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# Part 1: Introduction to Time Series Analysis

## The Nature of Time Series Data

- A time series is a sequence of observations indexed by time, often collected at regular intervals (e.g., hourly temperature, daily sales, intraday financial prices).
- Unlike cross-sectional data, observations are not independent; temporal ordering is fundamental.
- Dependence across time points is common and must be explicitly modeled.
- Key characteristics include trend, seasonality, cycles, and autocorrelation.
- These characteristics are essential for inference, modeling, and forecasting.

Time series analysis is used across a wide range of domains:

- Meteorology: forecasting weather and climate variables
- Business and operations: demand forecasting and inventory planning
- Finance: modeling asset prices, returns, and volatility
- Health and engineering: monitoring signals and system behavior over time
- The primary objective is to extract information from historical data to understand underlying processes and, when appropriate, generate forecasts under well-defined assumptions.

YouTube

## Time Series in Everyday Life

- Time series data surrounds us in everyday life.
- When you check the weather forecast, that's time series analysis at work.
- When businesses plan inventory for the holiday season, they're using time series methods.
- When your fitness tracker predicts your sleep patterns, it's analyzing time series data.
- The fundamental insight is simple yet powerful: the past contains patterns that help us understand the future.

## Components of a Time Series

Time series data are commonly modeled as the combination of several distinct components. Identifying and understanding these components is a prerequisite for appropriate model selection, interpretation, and forecasting.

### 1. Trend

The trend component represents the long-term progression of the series, reflecting persistent increases, decreases, or stability over time. It captures structural changes such as long-run economic growth, population change, or sustained improvements in business performance.

*Example:* A company's revenue exhibiting steady growth over multiple years.

### 2. Seasonality

Seasonality refers to systematic, calendar-related patterns that repeat at fixed and known intervals (e.g., daily, monthly, quarterly, or annually). These effects are driven by recurring factors such as weather, holidays, or institutional schedules.

*Examples:*

- Retail sales increasing every December
- Electricity demand rising during summer months

### 3. Cyclical Variations

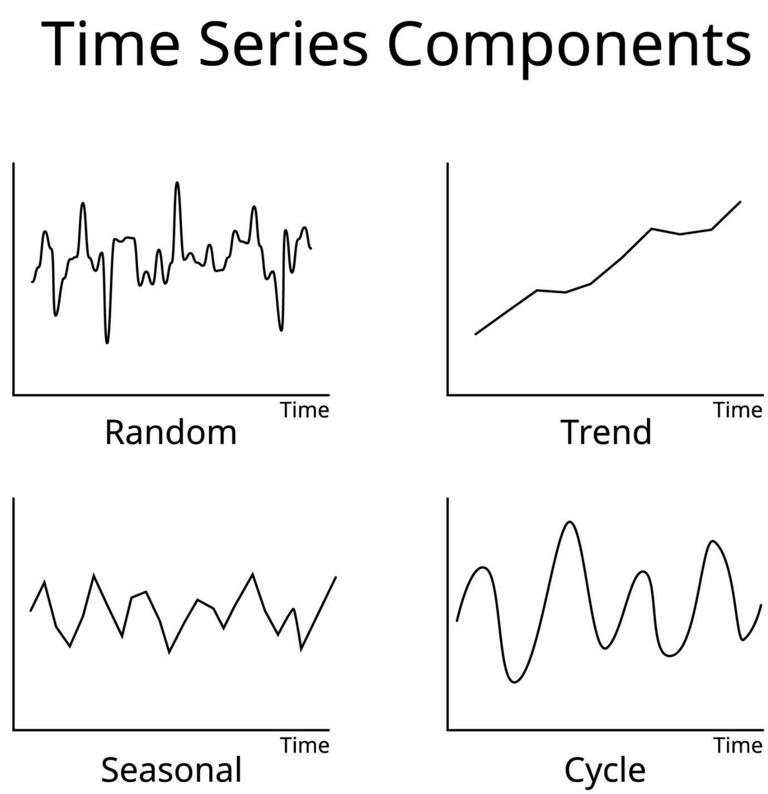
Cyclical variations describe medium- to long-term fluctuations associated with broader economic or system-wide dynamics. Unlike seasonality, cycles do not have a fixed period or regular frequency and are often influenced by external macroeconomic conditions.

*Example:* Business cycles characterized by expansions and recessions occurring over several years at irregular intervals.

### 4.Residual (Irregular) Component

The residual or noise component consists of random, unpredictable fluctuations that remain after removing trend, seasonal, and cyclical effects. These variations may arise from measurement error, unobserved factors, or one-time shocks.

This decomposition framework provides a structured basis for analyzing time series behavior and informs the choice of forecasting and statistical modeling techniques.




**Why Decomposition Matters for Forecasting**

- Understanding these components is not just academic; it directly informs how we approach forecasting.
- If we can identify and model each component separately, we can build more accurate and interpretable predictions.
- This decomposition approach is at the heart of many modern forecasting methods.

## Getting Started with Time Series in Pandas

Before conducting pattern analysis or developing forecasting models, time series data must be correctly represented and processed. Proper data handling ensures the validity of subsequent statistical analysis and modeling results.

The Python library **pandas** provides specialized data structures and functions for working with temporal data efficiently. Proficiency with these tools is a foundational requirement for effective time series analysis.

A central concept in pandas-based time series analysis is the **datetime index**.

- Unlike a default integer-based index, a datetime index explicitly represents time as the indexing dimension.
- This allows pandas to recognize the temporal ordering of observations and enables time-aware operations such as resampling, time-based slicing, shifting, and rolling-window calculations.

To illustrate these concepts, we begin by constructing a simple time series and examining how datetime indexing affects data manipulation and analysis in practice.

In [14]:
```python
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from datetime import datetime

# Set random seed for reproducibility
np.random.seed(42)

# Create a date range - this will be our time index
# We're creating daily data for one year
dates = pd.date_range(start='2023-01-01', end='2023-12-31', freq='D')

# Generate synthetic time series data with trend and seasonality
# This simulates realistic data: a random walk with an annual seasonal pattern
n_points = len(dates)
trend = np.linspace(100, 130, n_points)  # Linear upward trend
seasonal = 10 * np.sin(2 * np.pi * np.arange(n_points) / 365)  # Yearly seasonality
noise = np.random.randn(n_points) * 3  # Random fluctuations
values = trend + seasonal + noise

# Create DataFrame with datetime index
ts_data = pd.DataFrame({'value': values}, index=dates)

print("First few rows of our time series:")
print(ts_data.head())
print(f"\nDataFrame shape: {ts_data.shape}")
print(f"Date range: from {ts_data.index.min()} to {ts_data.index.max()}")
```

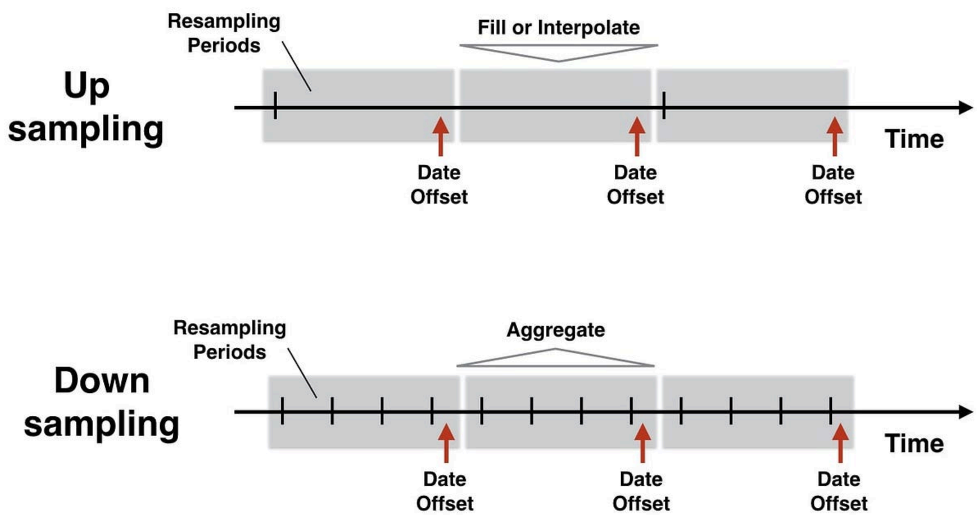
```
First few rows of our time series:
                 value
2023-01-01  101.490142
2023-01-02   99.839758
2023-01-03  102.452117
2023-01-04  105.332539
2023-01-05  100.315234

DataFrame shape: (365, 1)
Date range: from 2023-01-01 00:00:00 to 2023-12-31 00:00:00
```

# Resampling

Resampling enables the transformation of a time series from one temporal frequency to another by aggregating or interpolating observations over defined time intervals.

For example, high-frequency data recorded at hourly intervals can be resampled to a daily, weekly, or monthly frequency in order to examine broader temporal patterns. Pandas provides built-in resampling methods that make these frequency conversions systematic, reproducible, and computationally efficient.

Resampling is commonly used to examine patterns at broader time scales, reduce noise, or align data with other series recorded at different frequencies. In pandas, resampling provides a structured and efficient approach to frequency conversion for temporal data.




In [15]:

```
# Resampling: Converting daily data to different frequencies
# This is incredibly useful for getting different perspectives on your data

# Monthly averages - smooths out daily noise
monthly_avg = ts_data.resample('M').mean()
print("Monthly averages:")
print(monthly_avg.head())

# Weekly sums - useful if you're looking at cumulative effects
weekly_sum = ts_data.resample('W').sum()

# Quarterly statistics - common in business reporting
quarterly_stats = ts_data.resample('Q').agg(['mean', 'std', 'min', 'max'])
print("\nQuarterly statistics:")
print(quarterly_stats)
```

```
Monthly averages:
                 value
2023-01-31  103.155174
2023-02-28  110.104059
2023-03-31  115.680122
2023-04-30  118.185121
2023-05-31  118.084337

Quarterly statistics:
                 value
                  mean       std         min         max
2023-03-31  109.631198  6.072669   97.550803  121.065418
2023-06-30  117.881043  2.881317  112.793236  126.010152
2023-09-30  112.481356  3.434507  102.073468  124.378281
2023-12-31  120.039389  5.648432  107.819683  133.662940
```

```
C:\Users\me\AppData\Local\Temp\ipykernel_9924\3897107547.py:5: FutureWarning: 'M' is
deprecated and will be removed in a future version, please use 'ME' instead.
  monthly_avg = ts_data.resample('M').mean()
C:\Users\me\AppData\Local\Temp\ipykernel_9924\3897107547.py:13: FutureWarning: 'Q' i
s deprecated and will be removed in a future version, please use 'QE' instead.
  quarterly_stats = ts_data.resample('Q').agg(['mean', 'std', 'min', 'max'])
```

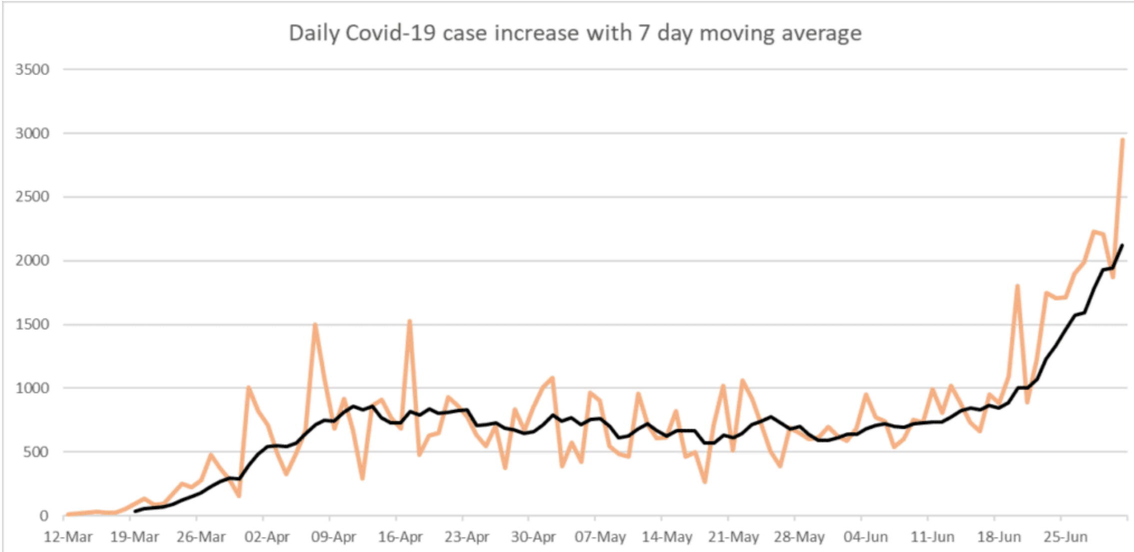
## Rolling Window

A **rolling window** is a technique in time series analysis that computes statistics over a fixed number of consecutive observations as the window advances through time.

