





307304 Data Mining

Association Rules Mining & Apriori Algorithm





Introduction to Association Rule Mining

What Is It?

Association Rule Mining is a data mining technique used to discover interesting relationships or patterns between variables in large datasets, typically in the form:

If A, then B

e.g.

If {Milk}, then {Bread})



Market Basket Analysis







Real Life Use Cases

Retail:

- Items

 Customers who buy Milk also buy Bread"
- Suggested Actions:
 - Place Milk close to Bread (in-store layout)
 - Recommend Bread when Milk is added to cart (cross-sell)
 - Offer Milk + Bread at a discounted price (bundle offer)
- Expected Results: Increase in Bread sales.

E-commerce Recommendation:

- Items

 Customers who buy iPhone also buy Phone Case"
- Action: Auto-suggest phone cases during iPhone checkout
- **Expected Results:** Many iPhone buyers also purchase cases

Healthcare:

Symptoms ← Diseases: "Patients with fever and headache often have flu"

- **Used for:** Diagnosis support
- Action: Flag patient records or prompt flu tests when both symptoms are present
- Expected Results: Faster diagnosis, better triage, improved patient outcomes

Drugs ↔ **Side Effects:** "Patients taking Drug A often experience nausea"

- Used for: Drug safety monitoring
- Action: Add nausea warnings and prescribe anti-nausea medication alongside
- Expected Results: Increased patient safety and compliance





Real Life Use Cases

Web Analytics:

- Pages ↔ Pages:
 - "Users who visit Product Page A also visit Product Page B"
 - Action: Add cross-links or suggest Page B under "Customers also viewed"
 - Expected Results: Increased session duration and conversion rates

Actions ↔ **Actions**:

- "Users who click 'Add to Cart' also click 'View Reviews' "
- Action: Automatically scroll or highlight reviews after cart interaction
- Expected Results: Higher purchase confidence, reduced cart abandonment





Basic Terminology

Key Terms

- Transaction: One customer's shopping trip
- Itemset: Group of items, e.g., {Bread, Milk}
- Association Rule: X → Y, e.g., {Bread} → {Milk}

Goal:

Find rules that are statistically significant, useful, and actionable



Market Basket Analysis







To evaluate the strength and relevance of rules, we use:

Support

Proportion of transactions that contain both A and B

$$\operatorname{Support}(A,B) = \frac{\operatorname{Count}(A \cap B)}{\operatorname{Total Transactions}}$$

Confidence

Likelihood that B is bought given that A is bought

$$\operatorname{Confidence}(A o B) = rac{\operatorname{Support}(A,B)}{\operatorname{Support}(A)}$$

Lift

Measures how much more likely B is bought with A compared to random chance

$$\operatorname{Lift}(A o B) = rac{\operatorname{Confidence}(A o B)}{\operatorname{Support}(B)}$$





The Support Metric - "How Popular Is This Pattern?"

Definition

Percentage of transactions containing the itemset

Formula

Support(X) = Count(transactions with X) / Total transactions

Calculations from Our Data

```
Support({Bread}) = 6/10 = 60%

Support({Milk}) = 5/10 = 50%

Support({Butter}) = 5/10 = 50%

Support({Cheese}) = 6/10 = 60%

Support({Bread, Milk}) = 3/10 = 30%

Support({Butter, Cheese}) = 4/10 = 40%
```

Why Support Matters

- **High support:** Common, reliable patterns
- Low support: Rare, potentially spurious patterns
- Threshold: Typically, 5-20% depending on business context

Transaction ID	Items Purchased
T1	{Bread, Milk, Eggs}
T2	{Bread, Butter, Cheese}
Т3	{Milk, Butter, Cheese, Yogurt}
T4	{Bread, Milk, Butter}
T5	{Bread, Milk, Eggs, Cheese}
Т6	{Butter, Cheese, Yogurt}
T7	{Bread, Butter, Eggs}
Т8	{Milk, Cheese, Yogurt}
Т9	{Bread, Milk, Butter, Cheese}
T10	{Eggs, Cheese, Yogurt}





The Confidence Metric – Direction Matters

- Support tells us how many times two items appear together:
 - Support(milk ∩ diapers) = 100 / 1000 = 10%
- But support is symmetric (have no direction) it does not tell us if we should recommend milk when diapers are bought or should we recommend milk when diapers are bought?
- Confidence answers this question by measuring the conditional probability.

