Evaluating Policy Counterfactuals: A “VAR-Plus” Approach Practical Implementation

Tomás E. Caravello MIT

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Outline

1. Introduction
2. Reduced-form Objects
3. "Plus" Step
4. Constructing the counterfactuals
5. Extra application: Financial Conditions Targeting

Outline

Plan for today: construct counterfactuals

Proposition

Suppose that the SVMA(∞) process (6) is **invertible***; i.e., that*

$$\varepsilon_t \in \operatorname{span}(\{y_\tau\}_{-\infty<\tau\leq t}).$$

Then knowledge of a) y_t and its autocovariance function $\Gamma_y(\ell)$ and b) policy causal effects $\boldsymbol{\Theta_\nu}$ suffices to construct the policy counterfactuals of interest.

1. How do we obtain $\Gamma_y(\ell)$ and $\boldsymbol{\Theta_\nu}$?

2. How do we exactly use them to arrive at the desired counterfactuals?

Objects of interest: counterfactuals under alternative policy rule

- **Average business cycle**: counterfactual second moments

$$\Gamma_y(\ell) = \sum_{m=0}^{\infty} \Theta_m \Theta'_{m+\ell} \quad \text{vs.} \quad \tilde{\Gamma}_y(\ell) = \sum_{m=0}^{\infty} \tilde{\Theta}_m \tilde{\Theta}'_{m+\ell}$$

Note: written in terms of structural IRFs $\{\Theta_m\}$, but under invertibility a rotation $\{\Theta_m P'\}$ suffices.

- Need: Policy Causal effects + Wold IRFs

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- **Condt'l forecast**: how is the economy predicted to evolve from t onwards? ▸ Init. Cond.

$$\mathbb{E}_t\left[y_{t+h}\right] = \sum_{\ell=0}^{\infty} \Theta_{\ell+h}\varepsilon_{t-\ell} \quad \text{vs.} \quad \mathbb{E}_t\left[\tilde{y}_{t+h}\right] = \sum_{\ell=0}^{\infty} \tilde{\Theta}_{\ell+h}\varepsilon_{t-\ell}$$

Note: under invertibility, the VAR recovers correct (FI) forecasts.

- Need: Policy Causal effects + Forecasts at t

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- **Historical evolution**: how would a given historical episode have unfolded? ▸ Init. Cond.

$$y_t = \sum_{\ell=0}^{\infty} \Theta_\ell \varepsilon_{t-\ell} \quad \text{vs.} \quad \tilde{y}_t = \sum_{\ell=0}^{\infty} \tilde{\Theta}_\ell \varepsilon_{t-\ell}$$

Note: what we need is forecasts and each date. Shock = forecast revision, $(E_t - E_{t-1})[y_{t+h}]$.

- Need: Policy Causal effects + Forecasts at $t \in [t_1, t_2]$.

Outline

Autocovariance Function: reduced-form VAR

- Start by Wold IRFs+Forecasts at t+Forecasts revisions.
- We get all this from a single VAR.

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- We get all this from a single VAR. ▸ Specification Details
  1. `run_var_wold.m`: get Wold IRFs.
     Estimate $y_t = \sum_{\ell=1}^{p} A_\ell y_{t-\ell} + u_t$,
     then use an arbitrary rotation (e.g. Cholesky with any ordering) to get Wold IRFs.

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  2. `run_var_fcst_scenario.m`: get forecasts at a given date t.
     Write VAR in companion form, $Y_t = \bar{A}Y_{t-1} + u_t$,
     then for $h > 0$, $E_t[Y_{t+h}] = AE_t[Y_{t+h-1}] = \bar{A}^h Y_t$

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     then for $h > 0$, $E_t[Y_{t+h}] = AE_t[Y_{t+h-1}] = \bar{A}^h Y_t$
  3. `run_var_fcst_evol.m`: get forecasts and their revisions for each $t \in [t_1, t_2]$
     Same as before, but do it for each $t \in [t_1, t_2]$

Policy Causal Effects

- We can get only a part of the $\boldsymbol{\Theta_{\nu}}$ matrix empirically.
- **Note: this can be enough for some applications.**
  Approximation with one shock might be quite close of the full counterfactual.
  In our paper: second moments and historical evolution.

