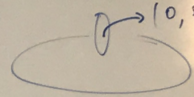
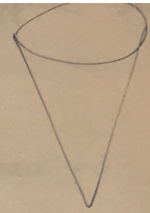
→ Parametrizasyon.

→ Yüzey.



$\to (0,R,0) + (0, r\cos\theta, r\sin\theta)$

$$\begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ R + r\cos\theta \\ r\sin\theta \end{bmatrix}$$



Örnek 4.5 S^2

$$S^2 = \left\{ (\sin\phi\cos\theta, \sin\phi\sin\theta, \cos\phi) \;\; \begin{array}{l} 0 \le \phi \le \pi \\ 0 \le \theta \le 2\pi \end{array} \right\}$$

Tanım 4.1 (Eğriler).

$$\Gamma = \{ \gamma(t) : t \in [a,b] \}$$

$\gamma : \mathbb{R} \longrightarrow \mathbb{R}^n$ türevlenebilir fonksiyon.

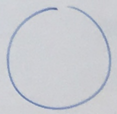
Γ: Eğri denir

γ: Eğri'nin parametrizasyon.

Örnek 4.2

$S^1 \subset \mathbb{R}^2$

$$\Gamma = \{ (\cos t, \sin t) : t \in [0, 2\pi] \}$$



Not: Aynı eğrinin bir den fazla parametrizasyon olabilir

$$S^1 = \{ (\cos t, \sin t) : t \in [0, 2\pi] \}$$

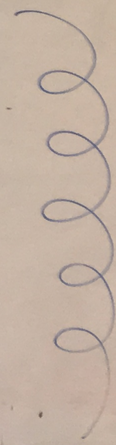
$$S^1 = \{ (\cos t^3, \sin t^3) : t \in [0, (2\pi)^{1/3}] \}$$

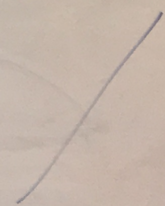
Görüntü aynı : Parametrizasyon farklı

Örnek 4.3

Heliks $H = \{ (\cos t, \sin t, t) : t \in \mathbb{R} \}$

Doğru $D = \{ (a_1 + b_1 t, a_2 + b_2 t, a_3 + b_3 t) : t \in \mathbb{R} \}$





Tanım 4.4 (Yüzeyler)

$S = \{ f(u,v) : (u,v) \in E \subset \mathbb{R}^2 \}$

$f(u,v) : E \subset \mathbb{R}^2 \longrightarrow \mathbb{R}^n$

f : Türevlenebilir

→ Parametrizasyon

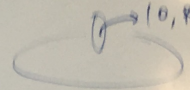
→ Yüzey

Örnek 4.5 S^2

$S^2 = \{ (\sin\phi\cos\theta, \sin\phi\sin\theta, \cos\phi) :$
$0 \le \phi \le \pi$
$0 \le \theta \le 2\pi \}$

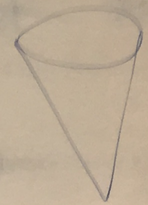
Örnek 4.6 Simit / [illegible]

$T = \{ (R\sin\phi + r\cos\theta\sin\phi, -R\sin\phi + r\cos\theta\cos\phi, r\sin\theta) :$
$0 \le \theta \le 2\pi$
$0 \le \phi \le 2\pi \}$



$(0, R, 0) + (0, r\cos\theta, r\sin\theta)$

$$\begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ R + r\cos\theta \\ r\sin\theta \end{bmatrix}$$



Tanım 4.1 (Eğriler).

$\Gamma = \{ \gamma(t) : t \in [a,b] \}$

$\gamma : \mathbb{R} \longrightarrow \mathbb{R}^n$ türevlenebilir fonksiyon.

