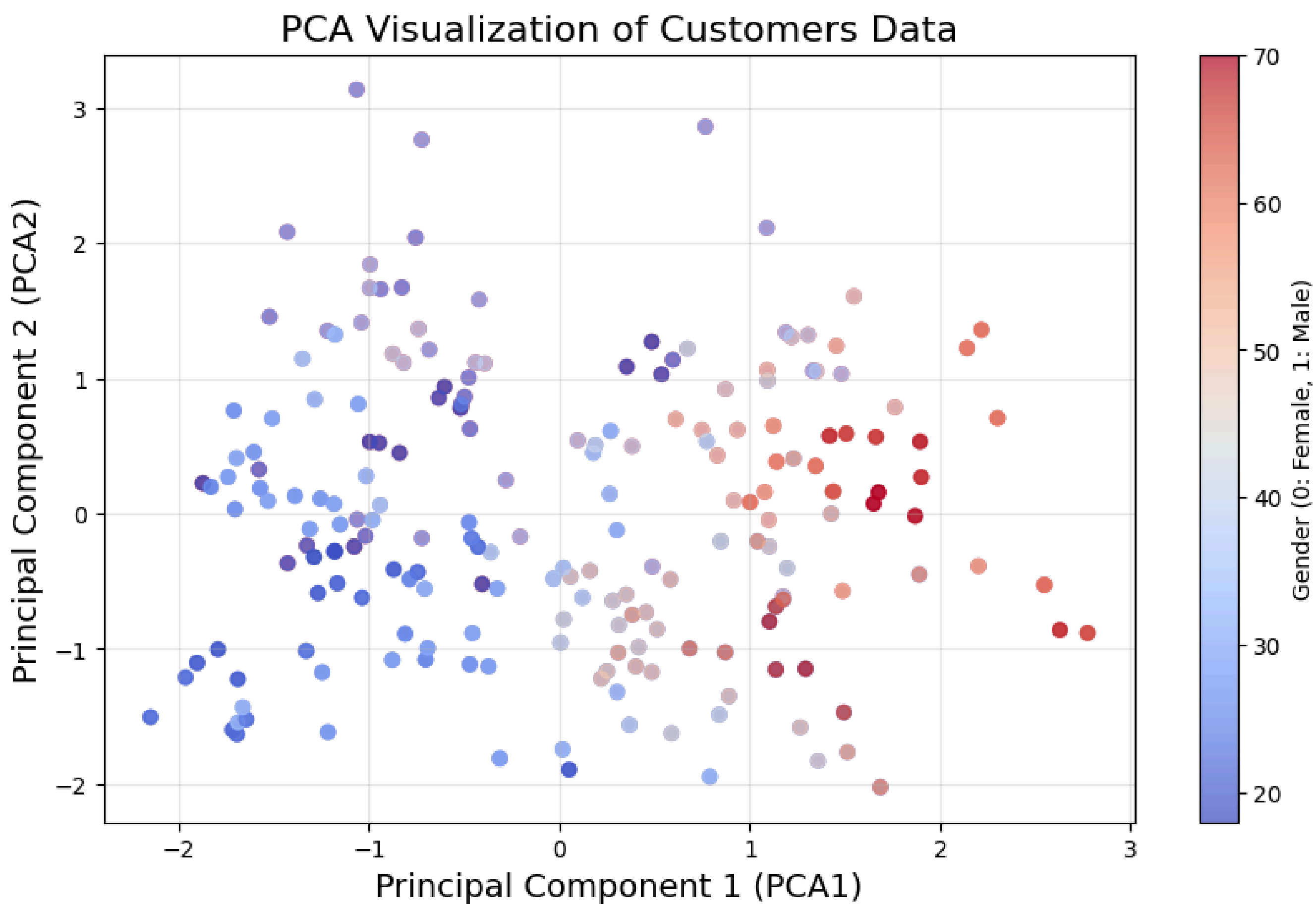
```
# Encode the Gender column (categorical to numerical)
label_encoder = LabelEncoder()
data['Gender'] = label_encoder.fit_transform(data['Gender'])

# Select features for PCA
features = ['Gender', 'Age', 'Annual Income (k$)', 'Spending Score (1-100)']

# Standardize the features
scaler = StandardScaler()
scaled_data = scaler.fit_transform(data[features])

# Apply PCA
pca = PCA(n_components=2)  # Reduce to 2 dimensions
pca_result = pca.fit_transform(scaled_data)

# Add the PCA result to the original data
data['PCA1'] = pca_result[:, 0]
data['PCA2'] = pca_result[:, 1]

# Display the first few rows of the transformed data
print(data[['PCA1', 'PCA2']].head())
```

```
       PCA1      PCA2
0 -0.406383 -0.520714
1 -1.427673 -0.367310
2  0.050761 -1.894068
3 -1.694513 -1.631908
4 -0.313108 -1.810483
```