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- `run_var_mp_ad.m` Run VAR with Aruoba and Drechsel (2024) shock ordered first, then output gap (Hamilton (2018)-filtered output), inflation, and FFR. Sample 1969Q1-2006Q4.

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- Use B-VAR (with loose priors) to get posterior draws, quantify IRF estimation uncertainty.

  Get posterior draws for B_ℓ, Σ_u in $y_t = \sum_{\ell=1}^{p} B_\ell y_{t-\ell} + u_t$, $u_t \sim N(0, \Sigma_u)$.

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- Estimate point estimate $\hat{\theta}_\nu$ and Variance-Covariance matrix $\hat{V}_{\theta_\nu}$ for IRFs.

  Follow Christiano et al. (2010). Different to most of the literature, use non-diagonal $\hat{V}_{\theta_\nu}$ to more accurately reflect informativeness of the data.

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- **Solution:** partial model structure. Only need models of MP transmission.
- We'll match those models to the empirical evidence on MP shocks, then the model extrapolates to full $\boldsymbol{\Theta_{\nu}}$.

  Aside: why many models?

  A: we want to be as robust to model mispecification as possible. Different models extrapolate differently.

Overview: "Plus" step in one slide

- **Notation:** model $\mathcal{M}_j$ has parameters ψ_j. Priors over both: $p(\mathcal{M}_j)$ and $p(\psi_j)$.
  In our case: 4 "models" RANK and HANK, both in "Ratex" and "Behavioral" versions.
  Parameters: prob. of price and wage adjustment, investment adj. cost, et.

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- Using asymptotic argument, empirical IRFs satisfy $\hat{\theta}_\nu \sim N(\theta_v, V_{\theta_\nu})$. Quasi-likelihood is:

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Let's dig deeper into each step.

Models

- 4 models:
  1. HANK + sticky information (similar to Auclert et al. (2020))
  2. RANK with habits (similar to Christiano et al. (2005); Smets and Wouters (2007))
  3. For both, a ratex and a behavioral version.

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  We need that in order to match delayed response to MP shock.
- **Key difference:** behavioral models are less forward-looking.
- We estimate habit parameter (RANK) or sticky information parameter (HANK), price and wage stickiness, inv. adj. cost, and capacity ut. elasticity.

Model Overview: RANK

- **Households & unions**
  - Representative household with preferences

$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t\left[u\left(c_t - hc_{t-1}\right) - v\left(\ell_t\right)\right]\right]$$

where $u(x) = \frac{x^{1-\gamma}}{1-\gamma}$ and $v(x) = \nu\frac{x^{1+\varphi}}{1+\varphi}$, and budget constraint

$$c_t + a_t^H = w_t(1-\tau_t^\ell)\ell_t + d_t^H - \tau_t + \frac{1+r_{t-1}^n}{1+\pi_t}a_{t-1}^H$$

  - Labor supply intermediated by sticky-wage unions (with indexation), gives

$$\pi_t^w - \pi_{t-1} = \kappa_w\chi_t + \beta\mathbb{E}_t\left[\pi_{t+1}^w - \pi_t\right]$$

where $\chi_t = \varphi\ell_t - (\lambda_t + w_t) + \frac{\bar{\tau}^\ell}{1-\bar{\tau}^\ell}\tau_t^\ell$

Model Overview: RANK, cont.

- **Production**
  - Standard layered structure: intermediate goods producers (capital & labor), retailers, aggregators, and capital goods producers
  - Features: variable capacity utilization, nominal rigidities (with indexation), inv. adj. costs
  - Pricing decisions give
    $$\pi_t - \pi_{t-1} = \kappa_p p_t^I + \beta \mathbb{E}_t \left[\pi_{t+1} - \pi_t\right]$$
    where p_t^I is intermediate goods price. Remaining conditions standard.
- **Policy**
  - Fiscal authority: nominal bonds with exponential maturity structure, spend constant fraction of GDP, adjust taxes gradually to maintain long-run budget balance
  - Monetary authority: close the model with an arbitrary (determinacy-inducing) rule.
    We'll see later that results are independent of the rule chosen.

