

$$\begin{aligned} & \left[\frac{\partial f}{\partial x_1}(a) \quad \cdots \quad \frac{\partial f}{\partial x_n}(a) \right]^\top \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix} = \sum_{i=1}^n b_i \frac{\partial f}{\partial x_i}(a) \\ \Rightarrow & \left. \frac{d}{dt} f(a+tb) \right|_{t=0} = \lim_{t \rightarrow 0} \frac{f(a+tb) - f(a)}{t} \\ & = \nabla f(a)^\top b \\ & = \sum_i \frac{\partial f}{\partial x_i}(a) b_i. \end{aligned}$$

①

$$\begin{aligned} & \text{Önerme 5.4} \\ & f: U \xrightarrow{\text{C}} \mathbb{R} \quad U \text{ açık} \\ & \text{Eğer } 'a' \text{ noktası } U \text{ 'nun içindedir} \\ & \quad \nabla f(a) = 0 \text{ olmak zorundadır.} \quad \left(\equiv \frac{\partial f}{\partial x_i}(a) = 0 \quad i = 1, \dots, n \right) \\ & \text{Işbu} \quad t \rightarrow f(a+t b) \\ & \text{Topluk 'değişkenli fonksiyenin} \\ & \text{'noktasında yerel maksima (min) vardır} \\ & \Rightarrow \left. \frac{d}{dt} f(a+tb) \right|_{t=0} = 0 \quad \left(\begin{array}{l} \text{sürekli tek değişkenli} \\ \text{fonksiyonları için} \\ \text{yerel maksima / minima} \\ \text{turev} = 0 \end{array} \right) \\ & \nabla f(a)^\top b = 0 \\ & \text{Bunun her biri için geçerli.} \quad \Rightarrow \nabla f(a) = 0 \quad i = 1, \dots, n \\ & \Rightarrow \frac{\partial f}{\partial x_i}(a) = 0 \end{aligned}$$

Önerme 5.4

Tanım 5.3 (Yerel max/min)

$f: A \rightarrow \mathbb{R}$, $a \in A$

Eğer $f(a) > f(b) \forall b \in B(a, \epsilon) \cap A$

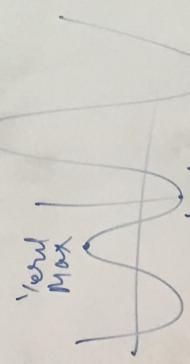
Sağlayın $\epsilon > 0$ varsa, ' a ' noktası

yerel max/min denir

Eğer $f(a) \leq f(b) \forall b \in B(a, \epsilon) \cap A$

Sağlayın $\epsilon > 0$ varsa, ' a ' noktası

yerel minima denir.



1.0
Topluk 'değişkenli fonksiyenin num
noktasında yerel maksima (min) vardır
 $\Rightarrow \left. \frac{d}{dt} f(a+tb) \right|_{t=0} = 0$ (sürekli tek
değişkenli
fonksiyonları için
yerel maksima / minima =)
 $\Rightarrow (\text{önerme 4.8})$

$\nabla f(a)^\top b = 0$
Bu her biri için geçerli. $\Rightarrow \nabla f(a) = 0$ $i = 1, \dots, n$
 $\Rightarrow \frac{\partial f}{\partial x_i}(a) = 0$

Öneme 4.4 (Habil adıma: Zinun kurallı)

$$\begin{aligned} f: \mathbb{R}^n &\rightarrow \mathbb{R}, \quad 'a' \text{ noktası türwensidir} \\ \Rightarrow \frac{df(a+t)}{dt} &= \sum \frac{\partial f}{\partial x_i}(a) b_i \quad \left(= \nabla f(a)^T b \right) \\ t=0 & \end{aligned}$$

Habil adıma

$$\begin{aligned} f(a+tb) &= f(a) + \nabla f(a)^T \cdot o(|tb|), \quad |tb| \rightarrow 0 \\ \Rightarrow f(a+tb) &= f(a) + \nabla f(a)^T \cdot b + o(t), \quad t \rightarrow 0 \\ \Rightarrow \frac{f(a+tb) - f(a)}{t} &= \nabla f(a)^T b + o(1), \quad t \rightarrow 0 \end{aligned}$$

