Gamilton-Cayley theorem

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Assume K is algebraicaly closed, but it isn't necessary.

Th. 1. $\chi_A(A) = 0$

e — Jordan basis for A, $J = [A]_e^e$ (i.e. A in e). Want to show that $\chi_A(J) = 0$ as a matrix, and it's enough to show that each Jordan block is annihilated.

$$\chi_A(t) = \pm \prod (t - \lambda_i)_i^\alpha$$

Note that α_i is the sum of blocks sizes with λ_i eigenvalue, so it's not less than any of them. Now consider a block with size k and eigenvalue λ_i:

$$\chi_A(J_k(\lambda_i)) = (J_k(\lambda_i) - \lambda_i)^{\alpha_i} \prod (\dots) = 0$$