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Time Series Analysis: Theory and Practice with Python

What Is a Time Series?

A **time series** is a sequence of observations collected **over time**, usually at regular intervals.

Examples:

- Daily stock prices
- Hourly electricity consumption
- Monthly sales revenue
- Sensor measurements every second

Why Time Series Are Different

- Observations are **dependent**
- The order of data points **cannot be shuffled**
- Past values influence future values

This dependency structure is the core challenge of time series analysis.

Time Series Decomposition

A time series can often be expressed as the combination of three interpretable components:

- **Trend** – the long-term direction or movement of the series
- **Seasonality** – systematic, calendar-related patterns that repeat over a fixed period
- **Noise (Residual)** – irregular, unsystematic variation not explained by trend or seasonality

Additive Decomposition

Used when seasonal fluctuations are approximately constant over time:

$$y_t = \text{Trend}_t + \text{Seasonality}_t + \text{Noise}_t$$

Multiplicative Decomposition

Used when seasonal effects scale with the level of the series:

$$y_t = \text{Trend}_t \times \text{Seasonality}_t \times \text{Noise}_t$$

Practical Interpretation

- Additive models assume **independent component magnitudes**
- Multiplicative models assume **proportional interactions** between components
- Taking logarithms converts the multiplicative form into an additive one, simplifying estimation: $\log(y_t) = \log(\text{Trend}_t) + \log(\text{Seasonality}_t) + \log(\text{Noise}_t)$

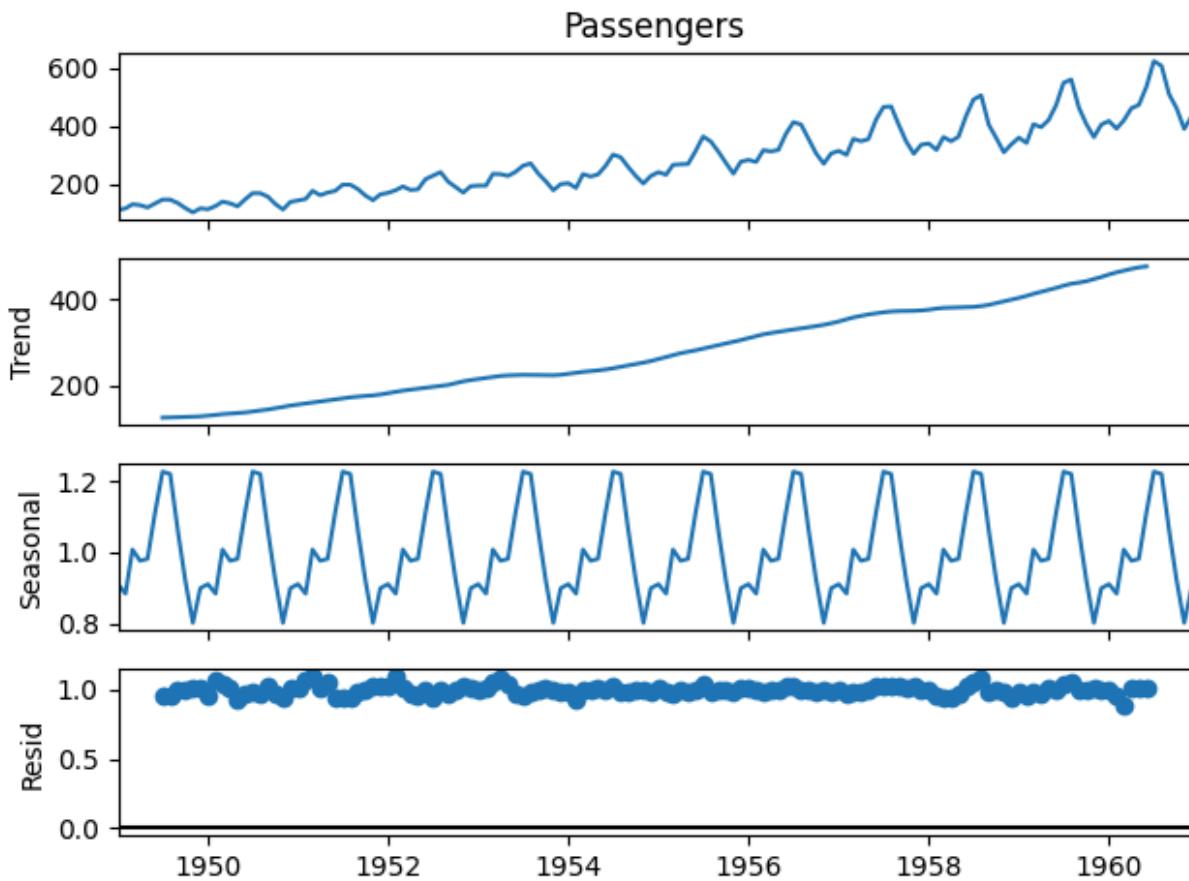
Python Example: Decomposition

```
In [ ]: import pandas as pd
import matplotlib.pyplot as plt
from statsmodels.tsa.seasonal import seasonal_decompose

# Load example data, the parse_dates and index_col arguments ensure the data is loaded as a datetime object
# The parse_dates argument ensures the data is loaded as a datetime object, and the index_col argument ensures the data is loaded as a column of dates
data = pd.read_csv(
    "https://raw.githubusercontent.com/jbrownlee/Datasets/master/airline-passengers.csv",
    parse_dates=["Month"],
    index_col="Month"
)

# Decompose
decomposition = seasonal_decompose(data["Passengers"], model="multiplicative")

# Plot
decomposition.plot()
plt.show()
```



Why this matters

- Makes patterns visible
- Makes seasonality explicit
- Separates structure from noise
- Helps decide which model to use

Stationarity (Critical Concept)

Many statistical models assume **stationarity**.

