Geometry in analysis

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Surfaces

Surface can be given by a function (U is usually open here):

$$(x_1 \dots x_{n-1}) \in U \subset \mathbb{R}^{n-1}, f : U \to \mathbb{R}, \Gamma_f = \{(x, y) \in \mathbb{R}^n \mid x \in U, y = f(x)\}$$

Coordinate lines are the images of sets $x_i = C, \forall i \in [n-1], C \in R$.

If we have path γ in U we can project it to the surface as $(\gamma, f \circ \gamma)$.

If f is smooth at x we can consider $y(x+h) = f(x) + \mathrm{d}f(x)(h)$ and call it affine tangent space. And without $f(x)$ it's tangent space. Tangent vectors are those lying in tangent space. Also these vectors are those being tangent (at x) to some curve in the surface.

Normal to the surface is $\bar{n}(x) = \begin{pmatrix} \nabla f(x) \\ -1 \end{pmatrix}$.

And $f : \mathbb{R}^n \to \mathbb{R}^m$ are also possible.