

# Geometry in analysis

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## Surfaces

Surface can be given by a function ( $U$  is usually open here):

$$(x_1 \dots x_{n-1}) \in U \subset \mathbb{R}^{n-1}, f: U \rightarrow \mathbb{R}, \Gamma_f = \{(x, y) \in \mathbb{R}^n \mid x \in U, y = f(x)\}$$

Coordinate lines are the images of sets  $x_i = C, \forall i \in [n-1], C \in \mathbb{R}$ .

If we have path  $\gamma$  in  $U$  we can project it to the surface as  $(\gamma, f \circ \gamma)$ .

If  $f$  is smooth at  $x$  we can consider  $y(x+h) = f(x) + df(x)(h)$  and call it affine tangent space. And without  $f(x)$  it's tangent space. Tangent vectors are those lying in tangent space. Also these vectors are those being tangent (at  $x$ ) to some curve in the surface.

Normal to the surface is  $\bar{n}(x) = \begin{pmatrix} \nabla f(x) \\ -1 \end{pmatrix}$ .

And  $f: \mathbb{R}^n \rightarrow \mathbb{R}^m$  are also possible.