Rotor / Wake Aerodynamics Assignment 2: Lifting Line

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Chapter 1

Introduction

With the global drive for more environmentally friendly aircraft transportation, aircraft manufactures are looking into new ways into improving engine efficiency, engine models such as the open rotor engine has been generating interest in the scientific community for their lower specific fuel consumption with respect to conventional engines. As such, rotor modelling is vital in engine design. This assignment looks into developing a frozen wake model of a rotor, and then performing an analysis on a two blade rotor model.

Chapter 2

Flow chart

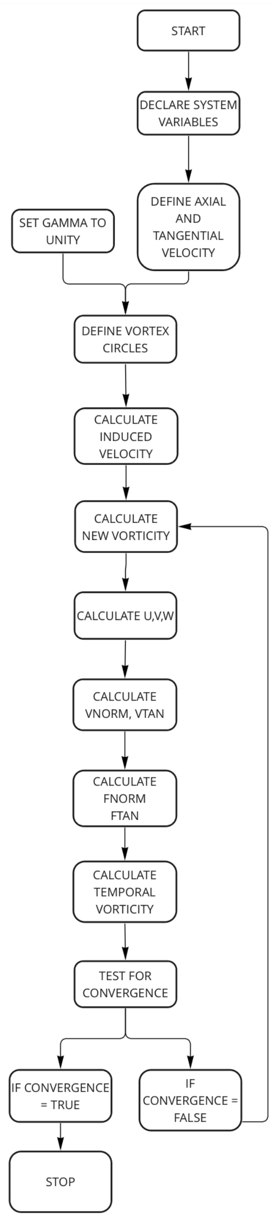


Figure 2.1: Flowchart of Code

Chapter 3

Main assumptions

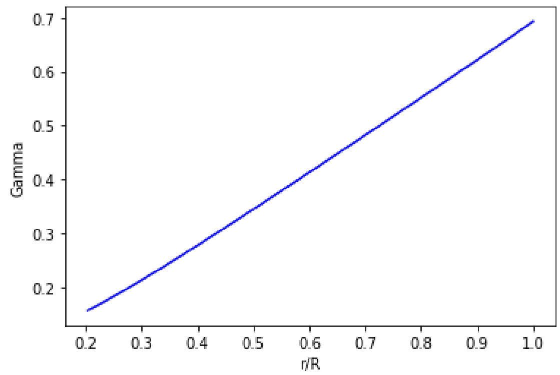
In order to apply the lifting model, some assumptions were made. Firstly, the incident airflow is considered steady, uniform, and axial. Moreover, the flow is assumed as incompressible. The blade element model is used for the calculation of the impact of each blade segment. Furthermore, it assumed that the lift generated by each cross-section is action at the quarter of the chord. The velocity induced by each vortex filament is calculated by the Biot-Savart Law [Katz and Plotkin, 2001]. In addition, Kelvin's second theorem implies that all the vortex segments must be part of a close vortex ring. Then the circulation of the blade is considered a function of the spanwise distribution by discretizing the blade into individual annuli. These assumptions allow for a numerical method to solve the blade parameters discretely. Regarding the wake, two assumptions are possible. Firstly, is the free wake assumption where the wake model takes into account the velocity induced by a rotor blade's circulation and the influence the wake has on itself. Thus, lead to a deformation of the wake. On the other hand, a simple assumption is employed, which is the frozen wake. In the case of frozen wake assumption, we assume that the wake is solely affected by the geometric constraint and the unperturbed flow. As such the wake's influence on itself is neglected and the wake is not considered to be deformed.