Model variants

- **HANK:** Replace RA by continuum of households subject to idiosyncratic earnings [process as in Kaplan-Moll-Violante] and tight borrowing constraint
  - To get sluggish aggregate consumption: inattention to macro conditions

$$V_t(a,e,s) = \max_{c,a'} \{u(c) - v(\ell_t) + \beta \mathbb{E}_{t-s}\left[\theta V_{t+1}(a',e',s+1) + (1-\theta)V_{t+1}(a',e',0)\right]\}$$

  subject to the budget constraint

$$c + a' = \left((1-\tau_{\ell,t})w_t\ell_t + d_t^H\right)e + \frac{1+r_{t-1}^n}{1+\pi_t}a + \tau_t, \quad a' \geq \underline{a}$$

  where θ is probability of updating, and s is time since last update

- **Behavioral frictions:** add cognitive discounting [Gabaix (2020)], wage and price PCs are:

$$\begin{aligned} \pi_t - \pi_{t-1} &= \kappa_p p_t^I + \beta^p \mathbb{E}_t\left[\pi_{t+1} - \pi_t\right], \quad \beta^p < \beta \\ \pi_t^w - \pi_{t-1} &= \kappa_w \chi_t + \beta^w \mathbb{E}_t\left[\pi_{t+1}^w - \pi_t\right], \quad \beta^w < \beta \end{aligned}$$

Solution Method - SSJ (Auclert et al., 2021)

- Given $\boldsymbol{X} = (\boldsymbol{y}', \boldsymbol{w}', \boldsymbol{\pi}')'$ and for monetary shocks $\boldsymbol{\nu}$, we can compute ($\approx$ up to 1st ord.) excess demand/supply in 3 markets: assets, labor and capital for all $t \leq T^{max}$.
- Solution is then written implicitly as:

$$\underbrace{F(\boldsymbol{X}, \boldsymbol{\nu})}_{3T^{max}\times 1} = \begin{pmatrix} \underbrace{\boldsymbol{a}^d(\boldsymbol{X}, \boldsymbol{\nu}) - \boldsymbol{a}^s(\boldsymbol{X}, \boldsymbol{\nu})}_{\text{Asset Market}} \\ \underbrace{\boldsymbol{\ell}^d(\boldsymbol{X}, \boldsymbol{\nu}) - \boldsymbol{\ell}^s(\boldsymbol{X}, \boldsymbol{\nu})}_{\text{Labor Market}} \\ \underbrace{\boldsymbol{k}^d(\boldsymbol{X}, \boldsymbol{\nu}) - \boldsymbol{k}^s(\boldsymbol{X}, \boldsymbol{\nu})}_{\text{Capital Market}} \end{pmatrix} = \boldsymbol{0}$$

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- IFT to get:

$$d\boldsymbol{X} = -(F_{\boldsymbol{X}})^{-1}(F_{\boldsymbol{\nu}})d\boldsymbol{\nu} \tag{2}$$

- Function `compute_irfs_model.m` does this to compute $\boldsymbol{\Theta}_{\boldsymbol{\nu}}(\psi_j, \mathcal{M}_j)$.

Evaluating the posterior density

- Function `solve_model.m` : first, get $\boldsymbol{\Theta}_{\boldsymbol{\nu}}(\psi_j, \mathcal{M}_j)$.
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Evaluating the posterior density

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- **Differ from literature:** we don't take the first column, instead:
- In order to obtain $\theta_\nu(\psi_j, \mathcal{M}_j)$, let $\tilde{\nu}^*$ be a $H \times 1$ vector of news shocks, and $\boldsymbol{\tilde{\nu}^*} = ((\tilde{\nu}^*)', \mathbf{0}')'$. Pick $\tilde{\nu}^*$ such that the fit is closest to $\hat{\theta}_\nu$.
- Abusing notation:

$$\tilde{\nu}^* = \underset{\tilde{\nu}^*}{\text{argmin}} \quad (\boldsymbol{\Theta_\nu}(\psi_j, \mathcal{M}_j)\tilde{\nu}^* - \hat{\theta}_\nu)' V_{\theta_\nu}^{-1} (\boldsymbol{\Theta_\nu}(\psi_j, \mathcal{M}_j)\tilde{\nu}^* - \hat{\theta}_\nu)$$

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- Closed form solution:

$$\tilde{\nu}^* = (\tilde{\Theta}'_{1:H,1:H} V_{\theta_v}^{-1} \tilde{\Theta}_{1:H,1:H})^{-1} (\tilde{\Theta}'_{1:H,1:H} V_{\theta_v}^{-1} \hat{\theta}_v) \tag{3}$$

Same algebra as GLS.