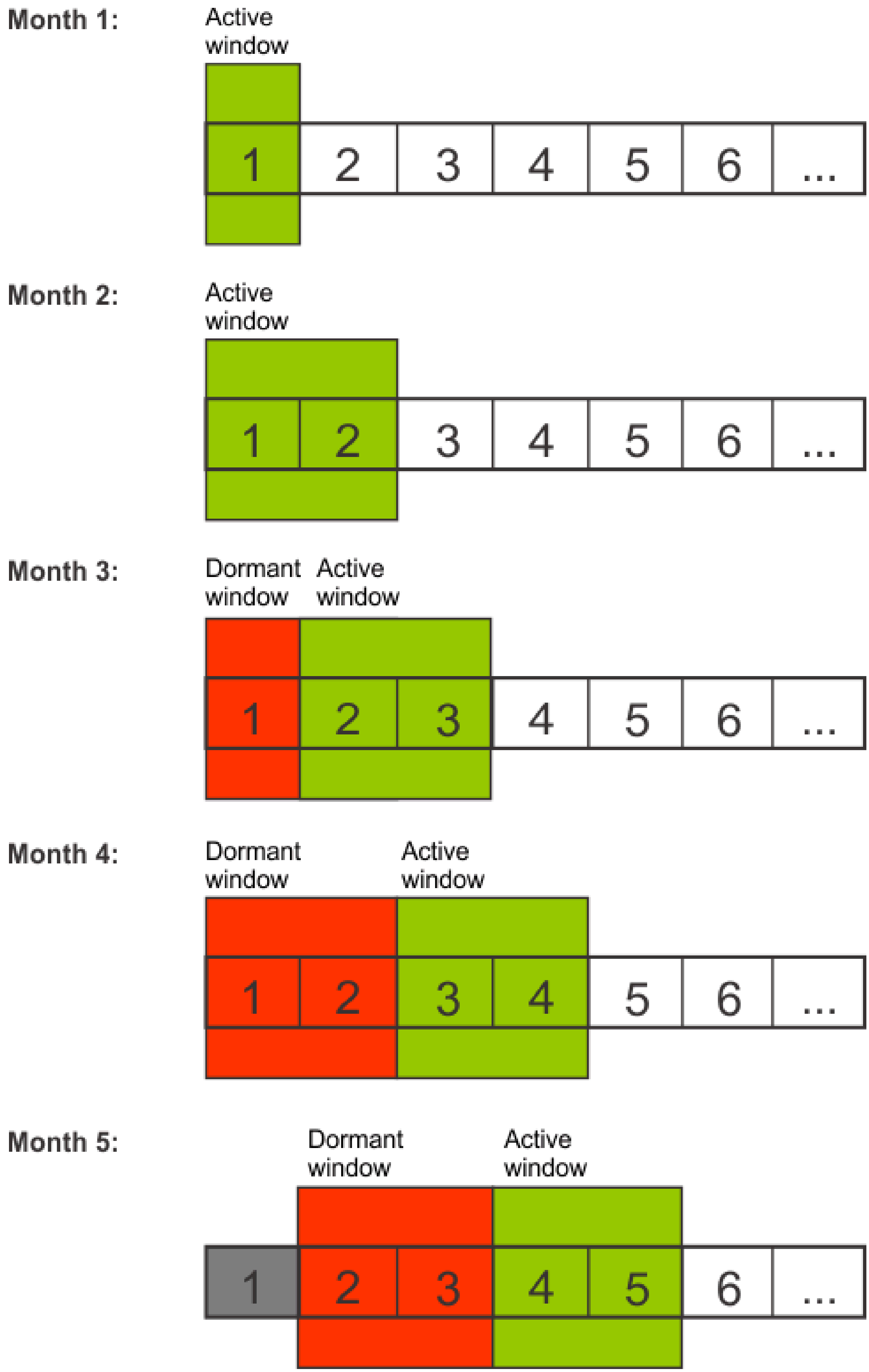
One of the most common rolling window statistics is the **moving average**. A moving average is calculated by taking the mean of the observations within each window position. As the window moves forward, the oldest observation is dropped and the newest observation is included, producing a sequence of averaged values over time.

Moving averages are used to smooth short-term fluctuations, reduce noise, and highlight longer-term trends in the data. They are widely applied in exploratory analysis, trend detection, and financial and economic time series modeling.

**Moving Average**




## Rolling Window

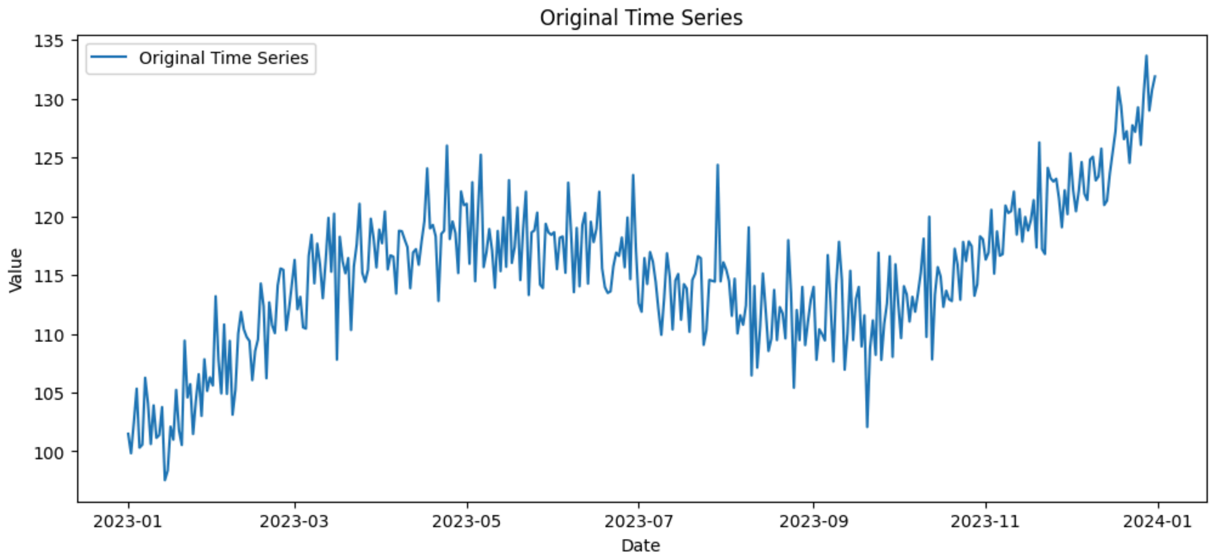

Month 1:
Active window
1
2
3
4
5
6
...
Month 2:
Active window
1
2
3
4
5
6
...
Month 3:
Dormant window
Active window
1
2
3
4
5
6
...
Month 4:
Dormant window
Active window
1
2
3
4
5
6
...
Month 5:
Dormant window
Active window
1
2
3
4
5
6
...

In [16]:
```
# Rolling window calculations - these help smooth data and identify trends
# The window parameter defines how many observations to include

# 7-day moving average - smooths weekly fluctuations
ts_data['rolling_mean_7'] = ts_data['value'].rolling(window=7).mean()

# 7-day rolling standard deviation - measures volatility
ts_data['rolling_std_7'] = ts_data['value'].rolling(window=7).std()

# 30-day moving average - shows longer-term trends
ts_data['rolling_mean_30'] = ts_data['value'].rolling(window=30).mean()

print("Time series with rolling statistics:")
print(ts_data.head(35))  # Note: first 6 rows of 7-day rolling will be NaN
```

```
Time series with rolling statistics:
                 value  rolling_mean_7  rolling_std_7  rolling_mean_30
2023-01-01  101.490142             NaN            NaN              NaN
2023-01-02   99.839758             NaN            NaN              NaN
2023-01-03  102.452117             NaN            NaN              NaN
2023-01-04  105.332539             NaN            NaN              NaN
2023-01-05  100.315234             NaN            NaN              NaN
2023-01-06  100.569325             NaN            NaN              NaN
2023-01-07  106.263161      102.323182       2.535268              NaN
2023-01-08  104.081308      102.693349       2.582098              NaN
2023-01-09  100.623705      102.805341       2.451395              NaN
2023-01-10  103.912527      103.013971       2.478321              NaN
2023-01-11  101.146854      102.416016       2.325939              NaN
2023-01-12  101.391671      102.569793       2.195846              NaN
2023-01-13  103.765943      103.026453       2.037137              NaN
2023-01-14   97.550803      101.781830       2.365109              NaN
2023-01-15   98.365820      100.965332       2.424777              NaN
2023-01-16  102.102934      101.176650       2.454320              NaN
2023-01-17  100.999770      100.760542       2.139952              NaN
2023-01-18  105.228665      101.343658       2.735906              NaN
2023-01-19  101.808657      101.403227       2.741659              NaN
2023-01-20  100.541720      100.942624       2.542139              NaN
2023-01-21  109.420527      102.638299       3.629004              NaN
2023-01-22  104.590202      103.527496       3.136856              NaN
2023-01-23  105.713017      104.043222       3.160287              NaN
2023-01-24  101.477994      104.111540       3.087869              NaN
2023-01-25  104.359754      103.987410       3.052742              NaN
2023-01-26  106.565143      104.666908       3.016099              NaN
2023-01-27  103.017632      105.020610       2.562821              NaN
2023-01-28  107.834662      104.794058       2.145097              NaN
2023-01-29  105.141279      104.872783       2.146480              NaN
2023-01-30  106.302367      104.956976       2.195919       103.073508
2023-01-31  105.605163      105.546572       1.571403       103.210675
2023-02-01  113.198489      106.809248       3.183255       103.655966
2023-02-02  107.831027      106.990089       3.202973       103.835263
2023-02-03  104.926699      107.262812       2.872592       103.821735
2023-02-04  110.794186      107.685601       3.172881       104.171033
```

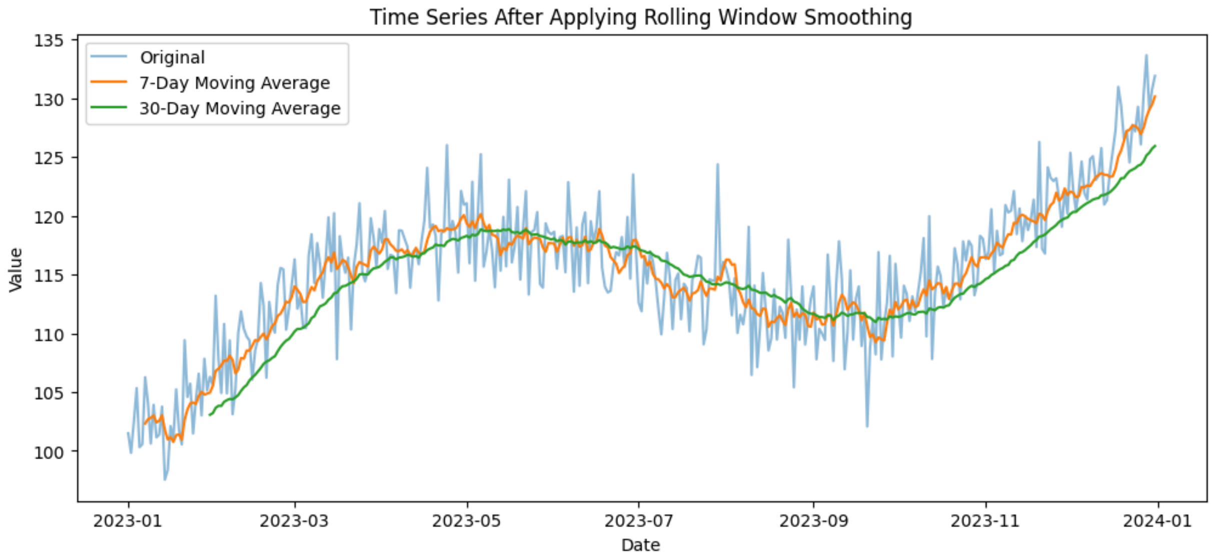
# Plot original time series

In [17]:
```
# ------------------------------
# 1. Plot original time series
# ------------------------------
plt.figure(figsize=(12, 5))
plt.plot(ts_data['value'], label='Original Time Series')
plt.title("Original Time Series")
plt.xlabel("Date")
plt.ylabel("Value")
plt.legend()
plt.show()
```




# Plot smoothed series

In [18]:
```
# ------------------------------
# 3. Plot smoothed series
# ------------------------------
plt.figure(figsize=(12, 5))
plt.plot(ts_data['value'], label='Original', alpha=0.5)
plt.plot(ts_data['rolling_mean_7'], label='7-Day Moving Average')
plt.plot(ts_data['rolling_mean_30'], label='30-Day Moving Average')
plt.title("Time Series After Applying Rolling Window Smoothing")
plt.xlabel("Date")
plt.ylabel("Value")
plt.legend()
plt.show()
```



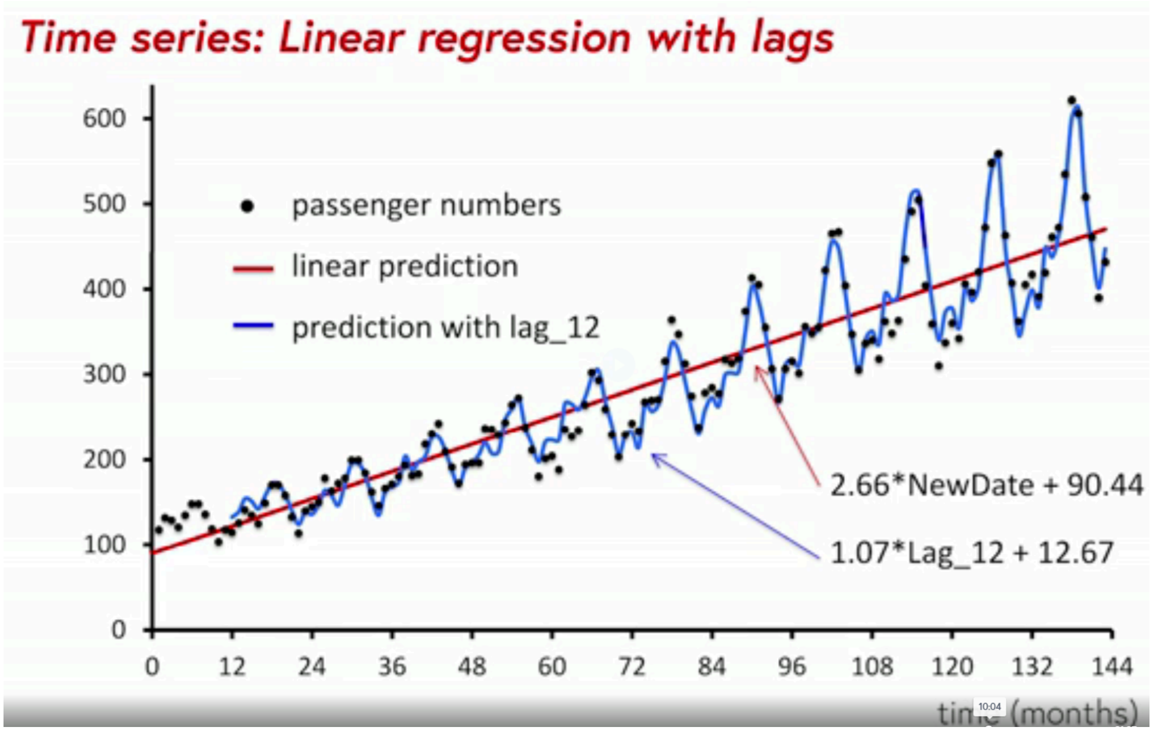

## Lagged Values

The concept of **lagged values** is fundamental in time series analysis. A lagged value represents a past observation of the same series at a specified time offset.