This code will output the new features (PCA1 and PCA2) that represent the reduced dimensions of the dataset.

Visualize the New Dimensions Against the Gender (Categorical Value)

```
import matplotlib.pyplot as plt

# Scatter plot for PCA components
plt.figure(figsize=(10, 6))
plt.scatter(data['PCA1'], data['PCA2'], c=data['Gender'], cmap='coolwarm', alpha=0.7)
plt.scatter(data['PCA1'], data['PCA2'], c=data['Age'], cmap='coolwarm', alpha=0.7)


# Add plot labels and title
plt.title('PCA Visualization of Customers Data', fontsize=16)
plt.xlabel('Principal Component 1 (PCA1)', fontsize=14)
plt.ylabel('Principal Component 2 (PCA2)', fontsize=14)

# Add a color bar to indicate gender
plt.colorbar(label='Gender (0: Female, 1: Male)')
plt.grid(alpha=0.3)

# Show plot
plt.show()
```



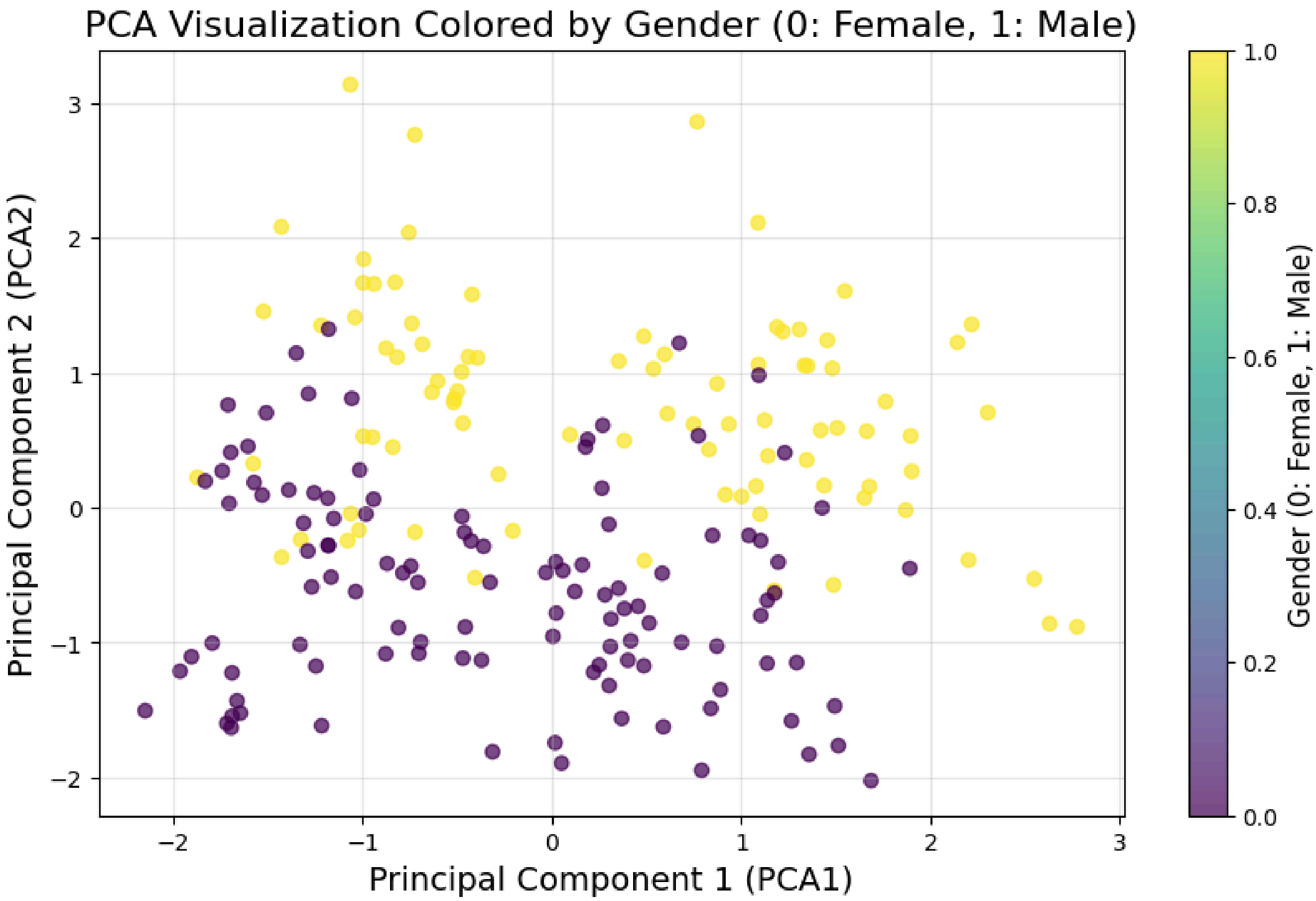
Color Mapping: The color of each point is determined by the "Gender" column (0 for Female, 1 for Male), which helps to observe any clustering or grouping patterns related to gender.

