Exercise 1: Lecture Questions (10 points)

Answer the following questions briefly and precisely:

a) We want to capture an MRI of a patients brain and especially make the fluids visible in the resulting scan. Do we need to capture a T1 weighted or T2 weighted image? Explain your answer and also describe what repetition time (TR) and echo time (TE) are and how different timings result in differently weighted images.

b) Create a sketch of the complex number $z := \exp(j\alpha) \in \mathbb{C}$. List both the real and the imaginary part of z

c) How does the Fourier transform of a linear combination of two functions f and g look like (i.e. $a \cdot f + b \cdot g$)?

d) What is the convolution theorem of the Fourier transform and how can it be applied when filtering signals?

e) We are sampling a signal with a sampling frequency of f_{sample}. If we want to avoid aliasing in the reconstruction of the signal, how do we need to set the frequency f_{limit} of the band-limit we apply to the signal before sampling?

f) What is the difference between the discrete-time Fourier transform (DTFT) and the discrete Fourier transform (DFT)? How do they relate to each other?

g) In what situations would a median filter improve the image quality? Is this a linear or nonlinear filter? Is this filter edge-preserving or not? Briefly explain your answers.

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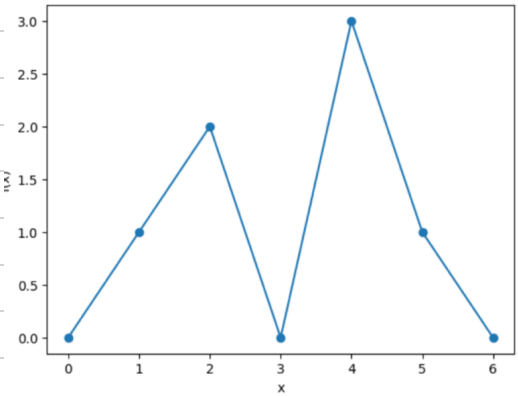
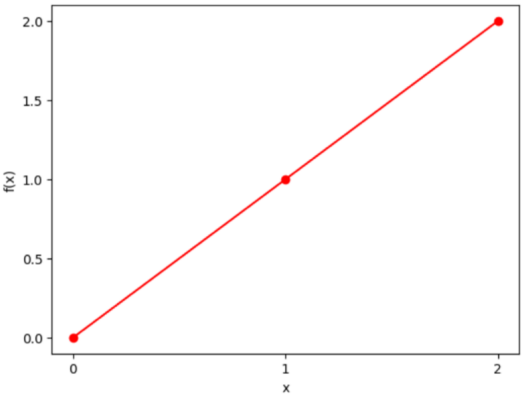
h) What is the difference between uncertainty and confidence? Explain both concepts and describe what aleatoric uncertainty and epistemic uncertainty are.

i) In machine learning, what does is mean to train in a supervised or unsupervised setting? Explain and name an example task for each setting.

j) Below in Fig 1a you can see a one-dimensional signal that has been sampled with $n = 7$ samples. Sketch the plot of the result of convolving this signal with the kernel given in Fig 1b. You may ignore the border cases and only compute in the regions fully covered by the kernel.



(a) Discrete Signal



(b) Kernel

a) > TR (Repetion Time): • time intervall between successive pulse seq. applied to same slice

↳ duration between application of RF (radio freq. pulses) MRI

• decides how much tissue can 'recover' before next pulse (T1 relaxation)

↳ affects images brighness & contrast

> TE (Echo Time): • time between delivery of RF pulse and peak of signal (echo) that is measured → determines T2 relaxation → diff. tissues look diff. in the image

→ for our MRI we need a T2-weighted image. It highlights fluids, because water has a long relaxation time

• long TR → less T1 weighting → favors T2

• long TE → more T2 decay to show differences in tissue

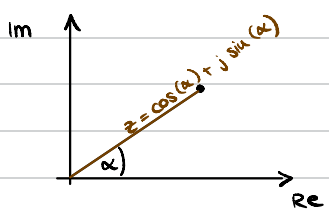
b) $z = e^{j\alpha}$

Note: Any Complex number can be written as exponential with j (img. unit) and angle α

→ Euler's formula: $z = \cos(\alpha) + j \sin(\alpha)$

• $Re(z) = \cos(\alpha)$

• $Im(z) = \sin(\alpha)$



c) $h(t) = a \cdot f(t) + b \cdot g(t)$

$\mathcal{F}(h(t)) = a \cdot \mathcal{F}(f(t)) + b \cdot \mathcal{F}(g(t))$

Fourier Properties: Linearity

$\mathcal{F}[\sum_i c_i x_i(t)] = \sum_i c_i \mathcal{F}[x_i(t)] = \sum_i c_i X_i(j\omega)$

↳ Scaling variable

• Addition & scaling are taken into consideration

• Constant applies also to Transformation

d) Convolution Theorem:

Fourier Transform of $(f \cdot g) = \mathcal{F}(f) \cdot \mathcal{F}(g)$

Eg.: · $f(t)$ is a signal } both signals
· $g(t)$ is a filter

Time domain: $y(t) = f(t) \cdot g(t)$ (Convolution)

→ Frequency domain: $\mathcal{F}(y(t)) = \mathcal{F}(f(t)) \cdot \mathcal{F}(g(t))$

→ Instead of doing the Convolution separately we
(1.) take the Fourier transforms of both signals and
(2.) multiply them in the frequency domain.
(3.) Compute the inverse Fourier Transform → get convoled function & filtered signal

e) According to the Nyquist criterion:

$$f_{limit} \leq \frac{f_{sample}}{2}$$

if we want to avoid aliasing, the signal must not have frequencies higher than half the sampling rate.