For example, when forecasting a variable observed daily, the value from the previous day corresponds to a lag of one period (lag-1), while the value from seven days earlier corresponds to a lag of seven periods (lag-7).

Lagged variables are central to many time series models because they capture **autocorrelation**, defined as the statistical dependence between current values and their historical observations.

By incorporating lagged values, forecasting models can exploit temporal structure and persistence present in the data.




In [19]:
```
# Creating lagged features - crucial for many forecasting models
# These show you what the value was N periods ago

ts_data['lag_1'] = ts_data['value'].shift(1)  # Yesterday's value
ts_data['lag_7'] = ts_data['value'].shift(7)  # Value from a week ago
ts_data['lag_30'] = ts_data['value'].shift(30)  # Value from a month ago

# Differences - showing change from previous period
# This is key for making non-stationary series stationary
ts_data['diff_1'] = ts_data['value'].diff(1)  # Day-to-day change
ts_data['diff_7'] = ts_data['value'].diff(7)  # Week-to-week change

print("Time series with lags and differences:")
print(ts_data.head(35))
```

```
Time series with lags and differences:
                 value  rolling_mean_7  rolling_std_7  rolling_mean_30  \
2023-01-01  101.490142             NaN            NaN              NaN
2023-01-02   99.839758             NaN            NaN              NaN
2023-01-03  102.452117             NaN            NaN              NaN
2023-01-04  105.332539             NaN            NaN              NaN
2023-01-05  100.315234             NaN            NaN              NaN
2023-01-06  100.569325             NaN            NaN              NaN
2023-01-07  106.263161      102.323182       2.535268              NaN
2023-01-08  104.081308      102.693349       2.582098              NaN
2023-01-09  100.623705      102.805341       2.451395              NaN
2023-01-10  103.912527      103.013971       2.478321              NaN
2023-01-11  101.146854      102.416016       2.325939              NaN
2023-01-12  101.391671      102.569793       2.195846              NaN
2023-01-13  103.765943      103.026453       2.037137              NaN
2023-01-14   97.550803      101.781830       2.365109              NaN
2023-01-15   98.365820      100.965332       2.424777              NaN
2023-01-16  102.102934      101.176650       2.454320              NaN
2023-01-17  100.999770      100.760542       2.139952              NaN
2023-01-18  105.228665      101.343658       2.735906              NaN
2023-01-19  101.808657      101.403227       2.741659              NaN
2023-01-20  100.541720      100.942624       2.542139              NaN
2023-01-21  109.420527      102.638299       3.629004              NaN
2023-01-22  104.590202      103.527496       3.136856              NaN
2023-01-23  105.713017      104.043222       3.160287              NaN
2023-01-24  101.477994      104.111540       3.087869              NaN
2023-01-25  104.359754      103.987410       3.052742              NaN
2023-01-26  106.565143      104.666908       3.016099              NaN
2023-01-27  103.017632      105.020610       2.562821              NaN
2023-01-28  107.834662      104.794058       2.145097              NaN
2023-01-29  105.141279      104.872783       2.146480              NaN
2023-01-30  106.302367      104.956976       2.195919       103.073508
2023-01-31  105.605163      105.546572       1.571403       103.210675
2023-02-01  113.198489      106.809248       3.183255       103.655966
2023-02-02  107.831027      106.990089       3.202973       103.835263
2023-02-03  104.926699      107.262812       2.872592       103.821735
2023-02-04  110.794186      107.685601       3.172881       104.171033

                 lag_1       lag_7      lag_30    diff_1     diff_7
2023-01-01         NaN         NaN         NaN       NaN        NaN
2023-01-02  101.490142         NaN         NaN -1.650384        NaN
2023-01-03   99.839758         NaN         NaN  2.612359        NaN
2023-01-04  102.452117         NaN         NaN  2.880422        NaN
2023-01-05  105.332539         NaN         NaN -5.017305        NaN
2023-01-06  100.315234         NaN         NaN  0.254091        NaN
2023-01-07  100.569325         NaN         NaN  5.693836        NaN
2023-01-08  106.263161  101.490142         NaN -2.181853   2.591165
2023-01-09  104.081308   99.839758         NaN -3.457602   0.783947
2023-01-10  100.623705  102.452117         NaN  3.288821   1.460410
2023-01-11  103.912527  105.332539         NaN -2.765672  -4.185685
2023-01-12  101.146854  100.315234         NaN  0.244817   1.076437
2023-01-13  101.391671  100.569325         NaN  2.374272   3.196618
2023-01-14  103.765943  106.263161         NaN -6.215140  -8.712358
2023-01-15   97.550803  104.081308         NaN  0.815017  -5.715487
2023-01-16   98.365820  100.623705         NaN  3.737114   1.479229
2023-01-17  102.102934  103.912527         NaN -1.103165  -2.912757
```

```
2023-01-18  100.999770  101.146854         NaN  4.228896   4.081811
2023-01-19  105.228665  101.391671         NaN -3.420009   0.416985
2023-01-20  101.808657  103.765943         NaN -1.266937  -3.224223
2023-01-21  100.541720   97.550803         NaN  8.878807  11.869724
2023-01-22  109.420527   98.365820         NaN -4.830325   6.224381
2023-01-23  104.590202  102.102934         NaN  1.122815   3.610083
2023-01-24  105.713017  100.999770         NaN -4.235023   0.478224
2023-01-25  101.477994  105.228665         NaN  2.881760  -0.868912
2023-01-26  104.359754  101.808657         NaN  2.205390   4.756487
2023-01-27  106.565143  100.541720         NaN -3.547511   2.475913
2023-01-28  103.017632  109.420527         NaN  4.817030  -1.585865
2023-01-29  107.834662  104.590202         NaN -2.693383   0.551077
2023-01-30  105.141279  105.713017         NaN  1.161088   0.589350
2023-01-31  106.302367  101.477994  101.490142 -0.697204   4.127169
2023-02-01  105.605163  104.359754   99.839758  7.593326   8.838735
2023-02-02  113.198489  106.565143  102.452117 -5.367462   1.265884
2023-02-03  107.831027  103.017632  105.332539 -2.904328   1.909067
2023-02-04  104.926699  107.834662  100.315234  5.867487   2.959523
```

## Extract Time-Based Features

- Pandas also makes it easy to extract time-based features, which are often valuable predictors in forecasting models.
- The month of the year, day of the week, or quarter can all be significant.
- For instance, retail sales patterns differ dramatically between weekdays and weekends, or between December and other months.

In [20]:
```python
# Extracting time components - these become features for modeling
ts_data['year'] = ts_data.index.year
ts_data['month'] = ts_data.index.month
ts_data['day_of_week'] = ts_data.index.dayofweek  # Monday=0, Sunday=6
ts_data['day_of_year'] = ts_data.index.dayofyear
ts_data['quarter'] = ts_data.index.quarter

# Cyclical encoding - this is important for circular time features
# Why? Because December (month 12) is actually close to January (month 1)
ts_data['month_sin'] = np.sin(2 * np.pi * ts_data['month'] / 12)
ts_data['month_cos'] = np.cos(2 * np.pi * ts_data['month'] / 12)

print("Time series with extracted features:")
print(ts_data[['value', 'month', 'day_of_week', 'quarter']].head(10))
```

```
Time series with extracted features:
                 value  month  day_of_week  quarter
2023-01-01  101.490142      1            6        1
2023-01-02   99.839758      1            0        1
2023-01-03  102.452117      1            1        1
2023-01-04  105.332539      1            2        1
2023-01-05  100.315234      1            3        1
2023-01-06  100.569325      1            4        1
2023-01-07  106.263161      1            5        1
2023-01-08  104.081308      1            6        1
2023-01-09  100.623705      1            0        1
2023-01-10  103.912527      1            1        1
```

## Visualize Time Series

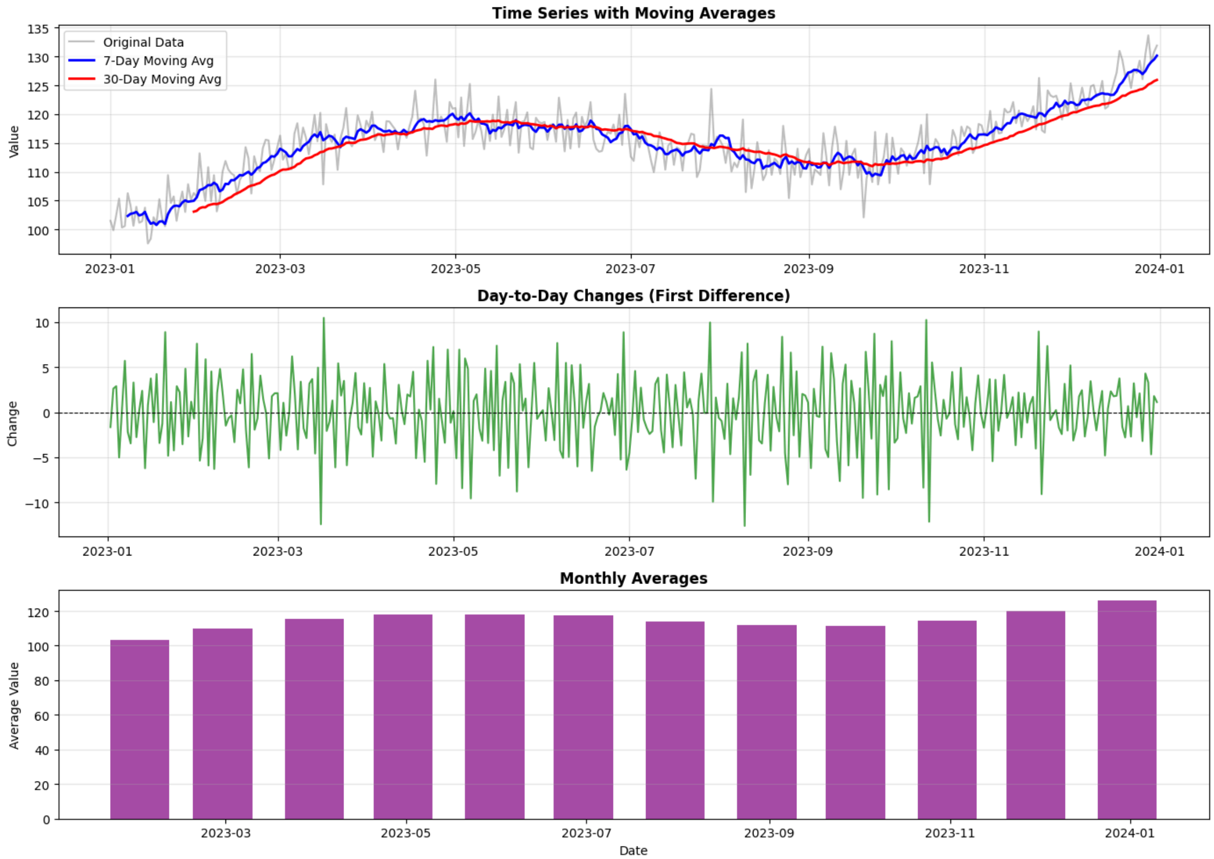
Let's visualize our time series to see these concepts in action.

Visualization is not just about making pretty pictures - it's about understanding your data's behavior, spotting anomalies, and getting intuition about what modeling approaches might work.

In [21]:

```python
# Create a comprehensive visualization
fig, axes = plt.subplots(3, 1, figsize=(14, 10))

# Original series with rolling mean
axes[0].plot(ts_data.index, ts_data['value'], label='Original Data', alpha=0.5, col
axes[0].plot(ts_data.index, ts_data['rolling_mean_7'], label='7-Day Moving Avg',
             linewidth=2, color='blue')
axes[0].plot(ts_data.index, ts_data['rolling_mean_30'], label='30-Day Moving Avg',
             linewidth=2, color='red')
axes[0].set_title('Time Series with Moving Averages', fontsize=12, fontweight='bold
axes[0].set_ylabel('Value')
axes[0].legend()
axes[0].grid(True, alpha=0.3)

# Day-to-day differences
axes[1].plot(ts_data.index, ts_data['diff_1'], color='green', alpha=0.7)
axes[1].axhline(y=0, color='black', linestyle='--', linewidth=0.8)
axes[1].set_title('Day-to-Day Changes (First Difference)', fontsize=12, fontweight=
axes[1].set_ylabel('Change')
axes[1].grid(True, alpha=0.3)

# Monthly averages
monthly_plot = ts_data.resample('M').mean()
axes[2].bar(monthly_plot.index, monthly_plot['value'], width=20, alpha=0.7, color='
axes[2].set_title('Monthly Averages', fontsize=12, fontweight='bold')
axes[2].set_ylabel('Average Value')
axes[2].set_xlabel('Date')
axes[2].grid(True, alpha=0.3, axis='y')

plt.tight_layout()
plt.show()
```

```
C:\Users\me\AppData\Local\Temp\ipykernel_9924\636133491.py:23: FutureWarning: 'M' is
deprecated and will be removed in a future version, please use 'ME' instead.
  monthly_plot = ts_data.resample('M').mean()
```




---

## Part 2: Statistical Methods for Time Series Forecasting

Now that we understand how to work with time series data, we need to address a fundamental question: how do we actually forecast future values?