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  Same algebra as GLS.
- Let $\theta_\nu(\psi_j, \mathcal{M}_j) = \boldsymbol{\Theta}_{\boldsymbol{\nu}}(\psi_j, \mathcal{M}_j)_{1:H,1:H} \tilde{\boldsymbol{\nu}}^*$, with that evaluate posterior as:

$$p(\psi_j | \mathcal{M}_j, \hat{\theta}_\nu) = \underbrace{p(\psi_j, \mathcal{M}_j)}_{\text{prior}} \times \underbrace{c \exp\left(-\frac{1}{2} (\theta_\nu(\psi_j, \mathcal{M}_j) - \hat{\theta}_\nu)' V_{\theta_v}^{-1} (\theta_\nu(\psi_j, \mathcal{M}_j) - \hat{\theta}_\nu) \right)}_{\text{likelihood}} \tag{4}$$

  In the code: compute log density, and omit constant c except for model posterior.

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- Why do it this way? Invariance to assumed MP rule. If we assumed a different rule, we'd get different $\tilde{\nu}^*$, but "fitted values" would be the same.

Sampling from the posterior and posterior model odds

Scripts `sample_posterior.m` and `posterior_model_probs.m`.

- Fix a model $\mathcal{M}_j$
  1. We know how to compute $p(\psi_j|\mathcal{M}_j, \hat{\theta}_\nu) \Rightarrow$ use RWMH to sample from the posterior.
  2. For posterior model odds, obtain marginal likelihood:

$$p(\hat{\theta}_\nu|\mathcal{M}_j) = \int p(\hat{\theta}_\nu|\mathcal{M}_j, \psi_j)p(\psi_j|\mathcal{M}_j)d\psi_j$$

Sampling from the posterior and posterior model odds

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Given posterior draws $\{\psi_j^i\}$, use harmonic mean estimator (Geweke, 1999):

$$p(\hat{\theta}_\nu|\mathcal{M}_j) \approx \left[\frac{1}{N_{use}}\sum_{i=1}^{N_{use}}\frac{f\left(\psi_j^i\right)}{p\left(\hat{\theta}_\nu \mid \psi_j^i, \mathcal{M}_j\right)p\left(\psi_j^i|\mathcal{M}_j\right)}\right]^{-1} \tag{5}$$

where

$$\begin{aligned} f(\psi) =& \tau^{-1}(2\pi)^{-d/2}\left|\bar{V}_\psi\right|^{-1/2}\exp\left[-0.5(\psi-\bar{\psi})'\bar{V}_\psi^{-1}(\psi-\bar{\psi})\right] \\ &\times \mathbb{I}\left\{(\psi-\bar{\psi})'\bar{V}_\psi^{-1}(\psi-\bar{\psi}) \leq F_{\chi_d^2}^{-1}(\tau)\right\} \end{aligned}$$

$\bar{\psi}$ is the posterior mean and $\bar{V}_\psi$ is the posterior covariance matrix. See Herbst and Schorfheide (2016) sec 4.6

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- Given $p(\hat{\theta}_\nu|\mathcal{M}_j)$, compute posterior model probabilites as:

$$p\left(\mathcal{M}_j \mid \hat{\theta}_\nu\right) = \frac{p\left(\hat{\theta}_\nu \mid \mathcal{M}_j\right) p\left(\mathcal{M}_j\right)}{\sum_{i=1}^{M} p\left(\hat{\theta}_\nu \mid \mathcal{M}_i\right) p\left(\mathcal{M}_i\right)} \tag{5}$$

- Can obtain sample across models with the posterior model probabilities.

Outline

Constructing the counterfactuals

- We have all we need construct counterfactuals!
  1. From VARs, we have Wold IRFs and Forecasts at each t of interest.
     Note: we also have a distrubtion for this. But for simplicity, fix all these objects at their medians.
  2. From the "Plus" step, we obtained a posterior distribution for $\boldsymbol{\Theta}_\nu$.
     All reported confidence sets account for this source of uncertainty only.
- **Building Block:** McKay and Wolf (2023)
  Recall: for a baseline IRF, we can get the counterfactual IRF if we have $\boldsymbol{\Theta}_\nu$

Second Moments

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- **Result:** full distribution of counterfactual second moments.