A time series is stationary if:

- Mean is constant over time
- Variance is constant
- Autocorrelation structure is stable

Why Stationarity Matters

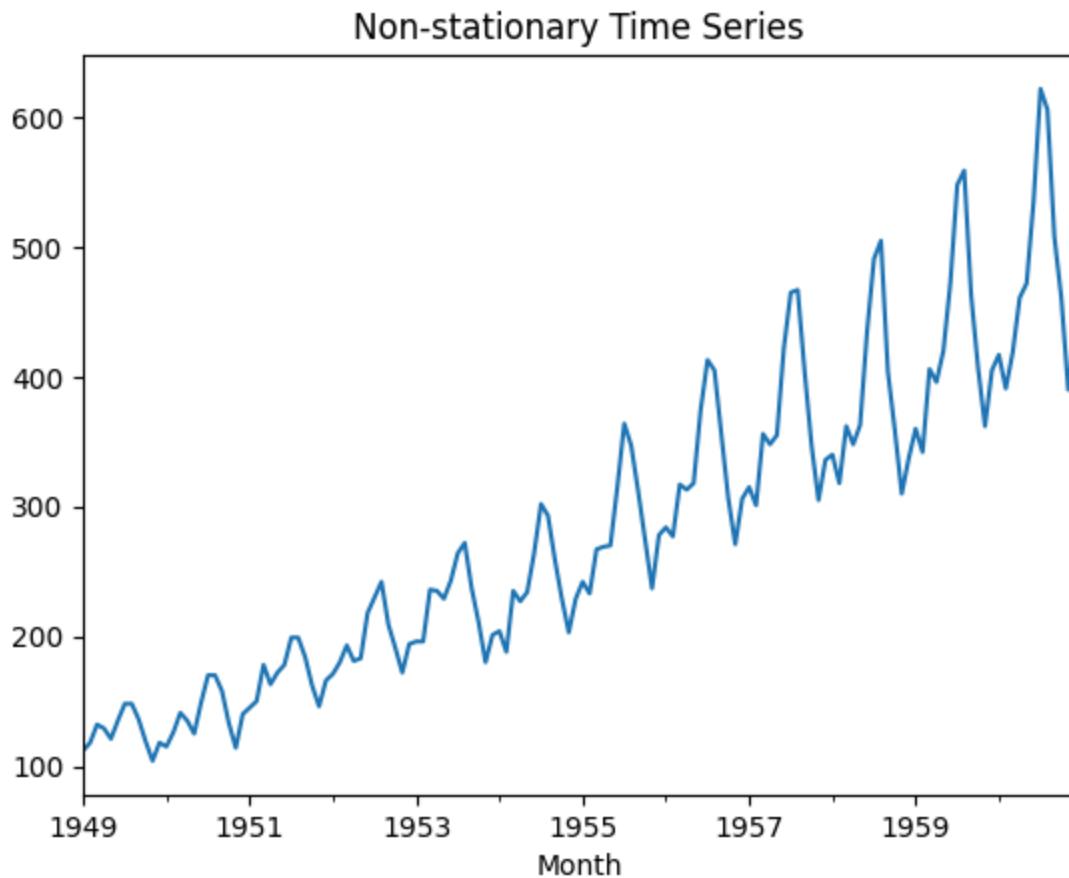
Many statistical models assume stable statistical properties. Without stationarity:

- Model parameters become unreliable

- Forecasts degrade quickly

Visual Example

```
In [4]: data["Passengers"].plot(title="Non-stationary Time Series")
plt.show()
```



Based on the plot, the time series data shows a clear upward trend, with some seasonal variations. The data also shows some volatility, with some periods of high volatility and some periods of low volatility.

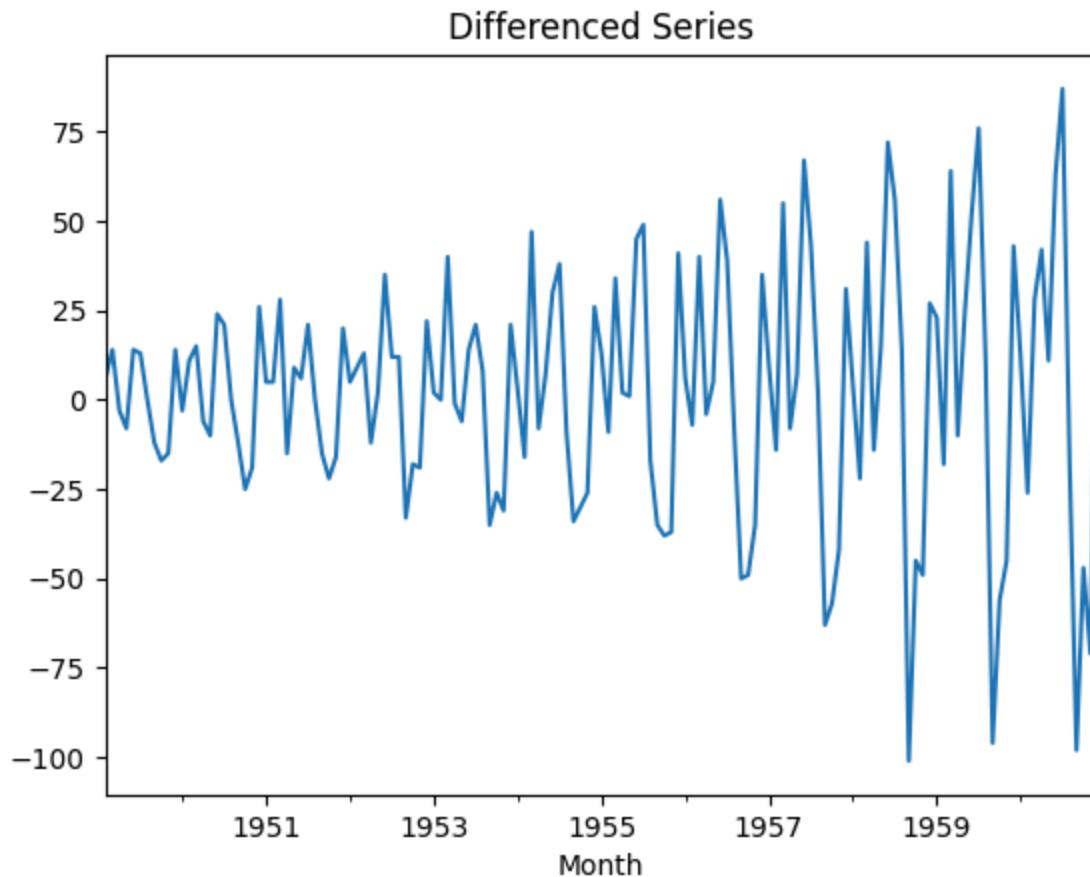
Differencing to Achieve Stationarity

Differencing is a common technique used in time series analysis to remove trends and seasonality from the data. It returns a new series by subtracting the previous value from the current value.