The statistical approach to time series has been refined over decades and provides both powerful methods and important theoretical insights.

These classical methods remain highly relevant today, often outperforming more complex machine learning approaches, especially when you have limited data or need interpretable models.

## The Concept of Stationarity

- Before building statistical models, we must understand **stationarity**, one of the most important concepts in time series analysis.

- A stationary time series has statistical properties—especially **mean** and **variance**—that remain constant over time.
- Any segment of a stationary series should look statistically similar to any other segment, even if the actual values differ.
- Stationarity matters because many classical time series models assume it.
  - If the series has a changing trend, shifting mean, or increasing volatility, these models break down.
  - It's like trying to hit a moving target that is also changing size; the mathematics becomes unstable.
- To check stationarity, we use the **Augmented Dickey-Fuller (ADF) test**.
  - Null hypothesis: the series is **non-stationary**.
  - A small p-value (typically below 0.05) means we can reject the null and conclude the series is stationary.

In [22]:
```
from statsmodels.tsa.stattools import adfuller

def test_stationarity(series, name="Series"):
    """
    Perform comprehensive stationarity test
    This helps us understand if we need to transform our data before modeling
    """
    # Drop any NaN values
    series = series.dropna()

    print(f"\n{'='*60}")
    print(f"Stationarity Test for: {name}")
    print(f"{'='*60}")

    # Perform Augmented Dickey-Fuller test
    result = adfuller(series)

    print(f'ADF Statistic: {result[0]:.6f}')
    print(f'p-value: {result[1]:.6f}')
    print(f'Number of lags used: {result[2]}')
    print(f'Number of observations: {result[3]}')
    print('\nCritical Values:')
    for key, value in result[4].items():
        print(f'  {key}: {value:.3f}')

    # Interpretation
    if result[1] <= 0.05:
        print(f"\n✓ Result: The series IS stationary (p-value = {result[1]:.6f})")
        print("  We can proceed with modeling without transformation.")
    else:
        print(f"\n✗ Result: The series is NOT stationary (p-value = {result[1]:.6f
        print("  We need to difference the series or remove trend/seasonality.")
    print(f"{'='*60}\n")
```

```python
# Test original series
test_stationarity(ts_data['value'], "Original Series")

# Test differenced series
test_stationarity(ts_data['diff_1'], "First Differenced Series")
```

```
============================================================
Stationarity Test for: Original Series
============================================================
ADF Statistic: -0.459863
p-value: 0.899614
Number of lags used: 8
Number of observations: 356

Critical Values:
  1%: -3.449
  5%: -2.870
  10%: -2.571

✗ Result: The series is NOT stationary (p-value = 0.899614)
  We need to difference the series or remove trend/seasonality.
============================================================


============================================================
Stationarity Test for: First Differenced Series
============================================================
ADF Statistic: -10.860314
p-value: 0.000000
Number of lags used: 7
Number of observations: 356

Critical Values:
  1%: -3.449
  5%: -2.870
  10%: -2.571

✓ Result: The series IS stationary (p-value = 0.000000)
  We can proceed with modeling without transformation.
============================================================
```

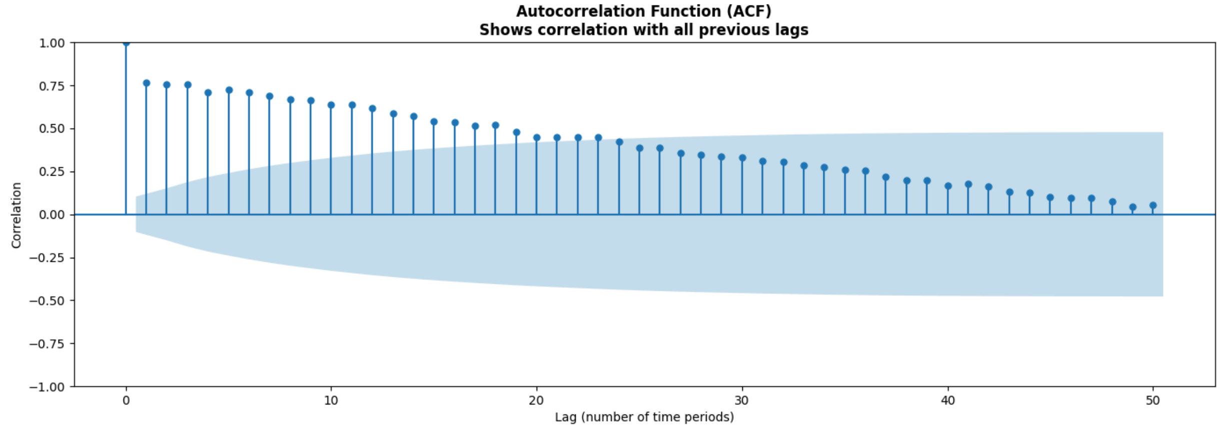
If our series is not stationary, we have several options. The most common approach is **differencing** - subtracting each value from the previous value. This removes trends and often makes the series stationary. For seasonal data, we might need seasonal differencing, where we subtract the value from the same season in the previous cycle.

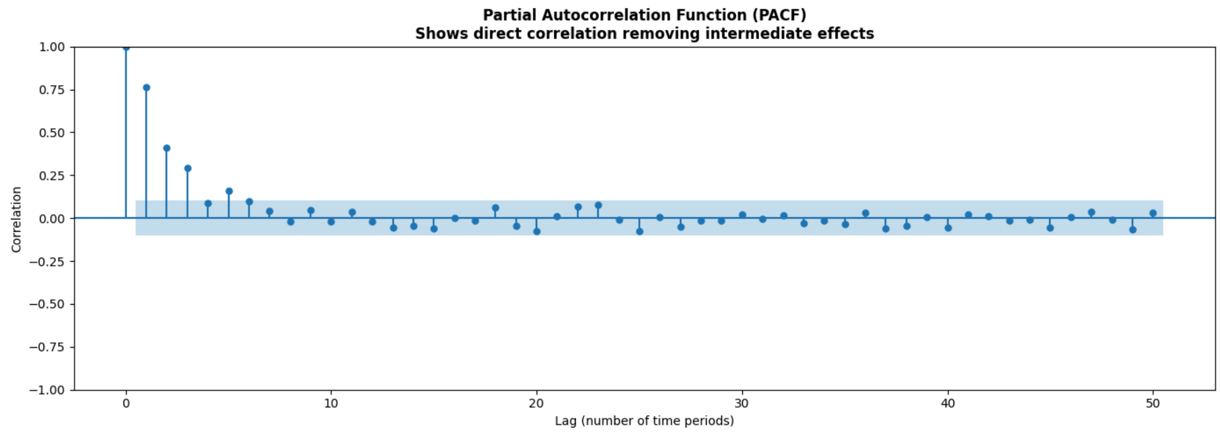
## Understanding Autocorrelation and Partial Autocorrelation

- After achieving stationarity, the next step is understanding the **autocorrelation structure** of the series.

- **Autocorrelation** measures how strongly the current value is related to its past values.
  - It helps answer questions like:
    - If you know today's temperature, how well can you predict tomorrow's?
    - How about a week from now?
  - The **Autocorrelation Function (ACF)** shows these relationships across different lags.
- The **Partial Autocorrelation Function (PACF)** provides a deeper view.
  - It measures the correlation between the current value and a specific lag **after removing the effects of all shorter lags**.
  - This allows us to isolate the direct relationship at each lag.
- Understanding the difference between ACF and PACF is essential for selecting appropriate time series models.

In [23]:
```
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf

# Prepare the data - use stationary series
series_for_analysis = ts_data['value'].dropna()

# Create ACF and PACF plots
fig, axes = plt.subplots(2, 1, figsize=(14, 10))

# Autocorrelation Function (ACF)
# This shows how current values relate to all previous values
plot_acf(series_for_analysis, lags=50, ax=axes[0], alpha=0.05)
axes[0].set_title('Autocorrelation Function (ACF)\nShows correlation with all previ
                  fontsize=12, fontweight='bold')
axes[0].set_xlabel('Lag (number of time periods)')
axes[0].set_ylabel('Correlation')

# Partial Autocorrelation Function (PACF)
# This shows direct relationship, removing indirect effects
plot_pacf(series_for_analysis, lags=50, ax=axes[1], alpha=0.05)
axes[1].set_title('Partial Autocorrelation Function (PACF)\nShows direct correlatio
                  fontsize=12, fontweight='bold')
axes[1].set_xlabel('Lag (number of time periods)')
axes[1].set_ylabel('Correlation')

plt.tight_layout()
plt.show()

print("\nInterpretation Guide:")
print("- Spikes outside the blue shaded area are statistically significant")
print("- ACF shows total correlation (direct + indirect)")
print("- PACF shows only direct correlation")
print("- These plots help us choose the right model parameters")
```




```
Interpretation Guide:
- Spikes outside the blue shaded area are statistically significant
- ACF shows total correlation (direct + indirect)
- PACF shows only direct correlation
- These plots help us choose the right model parameters
```

## 1. What problem are ACF and PACF trying to solve?

Both plots answer a single question:

"How much does the current value of a time series depend on its past values?"

They help you decide **what type of time-series model is appropriate**, especially AR, MA, or ARIMA models.

## 2. Autocorrelation Function (ACF) — the *memory* of the series

### What ACF measures

- ACF measures **correlation between the series and its past values**
- Lag 1: correlation with the previous time step
- Lag 2: correlation with two steps back
- And so on…

Importantly:

- ACF includes **both direct and indirect effects**
- If lag 1 influences lag 2, ACF at lag 2 will reflect that indirectly

#### How to read it

- Each vertical bar = correlation at that lag
- The blue shaded area = statistical “noise” range
- Bars **outside** the blue area = statistically significant correlation

#### What your ACF shows

- Very high correlation at lag 1
- Correlation **decays slowly** across many lags
- Many lags remain significant

#### Interpretation

This tells us:

- The series has **strong persistence**

- Past values influence the future for a long time

- This is typical of:

    - Autoregressive (AR) behavior
    - Or a non-stationary / trending series

### 3. Partial Autocorrelation Function (PACF) — the *direct* effect

#### What PACF measures

- PACF isolates the **direct correlation** between the current value and a specific lag
- It removes the effect of intermediate lags

Think of it as: “If I already know lag 1 and lag 2, does lag 3 add any *new* information?”

#### How to read it

- Same axes as ACF
- Only spikes outside the blue band matter

#### What your PACF shows

- A **large spike at lag 1**
- Smaller but still significant spikes at lag 2 (possibly lag 3)
- After that, values are mostly insignificant (near zero)

#### Interpretation

This tells us:

- Most of the predictive power comes from the **first one or two lags**
- Higher lags do not add much new information

### 4. Putting ACF and PACF together (the key insight)

This is the most important part.

| Observation | Meaning |
|---|---|
| ACF decays slowly | Strong persistence / memory |

| PACF cuts off after lag 1–2 | Few direct dependencies |c | Many ACF lags significant | Indirect effects dominate |

#### Classic interpretation

This pattern is **textbook autoregressive behavior**:

- Likely an **AR(1)** or **AR(2)** process
- Possibly ARIMA with:
  - p = 1 or 2
  - q ≈ 0
  - d = 0 or 1 (depending on stationarity)

### 5. Why these plots look complex but aren't

They look complex because:

- There are many lags
- Statistical confidence bands are shown
- Two similar-looking plots are stacked

But conceptually:

- **ACF = how long memory lasts**
- **PACF = how many past steps really matter**

That is all.

### 6. One-sentence summary of our figures

Our time series shows strong persistence, where the current value depends mainly on the last one or two observations, with longer-term correlations arising indirectly — a strong indicator of an autoregressive process.

## Translating our ACF & PACF into ARIMA parameters

An ARIMA model is written as:

ARIMA(p, d, q)

Where:

- **p** = number of autoregressive terms (from PACF)
- **d** = number of differences (from trend / persistence)
- **q** = number of moving-average terms (from ACF)

Let us map our plots **explicitly**.

**Step 1: Choosing `p` (AR order) — from PACF**

Our PACF shows:

- Strong, significant spike at **lag 1**
- Possibly a smaller but still meaningful spike at **lag 2**
- After that, values are near zero (insignificant)

Interpretation:

- Direct dependence exists mainly up to lag 1 or 2

**Candidate choices:**

- `p = 1` (most conservative, often sufficient)
- `p = 2` (if lag-2 spike is clearly significant)

In practice:

- Start with **AR(1)**, test AR(2) if residuals suggest it

**Step 2: Choosing `q` (MA order) — from ACF**

Our ACF shows:

- Very slow decay
- No sharp “cutoff” after a few lags

Interpretation:

- This is **not** MA behavior
- MA processes show ACF cutting off quickly

**Choice:**

- `q = 0`

**Step 3: Choosing `d` (differencing) — from persistence**

Our ACF:

- Starts very high
- Decays slowly over many lags

This often indicates:

- Trend
- Non-stationarity
- Accumulated effects over time

**Initial assumption:**

- `d = 1` (first difference)

**Final recommended starting models**

In order of preference:

- **ARIMA(1, 1, 0)** ← primary candidate
- **ARIMA(2, 1, 0)** ← if lag-2 is meaningful
- **ARIMA(1, 0, 0)** ← only if stationarity is confirmed

This is not guesswork; it is a direct reading of our plots.