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- **Result:** full distribution of counterfactual conditional forecast under new rule.

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     Note: for $h = 0$ this is $\tilde{y}_{t_1}$ so we have the first date done!

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- **Result:** full distribution of counterfactual evolution under new rule.

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- We know now how to implement the method!
- Moving Forward:
  1. Empirical evidence on policy shocks is the key missing object.
  2. Given that, it is particularly important to understand how different models extrapolate.

     Our results: RANK vs. HANK is not as important as behavioral vs ratex.

Outline

1. Introduction
2. Reduced-form Objects
3. "Plus" Step
4. Constructing the counterfactuals
5. Extra application: Financial Conditions Targeting

Caballero, Caravello, and Simsek (2024): FC Targeting

- *Non-fundamental* shocks move asset prices significantly. (De Long et al., 1990; Gabaix and Koijen, 2021)
- Also generate non-trivial macro fluctuations.
- **Q: how should monetary policy be conducted in this context?**

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- (Bernanke and Gertler, 2000, 2001): focus on macro.
  "There is no significant additional benefit to responding to asset prices."

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  - Lower risk ⇒ financial markets become more *elastic*
  - More *elastic* markets ⇒ less financial volatility
  - Less financial volatility ⇒ Less macro volatility.
- Policy Counterfactuals: FCT lowers financial *and* macro volatility.
- The model in the paper does not fit since noise shocks enter **nonlinearly** ▸ Details

$$p_0 = \ldots + \frac{\sigma_r^2}{\alpha}\mu_0$$

Methodological contribution: need to deal with this.

We generalize MW to allow for noise shocks

- Extended set-up:

$$
\begin{aligned}
\mathcal{F}_x \mathbf{x} + \mathcal{F}_z \mathbf{z} + \mathcal{F}_\mu(\sigma_r^2 \varepsilon_{\mu,0}) &= \mathbf{0} && \text{(Finance)}\\
\mathcal{H}_x \mathbf{x} + \mathcal{H}_z \mathbf{z} + \mathcal{H}_\varepsilon \varepsilon_0 &= \mathbf{0} && \text{(Macro)}\\
\mathcal{A}_z \mathbf{z} + \mathcal{A}_x \mathbf{x} + \mathcal{A}_v v_0 &= \mathbf{0} && \text{(Policy)}
\end{aligned}
$$

Restriction: σ_r^2 only affects transmission of $\varepsilon_{\mu,0}$ & does so proportionally

Result: Under invertibility, Wold rep + Policy IRFs + identified noise shocks $\{\varepsilon_{\mu,t}\}$ are sufficient for counterfactuals

S1. Use MW to obtain counterfactuals, including IRF to noise shock $\hat{\Theta}_{\ell,\mu}$

S2. Scale it by variance $\tilde{\Theta}_{\ell,\mu} = \hat{\Theta}_{\ell,\mu} \frac{\tilde{\sigma}_r^2}{\sigma_r^2}$ where $\tilde{\sigma}_r^2$ is solved as a fixed point.

FCI-augmented Taylor rules would have stabilized gaps

- Augmented Taylor rules: $i_t = \rho_i i_{t-1} + (1-\rho_i)(\phi_\pi \pi_t + \phi_y \tilde{y}_t + \psi(\overline{FCI}_t - FCI_t))$

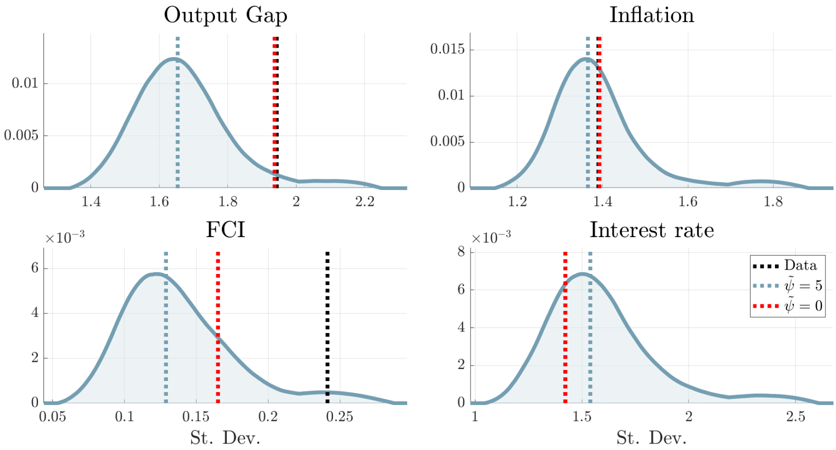


Figure: Benchmark Taylor $\psi = 0$ (red). **FCI-augmented Taylor** $\psi > 0$ (blue)