For example, if the series is [1, 2, 3, 4, 5], then the differenced series is [1, 1, 1, 1]

```
In [ ]: diff_series = data["Passengers"].diff().dropna()

diff_series.plot(title="Differenced Series")
plt.show()
```



The plot after differencing shows a little more stable pattern in terms of the mean, which is a characteristic of a stationary time series, but the variance is unstable i.e. increasing, which might require additional transformation e.g. log transform, we will leave that for now.

Understanding Temporal Dependence in Time Series Autocorrelation and Partial Autocorrelation

- Unlike cross-sectional data, **time series observations are ordered and interdependent**
- Past values often influence current behavior due to:
 - Momentum and inertia
 - Cycles and seasonality
 - Structural persistence
- Before building forecasting models, we must **quantify how the past relates to the present**

Auto Correlation Function ACF and Partial Auto Correlation Function PACF are the primary tools used for this purpose.

Motivation for ACF and PACF

- Raw time series plots show *patterns*, but not **lag-by-lag dependence**
- We need systematic tools to answer:
 - Does the series depend on its past?
 - How far back does this dependence persist?
 - Which lags matter directly vs. indirectly?

ACF and PACF provide a concise, visual summary of these relationships.

Autocorrelation Function (ACF)

The **ACF** measures how strongly a time series is correlated with **its own past values**.

At lag (k):

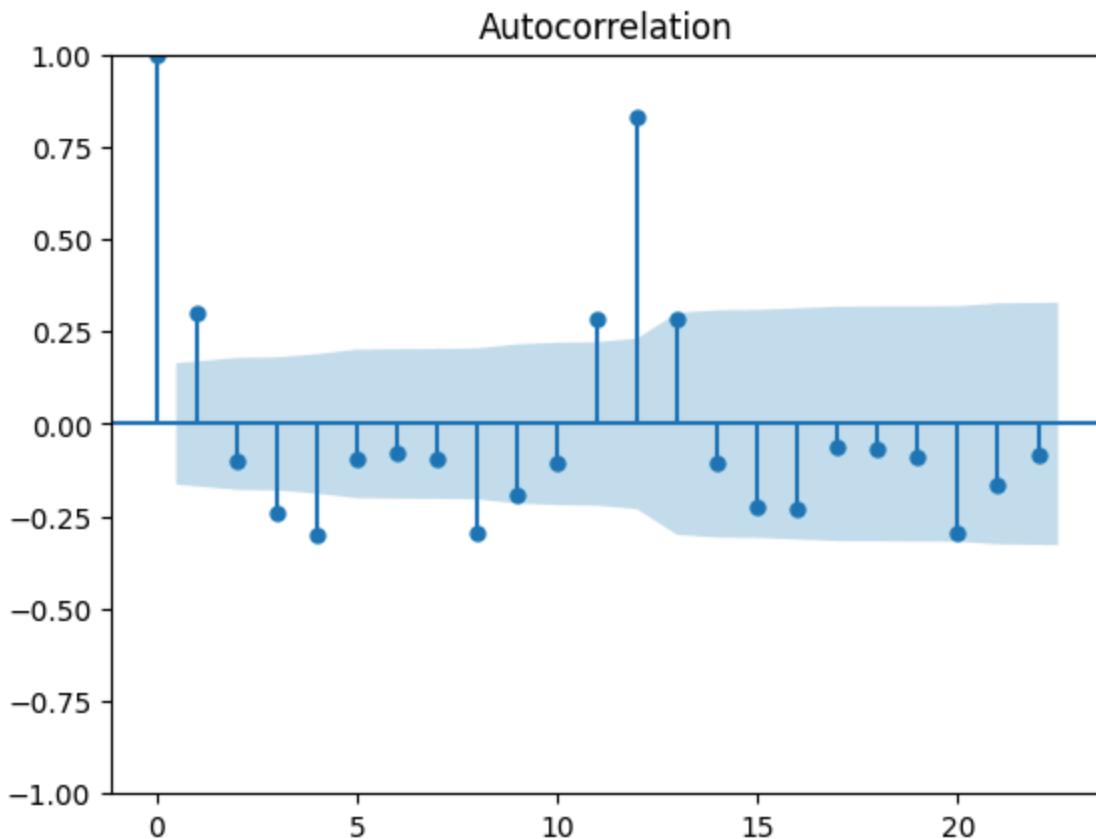
$$\text{ACF}(k) = \text{corr}(y_t, y_{t-k})$$

Intuition

- "How much does today look like k periods ago?"
- Measures **total correlation**, including indirect effects

Python Example

```
In [32]: from statsmodels.graphics.tsaplots import plot_acf  
plot_acf(diff_series)  
plt.show()
```



How to Interpret ACF

Pattern	Interpretation
Slow decay	Series is non-stationary
Sharp cutoff after lag q	MA(q) process
Spikes at seasonal lags	Seasonal MA component
All values near zero	White noise

Key Insight: ACF is primarily used to identify the **MA (Moving Average)** part of ARIMA.