Scenario: We have 1,000 transactions of **Itemsets:** {Milk, Diapers}

Raw Data:

- 500 bought milk
- 150 bought diapers
- 100 bought both milk and diapers

Which rule should we use

Diapers → Milk

Milk -> Diapers

Or Both?





The Confidence Metric – Direction Matters

Confidence (Conditional Probability):

$$\operatorname{Confidence}(X o Y) = rac{\operatorname{Support}(X \cap Y)}{\operatorname{Support}(X)} = P(Y \mid X)$$

Direction 1: Diapers → Milk

$$ext{Confidence(Milk | Diapers)} = \frac{100}{150} = 0.66 = 66\%$$

→ 66% of diaper-buyers also buy milk

Direction 2: Milk → Diapers

$$ext{Confidence(Diapers | Milk)} = rac{100}{500} = 0.20 = 20\%$$

→ 20% of milk-buyers also buy diapers

Therefore, Diapers → Milk is the rule that we should select





The Importance of Confidence, Direction Matters

Domain	Rule A → B (Strong)	Rule B → A (Weak or Misleading)	Reason for Asymmetry
Retail	Buys Smartphone → Buys Phone Case	Buys Phone Case → Buys Smartphone	Cases are often bought later or as replacements
Web Analytics	Visits Product Page → Adds to Cart	Adds to Cart → Visits Product Page	Visit is required, but not all visits lead to cart
Healthcare	Diagnosed with Diabetes → Prescribed Insulin	Prescribed Insulin → Diagnosed with Diabetes	Insulin used for other conditions too
Fraud Detection	Unusual Login → Account Lock	Account Lock → Unusual Login	Locks triggered by various issues
E-Learning	Watches Lecture Video → Submits Homework	Submits Homework → Watches Lecture Video	Some students skip the video
Streaming Services	Subscribes to Premium Plan → Watches Exclusive Shows	Watches Exclusive Shows → Subscribes to Premium Plan	Content may be shared or watched via someone else's account
Finance	Misses Loan Payment → Credit Score Drops	Credit Score Drops → Misses Loan Payment	Scores drop for many other financial behaviors
HR / Workforce	Attends Training → Improved Job Performance	Improved Job Performance → Attended Training	Performance may improve for unrelated reasons





The Lift Metric

The confidence metric can be misleading in some scenarios such as when the consequent is a common item.

The lift metric answers the questions whether this co-occurrence is truly meaningful or it occurred just by change.

Scenario: We analyze 100 transactions at a bakery.

Itemset: {Biscuits, Bread}

Observations:

- 40 people bought biscuits
- 100 people bought bread
- 32 of those who bought biscuits also bought bread

Confidence (Bread | Biscuits): = 32 / 40 = **80%**

But Bread is Bought in Every Transaction: Support(Bread) = 100 / 100 = 1.0

We can use Lift to expose this relation

Lift = Confidence / Support(Bread): = 0.80 / 1.0 = 0.8 (below 1)

Interpretation:

- Confidence is high (80%), suggesting a strong rule
- But since bread is in every basket, the 80% is not surprising
- Lift < 1 shows this is actually a weak or negative association

Conclusion:

- Lift adjusts for how common the consequent is.
- It reveals when a high confidence is simply a result of a **popular item**, not a meaningful relationship.





Possible Lift Values

- 1. Lift > 1: Items A and B are positively correlated (they appear together more than expected)
- Lift < 1: Items A and B are negatively correlated (they appear together less than expected)
- 3. Lift = 1: Items A and B are independent (no association)





Lift Examples: Lift > 1

Example: Analyzing convenience store purchases.

Items: {Chips, Soda}

From 100 transactions:

- 40 people bought chips
- 50 people bought soda
- 30 people bought both chips and soda

Metrics:

- Support (chips \cap soda) = 30 / 100 = 0.30
- Support (chips) = 40 / 100 = 0.40
- Support (soda) = 50 / 100 = 0.50
- Confidence (soda | chips):= 0.30 / 0.40 = 0.75
- Lift:= 0.75 / 0.50 = 1.5

Interpretation:

- A lift of 1.5 means that people who buy chips are 50% more likely to also buy soda than the average customer.
- This shows a positive association, which makes sense: chips and soda are often consumed together.





Lift Examples: Lift < 1 – Negative Relationship

Lift can be less than 1 in two cases:

- When the consequent is common like in the bread/butter example.
- When there is a negative relationship between the antecedent and the consequent as show next.

Example: Analyzing supermarket transactions.