## ARIMA Models: The Workhorses of Time Series Forecasting

- Now we can discuss **ARIMA**, one of the most widely used model families in time series analysis.

- ARIMA stands for **AutoRegressive Integrated Moving Average**, and each component describes a different part of how the model behaves.

- **AutoRegressive (AR)**
  - Uses the relationship between current values and previous observations.
  - An AR(1) model: today's value depends on yesterday's value plus noise.
  - An AR(p) model: uses the previous $p$ values.
- **Integrated (I)**
  - Refers to differencing applied to make the series stationary.
  - One difference → I(1); two differences → I(2).
  - Most real-world series require only one or two differences.
- **Moving Average (MA)**
  - Models dependency between the current value and past forecast errors.
  - An MA(1) model: today's value depends on yesterday's forecast error.
  - Useful for capturing short-term shocks.
- **Putting It Together: ARIMA(p, d, q)**
  - $p$ = number of AR lags

    - $d$ = number of differences
    - $q$ = number of MA terms
    - Model selection uses ACF/PACF patterns and comparison metrics such as AIC.
- Choosing the right p, d, and q values blends statistical guidance with analytical judgment.

In [24]:
```
from statsmodels.tsa.arima.model import ARIMA
from sklearn.metrics import mean_squared_error, mean_absolute_error
import warnings
warnings.filterwarnings('ignore')

# Prepare data: split into train and test
# We'll use 80% for training and reserve 20% for testing
train_size = int(len(ts_data) * 0.8)
train = ts_data['value'][:train_size]
test = ts_data['value'][train_size:]

print(f"Training set: {len(train)} observations")
print(f"Test set: {len(test)} observations")
print(f"Training period: {train.index.min()} to {train.index.max()}")
print(f"Test period: {test.index.min()} to {test.index.max()}")

# Fit ARIMA model
# We use (1,1,1) as a starting point - this often works reasonably well
# p=1: Use 1 lagged value
# d=1: Difference once to achieve stationarity
# q=1: Use 1 lagged forecast error

print("\nFitting ARIMA(1,1,1) model...")
arima_model = ARIMA(train, order=(1, 1, 1))
arima_fitted = arima_model.fit()

# The summary provides detailed statistics about the fit
print("\n" + "="*70)
print("ARIMA Model Summary")
print("="*70)
print(arima_fitted.summary())

# Make predictions on test set
print("\nGenerating forecasts...")
arima_forecast = arima_fitted.forecast(steps=len(test))

# Calculate performance metrics
rmse = np.sqrt(mean_squared_error(test, arima_forecast))
mae = mean_absolute_error(test, arima_forecast)
mape = np.mean(np.abs((test - arima_forecast) / test)) * 100

print("\n" + "="*70)
print("Model Performance on Test Set")
print("="*70)
print(f"RMSE (Root Mean Square Error): {rmse:.4f}")
print(f"  → Average prediction error magnitude")
print(f"MAE (Mean Absolute Error): {mae:.4f}")
print(f"  → Average absolute prediction error")
```

```python
print(f"MAPE (Mean Absolute Percentage Error): {mape:.2f}%")
print(f"  → Average percentage error")
print("="*70)
```

```
Training set: 292 observations
Test set: 73 observations
Training period: 2023-01-01 00:00:00 to 2023-10-19 00:00:00
Test period: 2023-10-20 00:00:00 to 2023-12-31 00:00:00

Fitting ARIMA(1,1,1) model...

======================================================================
ARIMA Model Summary
======================================================================
                               SARIMAX Results
==============================================================================
Dep. Variable:                  value   No. Observations:                  292
Model:                 ARIMA(1, 1, 1)   Log Likelihood                -750.077
Date:                Sun, 07 Dec 2025   AIC                           1506.154
Time:                        21:42:20   BIC                           1517.174
Sample:                    01-01-2023   HQIC                          1510.569
                         - 10-19-2023
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.L1         -0.1190      0.078     -1.526      0.127      -0.272       0.034
ma.L1         -0.8148      0.044    -18.729      0.000      -0.900      -0.730
sigma2        10.1032      0.796     12.693      0.000       8.543      11.663
===================================================================================
Ljung-Box (L1) (Q):                   0.08   Jarque-Bera (JB):                 0.74
Prob(Q):                              0.78   Prob(JB):                         0.69
Heteroskedasticity (H):               1.26   Skew:                             0.07
Prob(H) (two-sided):                  0.26   Kurtosis:                         3.21
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-ste
p).

Generating forecasts...

======================================================================
Model Performance on Test Set
======================================================================
RMSE (Root Mean Square Error): 9.5744
  → Average prediction error magnitude
MAE (Mean Absolute Error): 8.3366
  → Average absolute prediction error
MAPE (Mean Absolute Percentage Error): 6.71%
  → Average percentage error
======================================================================
```

## SARIMA — Extending ARIMA for Seasonal Patterns

- Many real-world time series contain **seasonal patterns** that repeat at regular intervals.
  - Retail sales often spike in December
  - Electricity use increases in summer
  - Flu cases rise in winter
- Standard ARIMA models struggle with these repeating patterns because they occur at fixed intervals.
- **SARIMA (Seasonal ARIMA)** extends ARIMA by adding seasonal terms that capture these repeating effects.
- A SARIMA model is written as: **ARIMA(p, d, q)(P, D, Q)m**
  - Lowercase letters (p, d, q) = non-seasonal components
  - Uppercase letters (P, D, Q) = seasonal components
  - *m* = number of periods per season
    - 12 → monthly data with yearly seasonality
    - 7 → daily data with weekly seasonality
- Although the notation looks more complex, the idea is simple:
  - SARIMA adds seasonal versions of AR, differencing, and MA terms that work just like the non-seasonal ones, but operate at seasonal lags.
- This extension makes SARIMA one of the most powerful tools for modeling series with strong seasonal structure.

In [25]:
```
from statsmodels.tsa.statespace.sarimax import SARIMAX

# Fit SARIMA model
# (1,1,1): non-seasonal components
# (1,1,1,7): seasonal components with 7-day seasonality (weekly pattern)
print("\nFitting SARIMA(1,1,1)(1,1,1,7) model...")
print("This captures both short-term dependencies and weekly patterns")

sarima_model = SARIMAX(train,
                       order=(1, 1, 1),
                       seasonal_order=(1, 1, 1, 7))
sarima_fitted = sarima_model.fit(disp=False)

print("\n" + "="*70)
print("SARIMA Model Summary")
print("="*70)
print(sarima_fitted.summary())

# Generate forecasts
print("\nGenerating forecasts with seasonal adjustments...")
sarima_forecast = sarima_fitted.forecast(steps=len(test))
```

```
# Evaluate
sarima_rmse = np.sqrt(mean_squared_error(test, sarima_forecast))
sarima_mae = mean_absolute_error(test, sarima_forecast)
sarima_mape = np.mean(np.abs((test - sarima_forecast) / test)) * 100

print("\n" + "="*70)
print("SARIMA Performance on Test Set")
print("="*70)
print(f"RMSE: {sarima_rmse:.4f}")
print(f"MAE: {sarima_mae:.4f}")
print(f"MAPE: {sarima_mape:.2f}%")
print("="*70)
```

```
Fitting SARIMA(1,1,1)(1,1,1,7) model...
This captures both short-term dependencies and weekly patterns

======================================================================
SARIMA Model Summary
======================================================================
                                     SARIMAX Results                                      
==========================================================================================
=====
Dep. Variable:                                value   No. Observations:                  
292
Model:             SARIMAX(1, 1, 1)x(1, 1, 1, 7)   Log Likelihood                  -74
5.247
Date:                            Sun, 07 Dec 2025   AIC                             150
0.495
Time:                                    21:42:21   BIC                             151
8.740
Sample:                                01-01-2023   HQIC                            150
7.810
                                     - 10-19-2023                                         
Covariance Type:                              opg                                         
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
ar.L1         -0.0942      0.078     -1.213      0.225      -0.246       0.058
ma.L1         -0.8464      0.043    -19.880      0.000      -0.930      -0.763
ar.S.L7        0.0042      0.063      0.066      0.947      -0.120       0.128
ma.S.L7       -0.9145      0.042    -21.549      0.000      -0.998      -0.831
sigma2        10.5759      0.891     11.863      0.000       8.829      12.323
===================================================================================
Ljung-Box (L1) (Q):                   0.09   Jarque-Bera (JB):                 0.72
Prob(Q):                              0.77   Prob(JB):                         0.70
Heteroskedasticity (H):               1.48   Skew:                             0.03
Prob(H) (two-sided):                  0.06   Kurtosis:                         3.24
===================================================================================

Warnings:
[1] Covariance matrix calculated using the outer product of gradients (complex-ste
p).

Generating forecasts with seasonal adjustments...

======================================================================
SARIMA Performance on Test Set
======================================================================
RMSE: 9.2924
MAE: 8.0670
MAPE: 6.49%
======================================================================
```

## Exponential Smoothing — A Different Perspective

- Exponential smoothing offers an alternative to ARIMA by using a different modeling philosophy.
- Instead of focusing on autocorrelation structures, it applies **exponentially decreasing weights** to past observations.
  - Recent data receives more influence
  - Older data gradually fades in importance
- The **Holt-Winters method (Triple Exponential Smoothing)** extends this idea to handle:
  - Level
  - Trend
  - Seasonality
- This makes it suitable for many real-world time series with both trend and seasonal patterns.
- Exponential smoothing models are often:
  - Faster to train than ARIMA
  - More intuitive and easier to interpret
  - Widely used in business settings where clarity and explainability matter
- This approach provides a smooth, adaptive way to forecast while still capturing essential structure in the data.

In [26]:
```
from statsmodels.tsa.holtwinters import ExponentialSmoothing

# Fit Holt-Winters Exponential Smoothing
# This method smooths level, trend, and seasonal components separately
print("\nFitting Holt-Winters Exponential Smoothing model...")
print("This gives more weight to recent observations")

ets_model = ExponentialSmoothing(
    train,
    seasonal_periods=7,     # Weekly seasonality
    trend='add',            # Additive trend
    seasonal='add',         # Additive seasonality
    damped_trend=True       # Trend dampens over time (more realistic)
)
ets_fitted = ets_model.fit()

# Generate forecasts
print("\nGenerating exponentially smoothed forecasts...")
ets_forecast = ets_fitted.forecast(steps=len(test))

# Evaluate
ets_rmse = np.sqrt(mean_squared_error(test, ets_forecast))
ets_mae = mean_absolute_error(test, ets_forecast)
ets_mape = np.mean(np.abs((test - ets_forecast) / test)) * 100
```

```
print("\n" + "="*70)
print("Exponential Smoothing Performance")
print("="*70)
print(f"RMSE: {ets_rmse:.4f}")
print(f"MAE: {ets_mae:.4f}")
print(f"MAPE: {ets_mape:.2f}%")
print("="*70)
```

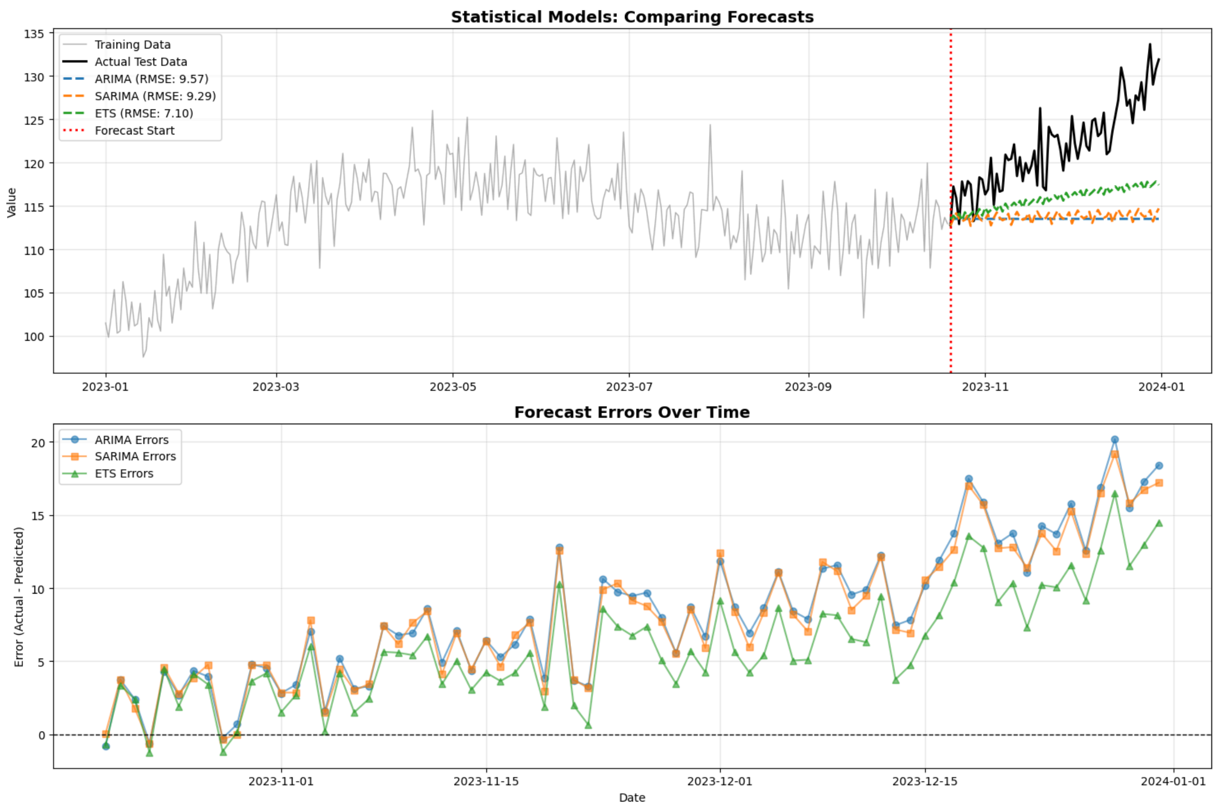
```
Fitting Holt-Winters Exponential Smoothing model...
This gives more weight to recent observations

Generating exponentially smoothed forecasts...

======================================================================
Exponential Smoothing Performance
======================================================================
RMSE: 7.0958
MAE: 6.0722
MAPE: 4.88%
======================================================================
```