Partial Autocorrelation Function (PACF)

What PACF Measures

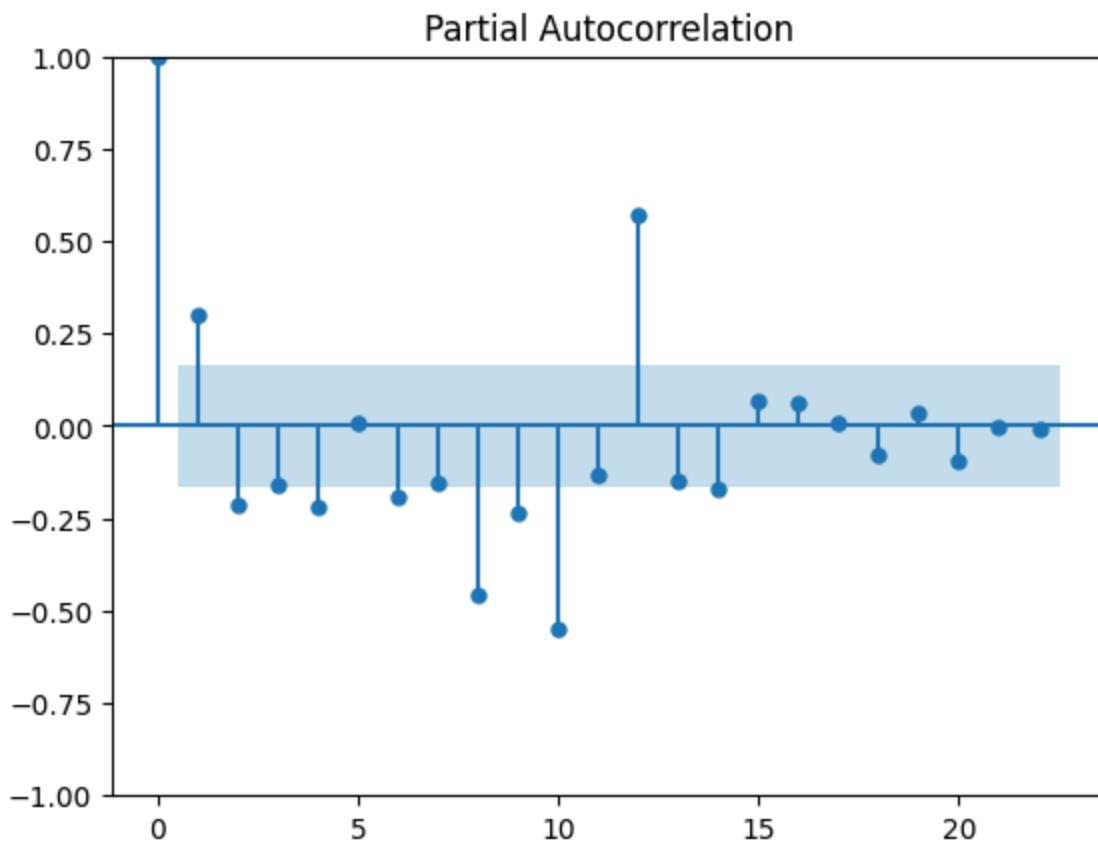
The **PACF** measures the **direct** relationship between (y_t) and (y_{t-k}) , **removing the influence of intermediate lags**.

Intuition

- “Does lag k matter *on its own*, or only because of closer lags?”

Python Example

```
In [33]: from statsmodels.graphics.tsaplots import plot_pacf
plot_pacf(diff_series)
plt.show()
```



How to Interpret PACF

Pattern	Interpretation
Sharp cutoff after lag p	AR(p) process
Gradual decay	MA behavior
Seasonal spikes	Seasonal AR component

Key Insight: PACF is primarily used to identify the **AR (Autoregressive)** part of ARIMA.

Why ACF and PACF Are Different (Critical Understanding)

- **ACF** includes **direct + indirect** correlations
- **PACF** isolates **direct** correlations only

Example: If:

- (y_t) depends on (y_{t-1})
- (y_{t-1}) depends on (y_{t-2})

Then:

- ACF at lag 2 may be high
- PACF at lag 2 may be near zero

This distinction is what allows AR and MA terms to be identified separately.

Summary

- **ACF** shows how similar the current month is to past months. → Strong spikes, especially at **12 months**, mean the data has a **clear yearly seasonal pattern**. → Correlations stay high for many lags, showing the series has a **trend** and is **not stationary**.
- **PACF** shows which past months **directly influence** the current month. → A strong spike at **lag 1** means this month depends on **last month**. → A strong spike at **lag 12** means this month also depends on **the same month last year**.
- **What we learn from both plots:** Passenger numbers depend on **recent history (last month)** and **seasonal history (last year)**.
- **Why SARIMA is a good choice:** SARIMA is built for time-series data that has a **trend** and **repeating seasonal patterns**. It can handle both **short-term changes** and **yearly seasonality**, which is exactly what the ACF and PACF plots reveal in the AirPassengers dataset.

CLASSICAL STATISTICAL MODELS ARIMA MODELS

ARIMA models are a foundational class of methods for analyzing and forecasting time series data when observations are dependent over time.

Unlike standard regression approaches, ARIMA explicitly models temporal structure by allowing the current value of a series to depend on its own past values and past random shocks.

The framework combines three ideas:

- The autoregressive (AR) component captures persistence by relating the present to previous observations.
- The moving average (MA) component models how past forecast errors propagate forward.
- The integrated (I) component addresses non-stationarity by differencing the series to remove trends or stochastic drift.

Together, these elements provide a flexible yet interpretable way to represent a wide range of time series behaviors.

ARIMA is typically applied in a diagnostic-driven workflow:

- The series is first explored and transformed to achieve stationarity, then its dependence structure is examined—often using ACF and PACF plots—to guide model specification.
- The emphasis is on parsimony and interpretability rather than black-box prediction, making ARIMA a standard baseline in time series analysis and forecasting.