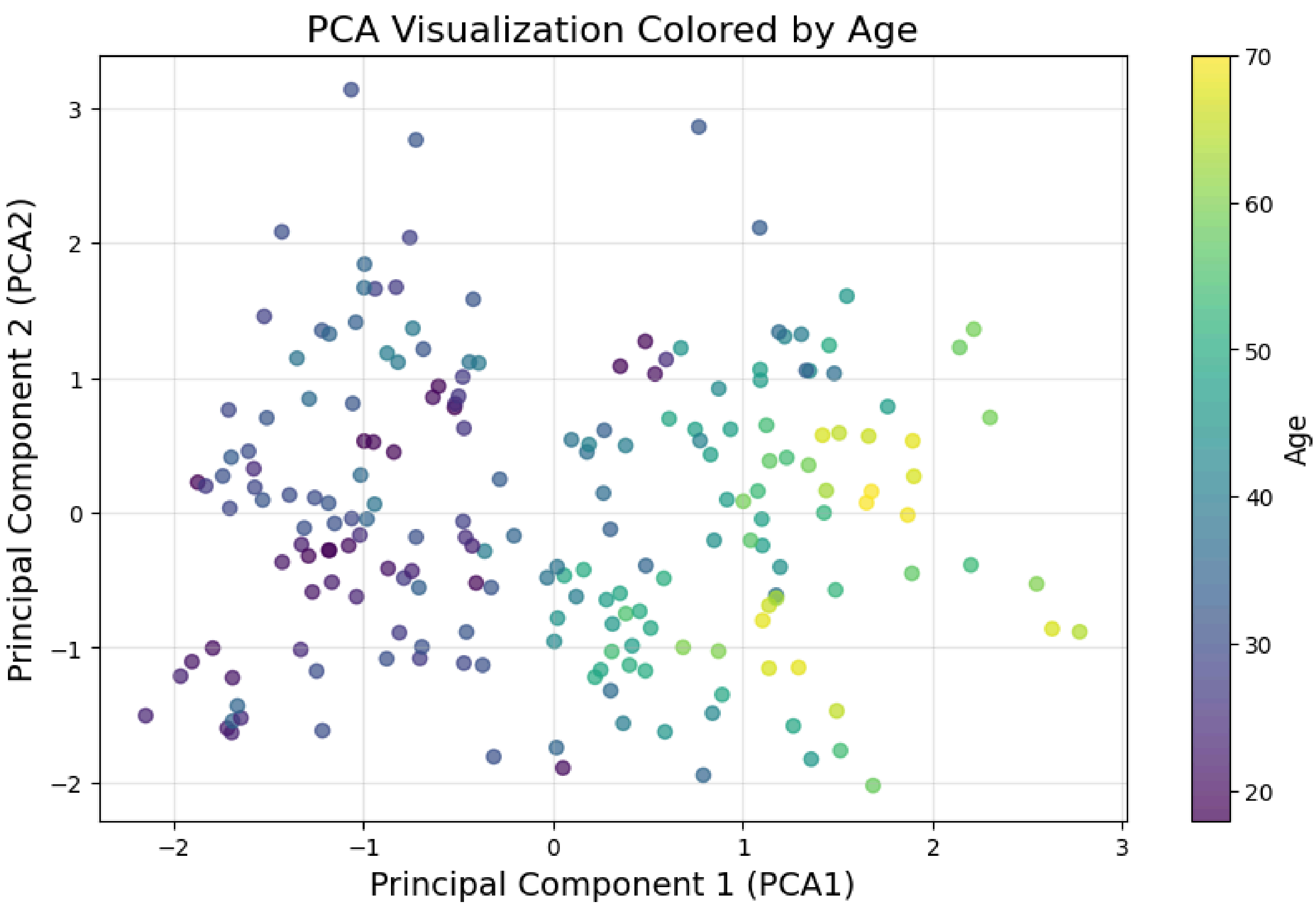
We can use the plot to identify patterns, clusters, or separations between genders or other characteristics

