MPC Control

Linearize Nonlinear Model

Problem description:

$$J = \sum_{i=0}^{N}(x - x_r)^2 + (y - y_r)^2 + \rho * (\phi - \phi_r)^2$$

$$s.t. -0.1 <= v_k <= v_{max}$$
$$|a_k| <= a_{max}$$
$$|\delta_k| <= \delta_{max}$$
$$|\delta_{k+1} - \delta_k|/dt <= d\delta_{max}$$

$$osqp\ interface:$$
$$minimize\ 0.5x^T Px + q^T x$$
$$subject\ to\ l <= Ax <= u$$
$$minimize\ 0.5x^T Px + q^T x$$
$$subject\ to\ l <= Ax <= u$$

将非线性模型泰勒展开后，整理成状态传递方程的形式如下：

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A_c}\mathbf{x} + \mathbf{B_c}\mathbf{u} + \mathbf{g_c} \\ \frac{\mathbf{x_{k+1}} - \mathbf{x_k}}{T_s} &= \mathbf{A_c}\mathbf{x_k} + \mathbf{B_c}\mathbf{u_k} + \mathbf{g_c} \\ \mathbf{x_{k+1}} &= (\mathbf{I} + T_s\mathbf{A}_c)\mathbf{x_k} + T_s\mathbf{B_c}\mathbf{u_k} + T_s\mathbf{g_c} \\ \mathbf{x_{k+1}} &= \mathbf{A_k}\mathbf{x_k} + \mathbf{B_k}\mathbf{u_k} + \mathbf{g_k}\end{aligned}$$

代价函数矩阵表达式如下：

$$J(\mathbf{z}, \mathbf{x_0}) = (\mathbf{X} - \mathbf{X_{ref}})^{\mathbf{T}}\bar{\mathbf{Q}}(\mathbf{X} - \mathbf{X_{ref}})$$

带入状态空间转移方程：

$$J(\mathbf{z}, \mathbf{x_0}) = (\bar{\mathbf{A}}_\mathbf{d}\mathbf{x_0} + \bar{\mathbf{B}}_\mathbf{d}\mathbf{z} + \bar{\mathbf{g}}_\mathbf{c} - \mathbf{X_{ref}})^{\mathbf{T}}\bar{\mathbf{Q}}(\bar{\mathbf{A}}_\mathbf{d}\mathbf{x_0} + \bar{\mathbf{B}}_\mathbf{d}\mathbf{z} + \bar{\mathbf{g}}_\mathbf{c} - \mathbf{X_{ref}})$$

其中X为$(\mathbf{x}_1^T, \mathbf{x}_2^T, \mathbf{x}_3^T, \ldots, \mathbf{x}_N^T)^T$，$\mathbf{x}_i$为某一离散时刻的状态向量。

控制序列$\mathbf{z}$为待优化变量，对其求J的梯度，由此可得到OSQP求解器需要的P和q系数矩阵：

$$\begin{aligned}\nabla_\mathbf{z} J &= \mathbf{P}\mathbf{z} + \mathbf{q} \\ &= 2\bar{\mathbf{B}}_\mathbf{d}^\mathbf{T}\bar{\mathbf{Q}}\bar{\mathbf{B}}_\mathbf{d}\mathbf{z} + 2\bar{\mathbf{B}}_\mathbf{d}^\mathbf{T}\bar{\mathbf{Q}}(\bar{\mathbf{A}}_\mathbf{d}\mathbf{x_0} + \bar{\mathbf{g}}_\mathbf{c} - \mathbf{X_{ref}})\end{aligned}$$

其中X_{ref}为跟踪的参考轨迹序列

With Delays

LInear model with delays:

$$\dot{\mathbf{x}} = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t - \tau)$$

Delay-free model:

$$\bar{\mathbf{x}}_\mathbf{0} = \mathbf{t} + \tau \approx \hat{\mathbf{x}}(t + \tau) = \mathbf{A}^\tau\mathbf{x}(t) + \sum_{j=0}^{\tau-1}\mathbf{A}^j\mathbf{B}\mathbf{u}(t - i - j)$$

我们只需要计算$\hat{x}(t+\tau)$即可。

HOW TO RUN

```
./install_tools.sh
catkin_make -j1
source devel/setup.bash
roslaunch mpc_car simulation.launch
```

HOW TO TURN PARAMETERS

```
./src/mpc_car/config/mpc_car.yaml -> mpc parameters
./src/car_simulator/config/car_simulator.yaml -> initial states (in simulation)
```

Simulation Result