Model Structure

$\text{ARIMA}(p, d, q)$

Parameter	Meaning	How Chosen
p	AR terms	From PACF
d	Differencing	From stationarity
q	MA terms	From ACF

$p = 2$ means we need to use the last 2 data points to predict the next data point

$d = 1$ means we need to difference the data once, for example if we have 100 data points, we will have 99 data points after differencing

$q = 2$ means we need to use the last 2 data points to predict the next data point

Mathematical Form

$$y_t = c + \sum_{i=1}^p \phi_i y_{t-i} + \sum_{j=1}^q \theta_j \varepsilon_{t-j} + \varepsilon_t$$

This equation shows that the current value y_t is made up of:

- a constant term (c), plus a contribution from **past values** of the series ($\sum_{i=1}^p \phi_i, y_{t-i}$) (where (y_{t-i}) are earlier observations and (ϕ_i) are their weights)
- a contribution from **past errors/shocks** ($\sum_{j=1}^q \theta_j, \varepsilon_{t-j}$) (where (ε_{t-j}) are previous forecast errors and (θ_j) are their weights)
- and finally the **current random noise** (ε_t).

This structure aligns with what we see in the ACF and PACF plots—dependence on recent history (e.g., lag 1) and seasonal repetition (e.g., lag 12)—which is why a seasonal extension such as SARIMA is appropriate for the AirPassengers series.

Python Example

The code below shows the application of an **ARIMA(2,1,2)** model to the AirPassengers time series.

The model first applies one level of differencing to remove the overall upward trend and achieve approximate stationarity.

It then captures short-term dependencies by modeling the influence of passenger counts from the previous two months (autoregressive terms) and the impact of recent shocks or errors from the past two periods (moving-average terms).

This approach focuses on modeling non-seasonal, month-to-month dynamics in the data and does not explicitly account for the strong yearly seasonal pattern characteristic of the AirPassengers series.

```
In [ ]: from statsmodels.tsa.arima.model import ARIMA
model = ARIMA(data["Passengers"], order=(2,1,2))
model_fit = model.fit()
print(model_fit.summary())
```

SARIMAX Results						
Dep. Variable:	Passengers	No. Observations:	144			
Model:	ARIMA(2, 1, 2)	Log Likelihood	-671.673			
Date:	Tue, 16 Dec 2025	AIC	1353.347			
Time:	23:10:20	BIC	1368.161			
Sample:	01-01-1949 - 12-01-1960	HQIC	1359.366			
Covariance Type:	opg					
	coef	std err	z	P> z	[0.025	0.975]
ar.L1	1.6850	0.020	83.059	0.000	1.645	1.725
ar.L2	-0.9548	0.017	-55.419	0.000	-0.989	-0.921
ma.L1	-1.8432	0.125	-14.798	0.000	-2.087	-1.599
ma.L2	0.9953	0.135	7.374	0.000	0.731	1.260
sigma2	665.9568	114.104	5.836	0.000	442.316	889.597
Ljung-Box (L1) (Q):	0.30	Jarque-Bera (JB):	1.84			
Prob(Q):	0.59	Prob(JB):	0.40			
Heteroskedasticity (H):	7.38	Skew:	0.27			
Prob(H) (two-sided):	0.00	Kurtosis:	3.14			

Warnings:

[1] Covariance matrix calculated using the outer product of gradients (complex-step).

```
c:\Users\me\venv\Lib\site-packages\statsmodels\tsa\base\tsa_model.py:473: ValueWarning: No frequency information was provided, so inferred frequency MS will be used.
    self._init_dates(dates, freq)
c:\Users\me\venv\Lib\site-packages\statsmodels\tsa\base\tsa_model.py:473: ValueWarning: No frequency information was provided, so inferred frequency MS will be used.
    self._init_dates(dates, freq)
c:\Users\me\venv\Lib\site-packages\statsmodels\tsa\base\tsa_model.py:473: ValueWarning: No frequency information was provided, so inferred frequency MS will be used.
    self._init_dates(dates, freq)
c:\Users\me\venv\Lib\site-packages\statsmodels\base\model.py:607: ConvergenceWarning: Maximum Likelihood optimization failed to converge. Check mle_retrvals
    warnings.warn("Maximum Likelihood optimization failed to "
```

What this *really* is (in one sentence)

ARIMA is just regression on time, where the predictors are **past values and past errors instead of columns in a table**.

Connect it to regular regression (intuition first)

In ordinary regression you write:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \text{error}$$

In ARIMA, you write:

$$y_t = \text{constant} + (\text{past } y\text{'s}) + (\text{past errors}) + \text{error}$$

Same idea. Different inputs.

Translate the output into regression language

1. The coefficients (ar.L1 , ar.L2 , ma.L1 , ma.L2)

These are just **regression coefficients**.

- `ar.L1 = 1.685` → "Last month strongly predicts this month"
- `ar.L2 = -0.955` → "Two months ago has a correcting (negative) effect"
- `ma.L1` and `ma.L2` → "If the model made errors recently, those errors help adjust today's prediction"

This is no different from saying:

$$\text{feature}_1 \text{ coefficient} = 1.68 \text{ feature}_2 \text{ coefficient} = -0.95$$

The "features" just happen to be **lags of the same variable**.

2. p-values and z-scores

Exactly the same as regression:

- Low p-value → coefficient matters
- High p-value → coefficient doesn't matter

Here, all p-values $\approx 0 \rightarrow$ **the model found strong structure in time.**

3. Residual diagnostics = model sanity checks

Think of these as **post-fit validation**, not magic.

- **Ljung–Box** → "Did we leave obvious patterns behind?" (No → good)
- **Jarque–Bera** → "Do residuals look roughly normal?" (Yes → OK)
- **Heteroskedasticity test** → "Does variance change over time?" (Yes → data grows over time, known issue with AirPassengers)

Same role as:

- residual plots
- normality checks
- constant variance checks

Why it *feels* alien

1. No feature matrix

- Features are created internally from time lags.

2. Statistical language

- Econometrics emphasizes inference, not prediction accuracy.