Items: {Toothpaste, Candy}

Data from 100 transactions:

- 40 people bought toothpaste
- 30 people bought candy
- Only 4 people bought both toothpaste and candy

Metrics:

- Support (toothpaste ∩ candy) = 4 / 100 = 0.04
- Support (toothpaste) = 40 / 100 = 0.40
- Support (candy) = 30 / 100 = 0.30
- Confidence (candy | toothpaste):= Support (toothpaste ∩ candy) / Support (toothpaste)= 0.04 / 0.40 = 0.10
- Lift:= Confidence / Support (candy)= 0.10 / 0.30 = 0.33

Interpretation:

- A lift of 0.33 means people who buy toothpaste are less likely to buy candy than the average customer.
- This negative association makes sense intuitively: people buying dental hygiene products might avoid sugary items like candy.





Lift Examples:- Lift = 1

Example: Analyzing grocery store data.

Items: {Milk, Bread}

From 100 transactions:

- 40 people bought milk
- 50 people bought bread
- 20 people bought both milk and bread

Metrics:

- Support (milk ∩ bread) = 20 / 100 = 0.20
- Support (milk) = 40 / 100 = 0.40
- Support (bread) = 50 / 100 = 0.50
- Confidence (bread | milk):= 0.20 / 0.40 = 0.50
- Lift:= 0.50 / 0.50 = 1.0

Another Example: Analyzing grocery store data.

Items: {Milk, Bread}

From 100 transactions:

- 40 people bought milk
- 40 people bought bread
- 16 people bought both milk and bread

Metrics:

- Support (milk ∩ bread) = 16 / 100 = 0.16
- Support (milk) = 40 / 100 = 0.40
- Support (bread) = 40 / 100 = 0.40
- Confidence (bread | milk):= 0.16 / 0.40 = 0.40
- Lift:= 0.40 / 0.40 = 1.0

Interpretation:

- A lift of 1 means people who buy milk are neither more nor less likely to buy bread than the general population.
- This suggests no real association between milk and bread in this dataset.





Apriori Algorithm Overview

Core Principle: Apriori Property

"All subsets of a frequent itemset must be frequent"

Example

If {Bread, Milk, Butter} is frequent, then:

- {Bread, Milk} must be frequent
- {Bread, Butter} must be frequent
- {Milk, Butter} must be frequent
- {Bread}, {Milk}, {Butter} must be frequent

Pruning Power

If {Bread, Milk} is NOT frequent, then:

- {Bread, Milk, Butter} cannot be frequent
- {Bread, Milk, Cheese} cannot be frequent
- Any superset containing {Bread, Milk} can be eliminated





Apriori Algorithm Steps

Algorithm Workflow

- **1.** Find L₁: Count all items, keep frequent ones
- **2. Generate candidates:** Combine frequent (k-1)-itemsets
- **3. Prune candidates:** Remove those with infrequent subsets
- 4. Count support: Scan database for remaining candidates
- **5. Filter:** Keep only frequent k-itemsets
- **6. Repeat:** Until no new frequent itemsets found
- **7. Generate rules:** From all frequent itemsets

Parameters for Our Example

- Minimum Support: 30% (3 out of 10 transactions)
- Minimum Confidence: 60%
- Minimum Lift: 1.2





Iteration 1 - Find Frequent Items

Count Individual Items

Item	Count	Support	Status
Bread	6	60%	\checkmark
Milk	5	50%	\checkmark
Butter	5	50%	\checkmark
Cheese	6	60%	\checkmark
Eggs	4	40%	\checkmark
Yogurt	3	30%	\checkmark

Result: L₁ (Frequent 1-itemsets)

All items meet 30% minimum support threshold

 $L_1 = \{\{Bread\}, \{Milk\}, \{Butter\}, \{Cheese\}, \{Eggs\}, \{Yogurt\}\}\}$





Iteration 2 - Find Frequent Pairs

Generate All Possible Pairs

6 items \rightarrow 15 possible pairs

Count Support for Each Pair

Itemset	Transactions	Count	Support	Status
{Bread, Milk}	T1,T4,T5,T9	4	40%	✓
{Bread, Butter}	T2,T4,T7,T9	4	40%	✓
{Bread, Cheese}	T2,T5,T9	3	30%	✓
{Bread, Eggs}	T1,T5,T7	3	30%	✓
{Bread, Yogurt}	None	0	0%	Х
{Milk, Butter}	T3,T4,T9	3	30%	✓
{Milk, Cheese}	T3,T5,T8,T9	4	40%	✓
{Milk, Eggs}	T1,T5	2	20%	Х
{Milk, Yogurt}	Т3,Т8	2	20%	Х
{Butter, Cheese}	T2,T3,T6,T9	4	40%	✓
{Butter, Eggs}	T7	1	10%	Х
{Butter, Yogurt}	Т3,Т6	2	20%	Х
{Cheese, Eggs}	T5,T10	2	20%	Х
{Cheese, Yogurt}	T3,T6,T8,T10	4	40%	✓
{Eggs, Yogurt}	T10	1	10%	Х