Ornek

$$\begin{aligned} \left[\begin{array}{c} \frac{\partial f}{\partial x_1}(a) \\ \vdots \\ \frac{\partial f}{\partial x_n}(a) \end{array} \right]^T \left[\begin{array}{c} b_1 \\ \vdots \\ b_n \end{array} \right] &= \sum b_i \frac{\partial f}{\partial x_i}(a) \\ \Rightarrow \frac{d}{dt} f(a+tb) \Big|_{t=0} &= \lim_{t \rightarrow 0} \frac{f(a+tb) - f(a)}{t} \\ &= \nabla f(a)^T b \\ &= \sum \frac{\partial f}{\partial x_i}(a) b_i \end{aligned}$$

Analiz II

Tanım 5.3 (Yerel max/min)

$$f: A \rightarrow \mathbb{R}, \quad a \in A$$

Eğer $f(a) > f(b) \quad \forall b \in B(a, \varepsilon) \cap A$
 $\nabla f(a) = 0$ olmak zorundadır.

$$\text{Sügüt } \nabla f(a) > 0 \text{ varsa, } 'a' \text{ noktası}$$

i)

$$f: J \rightarrow \mathbb{R} \quad U \xrightarrow{\text{opt}} \underline{\text{yerel max/min ise}}$$

$$\begin{aligned} \text{Eğer } 'a' \text{ noktası} \\ \nabla f(a) = 0 \quad \text{olmak zorundadır.} \quad \left(\frac{\partial f}{\partial x_i}(a) = 0 \quad i = 1, \dots, n \right) \end{aligned}$$

Övning 5.5

$$f(x, y) = (x^2 + y^4) e^{-x^2 - y^2}$$

Vad maksima / minima nedan?

$$\frac{\partial f}{\partial x} = e^{-x^2-y^2} \cdot 2x(1-x^2-y^2)$$

$$\frac{\partial f}{\partial y} = e^{-x^2-y^2} \left[4y^3 - 2y(x^2+y^4) \right]$$

$$\text{Vad max/min} \rightarrow 2x(1-x^2-y^2) = 0 \quad \begin{cases} x^2+y^4=1 \\ x+y=0 \end{cases}$$

$$\Rightarrow 4y^3 - 2y(x^2+y^4) = 0 \quad \begin{cases} y=0 \\ x^2+y^2=1 \end{cases}$$

$$2y^2 - x^2 - y^4 = 0$$

Steg 1 : $x=0, y=0$

$$\text{Sej, } x^2+4y^4=1, y=0 \Rightarrow x \in \left\{ \pm 1 \right\}$$

$$\text{Sej, } 2y^2 - x^2 - y^4 = 0, x=0 \Rightarrow y \in \left\{ 0, \pm \sqrt{2} \right\}$$

$$\text{Sej, } x^2+y^4=1, y^2(2-y^2)=x^2$$

$$\Rightarrow y^2(8-y^2)+y^4=4 \Rightarrow y \in \left\{ \pm \frac{1}{\sqrt{2}} \right\}$$

$$\Rightarrow (x, y) \in \left\{ \left(\pm \frac{\sqrt{3}}{2}, \pm \frac{1}{\sqrt{2}} \right) \right\}$$

$$(0, 0), (-1, 0)$$

$$(0, \sqrt{2}), (0, -\sqrt{2})$$

$$\left(\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}} \right), \left(-\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}} \right), \left(\frac{\sqrt{3}}{2}, -\frac{1}{\sqrt{2}} \right), \left(-\frac{\sqrt{3}}{2}, -\frac{1}{\sqrt{2}} \right)$$

9 alternativa att slå om. Värd max/min.

a) $(0, 0)$: Mudlak Min + Värd min
Givet $f(x, y) > 0$ för $(x, y) \neq (0, 0)$
 $f(0, 0) = 0$.