3. Diagnostics-heavy

- Much more focus on *why* a model works than ML usually gives.

How this fits into machine learning

ARIMA is:

- ✓ supervised learning
- ✓ parametric
- ✓ linear model
- ✓ maximum likelihood-based

It sits closer to:

- linear regression
- ridge regression
- Kalman filters

than to:

- neural networks
- random forests

One grounding sentence you can remember

ARIMA is just linear regression where the predictors are the past of the same variable, and the output is wrapped in statistical diagnostics instead of ML metrics.

Forecasting with ARIMA

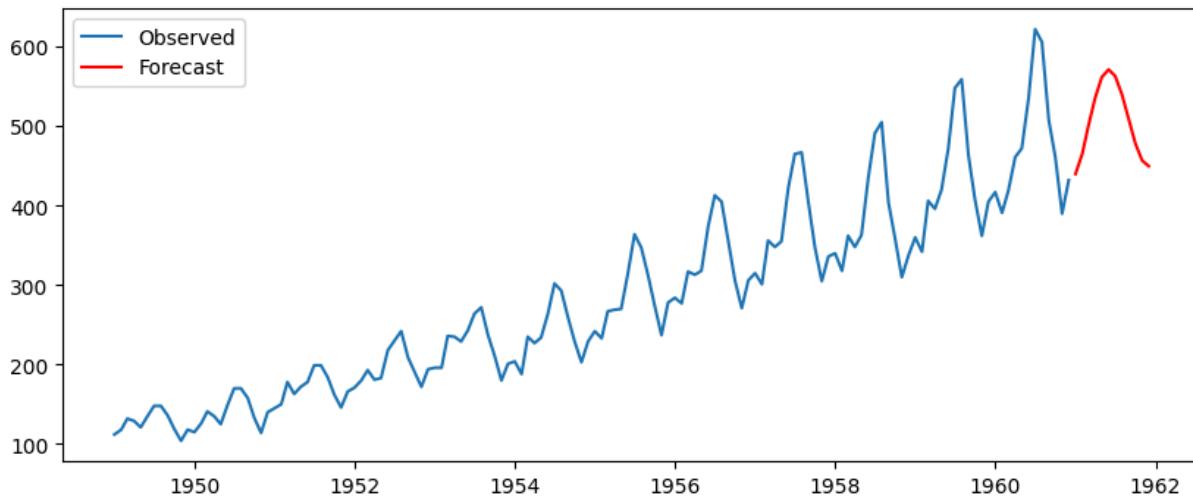
The code below uses the fitted ARIMA model to generate a 12-month forecast of airline passenger numbers and then visualizes the results.

The historical observations are plotted alongside the model's out-of-sample predictions to show how the learned time-series patterns are extrapolated into the future.

This comparison helps assess how well the model captures the overall trend and highlights any limitations, such as the absence of explicit seasonal behavior in the forecast.

```
In [35]: forecast = model_fit.forecast(steps=12)

plt.figure(figsize=(10,4))
plt.plot(data.index, data["Passengers"], label="Observed")
plt.plot(forecast.index, forecast, label="Forecast", color="red")
plt.legend()
plt.show()
```



The forecast shows that the ARIMA model captures the overall upward trend in passenger numbers, but because it does not include seasonal components, it fails to reproduce the strong yearly fluctuations present in the data.

What this tells you scientifically

- The model is not wrong
- It is simply incomplete for this dataset
- The plot visually proves why seasonal models (SARIMA) or log-transformed models are more appropriate for AirPassengers

ARIMA Strengths

- Interpretable
- Statistically grounded

ARIMA Limitations

- **No built-in seasonality** Standard ARIMA does not explicitly model repeating seasonal patterns (e.g., yearly or weekly cycles). Seasonal effects must be handled through preprocessing or by using a seasonal extension such as **SARIMA**.
- **Linear model** Assumes linear relationships between past values and errors.
- **Sensitive to assumptions** Requires correct differencing and approximate stationarity.
- **Limited handling of external variables** Does not naturally incorporate many exogenous factors (e.g., holidays, promotions) without extensions like **ARIMAX/SARIMAX**.

Simple one-line takeaway

ARIMA is effective for trend and short-term dynamics but is limited when strong seasonal patterns or external drivers are present.

Seasonal ARIMA (SARIMA)

The code below fits a SARIMA model to the AirPassengers time series in order to capture both non-seasonal and seasonal patterns in the data.

The non-seasonal component ((1,1,1)) models short-term month-to-month dependence after removing the overall trend through differencing, while the seasonal component ((1,1,1,12)) explicitly models yearly seasonality with a 12-month period.

By combining these components, the SARIMAX framework allows the model to account for trend, short-term dynamics, and recurring annual fluctuations, providing a more appropriate representation of the AirPassengers series than a non-seasonal ARIMA model.

```
In [36]: from statsmodels.tsa.statespace.sarimax import SARIMAX
```

```

model = SARIMAX(
    data["Passengers"],
    order=(1,1,1),
    seasonal_order=(1,1,1,12)
)

fit = model.fit()
fit.summary()

```

```

c:\Users\me\venv\Lib\site-packages\statsmodels\tsa\base\tsa_model.py:473: ValueWarning: No frequency information was provided, so inferred frequency MS will be used.
    self._init_dates(dates, freq)
c:\Users\me\venv\Lib\site-packages\statsmodels\tsa\base\tsa_model.py:473: ValueWarning: No frequency information was provided, so inferred frequency MS will be used.
    self._init_dates(dates, freq)

```

Out[36]:

SARIMAX Results

Dep. Variable:	Passengers	No. Observations:	144
Model:	SARIMAX(1, 1, 1)x(1, 1, 1, 12)	Log Likelihood	-506.149
Date:	Tue, 16 Dec 2025	AIC	1022.299
Time:	23:28:55	BIC	1036.675
Sample:	01-01-1949 - 12-01-1960	HQIC	1028.140

Covariance Type: opg

	coef	std err	z	P> z	[0.025	0.975]
ar.L1	-0.1272	0.356	-0.358	0.721	-0.825	0.570
ma.L1	-0.2149	0.325	-0.660	0.509	-0.853	0.423
ar.S.L12	-0.9272	0.214	-4.341	0.000	-1.346	-0.509
ma.S.L12	0.8394	0.309	2.717	0.007	0.234	1.445
sigma2	130.7826	15.420	8.481	0.000	100.559	161.006

Ljung-Box (L1) (Q): 0.00 Jarque-Bera (JB): 7.05

Prob(Q): 0.99 Prob(JB): 0.03

Heteroskedasticity (H): 2.65 Skew: 0.13

Prob(H) (two-sided): 0.00 Kurtosis: 4.11

Warnings:

[1] Covariance matrix calculated using the outer product of gradients (complex-step).