Not: Aynı eğrinin birden fazla parametrizasyon olabilir

$S^1 = \{ (\cos t, \sin t) : t \in [0, 2\pi] \}$

$S^1 = \{ (\cos t^3, \sin t^3) : t \in [0, (2\pi)^{1/3}] \}$

Goruntu aynı : Parametrizasyon farklı

Tanım 4.8

$$F : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$$

vektör alan denir.

$F(a)$: 'a' noktada bir vektör.

Fizik'ten gelen temel gerçek: şarjı 'a' noktadan 'b' noktaya Γ üzerinde taşarsak, harcanan enerji

$$\int_a^b F(\gamma(t)) \cdot \gamma'(t)\, dt$$

Not γ_1, γ_2 Γ için farklı parametrizasyon olsun

$$\Gamma = \{\gamma(t) : t \in [a,b]\}$$

$$= \{\delta(t) : t \in [c,d]\}$$

O zaman

$$\int_a^b F(\gamma(t)) \cdot \gamma'(t)\, dt = \int_c^d F(\delta(t)) \cdot \delta'(t)\, dt$$

$$F(x,y,z) = (f(x,y,z), g(x,y,z), h(x,y,z)) \qquad \gamma(t) = (\gamma_1(t), \gamma_2(t), \gamma_3(t))$$

$$I_1 = \int_a^b \left[f(\gamma(t))\, \gamma_1'(t) + g(\gamma(t))\, \gamma_2'(t) + h(\gamma(t))\, \gamma_3'(t) \right] dt$$

$$\delta(t) = (\delta_1(t), \delta_2(t), \delta_3(t))$$

$$I_2 = \int_c^d \left[f(\delta(t))\, \delta_1'(t) + f(\delta(t))\, \delta_2'(t) + h(\delta(t))\, \delta_3'(t) \right] dt$$

④

İddia

$$\int_a^b f(\gamma(t))\, \gamma_1'(t)\, dt = \int_c^d f(\delta(t))\, \delta_1'(t)\, dt$$

$$\gamma_1(a) = \delta_1(c)$$

$$\gamma_1(b) = \delta_1(d)$$

$$t = \gamma^{-1}\delta(s) \quad \sim \quad \gamma(t) = \delta(s)$$

$$\gamma'(t)\, dt = \delta'(s)\, ds$$

$$\Rightarrow \gamma_1'(t)\, dt = \delta_1'(s)\, ds$$

$$\int_c^d f(\delta(s))\, \delta_1'(s)\, ds$$

Dolayısıyla, ~~dur~~ eşit.

Örnek 4.7

(Cone)

$$C = \left\{ (x, y, \sqrt{x^2+y^2}) : x, y \in \mathbb{R} \right\}$$

Tanım 4.8

$F : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$

vektör alan denir.

$F(a)$: 'a' noktada bir vektör.

Tartışma 4.9

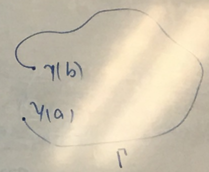
③

$E : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ Elektrik alan

q : Bir şarj.

Γ : Bir eğri.

$\Gamma = \{ \gamma(t) : t \in [a,b] \}$



Fizik'tan gelen temel gerçek : şarjı 'a' noktadan 'b' noktaya Γ üzerinde taşırsak, harcanan enerji

$$\int_a^b F(\gamma(t)) \cdot \gamma'(t)\, dt$$

Not γ_1, γ_2 Γ için farklı parametrizasyon olsun

$$\Gamma = \{ \gamma(t) : t \in [a,b] \}$$

$$= \{ \delta(t) : t \in [c,d] \}$$

$\delta(t) = (\delta_1(t), \delta_2(t), \delta_3(t))$

④

$$I_2 = \int_c^d \left[f(\delta(t))\, \delta_1'(t) + f(\delta(t))\, \delta_2'(t) + h(\delta(t))\, \delta_3'(t) \right] dt$$

İddia

$$\int_a^b f(\gamma(t))\, \gamma_1'(t)\, dt = \int_c^d f(\delta(t))\, \delta_1'(t)\, dt$$

(5)

Özet 4.11

Eğri $\Gamma = \{ \gamma(t) : t \in [a,b] \} \subset \mathbb{R}^n$ türevlenebilir

Vektör Alan : $F : \mathbb{R}^n \longrightarrow \mathbb{R}^n$

Eğri İntegral $\int_\Gamma F \cdot d\Gamma := \int_a^b F(\gamma(t)) \cdot \gamma'(t)\, dt$

↳ Parametrizasyonla bağımsız.