$$\approx e^{-1} + e^{-1} \left[-k^2 - 2h(2ht+k^2) \right] \quad (1)$$

$$\approx e^{-1} + e^{-1} \left[-k^2(1+2h) - 4h^2 \right]$$

$$\approx e^{-1} + e^{-1} \left[-k^2 - 4h^2 \right] \quad \rightarrow \text{Här notatda < 0}$$

$$\begin{aligned} \text{③ } x & \\ \frac{\partial f}{\partial y} &= e^{-x^2-y^2} \left[4y^3 - 2y(x + y^4) \right] \\ &= 0 \\ \text{V end Max / Min} &\Rightarrow 2x(1-x^2-y^4)=0 \\ &\quad \left. \begin{array}{l} x^2+y^4=1 \\ y=0 \end{array} \right\} \\ &\Rightarrow 4y^3-2y(x^2+y^4)=0 \\ &\quad \left. \begin{array}{l} y=0 \\ x^2+y^2=1 \end{array} \right\} \end{aligned}$$

a) $(10, 0)$: Mudah diambil + yang lain
 Cukup $f(x, y) > 0$ juga ($x \neq 10, y \neq 0$)

$$\begin{aligned}
 b) (1,0) &: f(1,0) = e^{-1} \\
 f(1+h, k) &= \frac{(1+kh+h^2+k^4)e^{-1}}{(1+2h+k^4)e^{-1}} \\
 (h,k \neq 0) &\quad \approx \frac{(1+2h+k^4)(1-2h-k^2)e^{-1}}{(2h+k^4)^2 - (2h+k^4)(2h+k^2)} \\
 &= e^{-1} + e^{-1} \left[\frac{(1+2h+k^4)(1-2h-k^2)e^{-1}}{(2h+k^4)^2 - (2h+k^4)(2h+k^2)} - 1 \right] \\
 &\quad \boxed{(h,k) \neq 0 \quad (h,k) \neq 0} \\
 &\quad \text{Hab positiv } (\Rightarrow) \text{ Vord min} \\
 &\quad \text{Hab negativ } (\Rightarrow) \text{ Vord max}
 \end{aligned}$$

$$= e^{-k^2} + e^{-k^2} \sqrt{k^4 - k^2 - (2k+4)(2k+k^2)}$$

$$\begin{aligned} & (0, 0) \quad (1, 0) \quad (-1, 0) \\ & \left(0, \frac{1}{\sqrt{2}}\right) \quad \left(0, -\frac{1}{\sqrt{2}}\right) \\ & \left(\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}\right) \quad \left(-\frac{\sqrt{3}}{2}, \frac{1}{\sqrt{2}}\right) \quad \left(\frac{\sqrt{3}}{2}, -\frac{1}{\sqrt{2}}\right) \quad \left(-\frac{\sqrt{3}}{2}, -\frac{1}{\sqrt{2}}\right) \\ & \Rightarrow (x, y) \in \left\{ \left(\pm \frac{\sqrt{3}}{2}, \pm \frac{1}{\sqrt{2}} \right) \right\} \end{aligned}$$

$$\approx e^{-1} + e^{-1} \left[-k^2 - 2k(2k+k^2) \right] \\ \approx e^{-1} + e^{-1} \left[-k^2 (1+2k) - 4k^2 \right]$$

$$\begin{aligned}
 & \text{01} \quad \frac{\text{Yerel Maksimam}}{\text{Yerel Maksimim}} \\
 & f(0, \sqrt{2}) = \frac{4}{e^2} - h^2 - 2\sqrt{2}k \\
 & \Rightarrow \left(h^2 + 4 + 8\sqrt{2}k \right) e^{-h^2 - 2\sqrt{2}k} \\
 & \approx \left(4 + h^2 + 8\sqrt{2}k \right) \left(1 - h^2 - 2\sqrt{2}k \right) e^{-2} \\
 & \approx \frac{4}{e^2} + \left[h^2 + 8\sqrt{2}k - 4h^2 - 8\sqrt{2}k \right] e^{-2} \\
 & \qquad \qquad \qquad - 4h - \frac{3h^2}{e^2} - \frac{3h}{e} \\
 & = \frac{4}{e^2} - \frac{3h^2}{e^2} - \frac{3h}{e}
 \end{aligned}$$

d) $f\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right) = e^{-\frac{5}{4}}$