Chapter 4

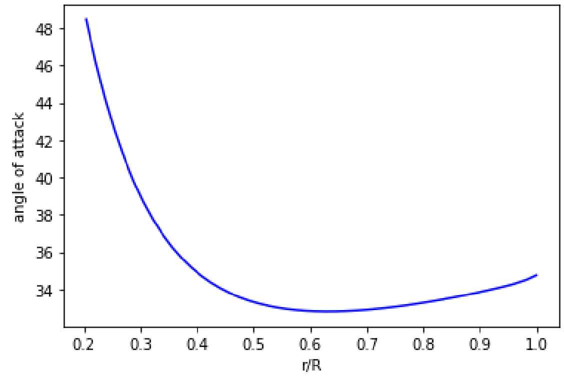
Results

4.1 Evaluation of a single rotor

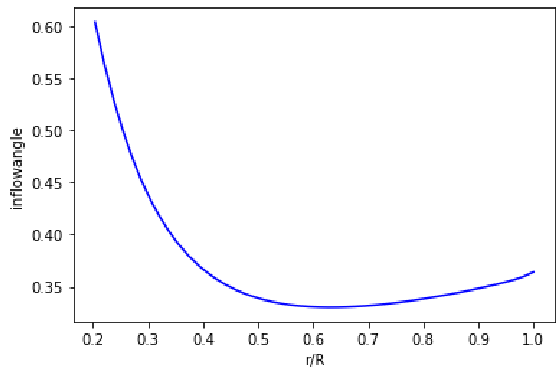
In this chapter, the evaluation of the single rotor case is taking place. All the plots are non-dimensionalized. The non-dimensioning is related to the radial position, which represents the ratio between the current distance from the center and the total radius of the blade. Furthermore, the results of the Blade Element Momentum theory are illustrated, to verify the two models. The demonstration of circulation, angle of attack, inflow angle, tangential load, and the normal load over the wind turbine are following in Figure 4.1:



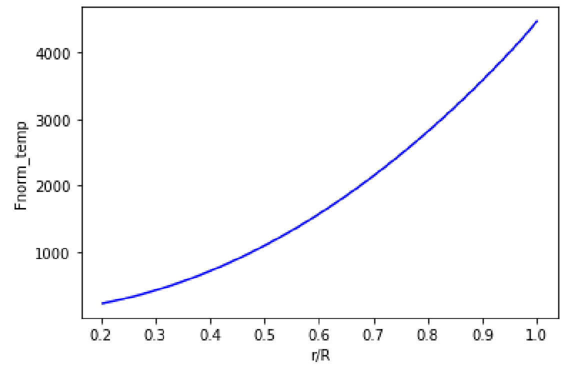
(a) circulation



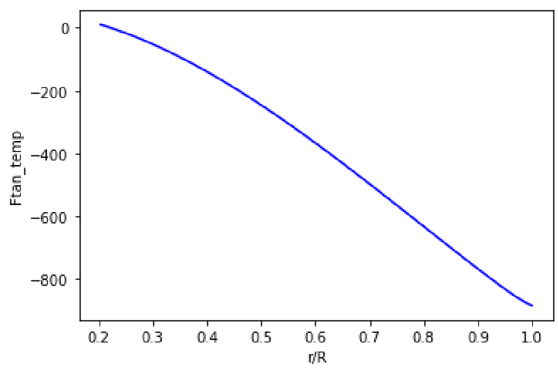
(b) alpha



(c) inflowangle



(d) F-norm



(e) F-tan

Figure 4.1: single rotor

As one can obtain based on Figure 4.1 is that the circulation increases as it's radial position increases. This phenomenon is caused by the fact that the velocity of each segment induces a velocity to the tip direction.

Regarding the distribution angle of attack and the inflow angle, we noticed a similar behavior. As the ratio r/R increases, the average value of angle of attack and inflow angle decrease, as obtained by Figure 4.1b and Figure 4.1c. If the r/R increase, the induced velocity increase. As an effect, the induced angle of attack increases (small-angle approximation), which leads to a decrease in the effective angle of attack. This remark justifies the reduction of inflow angle, as the r/Rincrease.

To continue with, the normal force increase as the r/R increase. This behavior is expected, because as the radial position increase the total perceived velocity of the blade increase, which leads to high lift and drag forces at each blade element. As a result of the average normal force action on the blade grow. On the other hand, the tangential force is noticed to decrease as the radial position increase, as expected.

In conclusion, the total thrust and power coefficients on the blade are represented in Table 4.1:

Table 4.1: Coefficients

Thrust coefficient (C_T)	27.7207
Power coefficient (C_P)	-1758.25407

4.2 Evaluation of the two-rotor case

In this section, the evaluation of the operation with two-rotors side-by-side of is appearing. First of all, to make the calculations feasible, we consider that the wakes of the wind turbines do not interact. Of course, this assumption is incorrect but necessary in order to derive some reasonable results. Before the results are discussed, a brief section detailing the expectations is presented hypotheses based on heuristics and scientific literature are presented.