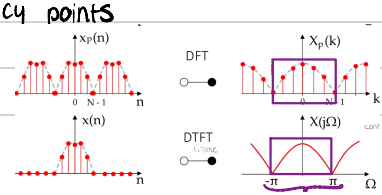
· sampling rate = f_{sample} → filter signal: $\leq f_{sample}/2$
↳ ensuring that high frequencies won't fold into lower ones (aliasing) → using low-pass filter

f)

	DFT	DTFT
Input:	discrete-time signal of infinite length	discrete time signal, finite length
Output:	continous frequency spectrum ↳ periodic: period 2π	discrete frequency spectrum → signal & spectrum periodic

→ DFT samples the DTFT at N equally spaced frequency points

→ take an infinite length DTFT and sample it in discrete frequencies, we get the DTF.



g) > Linear Filter: Output is weighted sum of input values (e.g. Gaussian blur)

> Non-linear Filter: Output non-weighted sum

> Edge-Preserving Filter: reduces noise but keep sharp edges in the filter

→ A median filter is good when an image has salt and pepper noise. It removes the noise well without blurring important details.

- It is then a non-linear filter: takes median value of neighborhood, instead of weighted sum
- Yes it would be edge-preserving. Median don't average across edges like a linear blur would, so edge are still sharp.

h) > Uncertainty: how unsure is the model about the prediction

> Confidence: how sure the model thinks it is

→ high confidence: model thinks strongly the answer is right

→ high uncertainty: model unsure or lacks information

Aleatoric uncertainty:
- uncertainty of the data itself
- comes from noise in the data & cannot be reduced
- e.g.: blurry images, sensor noise, low-light fotos

Epistemic uncertainty:
- ≙ model uncertainty
- comes from lack of data / knowledge
  ↳ can be reduced by collecting more/better data
- eg: model sees something for the first time and wasn't trained on it

i) > Supervised learning : · model trained with labeled data (→ correct answers / outcomes are known)

· model learns to map inputs → outputs

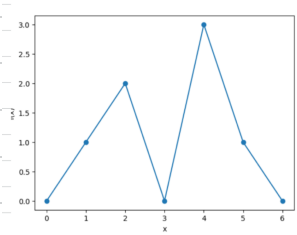
Example: Image classification (e.g. of organs "brain" vs. "heart")

> Unsupervised learning: · model trained with unlabeled data → no correct answers / outputs given

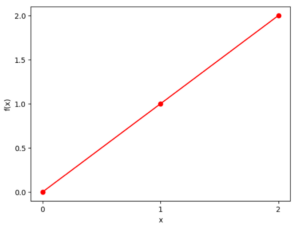
· model has to find groups / patterns on its own

Example: Clustering (grouping samples according to criterion / relatedness)

j)



(a) Discrete Signal



(b) Kernel

· Discrete signal: $x = [x_1, x_2, x_3, x_4, x_5, x_6, x_7]$

· Kernel: $h = [h_1, h_2, h_3]$

<u>Convolution</u>:

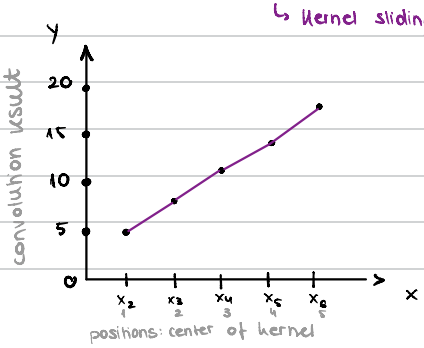
general: $y[i] = x[i] \cdot h_1 + x[i+1] \cdot h_2 + x[i+2] \cdot h_3$

· $y_1 = x_1 \cdot h_1 + x_2 \cdot h_2 + x_3 \cdot h_3 = 0 \cdot 0 + 1 \cdot 1 + 2 \cdot 2 = 5$

· $y_2 = x_2 \cdot h_1 + x_3 \cdot h_2 + x_4 \cdot h_3 = 1 \cdot 0 + 2 \cdot 1 + 3 \cdot 2 = 8$

· $y_3 = x_3 \; h_1 + x_4 \cdot h_2 + x_5 \cdot h_3 = 2 \cdot 0 + 3 \cdot 1 + 4 \cdot 2 = 11$

· $y_4 = x_4 \; h_1 + x_5 \cdot h_2 + x_6 \cdot h_3 = 3 \cdot 0 + 4 \cdot 1 + 5 \cdot 2 = 14$

· $y_5 = x_5 \; h_1 + x_6 \cdot h_2 + x_7 \cdot h_3 = 4 \cdot 0 + 5 \cdot 1 + 6 \cdot 2 = 17$

↳ kernel sliding across signal