$$f\left(\frac{\sqrt{3}+h}{2}, \frac{1+h}{2}\right) = e^h \approx 1+h$$

$$\begin{aligned} & \int \left(\frac{\sqrt{3}+h}{2}\right)^2 + \left(\frac{1+h}{2}\right)^2 e^{-\left[\frac{5}{4} + h\sqrt{3} + h\sqrt{2}\right]} \\ & \approx \left(1 + h\sqrt{3} + h\sqrt{2}\right) e^{-\left[\frac{5}{4} + h\sqrt{3} + h\sqrt{2}\right]} \\ & \sim \left(1 + h\sqrt{3} + h\sqrt{2}\right) e^{-\frac{5}{4}} \quad (1 - h\sqrt{3} - h\sqrt{2}) \\ & = \left[1 - (h\sqrt{3} + h\sqrt{2})^2\right] e^{-\frac{5}{4}} \end{aligned}$$

$\frac{-5}{4}$
 $(h, k) \neq (0, 0)$
 $h, k \approx 0$

very max

Endigt 11

($a+h$) $\approx a+n \cdot h$!
 $f\left(\frac{\sqrt{3}+h}{2}, \frac{1+h}{2}\right) \approx f(a, 0)$

$f : U \rightarrow \mathbb{R}$ $U \subset \mathbb{R}^2$

iki deyin türvelenbozin olsun.

$\nabla^2 f(a) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ herman matrix.

$\nabla^2 f(a) \succ 0$ ($\text{Tüm özdögelen} > 0$)

0 2 anan 'a' very minima dan.

2) Eigen $\nabla^2 f(a) \prec 0$ ($\text{Tüm özdögelen} < 0$)

0 Zaman 'a' very maksimada.

6

$f(a) = A^T Q M_n(R)$

$\sigma(A) \subset C([0, \infty))$ ($\text{Tüm özdögelen} > 0$)

$\nabla^2 f(a) \succ 0$ ($A h, h \succ 0$.
 $\Leftrightarrow A h \perp h \backslash \{0\}$)

İşpat $\left| \frac{d^2 f(a+t b)}{dt^2} \right|_{t=0} = \langle \nabla^2 f(a), b \rangle = b^\top \nabla^2 f(a) b$ Hahn-Latura

Neden? (Basisit)

İşten $\nabla^2 f(a) \succ 0$, (lineer) $\langle \nabla^2 f(a) b, b \rangle > 0$ + b
 Cebir (PSD)

Very busy.

$$\frac{\partial^2}{\partial t^2} f(a+tb) \Big|_{t=0} = \langle \nabla^2 f(a), b, b \rangle = b^\top \nabla^2 f(a) b$$

Nededen? (Basit!)

Neder? (Basit)

$$\nabla^2 f(\alpha) > 0 \quad , \quad \begin{array}{l} \text{(linear)} \\ \text{Cebir} \end{array} \quad \nabla^2 f(\alpha | b, b) > 0 \quad , \quad b$$

=> Her doğru parça üzerinde flatb / run yerel minimum vardır.

Yew May : Isbat awm.

$$g(t) := f(a+tb)$$

Türen im Tannen
Kuhmattark.

\Rightarrow 'a' notasi you minuman.

② $f: U \rightarrow \mathbb{R}$

deja für verlängbar

$$\frac{\partial^2 g(t)}{\partial t^2} = \langle \nabla^2 f(a) \rangle_{\theta, b}$$

at-
source Tires in Tannin
Ruhmataar.

emco

6
Fatu Latina

$$t = A + e^{M_n(R)}$$

$\sigma(A) = C(0, \infty)$ (Tüm özdeğerler > 0)
 $\Leftrightarrow A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$.

1.1.1. $\exists x \forall y \exists z (P(x) \wedge Q(y) \wedge R(z))$

=> Hoy alegreamente se celebra el aniversario de la fundación de la Universidad.

Yerw May : Isbat ayri .

Quiz 1 10 / 100

$$A = \overline{At} \in M_n(\mathbb{R}) \quad \text{simetrik Matris}$$

Ergebnisse der Biometrie werden

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1100 N. 8th Street

Systematic sampling: N_1, \dots, N_m , $\lambda_1, \dots, \lambda_m$

10. $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$