Visualize the New Dimensions Against All the Data Features

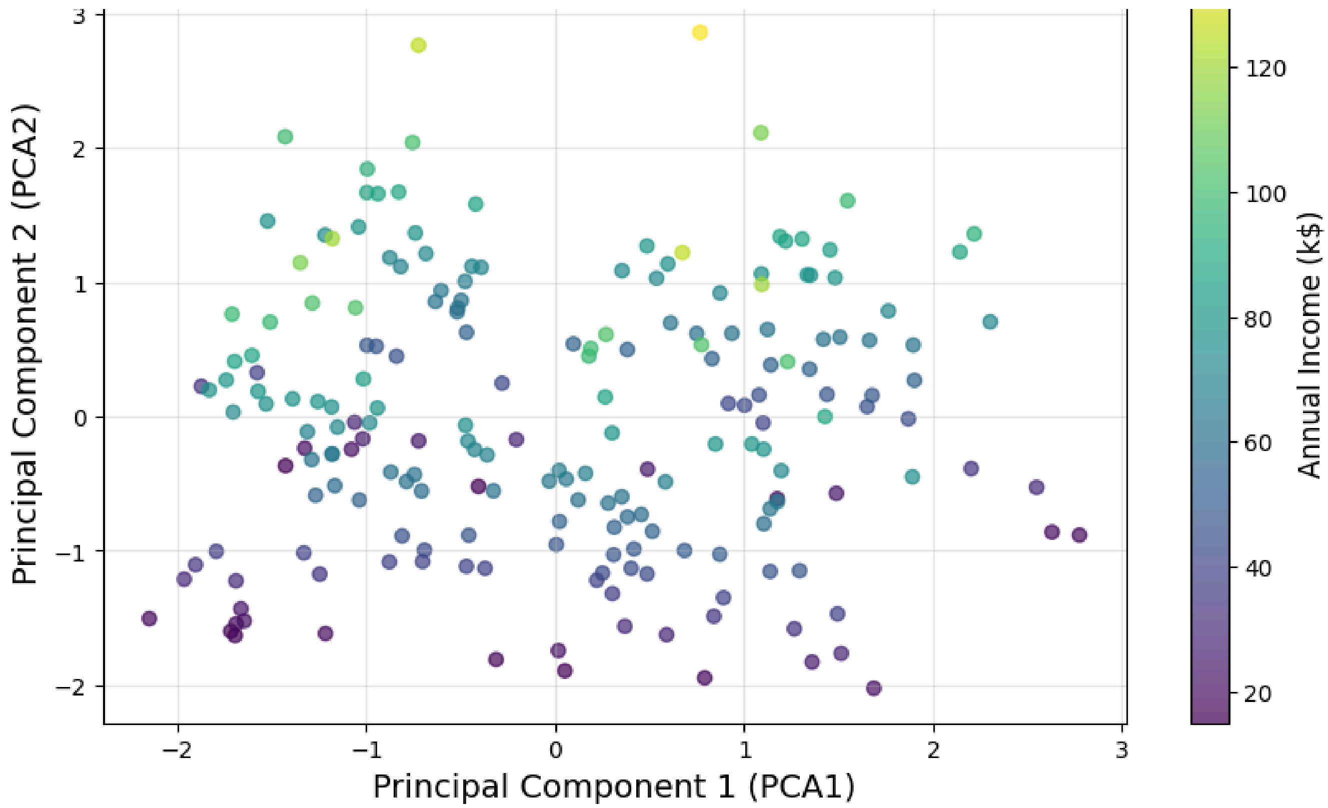
```
import matplotlib.pyplot as plt

# Function to create scatter plots with different color mappings
def plot_pca(data, color_feature, color_label):
    plt.figure(figsize=(10, 6))
    scatter = plt.scatter(data['PCA1'], data['PCA2'], c=data[color_feature], cmap='viridis',
    plt.title(f'PCA Visualization Colored by {color_label}', fontsize=16)
    plt.xlabel('Principal Component 1 (PCA1)', fontsize=14)
    plt.ylabel('Principal Component 2 (PCA2)', fontsize=14)
```