## Visualize all our statistical models together to compare their performance

In [27]:

```
# Comprehensive comparison visualization
fig, axes = plt.subplots(2, 1, figsize=(15, 10))

# Plot 1: All forecasts together
axes[0].plot(train.index, train.values, label='Training Data',
             color='gray', alpha=0.6, linewidth=1)
axes[0].plot(test.index, test.values, label='Actual Test Data',
             color='black', linewidth=2)
axes[0].plot(test.index, arima_forecast, label=f'ARIMA (RMSE: {rmse:.2f})',
             linestyle='--', linewidth=2)
axes[0].plot(test.index, sarima_forecast, label=f'SARIMA (RMSE: {sarima_rmse:.2f})'
             linestyle='--', linewidth=2)
axes[0].plot(test.index, ets_forecast, label=f'ETS (RMSE: {ets_rmse:.2f})',
             linestyle='--', linewidth=2)
axes[0].axvline(x=test.index[0], color='red', linestyle=':', linewidth=2,
                label='Forecast Start')
axes[0].set_title('Statistical Models: Comparing Forecasts',
                  fontsize=14, fontweight='bold')
axes[0].set_ylabel('Value')
axes[0].legend(loc='best')
axes[0].grid(True, alpha=0.3)

# Plot 2: Forecast errors
arima_error = test.values - arima_forecast
sarima_error = test.values - sarima_forecast
ets_error = test.values - ets_forecast

axes[1].plot(test.index, arima_error, label='ARIMA Errors',
             marker='o', linestyle='-', alpha=0.6)
```

```python
axes[1].plot(test.index, sarima_error, label='SARIMA Errors',
             marker='s', linestyle='-', alpha=0.6)
axes[1].plot(test.index, ets_error, label='ETS Errors',
             marker='^', linestyle='-', alpha=0.6)
axes[1].axhline(y=0, color='black', linestyle='--', linewidth=1)
axes[1].set_title('Forecast Errors Over Time',
                  fontsize=14, fontweight='bold')
axes[1].set_xlabel('Date')
axes[1].set_ylabel('Error (Actual - Predicted)')
axes[1].legend()
axes[1].grid(True, alpha=0.3)

plt.tight_layout()
plt.show()

print("\nKey Insights:")
print("- Consistent errors suggest systematic bias in the model")
print("- Random scattered errors around zero suggest good model fit")
print("- Increasing error magnitude over time suggests model degradation")
```




```
Key Insights:
- Consistent errors suggest systematic bias in the model
- Random scattered errors around zero suggest good model fit
- Increasing error magnitude over time suggests model degradation
```

---

# Part 3: Machine Learning Approaches to Time Series

## From Statistical Models to Machine Learning

- Statistical models like ARIMA are powerful, well-understood, and often perform excellently.
- However, they come with important limitations:
  - Assume mostly linear relationships
  - Struggle with multiple seasonalities
  - Do not naturally handle many external variables (features)
- This is where **machine learning** becomes valuable:
  - Can capture complex non-linear patterns
  - Naturally supports multiple features (covariates)
  - Often adapts to different data types with less manual tuning
- We will explore two main ML-oriented approaches for time series:
  - **Facebook Prophet** — a model that bridges statistical and ML ideas
  - **Traditional supervised learning algorithms** adapted to forecasting problems

## Facebook Prophet — Automated Forecasting at Scale

- **Prophet** was developed by Facebook's Core Data Science team to solve a practical problem:
  - How to let analysts without deep time series expertise generate good forecasts quickly.
- Prophet takes a different philosophical approach than ARIMA:
  - Rather than focusing on autocorrelations, it models the series as a sum of components:
    - **Trend + Seasonality + Holidays + Error**
- Key strengths of Prophet:
  - Flexible and robust for real-world business data
  - Handles missing data gracefully
  - Works well in the presence of outliers
  - Makes it easy to incorporate domain knowledge via holidays and special events
- Because of its **additive component structure**:
  - Each component (trend, seasonality, holidays) can be inspected separately
  - The model is highly interpretable

- This interpretability is crucial in business settings where stakeholders need to understand *why* a forecast looks the way it does.

In [28]:
```
# Note: You may need to install prophet first
# pip install prophet

from prophet import Prophet

# Prepare data for Prophet
# Prophet requires columns named 'ds' (datestamp) and 'y' (value)
print("Preparing data for Prophet...")
prophet_train = pd.DataFrame({
    'ds': train.index,
    'y': train.values
})

prophet_test = pd.DataFrame({
    'ds': test.index,
    'y': test.values
})

print(f"Prophet training data: {len(prophet_train)} observations")
print("Sample of prepared data:")
print(prophet_train.head())

# Initialize Prophet model
# These parameters control how flexible the model is
print("\nInitializing Prophet model...")
prophet_model = Prophet(
    yearly_seasonality=True,      # Model yearly patterns
    weekly_seasonality=True,       # Model weekly patterns
    daily_seasonality=False,       # No daily patterns in our data
    changepoint_prior_scale=0.05,  # Controls trend flexibility (lower = more stabl
    seasonality_prior_scale=10.0,  # Controls seasonality flexibility
    interval_width=0.95            # 95% confidence intervals
)

# Fit the model
print("Fitting Prophet model (this may take a moment)...")
prophet_model.fit(prophet_train)

# Create future dataframe for predictions
# This includes both historical dates and future dates
future = prophet_model.make_future_dataframe(periods=len(test), freq='D')
print(f"\nCreated future dataframe with {len(future)} dates")

# Generate forecast
print("Generating forecast...")
prophet_forecast = prophet_model.predict(future)

# Extract predictions for test period
# We need to match the dates
prophet_test_predictions = prophet_forecast[prophet_forecast['ds'].isin(test.index)

# Evaluate
```

```
prophet_rmse = np.sqrt(mean_squared_error(test, prophet_test_predictions))
prophet_mae = mean_absolute_error(test, prophet_test_predictions)
prophet_mape = np.mean(np.abs((test - prophet_test_predictions) / test)) * 100

print("\n" + "="*70)
print("Prophet Performance")
print("="*70)
print(f"RMSE: {prophet_rmse:.4f}")
print(f"MAE: {prophet_mae:.4f}")
print(f"MAPE: {prophet_mape:.2f}%")
print("="*70)
```

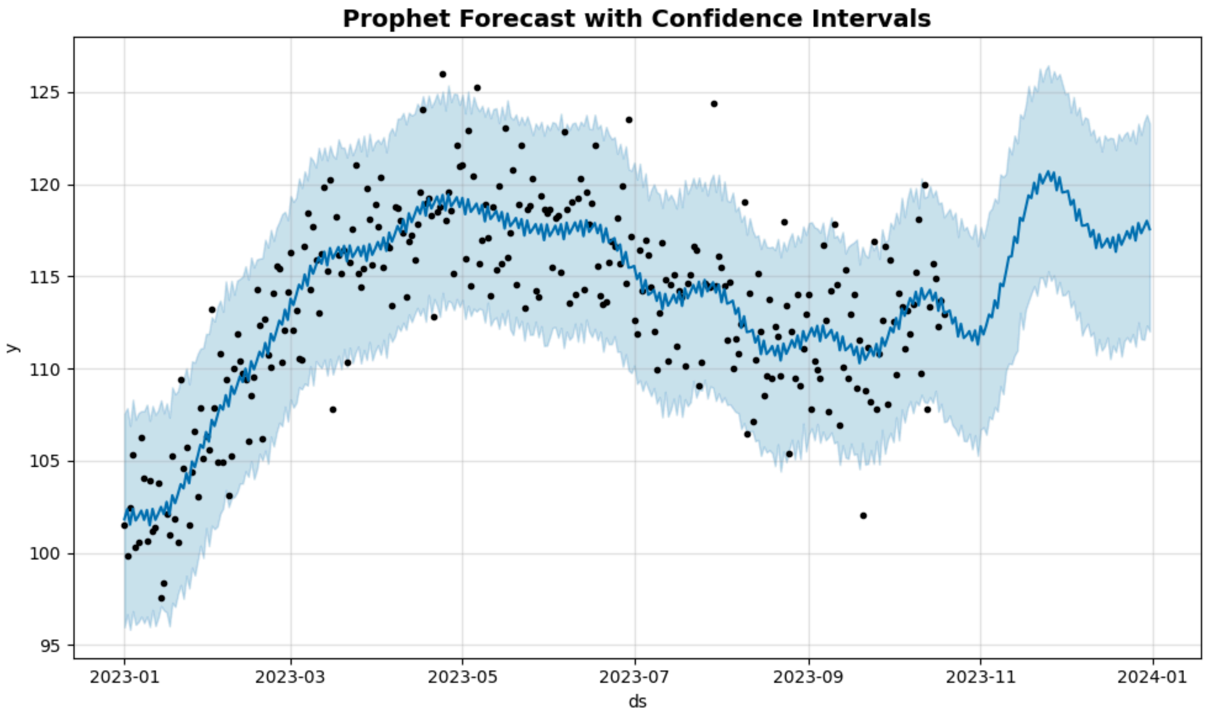
```
Importing plotly failed. Interactive plots will not work.
21:42:21 - cmdstanpy - INFO - Chain [1] start processing
Preparing data for Prophet...
Prophet training data: 292 observations
Sample of prepared data:
          ds           y
0 2023-01-01  101.490142
1 2023-01-02   99.839758
2 2023-01-03  102.452117
3 2023-01-04  105.332539
4 2023-01-05  100.315234

Initializing Prophet model...
Fitting Prophet model (this may take a moment)...
21:42:21 - cmdstanpy - INFO - Chain [1] done processing
Created future dataframe with 365 dates
Generating forecast...

======================================================================
Prophet Performance
======================================================================
RMSE: 6.7123
MAE: 5.5248
MAPE: 4.45%
======================================================================
```

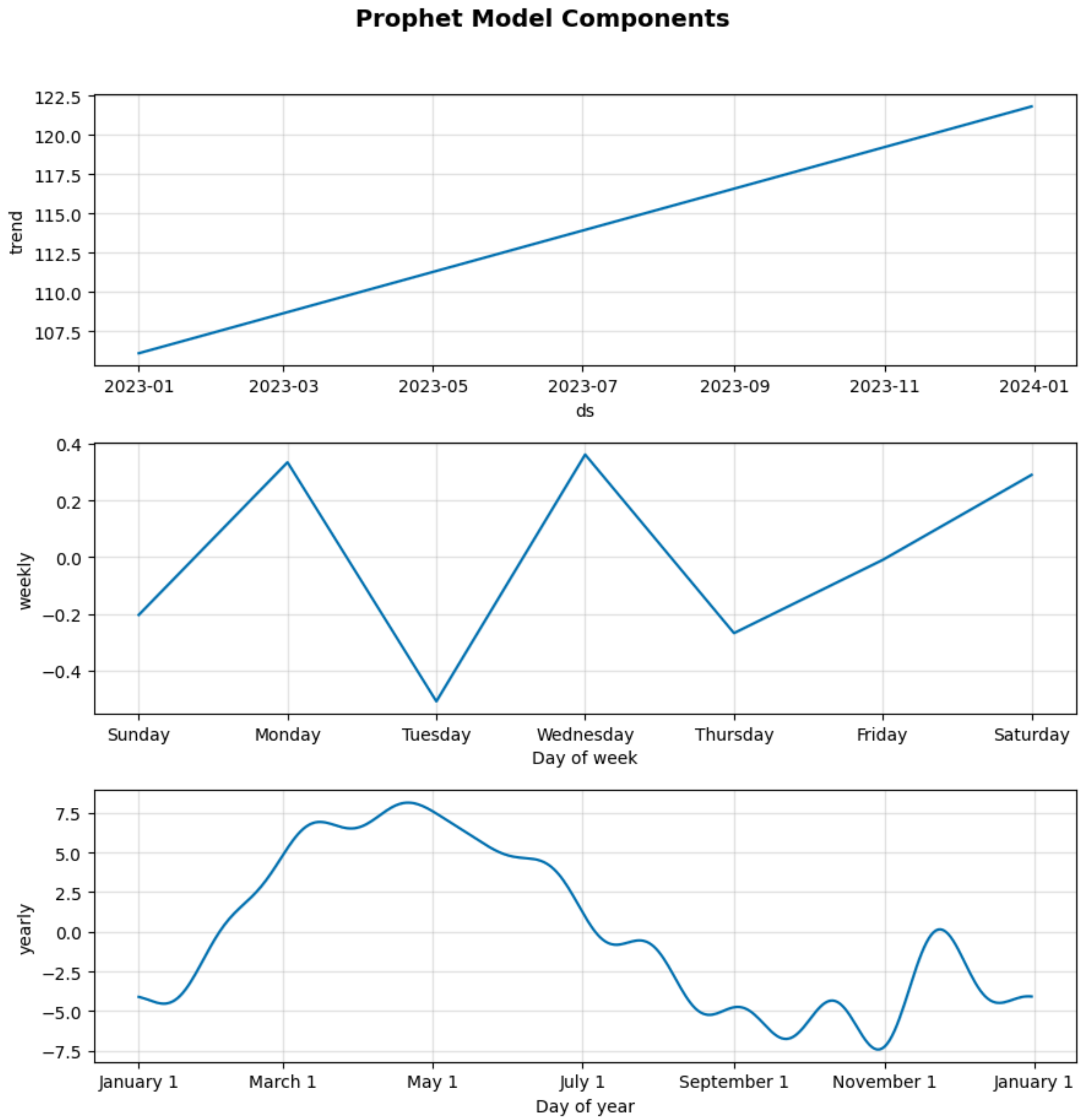
One of Prophet's most powerful features is its ability to visualize the decomposition of the forecast into its constituent parts. This helps us understand what's driving the predictions.