4.2.1 Double rotor case expectations

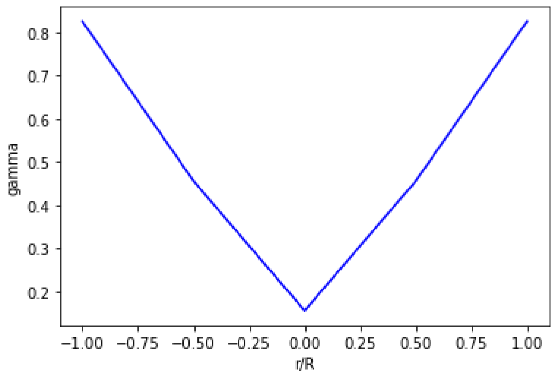
Scientific literature on wind farms suggest that keeping windmills spread over a larger area improves their efficiency, they also find that the larger C_T, the higher the losses [Hu et al., 2021] [Bhatia, 2014]. These findings are however, largely based on wake influence which is ignored in the upcoming double rotor analysis; hence contrasting findings are plausible.

A different way of thought is modelling the two rotors as point vortices so that their influence on one another can be explained using potential flow as in Equation 4.1 [Katz and Plotkin, 2001]. Note that in the equation the rotors are counterrotating thus weakening each other's effect and thus when co-rotating, they would boost (higher C_p and C_t) each other. A higher tip speed ratio could interpreted as a higher vortex strength and thus higher C_p and C_t and an increased distance a lower boost (hence distance infinite has least effect and D highest). Finally regarding phase, one would expect that from 0 to 90, the influence would drop from maximum to zero and then from 90 to 180 the influence would be negative with a minimum at -180.

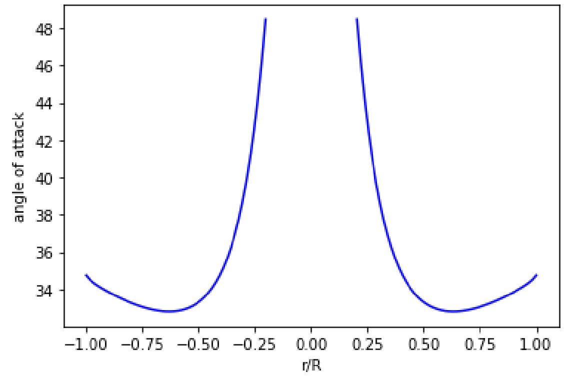
$$F(Y) = \frac{i\Gamma}{2\pi} \ln Y - \frac{i\Gamma}{2\pi} \ln(Y + 2ih) \tag{4.1}$$

4.2.2 double rotor case results

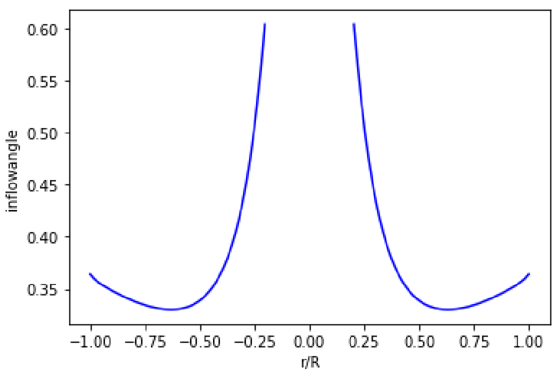
This utilization of the same code brings some various issues into our results as happened before. The presentation of the requested radials distributions are following in Figure 4.2:



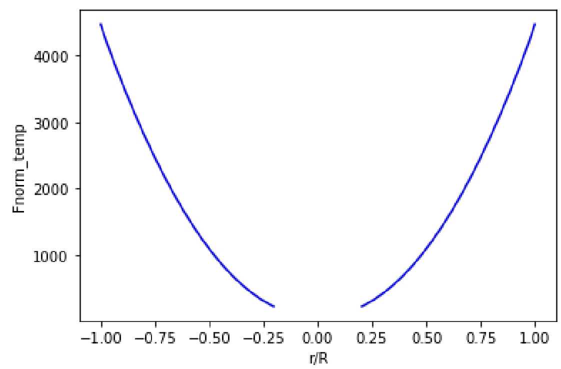
(a) circulation



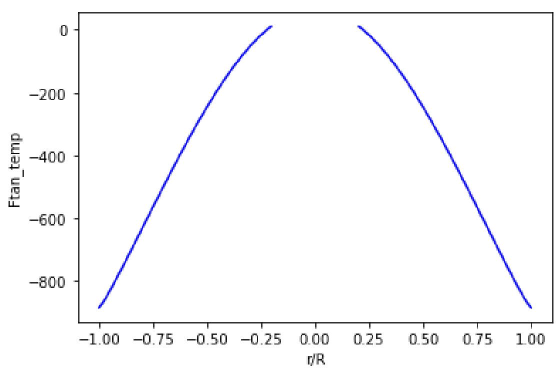
(b) alpha2



(c) inflow angle



(d) F-norm



(e) F-tan

Figure 4.2: 2 rotor

As one can observe, the results are similar to in the single rotor case, in the positive values of radial position. The graphs are mirrored around the y-axis, which is expected and caused by the assumption of this configuration. The correct approach would be to either take the second rotor as a point source and model its effect on the individual elements of the first rotor or take all the elements of the second rotor and see their effect on all the elements of the first rotor (using Biot-Savart's law for example). The validity of the results is the same as the single rotor case. Hence, the total thrust and power coefficients on the blade are identical with corresponding values that represented in Table 4.1. Note that this is blatantly wrong since rotors do not affect each other (in terms of all parameters displayed and performance coefficients) whatsoever. These results would also imply distance does not matter which seems highly unlikely.

4.3 BEM vs lifting line

The comparison between the lifting line and the BEM models is following. Taking into consideration Figure 4.1, we can derive the following conclusions about the two methods:

• The range of circulation is similar between the two models.

• The gamma distribution of the two models varies.

• The axial and normal force between the two models deviate a lot. This phenomenon is caused by the miscalculation of the normal and the axial velocities in our code and the wrong circulation distribution.

• In general, our model gives valid results for the circulation and failed to calculate the forces over the blade. The reasoning for this is taking place in the last chapter.

The reasoning between these discrepancies can be attributed to the different assumptions made with respect to the BEM and lifting line models. Whereas the BEM model does not take into account any radial influences (Streamtube/Annuli independent) the lifting line model does (fully dependent solution). Moreover, the BEM is an 1-D or a pseudo 2-D model, while the lift line is a fully 3-D model. One significant discrepancy is the tip effect, which is explicit in the lifting line model and it is corrected by the Prandl tip correction in BEM.

Furthermore whereas for BEM we do take into account loading and thickness effects, this is not the case for the lifting line theorem. Moreover, the number of blades is an intrinsic consideration for the lifting line whereas for the annular model of the BEM theorem that is not the case.

Chapter 5

Validity of wake geometry assumption

One of the main assumptions regarding the wake geometry, frozen wake, is that there are no deformations in the wake. This seems however, unrealistic in the dual rotor scenario since the two rotor rotate in opposite direction. Inevitably, the wakes of the two rotors will clash and thus deform making the frozen wake assumption invalid.

Chapter 6

Discussion and Conclusion

The first important conclusion is that the correct integration of the code wasn't feasible. The steps described in [Ferreira, 2022] and the lecture videos/PowerPoint, were followed but gives us some strange results. During the adaptation of the lifting line theory into our model, some major error occurs that was not able to solve. The inaccuracy of the induced velocity was identified as the main source of this issue. The implementation of the velocity-induced function can be improved, in order to derive more reasonable results. In addition, the time integration could shorten the errors. If we circumvent this phenomenon, we can evaluate the influence of the requested factors. The increase in the number of rings, increase the quality of the results but it doesn't reduce the error. Hence, fifty rings are selected to reduce the computational time. Finally, if the number of interactions is higher than 200, the solution converges.

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