FCI-augmented dual mandate would have stabilized gaps

- $\min \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \tilde{y}_t^2 + \lambda_{\Delta i}(i_t - i_{t-1}) + \psi(\overline{FCI}_t - FCI_t)\right)^2\right]$ s.t implementation lag

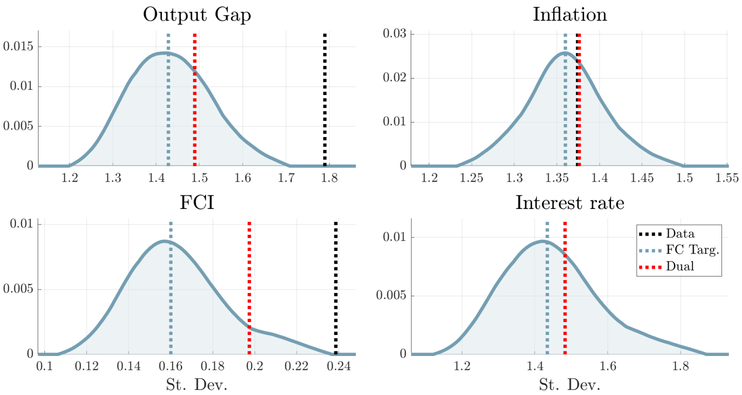
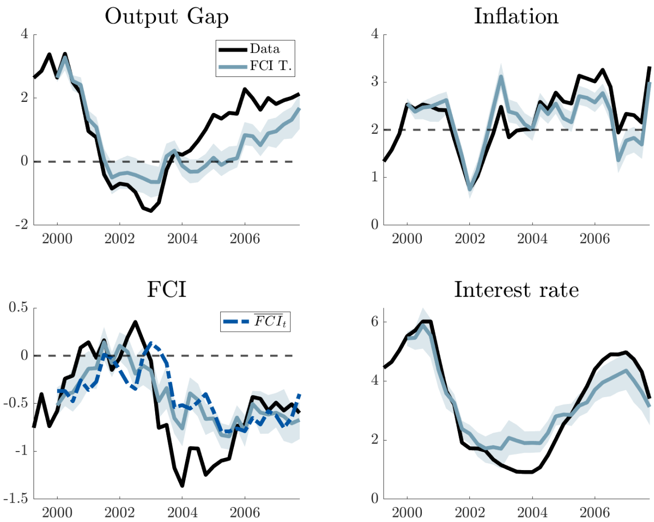


Figure: Flexible Dual Mandate $\psi = 0$ (red). Optimal **FCI targeting** ψ^* (blue)

FCI targeting would have stabilized gaps before GFC



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Appendix

Wold VAR: specification details ▸ Back

- Use unemplyoment, GDP, Investment, Consumption, Hours, TFP, labor productivity, labor share, inflation (GDP deflator), Fed Funds rate.
- Unemployment, inflation and Fed Funds rate are in levels. Everything else is detrended using Hamilton (2018) filter.
- Sample 1960Q1:2019Q4, extend for Covid counterfactual.

Model: variant of Caballero and Simsek (2023) ▸ Back

Finance Block

- Assets: stock, risk-free bond
- Traders: arbitrageurs (α), noise ($1-\alpha$)

$$\omega_t^A = \frac{E_t[r_{t+1}] + \frac{\sigma^2_{t,r_{t+1}}}{2} - r_t^f}{\sigma^2_{t,r_{t+1}}}$$

$$\omega_t^N = 1 + \frac{\mu_t}{1-\alpha}$$

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$$\Rightarrow E_t[r_{t+1}] = r_t^f + \frac{1}{2}\sigma^2_{t,r_{t+1}} - \frac{\mu_t}{\alpha}\sigma^2_{t,r_{t+1}}$$

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The impact of noise shocks is increasing in return variance.