Örnek 4.12

$F(x,y) = (xy,\ y-x)$

Eğri : $(1,1)$ ve $(3,9)$ arasındaki doğru parçası.

$\int_\Gamma F \cdot d\Gamma$ hesaplayınız.

Cevap $\Gamma = \{ \underbrace{(2t+1,\ 8t+1)}_{\gamma(t)} : t \in [0,1] \}$

$$\int_0^1 F(\gamma(t)) \cdot \gamma'(t)\, dt$$

$$= \int_0^1 \big((2t+1)(8t+1),\ 8t+1-2t-1 \big) \cdot (2,8)\, dt$$

(6)

$$= \int_0^1 \big[2(2t+1)(8t+1) + 8(6t) \big]\, dt$$

$$= \int_0^1 (\ldots) \, dt$$

Örnek 4.13

Γ : $y^2 + 2z^2 = 1 \ \cap \ x = -1$

twe x-eksene göre saat yelkovanın ters yönünde

$$= \int_0^1 \left((2t+1)(8t+1),\ 8t+1-2t-1 \right) \cdot (2,8)\, dt$$

$$= \int_0^1 \left[2(2t+1)(8t+1) + 8(6t) \right] dt$$

$$= \int_0^1 \left(32t^2 + 68t + 2 \right) dt$$

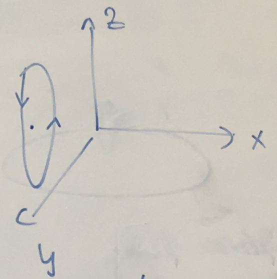
$$= \frac{32}{3} + \frac{68}{2} + 2$$

6

Örnek 4.13

$\Gamma : \quad y^2 + 2z^2 = 1 \ \cap \ x = -1$

+ve x-eksen e göre saat yelkovanın ters yönünde.



$$\Gamma = \left\{ r(t) = \left(-1,\ \cos t,\ \frac{\sin t}{\sqrt{2}}\right) : t \in [0, 2\pi] \right\}$$

$$F(x,y,z) = \left(\sqrt{x^3+y^3+5},\ z,\ x^2 \right)$$

Cevap $\gamma'(t) = \left(0, -\sin t, \frac{\cos t}{\sqrt{2}}\right)$

$F(\gamma(t)) = \left(*, \frac{\sin t}{\sqrt{2}}, 1\right)$

$$\int_\Gamma F\cdot dT = \int_0^{2\pi}\left(-\frac{\sin^2 t}{\sqrt{2}} + \frac{\cos t}{\sqrt{2}}\right)dt$$

$$= -\frac{\pi}{\sqrt{2}} \ \square$$

Örnek 4.14 (7)

$\Gamma: \ z = x^2+y^2 \ \cap \ z^2+y^2+x^2 = 1$

+z eksen'e göre saat yelkovanın yönü.

$F = (1, x+y, x^2+xy+y^2)$

$$\Gamma = \left\{ (\alpha\cos t, \alpha\sin t, \alpha^2) : t: 2\pi \to 0 \right\}$$