Iteration 2 Results

L₂ (Frequent 2-itemsets)

• {Bread, Milk} - 40% support {Bread, Butter} - 40% support {Bread, Cheese} - 30% support {Bread, Eggs} - 30% support {Milk, Butter} - 30% support {Milk, Cheese} - 40% support {Butter, Cheese} - 40% support {Cheese, Yogurt} - 40% support

Key Observations

- 8 frequent pairs out of 15 possible
- {Bread, Yogurt} has 0% support strong negative correlation
- {Butter, Cheese} has highest support (40%) good combination





Iteration 3 - Generate 3-itemset Candidates

Step 1: Candidate Generation (Join Step)

Combine frequent 2-itemsets that share exactly one item:

- {Bread, Milk} + {Bread, Butter} → candidate {Bread, Milk, Butter}
- {Bread, Milk} + {Bread, Cheese} → candidate {Bread, Milk, Cheese}
- {Bread, Butter} + {Bread, Cheese} → candidate {Bread, Butter, Cheese}
- {Milk, Butter} + {Milk, Cheese} → candidate {Milk, Butter, Cheese}

Step 2: Prune Candidates (Based on Apriori Property)

• For each candidate, check if ALL its 2-subsets are frequent:

{Bread, Milk, Butter}:

- {Bread, Milk} √ (in L₂), {Bread, Butter} √ (in L₂), {Milk, Butter} √ (in L₂)
- Keep candidate

{Bread, Milk, Cheese}:

- {Bread, Milk} √ (in L₂), {Bread, Cheese} √ (in L₂), {Milk, Cheese} √ (in L₂)
- Keep candidate

Step 3: Ready for Database Scan

4 candidates survive pruning - now we check which ones actually exist in transactions!





Iteration 3 - Count Actual Support in Database

- Step 4: Database Scan (The Crucial Step!)
- Now we check: Do these 3 items actually appear together in real transactions?

Candidate	Which Transactions?	Count	Support	Status
{Bread, Milk, Butter}	T4, T9	2	20%	X
{Bread, Milk, Cheese}	T5, T9	2	20%	X
{Bread, Butter, Cheese}	T2, T9	2	20%	X
{Milk, Butter, Cheese}	T3, T9	2	20%	Х

Let's Verify One Example:

{Bread, Milk, Butter} appears in:

- **T4:** {Bread, Milk, Butter} ✓
- **T9:** {Bread, Milk, Butter, Cheese} √
- Count: 2 transactions out of 10 = 20% support

Result: $L_3 = \{\}$ (Empty Set)

No 3-itemsets meet our 30% minimum support threshold

Algorithm Terminates

Cannot generate any 4-itemsets since L₃ is empty





Generate Association Rules

From Each Frequent Itemset, Generate All Possible Rules

Rules from {Butter, Cheese} (40% support):

Rule	Confidence	Lift	Evaluation
$\{Butter\} \rightarrow \{Cheese\}$	80%	1.33	EXCELLENT√
{Cheese} → {Butter}	67%	1.33	GOOD √
4			▶

Rules from {Bread, Milk} (40% support):

Rule	Confidence	Lift	Evaluation
$\{Bread\} \rightarrow \{Milk\}$	67%	1.33	GOOD √
${Milk} \rightarrow {Bread}$	80%	1.33	EXCELLENT √
4			▶

Rules from {Cheese, Yogurt} (40% support):

Rule	Confidence	Lift	Evaluation
$\{Cheese\} \rightarrow \{Yogurt\}$	67%	2.22	EXCELLENT√
{Yogurt} → {Cheese}	100%	1.67	PERFECT √
4	•		▶





Final Rule Ranking

Applying Our Thresholds (Confidence ≥ 60%, Lift ≥ 1.2)