These SARIMAX results show that the model successfully captures the **seasonal structure** of the AirPassengers data.

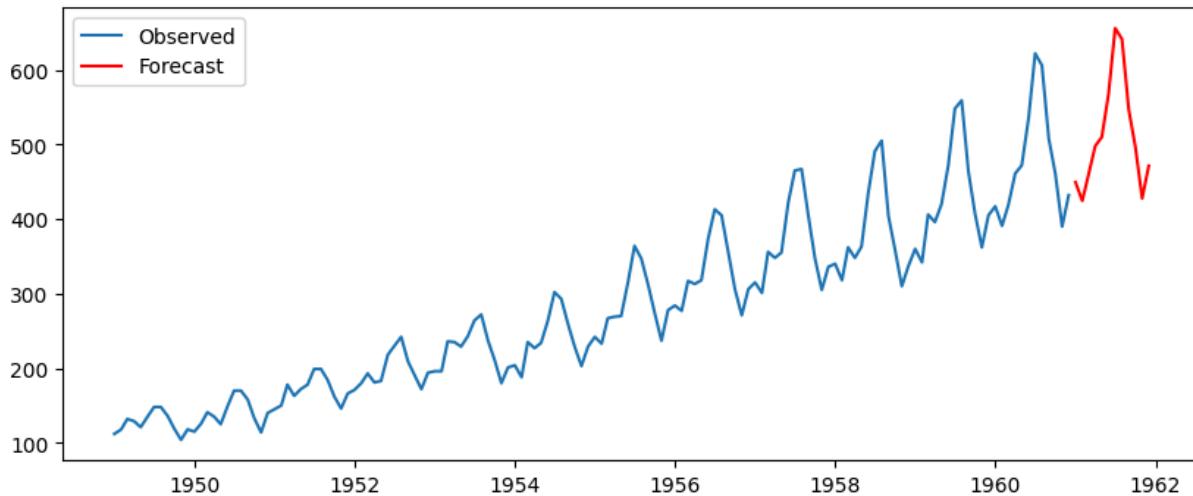
The **seasonal autoregressive (AR.S.12)** and **seasonal moving-average (MA.S.12)** terms are statistically significant, confirming a strong yearly dependency, while the non-seasonal AR and MA terms are not significant, indicating that most of the explanatory power comes from the seasonal component rather than short-term month-to-month effects.

The low AIC value reflects an improved fit compared to a non-seasonal ARIMA model, and the Ljung–Box test suggests no remaining autocorrelation in the residuals.

Overall, this confirms that explicitly modeling seasonality is critical for accurately representing the AirPassengers time series.

```
In [15]: forecast = fit.forecast(steps=12)
```

```
plt.figure(figsize=(10,4))
plt.plot(data.index, data["Passengers"], label="Observed")
plt.plot(forecast.index, forecast, label="Forecast", color="red")
plt.legend()
plt.show()
```



PART II — MACHINE LEARNING APPROACHES

Why Machine Learning for Time Series?

Statistical models:

- Focus on *past values only*
- Require strong assumptions

ML models:

- Handle non-linearity
- Incorporate many features
- Scale well with data

Feature Engineering for ML

ML models **do not understand time**. Time information must be converted into features.

Lag Features

```
In [17]: df = data.copy()

df["lag_1"] = df["Passengers"].shift(1)
df["lag_12"] = df["Passengers"].shift(12)

df.dropna(inplace=True)
```

Display new dataframe with lags

```
In [20]: df
```

```
Out[20]:
```

Month	Passenger	lag_1	lag_12
1950-01-01	115	118.0	112.0
1950-02-01	126	115.0	118.0
1950-03-01	141	126.0	132.0
1950-04-01	135	141.0	129.0
1950-05-01	125	135.0	121.0
...
1960-08-01	606	622.0	559.0
1960-09-01	508	606.0	463.0
1960-10-01	461	508.0	407.0
1960-11-01	390	461.0	362.0
1960-12-01	432	390.0	405.0

132 rows × 3 columns

Train-Test Split (Time-Aware)

```
In [ ]: train = df.iloc[:-12] # All data except last 12 months
          test = df.iloc[-12:] # Last 12 months for testing

          X_train = train.drop("Passengers", axis=1) # Drop the target column
          y_train = train["Passengers"] # Target column

          X_test = test.drop("Passengers", axis=1)
          y_test = test["Passengers"]
```

Regression Model (Baseline ML)