In [29]:

```
# Visualize the Prophet forecast
fig1 = prophet_model.plot(prophet_forecast)
plt.title('Prophet Forecast with Confidence Intervals',
          fontsize=14, fontweight='bold')
plt.tight_layout()
plt.show()

# Visualize the components
# This shows how trend, weekly, and yearly seasonality contribute
fig2 = prophet_model.plot_components(prophet_forecast)
plt.suptitle('Prophet Model Components', fontsize=14, fontweight='bold', y=1.02)
plt.tight_layout()
plt.show()
```

```
print("\nUnderstanding the Component Plot:")
print("- Trend: Shows the overall direction (up/down) over time")
print("- Weekly: Shows which days of the week are typically higher/lower")
print("- Yearly: Shows seasonal patterns throughout the year")
print("- The final forecast is the sum of all these components")
```






```
Understanding the Component Plot:
- Trend: Shows the overall direction (up/down) over time
- Weekly: Shows which days of the week are typically higher/lower
- Yearly: Shows seasonal patterns throughout the year
- The final forecast is the sum of all these components
```

Prophet also allows us to add domain-specific information. For instance, if we're forecasting retail sales, we can explicitly model the impact of holidays or promotional events.

In [30]:
```
# Advanced Prophet with custom seasonality and holidays
print("\n" + "="*70)
print("Building Advanced Prophet Model with Custom Features")
print("="*70)

# Create holiday dataframe
# In reality, you'd define holidays relevant to your domain
holidays = pd.DataFrame({
    'holiday': 'special_event',
```

```
    'ds': pd.to_datetime(['2023-07-04', '2023-12-25']),  # Example holidays
    'lower_window': -1,  # Start considering effect 1 day before
    'upper_window': 1,   # Effect lasts 1 day after
})

# Initialize advanced model
prophet_advanced = Prophet(
    yearly_seasonality=True,
    weekly_seasonality=True,
    daily_seasonality=False,
    holidays=holidays,
    changepoint_prior_scale=0.05
)

# Add custom monthly seasonality
# This can capture patterns that recur monthly
prophet_advanced.add_seasonality(
    name='monthly',
    period=30.5,
    fourier_order=5  # Higher order = more complex patterns
)

# Fit and predict
print("Fitting advanced model...")
prophet_advanced.fit(prophet_train)

future_advanced = prophet_advanced.make_future_dataframe(periods=len(test), freq='D
forecast_advanced = prophet_advanced.predict(future_advanced)

print("✓ Advanced Prophet model trained successfully")
print("  Includes: yearly seasonality, weekly seasonality, monthly patterns, and ho
```

```
21:42:22 - cmdstanpy - INFO - Chain [1] start processing
21:42:22 - cmdstanpy - INFO - Chain [1] done processing
======================================================================
Building Advanced Prophet Model with Custom Features
======================================================================
Fitting advanced model...
✓ Advanced Prophet model trained successfully
  Includes: yearly seasonality, weekly seasonality, monthly patterns, and holidays
```

# Traditional Machine Learning for Time Series

- Beyond specialized tools like Prophet, we can adapt traditional ML models such as:
  - Random Forest
  - Gradient Boosting
  - Neural Networks
- The key enabler is **feature engineering**:
  - Transform the time series problem into a standard supervised learning task
  - Build meaningful input features from historical data

- Core idea: turn forecasting into **regression** by constructing features from:
    - Lagged values (e.g., “yesterday’s value”, “value 7 days ago”)
    - Rolling statistics (e.g., “average of last week”, “rolling standard deviation”)
    - Time-based variables (e.g., “day of week”, “month”, “is holiday?”)
- To predict tomorrow’s value, we feed these engineered features into a regression model and let the algorithm learn the relationships.

In [31]:
```
def create_comprehensive_features(df, target_col='value'):
    """
    Transform time series into rich feature set for ML models
    This is where domain knowledge and creativity matter
    """
    df = df.copy()

    # Time-based features
    # These capture calendar effects
    df['dayofweek'] = df.index.dayofweek
    df['month'] = df.index.month
    df['quarter'] = df.index.quarter
    df['dayofyear'] = df.index.dayofyear
    df['weekofyear'] = df.index.isocalendar().week.astype(int)
    df['is_weekend'] = (df['dayofweek'] >= 5).astype(int)

    # Cyclical encoding for periodic features
    # Why? Because December (12) is actually close to January (1)
    df['month_sin'] = np.sin(2 * np.pi * df['month'] / 12)
    df['month_cos'] = np.cos(2 * np.pi * df['month'] / 12)
    df['dayofweek_sin'] = np.sin(2 * np.pi * df['dayofweek'] / 7)
    df['dayofweek_cos'] = np.cos(2 * np.pi * df['dayofweek'] / 7)

    # Lag features - past values as predictors
    for lag in [1, 2, 3, 7, 14, 21, 30]:
        df[f'lag_{lag}'] = df[target_col].shift(lag)

    # Rolling window features - capturing trends
    for window in [7, 14, 30]:
        df[f'rolling_mean_{window}'] = df[target_col].rolling(window=window).mean()
        df[f'rolling_std_{window}'] = df[target_col].rolling(window=window).std()
        df[f'rolling_min_{window}'] = df[target_col].rolling(window=window).min()
        df[f'rolling_max_{window}'] = df[target_col].rolling(window=window).max()

        # Rate of change
        df[f'rolling_mean_{window}_diff'] = df[f'rolling_mean_{window}'].diff()

    # Expanding features - cumulative statistics
    df['expanding_mean'] = df[target_col].expanding().mean()
    df['expanding_std'] = df[target_col].expanding().std()

    return df

# Apply feature engineering
print("Applying comprehensive feature engineering...")
ml_data = create_comprehensive_features(ts_data[['value']])
```

```
# Remove rows with NaN (from lag/rolling windows)
ml_data = ml_data.dropna()

print(f"\nFeature engineering complete!")
print(f"Original features: 1 (just 'value')")
print(f"Total features created: {len(ml_data.columns) - 1}")
print(f"Samples after removing NaN: {len(ml_data)}")
print("\nSample of engineered features:")
print(ml_data.head())
```

```
Applying comprehensive feature engineering...

Feature engineering complete!
Original features: 1 (just 'value')
Total features created: 34
Samples after removing NaN: 335

Sample of engineered features:
                 value  dayofweek  month  quarter  dayofyear  weekofyear  \
2023-01-31  105.605163          1      1        1         31           5   
2023-02-01  113.198489          2      2        1         32           5   
2023-02-02  107.831027          3      2        1         33           5   
2023-02-03  104.926699          4      2        1         34           5   
2023-02-04  110.794186          5      2        1         35           5   

            is_weekend  month_sin  month_cos  dayofweek_sin  ...  \
2023-01-31           0   0.500000   0.866025       0.781831  ...   
2023-02-01           0   0.866025   0.500000       0.974928  ...   
2023-02-02           0   0.866025   0.500000       0.433884  ...   
2023-02-03           0   0.866025   0.500000      -0.433884  ...   
2023-02-04           1   0.866025   0.500000      -0.974928  ...   

            rolling_min_14  rolling_max_14  rolling_mean_14_diff  \
2023-01-31      100.541720      109.420527              0.328957   
2023-02-01      100.541720      113.198489              0.569273   
2023-02-02      100.541720      113.198489              0.430169   
2023-02-03      101.477994      113.198489              0.313213   
2023-02-04      101.477994      113.198489              0.098118   

            rolling_mean_30  rolling_std_30  rolling_min_30  rolling_max_30  \
2023-01-31       103.210675        2.843392       97.550803      109.420527   
2023-02-01       103.655966        3.305723       97.550803      113.198489   
2023-02-02       103.835263        3.383141       97.550803      113.198489   
2023-02-03       103.821735        3.377755       97.550803      113.198489   
2023-02-04       104.171033        3.540538       97.550803      113.198489   

            rolling_mean_30_diff  expanding_mean  expanding_std  
2023-01-31              0.137167      103.155174       2.812627  
2023-02-01              0.445291      103.469028       3.287524  
2023-02-02              0.179297      103.601209       3.323650  
2023-02-03             -0.013528      103.640194       3.280789  
2023-02-04              0.349298      103.844594       3.450981  

[5 rows x 35 columns]
```

Now we can train traditional ML models on this engineered feature set:

In [32]:
```
from sklearn.ensemble import RandomForestRegressor, GradientBoostingRegressor
from sklearn.linear_model import Ridge

# Prepare features and target
feature_cols = [col for col in ml_data.columns if col != 'value']
X = ml_data[feature_cols]
y = ml_data['value']

# Create time-based split (must respect temporal order!)
# We can't randomly shuffle time series data
train_size = int(len(X) * 0.8)
X_train, X_test = X[:train_size], X[train_size:]
y_train, y_test = y[:train_size], y[train_size:]

print(f"ML Training samples: {len(X_train)}")
print(f"ML Test samples: {len(X_test)}")
print(f"Number of features: {X_train.shape[1]}")

# Train Random Forest
print("\nTraining Random Forest...")
rf_model = RandomForestRegressor(
    n_estimators=100,
    max_depth=15,
    min_samples_split=5,
    random_state=42,
    n_jobs=-1
)
rf_model.fit(X_train, y_train)
rf_predictions = rf_model.predict(X_test)

# Train Gradient Boosting
print("Training Gradient Boosting...")
gb_model = GradientBoostingRegressor(
    n_estimators=100,
    learning_rate=0.1,
    max_depth=5,
    random_state=42
)
gb_model.fit(X_train, y_train)
gb_predictions = gb_model.predict(X_test)

# Train Ridge Regression (Linear model with regularization)
print("Training Ridge Regression...")
ridge_model = Ridge(alpha=1.0)
ridge_model.fit(X_train, y_train)
ridge_predictions = ridge_model.predict(X_test)

# Evaluate all models
print("\n" + "="*70)
print("Machine Learning Models Performance")
print("="*70)

ml_models = {
```

```
    'Random Forest': rf_predictions,
    'Gradient Boosting': gb_predictions,
    'Ridge Regression': ridge_predictions
}

for name, predictions in ml_models.items():
    rmse = np.sqrt(mean_squared_error(y_test, predictions))
    mae = mean_absolute_error(y_test, predictions)
    mape = np.mean(np.abs((y_test - predictions) / y_test)) * 100
    print(f"\n{name}:")
    print(f"  RMSE: {rmse:.4f}")
    print(f"  MAE:  {mae:.4f}")
    print(f"  MAPE: {mape:.2f}%")

print("="*70)
```

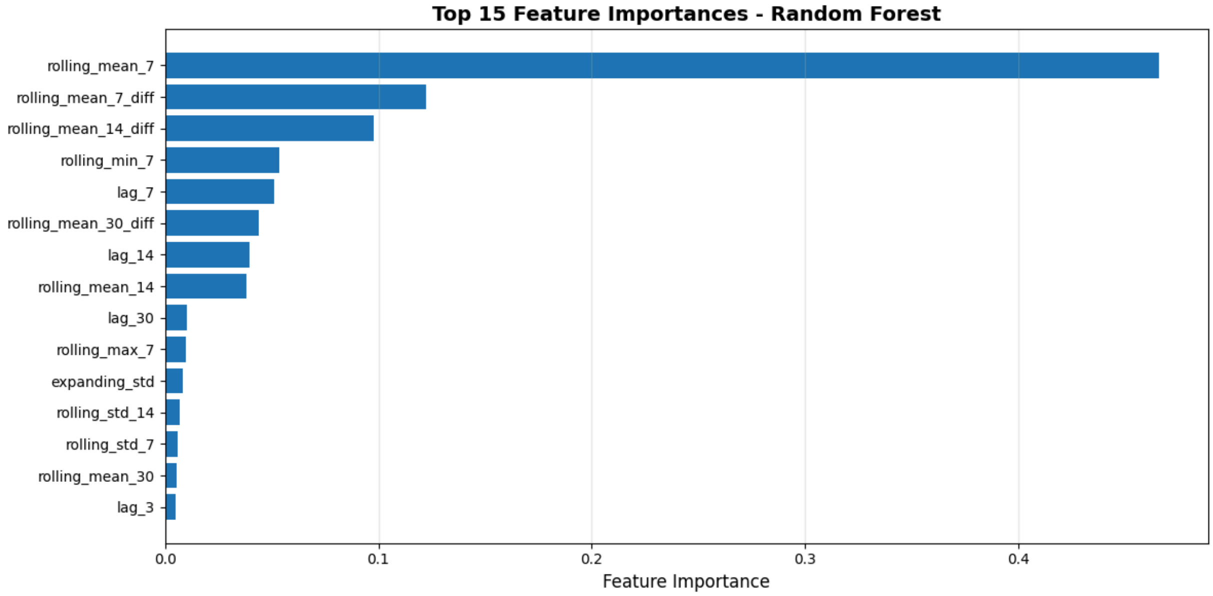
```
ML Training samples: 268
ML Test samples: 67
Number of features: 34

Training Random Forest...
Training Gradient Boosting...
Training Ridge Regression...

======================================================================
Machine Learning Models Performance
======================================================================

Random Forest:
  RMSE: 3.9757
  MAE:  2.9650
  MAPE: 2.36%

Gradient Boosting:
  RMSE: 3.3866
  MAE:  2.4106
  MAPE: 1.91%

Ridge Regression:
  RMSE: 0.0666
  MAE:  0.0543
  MAPE: 0.04%
======================================================================
```