$: \alpha^4 + \alpha^2 = 1$

$\alpha^2 = \frac{-1\pm\sqrt{5}}{2}$

$\frac{-1-\sqrt{5}}{2}$ olmaz

$\alpha^2 = \frac{\sqrt{5}-1}{2}$

$\alpha = \sqrt{\frac{\sqrt{5}-1}{2}}$

$\gamma'(t) = (-\alpha\sin t, \alpha\cos t, 0)$

$F(\gamma(t)) = (1, \alpha\sin t + \alpha\cos t, *)$

$$\int F\cdot dT = \int_{2\pi}^{0} \alpha^2\sin t\cos t + \alpha^2\cos^2 t - \alpha\sin t \, dt$$

$$= \alpha^2\pi = \pi\left(\frac{\sqrt{5}-1}{2}\right) \ \square$$

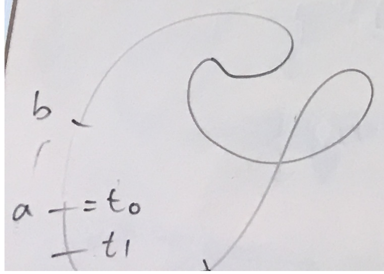
$\cdot \int_0^{2\pi} \sin t \, dt = 0$

$\cdot \int_0^{2\pi} \sin^2 t \, dt = \pi$

Tanım 4.15 Eğrinin Uzunluğu:

$\Gamma = \{\gamma(t) : t\in[a,b]\}$

$$\text{Uzunluk}(\Gamma) \approx \sum |\gamma(t_{i+1}) - \gamma(t_i)|$$



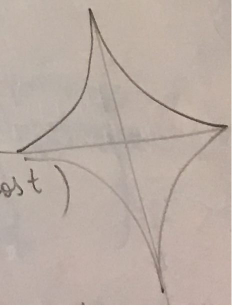
Ortalama değer Thm

$$\gamma(t_{i+1}) - \gamma(t_i) \approx \gamma'(t_i)(t_{i+1}-t_i)$$

4.16 örnek Uzunluk hesaplayınız. (8)

a) $\Gamma = \left\{ (\cos^3(t), \sin^3(t)) : t\in[0,2\pi] \right\}$

$x^{2/3} + y^{2/3} = 1$



$\gamma'(t) = (3\cos^2 t\sin(t), -3\sin^2(t)\cos t)$

$9|\sin t \cos t|$

Cevap $\gamma'(t) = \left(0, -\sin t, \frac{\cos t}{\sqrt{2}}\right)$

$F(\gamma(t)) = \left(*, \frac{\sin t}{\sqrt{2}}, 1\right)$

$$\int_\Gamma F \cdot dT = \int_0^{2\pi} \left(-\frac{\sin^2 t}{\sqrt{2}} + \frac{\cos t}{\sqrt{2}} \right) dt$$

$$= -\frac{\pi}{\sqrt{2}} \quad \square$$

Örnek 4.14 ⑦

$\Gamma: \quad z = x^2 + y^2 \quad \cap \quad z^2 + y^2 + x^2 = 1$

$+z$ eksen'e göre saat yelkovanın yönü.

$F = (1, x+y, x^2+xy+y^2)$

$$\Gamma = \left\{ (\alpha \cos t, \alpha \sin t, \alpha^2) : t : 2\pi \to 0 \right\}$$

$: \alpha^4 + \alpha^2 = 1$

$\alpha^2 = \frac{-1 \pm \sqrt{5}}{2}$

$\frac{-1-\sqrt{5}}{2}$ olmaz

$\alpha^2 = \frac{\sqrt{5}-1}{2}$

$\alpha = \sqrt{\frac{\sqrt{5}-1}{2}}$

$\gamma'(t) = (-\alpha \sin t, \alpha \cos t, 0)$

$F(\gamma(t)) = (1, \alpha \sin t + \alpha \cos t, *)$

$$\int F \cdot dT = \int_{2\pi}^{0} \alpha^2 \sin t \cos t + \alpha^2 \cos^2 t - \alpha \sin t \, dt$$

$$= \alpha^2 \pi = \pi \left(\frac{\sqrt{5}-1}{2} \right) \quad \square$$

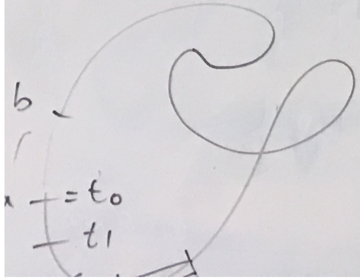
$\cdot \int_0^{2\pi} \sin t \, dt = 0$

$\cdot \int_0^{2\pi} \sin^2 t \, dt = \pi$

Tanım 4.15 Eğrinin uzunluğu:

$\Gamma = \{ \gamma(t) : t \in [a,b] \}$

$$\text{uzunluk}(\Gamma) \approx \sum |\gamma(t_{i+1}) - \gamma(t_i)|$$



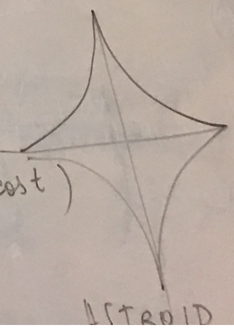
Ortalama değer Thm

$\gamma(t_{i+1}) - \gamma(t_i) \approx \gamma'(t_i)(t_{i+1} - t_i)$

4.16 örnek Uzunluk hesaplayınız. ⑧

a) $\Gamma = \left\{ (\cos^3(t), \sin^3(t)) : t \in [0, 2\pi] \right\}$

$x^{2/3} + y^{2/3} = 1$



$\gamma'(t) = (3\cos^2 t \sin(t), -3\sin^2(t) \cos t)$

$|\gamma'(t)| = 9 |\sin t \cos t|$

$$\gamma'(t) = (1,\ \alpha \sin t + \alpha \cos t,\ *)$$

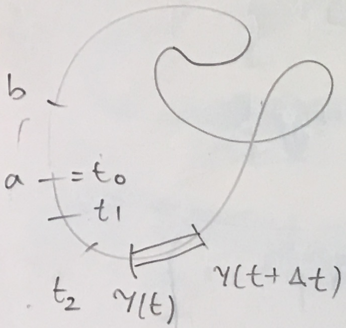
$$\int F \cdot dT = \int_{2\pi}^{0} \alpha^2 \sin t \cos t + \alpha \cos^2 t - \alpha \sin t \, dt \qquad \alpha^2 = \frac{\sqrt{5}-1}{2} \qquad \alpha = \sqrt{\frac{\sqrt{5}-1}{2}}$$

$$\cdot \int_0^{2\pi} \sin t \, dt = 0 \qquad \cdot \int_0^{2\pi} \sin^2 t \, dt = \pi$$

$$= \alpha^2 \pi = \pi \left(\frac{\sqrt{5}-1}{2} \right) \square$$

Tanım 4.15 Eğrinin Uzunluğu:

$$\Gamma = \{ \gamma(t) : t \in [a,b] \}$$



$$\text{Uzunluk}(\Gamma) \approx \sum |\gamma(t_{i+1}) - \gamma(t_i)|$$

Ortalama değer Thm

$$\gamma(t_{i+1}) - \gamma(t_i) \approx \gamma'(t_i)(t_{i+1} - t_i)$$

$$\approx \sum |\gamma'(t_i)| (t_{i+1} - t_i)$$

$$\to \int_a^b |\gamma'(t)| \, dt$$

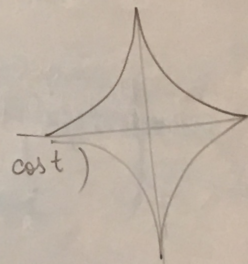
Tanım (Yay Uzunluğu)

$$|\Gamma| := \int_a^b |\gamma'(t)| \, dt$$

4.16 Örnek Uzunluk hesaplayınız. (8)

a) $\Gamma = \{ (\cos^3(t), \sin^3(t)) : t \in [0, 2\pi] \}$

$$x^{2/3} + y^{2/3} = 1$$



ASTROID

$$\gamma'(t) = (3\cos^2 t \sin(t),\ -3\sin^2(t) \cos t)$$

$$|\gamma'(t)| = 9 |\sin t \cos t|$$

$$|\Gamma| = 9 \int_0^{2\pi} |\sin t \cos t| \, dt$$

$$= 36 \ \square$$

Yüzey Integral, Fluks Integraller
Tanım 4.17.