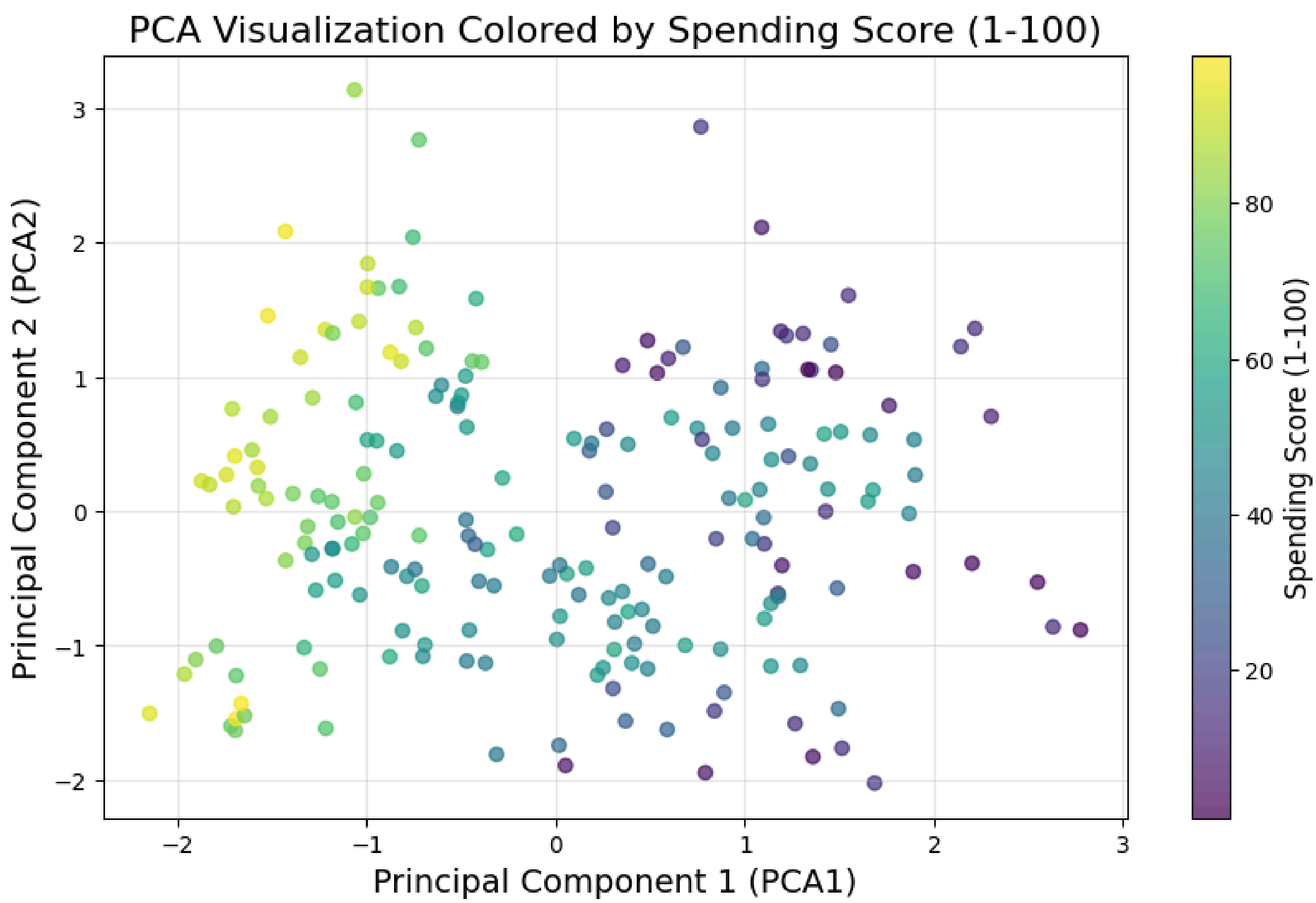
```
        cbar = plt.colorbar(scatter)
        cbar.set_label(color_label, fontsize=12)
        plt.grid(alpha=0.3)
        plt.show()

    # Plot colored by Gender
    plot_pca(data, 'Gender', 'Gender (0: Female, 1: Male)')

    # Plot colored by Age
    plot_pca(data, 'Age', 'Age')

    # Plot colored by Annual Income (k$)
    plot_pca(data, 'Annual Income (k$)', 'Annual Income (k$)')

    # Plot colored by Spending Score
    plot_pca(data, 'Spending Score (1-100)', 'Spending Score (1-100)')
```


PCA Visualization Colored by Gender (0: Female, 1: Male)
Principal Component 1 (PCA1)
Principal Component 2 (PCA2)
Gender (0: Female, 1: Male)


PCA Visualization Colored by Age
Principal Component 1 (PCA1)
Principal Component 2 (PCA2)
Age


PCA Visualization Colored by Annual Income (k$)


Principal Component 2 (PCA2)
Principal Component 1 (PCA1)
Annual Income (k$)
−2
−1
0
1
2
3
20
40
60
80
100
120


PCA Visualization Colored by Spending Score (1-100)
Principal Component 2 (PCA2)
Principal Component 1 (PCA1)
Spending Score (1-100)
−2
−1
0
1
2
3
20
40
60
80

How Can PCA Benifit The Analysis of Customers Dataset?

PCA can be leveraged for various purposes in a customer dataset to gain insights, enhance analytics, and improve machine learning models. Here are some additional uses of PCA for the given customer dataset:

a. **Data Visualization**

- **Exploration**: PCA helps in visualizing high-dimensional data in 2D or 3D spaces. This makes it easier to observe clustering patterns or separations in customer behavior based on features like age, income, or spending.
- **Cluster Validation**: By visualizing reduced dimensions, you can assess whether natural clusters (e.g., low-income vs. high-income groups) exist in the data.