Rank	Rule	Support	Confidence	Lift	Business Action
1	{Yogurt} → {Cheese}	40%	100%	1.67	Always suggest cheese with yogurt
2	$\{Milk\} \mathop{\rightarrow} \{Bread\}$	40%	80%	1.33	Promote bread to milk buyers
3	{Butter} → {Cheese}	40%	80%	1.33	Bundle butter with cheese
4	{Cheese} → {Yogurt}	40%	67%	2.22	Strong yogurt promotion to cheese buyers
5	{Cheese} → {Butter}	40%	67%	1.33	Cross-sell butter with cheese
6	$\{Bread\} \rightarrow \{Milk\}$	40%	67%	1.33	Promote milk to bread buyers

Top Business Insights

- 1. Perfect reliability: Every yogurt buyer also buys cheese
- 2. Strong asymmetry: Milk buyers more likely to buy bread than vice versa
- 3. Dairy synergy: Butter and cheese are strongly correlated





Why Each Metric Was Essential

Support Eliminated Rare Patterns

- **{Eggs, Yogurt}:** Only 10% support → Too rare to be reliable
- {Bread, Yogurt}: 0% support → Strong negative correlation discovered

Confidence Revealed Direction

- {Butter} → {Cheese}: 80% confidence
- {Cheese} → {Butter}: 67% confidence
- Action: Prioritize cheese recommendations to butter buyers

Lift Identified True Correlations

- {Cheese} → {Yogurt}: 67% confidence seems modest
- But lift = 2.22: Cheese buyers are 122% more likely to buy yogurt!
- Insight: This is actually a very strong relationship

Combined Power

 Without all three metrics, we would miss the strongest business opportunities and waste resources on weak relationships





Computational Complexity and Limitations

Computational Challenges

· Database scans: Multiple passes through data

• Candidate generation: Can grow exponentially

· Memory usage: Storing all frequent itemsets

Performance Factors

Factor	Impact on Performance	
Low support threshold	Exponential candidate growth	
Dense datasets	More frequent itemsets	
Many items	Larger search space	
Large transactions	Expensive database scans	
4		

When Apriori Struggles

- Very low support thresholds (< 1%)
- High-dimensional data (> 1000 items)
- · Dense datasets (most items frequent)





Alternative Algorithms

FP-Growth

- Advantage: Only 2 database scans
- Method: Compact FP-tree data structure
- **Best for:** Dense datasets, low support thresholds

ECLAT

- Advantage: Vertical data representation
- **Method:** Set intersection operations
- Best for: Sparse datasets, many unique items

Modern Approaches

- Parallel processing: Distributed Apriori implementations
- Approximate algorithms: Trade accuracy for speed
- Streaming algorithms: Handle continuous data flows





Practical Business Applications

Supermarket Chain Strategy

Discovered Rules:

- {Yogurt} → {Cheese} (100% confidence, 1.67 lift)
- {Butter} → {Cheese} (80% confidence, 1.33 lift)

Business Actions:

- Place cheese display near yogurt section
- Create butter-cheese bundle promotions
- Stock cheese heavily when yogurt sales increase

E-commerce Recommendations

Discovered Rules:

• {Milk} → {Bread} (80% confidence, 1.33 lift)

Implementation:

- "Customers who bought milk also bought bread"
- Automatic cross-sell suggestions
- Email marketing campaigns

Inventory Management

Negative Correlations:

{Bread, Yogurt} (0% support)

Insight: Customers segment into different dietary preferences Action: Separate promotional campaigns for different customer segments





Summary & Key Takeaways

What We Learned

- Support filters rare, unreliable patterns
- Confidence reveals directional strength and asymmetry
- Lift distinguishes real correlation from coincidence
- All three metrics together prevent false insights

Algorithm Insights

- Apriori property enables efficient search space pruning
- Multiple database scans are the main performance bottleneck
- Parameter tuning dramatically affects both performance and results

Business Value

- Actionable insights for cross-selling and recommendations
- Customer behavior understanding beyond simple popularity
- Data-driven decisions for product placement and marketing

Critical Success Factors

- Domain expertise to interpret and validate rules
- Proper preprocessing to clean and prepare data
- Continuous monitoring to ensure rules remain relevant





Next Steps & Advanced Topics

Immediate Applications

- Implement basic Apriori on your own datasets
- **Experiment with thresholds** to understand trade-offs
- Validate discovered rules with domain experts

Advanced Techniques

- **Sequential pattern mining:** Time-ordered associations
- Multi-level associations: Category hierarchies
- Constraint-based mining: User-specified business rules

Modern Developments

- Real-time association mining for streaming data
- Privacy-preserving techniques for sensitive data
- Deep learning approaches for complex pattern discovery

Tools to Explore

- **Python:** mlxtend, apyori libraries
- R: arules, arulesViz packages
- Spark: MLlib for large-scale mining