$$S = \{ \phi(u,v) : (u,v) \in E \subset \mathbb{R}^2 \}$$

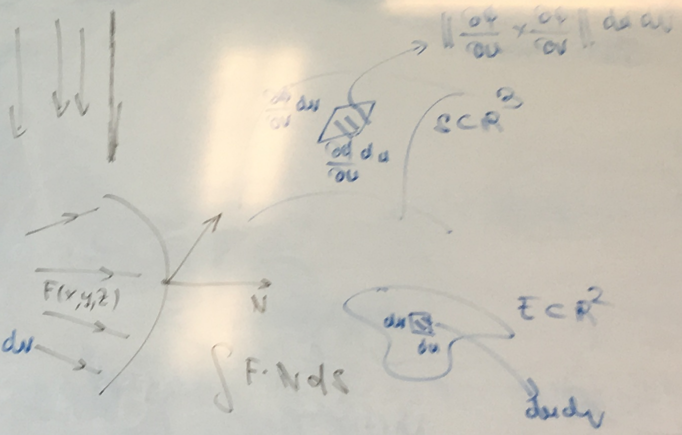
yüzey olsun. ($\mathbb{R}^3$ te)

$f : S \longrightarrow \mathbb{R}$ sürekli

$$\int_S f := \int f(\phi(u,v)) \left\| \frac{\partial \phi}{\partial u} \times \frac{\partial \phi}{\partial v} \right\| du\,dv$$

$F : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ vektor alan.

$$\int_S F \cdot dS := \int F \cdot \left(\frac{\partial \phi}{\partial u} \times \frac{\partial \phi}{\partial v} \right) du\,dv$$



Özet

$\Gamma = \{ \gamma(t) : t \in [a,b] \} \subset \mathbb{R}^n$

$S = \{ \phi(u,v) : u,v \in E \}$ $\phi : E \longrightarrow \mathbb{R}^3$

10

$$\int_S \ldots \int f(\phi(u,v)) \left\| \frac{\partial \phi}{\partial u} \times \frac{\partial \phi}{\partial v} \right\| du\,dv$$

$\int F \cdot N\,ds$

$du\,dv$

$F : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ vektor alan.

$$\int_S \bar{F} \cdot ds := \int F \cdot \left(\frac{\partial \phi}{\partial u} \times \frac{\partial \phi}{\partial v} \right) du\,dv$$

Özet

$\Gamma = \{ \gamma(t) : t \in [a,b] \} \subset \mathbb{R}^n$

Eğri

Yay uzunluğu $\int_a^b |\gamma'(t)|\,dt$

Eğri Integral $\int_\Gamma f(t)\,|\gamma'(t)|\,dt$

$f : \Gamma \to \mathbb{R}$ sürekli

Yonlu Eğri Integral $\int_a^b F \cdot \gamma'\,dt$

F : vektor alan

$S = \{ \phi(u,v) : u,v \in E \}$ $\phi : E \longrightarrow \mathbb{R}^3$

yüzey.

Yüzey Alan $\int \left| \frac{\partial \phi}{\partial u} \times \frac{\partial \phi}{\partial v} \right|$

Yuzey $\int$ $\int f \left| \frac{\partial \phi}{\partial u} \times \frac{\partial \phi}{\partial v} \right| \rightsquigarrow \int f(\phi(u,v)) \left| \frac{\partial \phi}{\partial u} \times \frac{\partial \phi}{\partial v} \right| du\,dv$

Fluxs $\int$ $\int F \cdot \left(\frac{\partial \phi}{\partial u} \times \frac{\partial \phi}{\partial v} \right)$

10