An important aspect of tree-based models is understanding which features matter most:

In [33]:
```
# Feature importance analysis
feature_importance = pd.DataFrame({
    'feature': feature_cols,
    'importance': rf_model.feature_importances_
}).sort_values('importance', ascending=False)

print("\n" + "="*70)
print("Top 15 Most Important Features (Random Forest)")
print("="*70)
print(feature_importance.head(15).to_string(index=False))
```

```python
print("\nInsights:")
print("- Recent lags (lag_1, lag_2) are often most important")
print("- Rolling statistics capture trend information")
print("- Time features (month, dayofweek) capture seasonality")

# Visualize feature importance
plt.figure(figsize=(12, 6))
top_features = feature_importance.head(15)
plt.barh(range(len(top_features)), top_features['importance'])
plt.yticks(range(len(top_features)), top_features['feature'])
plt.xlabel('Feature Importance', fontsize=12)
plt.title('Top 15 Feature Importances - Random Forest',
          fontsize=14, fontweight='bold')
plt.gca().invert_yaxis()
plt.grid(True, alpha=0.3, axis='x')
plt.tight_layout()
plt.show()
```

```
======================================================================
Top 15 Most Important Features (Random Forest)
======================================================================
             feature  importance
       rolling_mean_7    0.465811
  rolling_mean_7_diff    0.122518
rolling_mean_14_diff    0.098017
        rolling_min_7    0.053560
                lag_7    0.051006
rolling_mean_30_diff    0.044071
               lag_14    0.039371
      rolling_mean_14    0.038079
               lag_30    0.010374
        rolling_max_7    0.009588
        expanding_std    0.008237
       rolling_std_14    0.006775
        rolling_std_7    0.006016
      rolling_mean_30    0.005209
                lag_3    0.005153

Insights:
- Recent lags (lag_1, lag_2) are often most important
- Rolling statistics capture trend information
- Time features (month, dayofweek) capture seasonality
```




# Comparing All Approaches

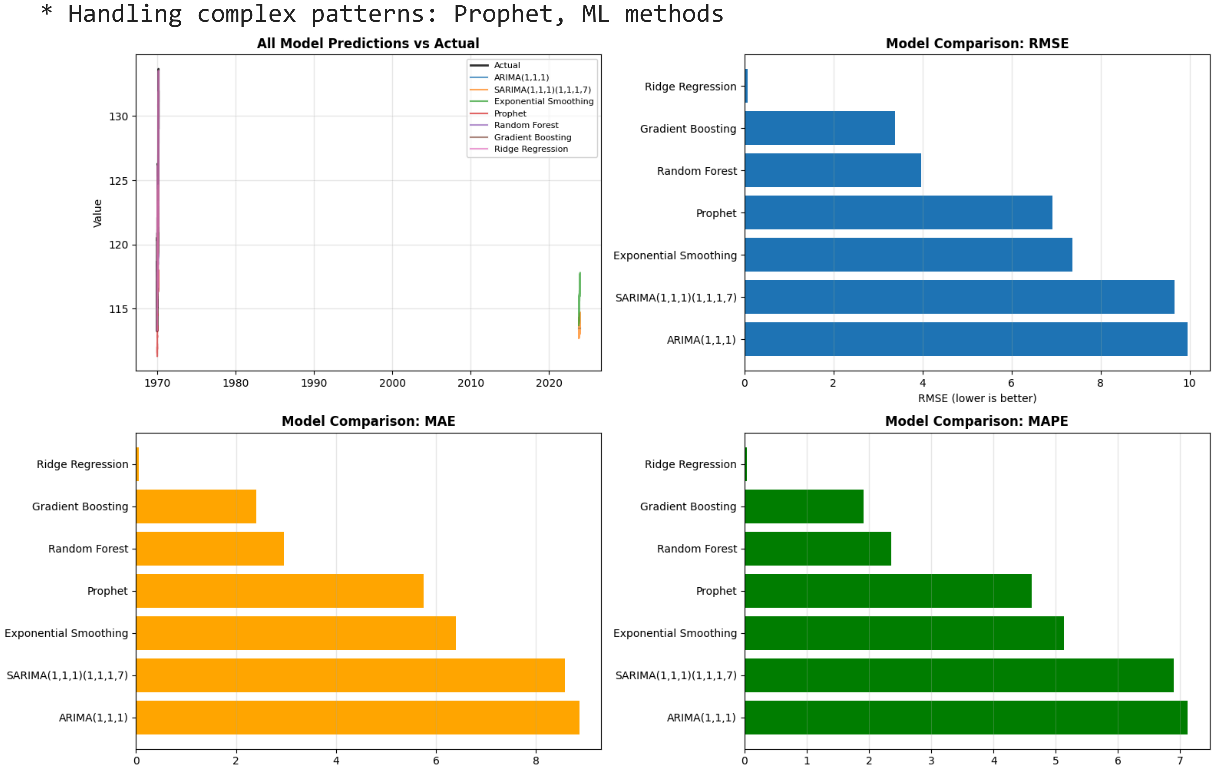
Now let's bring everything together and compare all the methods we've explored:

In [34]:
```
# Create comprehensive comparison
comparison_results = {
    'Model': [],
    'RMSE': [],
    'MAE': [],
    'MAPE (%)': []
}

# We need to align test sets for fair comparison
# Use the shortest common test period
min_test_length = min(len(test), len(y_test))

# Align all predictions to same test period
test_aligned = test[-min_test_length:].values
arima_aligned = arima_forecast[-min_test_length:]
sarima_aligned = sarima_forecast[-min_test_length:]
ets_aligned = ets_forecast[-min_test_length:]
prophet_aligned = prophet_test_predictions[-min_test_length:]
y_test_aligned = y_test[-min_test_length:].values

# Calculate metrics for each model
all_models = {
    'ARIMA(1,1,1)': arima_aligned,
    'SARIMA(1,1,1)(1,1,1,7)': sarima_aligned,
    'Exponential Smoothing': ets_aligned,
    'Prophet': prophet_aligned,
    'Random Forest': rf_predictions[-min_test_length:],
    'Gradient Boosting': gb_predictions[-min_test_length:],
    'Ridge Regression': ridge_predictions[-min_test_length:]
}
```

```
for model_name, predictions in all_models.items():
    rmse = np.sqrt(mean_squared_error(test_aligned, predictions))
    mae = mean_absolute_error(test_aligned, predictions)
    mape = np.mean(np.abs((test_aligned - predictions) / test_aligned)) * 100

    comparison_results['Model'].append(model_name)
    comparison_results['RMSE'].append(rmse)
    comparison_results['MAE'].append(mae)
    comparison_results['MAPE (%)'].append(mape)

# Create comparison dataframe
comparison_df = pd.DataFrame(comparison_results)
comparison_df = comparison_df.sort_values('RMSE')

print("\n" + "="*70)
print("FINAL MODEL COMPARISON - Ranked by RMSE")
print("="*70)
print(comparison_df.to_string(index=False))
print("="*70)
print("\nKey Takeaways:")
print("- Lower values are better for all metrics")
print("- RMSE penalizes large errors more than MAE")
print("- MAPE shows percentage error (easier to interpret)")
print("- Best model depends on your specific needs:")
print("  * Interpretability: ARIMA, SARIMA, Prophet")
print("  * Accuracy: Often ML methods, but not always")
print("  * Speed: Exponential Smoothing, Ridge Regression")
print("  * Handling complex patterns: Prophet, ML methods")

# Visualization comparison
fig, axes = plt.subplots(2, 2, figsize=(16, 10))

# Plot 1: All predictions
axes[0, 0].plot(test_aligned, label='Actual', color='black', linewidth=2, alpha=0.8
colors = plt.cm.tab10(np.linspace(0, 1, len(all_models)))
for (name, pred), color in zip(all_models.items(), colors):
    axes[0, 0].plot(pred, label=name, alpha=0.7, linewidth=1.5)
axes[0, 0].set_title('All Model Predictions vs Actual', fontsize=12, fontweight='bo
axes[0, 0].set_ylabel('Value')
axes[0, 0].legend(loc='best', fontsize=8)
axes[0, 0].grid(True, alpha=0.3)

# Plot 2: RMSE comparison
axes[0, 1].barh(comparison_df['Model'], comparison_df['RMSE'])
axes[0, 1].set_xlabel('RMSE (lower is better)')
axes[0, 1].set_title('Model Comparison: RMSE', fontsize=12, fontweight='bold')
axes[0, 1].invert_yaxis()
axes[0, 1].grid(True, alpha=0.3, axis='x')

# Plot 3: MAE comparison
axes[1, 0].barh(comparison_df['Model'], comparison_df['MAE'], color='orange')
axes[1, 0].set_xlabel('MAE (lower is better)')
axes[1, 0].set_title('Model Comparison: MAE', fontsize=12, fontweight='bold')
axes[1, 0].invert_yaxis()
axes[1, 0].grid(True, alpha=0.3, axis='x')
```

```
# Plot 4: MAPE comparison
axes[1, 1].barh(comparison_df['Model'], comparison_df['MAPE (%)'], color='green')
axes[1, 1].set_xlabel('MAPE % (lower is better)')
axes[1, 1].set_title('Model Comparison: MAPE', fontsize=12, fontweight='bold')
axes[1, 1].invert_yaxis()
axes[1, 1].grid(True, alpha=0.3, axis='x')

plt.tight_layout()
plt.show()
```

```
======================================================================
FINAL MODEL COMPARISON - Ranked by RMSE
======================================================================
                Model      RMSE       MAE  MAPE (%)
     Ridge Regression  0.066600  0.054305  0.043915
    Gradient Boosting  3.386581  2.410556  1.911511
        Random Forest  3.975707  2.965002  2.355508
              Prophet  6.920470  5.753497  4.615395
 Exponential Smoothing  7.363248  6.405526  5.135082
SARIMA(1,1,1)(1,1,1,7)  9.663778  8.586591  6.894476
         ARIMA(1,1,1)  9.958941  8.866136  7.119518
======================================================================

Key Takeaways:
- Lower values are better for all metrics
- RMSE penalizes large errors more than MAE
- MAPE shows percentage error (easier to interpret)
- Best model depends on your specific needs:
  * Interpretability: ARIMA, SARIMA, Prophet
  * Accuracy: Often ML methods, but not always
  * Speed: Exponential Smoothing, Ridge Regression
  * Handling complex patterns: Prophet, ML methods
```



## Conclusion — Choosing the Right Approach

- We've covered a full spectrum of time series methods, from foundational statistical models to modern machine learning approaches.
- Choosing the right technique depends on the problem, the data, and the constraints of your project.
- **When to Use ARIMA / SARIMA**
  - Ideal for well-behaved univariate series
  - Useful when interpretability and statistical inference matter
  - Works well with limited data
  - Downsides: more manual tuning and limited ability to include external variables
- **When to Use Prophet**
  - Excellent for quick, robust forecasts with minimal tuning
  - Handles multiple seasonalities, missing data, and outliers gracefully
  - Easy to incorporate domain knowledge (holidays, events)
  - Highly interpretable for business and stakeholder communication
- **When to Use Machine Learning Methods**
  - Best when external variables strongly influence the target
  - Captures non-linear and complex relationships
  - Useful with large datasets
  - Needs thoughtful feature engineering and careful validation
  - Harder to interpret and prone to overfitting if unchecked
- **Practical Advice**
  - Don’t commit to a single method too early
  - Begin with exploratory data analysis
  - Compare multiple models using proper time-series cross-validation
  - Ensembles often outperform any single method
  - A simpler, trusted model can be more valuable than a complex black box
- As the field evolves, deep learning approaches like LSTMs and Transformers are becoming increasingly influential, but the fundamentals remain essential.

## Key Takeaways and Best Practices

- **Data Preparation**

    - Start with clean data, proper datetime indexing, and thorough exploration
    - Check for missing values, outliers, and structural breaks
    - Visualize extensively before modeling
- **Stationarity**
    - Test for stationarity and apply transformations (differencing, detrending) when needed
    - Many statistical models rely on this prerequisite
- **Validation**
    - Never use random splits for time series
    - Use walk-forward or time-aware cross-validation methods
    - Maintain proper temporal order at all times
- **Feature Engineering (for ML Approaches)**
    - Create meaningful features: lags, rolling stats, time-based indicators, domain-specific variables
    - Feature quality often matters more than model selection
- **Model Selection**
    - Evaluate accuracy, interpretability, speed, maintainability, and stakeholder needs
    - Don't focus exclusively on numerical metrics
- **Monitoring**
    - Patterns change — models degrade over time
    - Implement monitoring to detect drift and decide when retraining is needed
- **Documentation**
    - Document assumptions, transformations, and model choices
    - Ensures the project remains understandable months later
- This concludes our comprehensive lecture on time series analysis.
    - You now have a systematic framework from data preparation to model evaluation.
    - The key is practice — apply these methods, experiment, and build intuition through real-world experience.