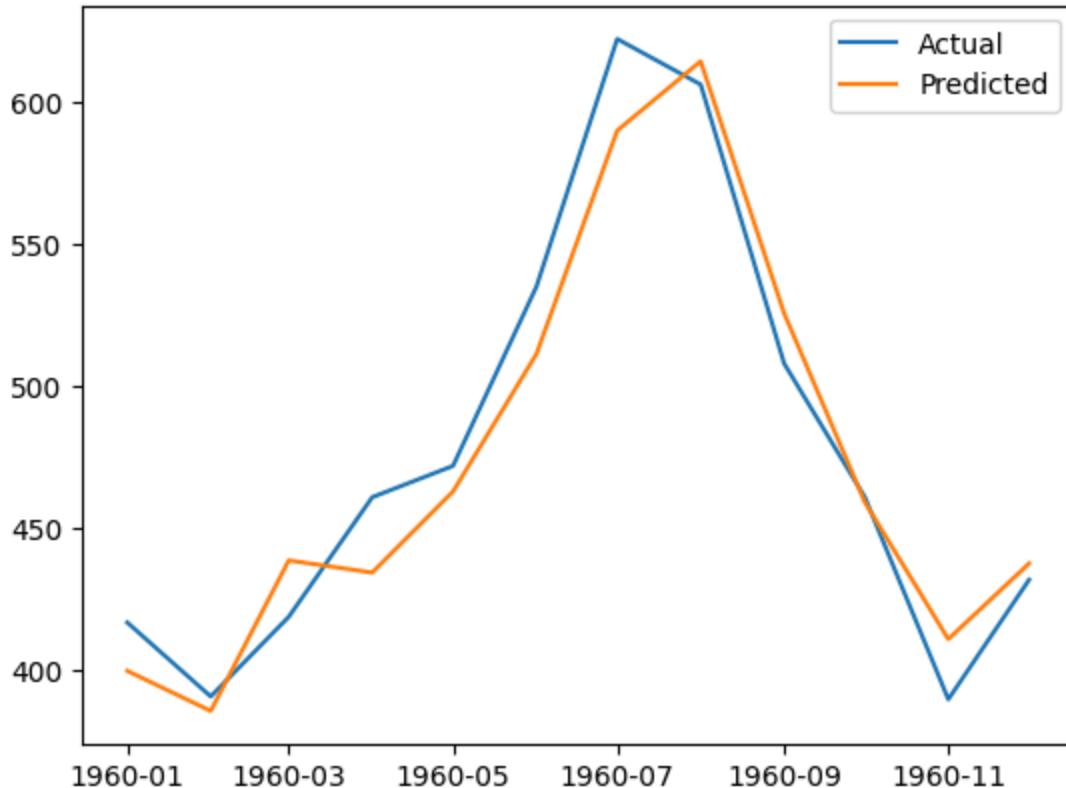
```
In [21]: from sklearn.linear_model import LinearRegression

model = LinearRegression()
model.fit(X_train, y_train)

predictions = model.predict(X_test)
```

Plot the actual vs predicted values

```
In [22]: plt.plot(test.index, y_test, label="Actual")
          plt.plot(test.index, predictions, label="Predicted")
          plt.legend()
          plt.show()
```



Evaluate the performance of the model

```
In [29]: import numpy as np

# Compute R-Squared
from sklearn.metrics import r2_score
r2 = r2_score(y_test, predictions)
print(f'R-squared: {r2:.4f}')

# Compute RMSE
from sklearn.metrics import mean_squared_error
rmse = np.sqrt(mean_squared_error(y_test, predictions))
print(f'RMSE: {rmse:.4f}'')
```

R-squared: 0.9406

RMSE: 18.1356

Tree-Based Models (Nonlinear Patterns)

```
In [24]: from sklearn.ensemble import RandomForestRegressor

rf = RandomForestRegressor(n_estimators=200, random_state=42)
rf.fit(X_train, y_train)

rf_preds = rf.predict(X_test)
```

Plot the time series data

```
In [25]: plt.plot(test.index, y_test, label="Actual")
plt.plot(test.index, rf_preds, label="Predicted")
plt.legend()
plt.show()
```



Evaluate the performance of the model

```
In [28]: import numpy as np

# Compute R-Squared
from sklearn.metrics import r2_score
r2 = r2_score(y_test, rf_preds)
print(f'R-squared: {r2:.4f}')

# Compute RMSE
from sklearn.metrics import mean_squared_error
rmse = np.sqrt(mean_squared_error(y_test, rf_preds))
print(f'RMSE: {rmse:.4f}')
```

R-squared: 0.8184

RMSE: 31.7142

Gradient Boosting (XGBoost-style Logic)

Gradient Boosting Regressor

```
In [30]: from sklearn.ensemble import GradientBoostingRegressor

gbr = GradientBoostingRegressor()
gbr.fit(X_train, y_train)

gbr_preds = gbr.predict(X_test)
```

```
import numpy as np

# Compute R-Squared
from sklearn.metrics import r2_score
r2 = r2_score(y_test, gbr_preds)
print(f'R-squared: {r2:.4f}')

# Compute RMSE
from sklearn.metrics import mean_squared_error
rmse = np.sqrt(mean_squared_error(y_test, gbr_preds))
print(f'RMSE: {rmse:.4f}')
```

R-squared: 0.8396

RMSE: 29.8079

Comparing Models

```
In [31]: from sklearn.metrics import mean_absolute_error

print("Linear:", mean_absolute_error(y_test, predictions))
print("RF:", mean_absolute_error(y_test, rf_preds))
print("GBR:", mean_absolute_error(y_test, gbr_preds))
```

Linear: 15.640331783370726

RF: 23.02416666666677

GBR: 24.80970757043872

DEEP LEARNING (CONCEPTUAL)

Recurrent Neural Networks (LSTM)

Deep learning models:

- Learn sequences directly
- Capture long-term dependencies

Key requirements:

- Large datasets
- Careful scaling
- Computational resources

Typical workflow:

1. Window the series
2. Normalize
3. Train LSTM
4. Forecast iteratively

(Implementation usually done with TensorFlow or PyTorch.)

Statistical vs ML: When to Use What

Scenario	Best Approach
Small dataset, interpretability	ARIMA / SARIMA
Strong seasonality	SARIMA
Many external variables	ML
Non-linear patterns	Tree models
Large-scale forecasting	ML / Deep Learning
Regulatory / explainable needs	Statistical

Key Takeaways

- Time series analysis is about **structure + dependency**
- Statistical models explain *why*
- ML models focus on *accuracy*
- Feature engineering is essential for ML
- No single model is universally best