Model: variant of Caballero and Simsek (2023)

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Macro Block

- Output-Asset Price
  $y_t = m + p_t + \delta_t$
- Potential Output
  $y_t^* = y_{t-1}^* + z_t$
- C-S decomposition
  $r_{t+1} = \rho - (1-\beta)\, m + (1-\beta)\, y_{t+1} + \beta p_{t+1} - p_t$
- Policy closes the model

Optimal Policy ▸ Back

- Fed chooses r_t^f *before* observing μ_t. Denote $\underline{E}_t[x] \equiv E[x|\mathcal{I}_t - \{\mu_t\}]$.

Optimal Policy

Back

- Fed chooses r_t^f *before* observing μ_t. Denote $\underline{E}_t[x] \equiv E[x|\mathcal{I}_t - \{\mu_t\}]$.
- Operational CB does FC Targeting:

$$G^{FCI}(x_t; \bar{p}_t) = \min_{r_t^f, \bar{p}_{t+1}} \underline{E}_t \left[(\tilde{y}_t)^2 + \psi(p_t - \bar{p}_t)^2 \right] + \beta \underline{E}_t [G^{FCI}(x_{t+1}; \bar{p}_{t+1})] \tag{6}$$

where $\bar{p}_{t+1}$ denotes an FC target committed one period in advance.

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where $\bar{p}_{t+1}$ denotes an FC target committed one period in advance.

- We evaluate losses using the *original* objective function.

$$G(\psi) = \underline{E}_t\left[\sum_{h=0}^{\infty} \beta^h \left(y_{t+h}(\psi) - y^*_{t+h}\right)^2\right] \tag{7}$$

Proposition (Stabilization Effects of FC Targeting)

Suppose the planner follows the FCI targeting policy in (6) with $\psi \geq 0$. Then in equilibrium, the output gap is

$$\tilde{y}_t = \left(\varepsilon_{\delta,t} - \varepsilon_{z,t}\right) \frac{\psi}{1+\psi} + \varepsilon_{\mu,t} \frac{\sigma_r^2(\psi)}{\alpha}. \tag{8}$$

The present discounted value of squared gaps defined in (7) is

$$G_t(\psi) = \frac{\left(\sigma_z^2 + \sigma_\delta^2\right)\left(\frac{\psi}{1+\psi}\right)^2}{1-\beta} + \frac{\sigma_\mu^2 \left(\frac{\sigma_r^2}{\alpha}\right)^2}{1-\beta} \tag{9}$$

We have $\frac{dG_t(\psi)}{d\psi}\Big|_{\psi=0} < 0$. *Thus,* $\psi^* = \arg\min_{\psi \geq 0} G_t(\psi) > 0$.

- Starting from $\psi = 0$, macro stabilization losses are second order.
- **Macro** gains of financial stabilization are first order $\Rightarrow$ $\psi^* > 0$ Back

Definitions of counterfactuals

- Unconditional counterfactual already defined. For conditional counterfactuals:

$$\tilde{y}_t = \underbrace{\sum_{\ell=0}^{t-t^*} \tilde{\Theta}_\ell \varepsilon_{t-\ell}}_{\text{new shocks after } t^*} + \underbrace{\tilde{y}_t^*}_{\text{initial conditions}}$$

  where the counterfactual rule is in place after t^*

- Let $y_t^* = \mathbb{E}_{t^*-1}[y_t]$. The initial conditions term $\tilde{y}_t^*$ solves the system

$$\begin{aligned} \mathcal{H}_x(\tilde{\boldsymbol{x}}^* - \boldsymbol{x}^*) + \mathcal{H}_z(\tilde{\boldsymbol{z}}^* - \boldsymbol{z}^*) &= \boldsymbol{0}, \\ \tilde{\mathcal{A}}_x \tilde{\boldsymbol{x}}^* + \mathcal{A}_z \tilde{\boldsymbol{z}}^* &= \boldsymbol{0}. \end{aligned}$$

- We then get the conditional counterfactuals as:
  (i) $\tilde{y}_t = \sum_{\ell=0}^{t-t_1} \tilde{\Theta}_\ell \varepsilon_{t-\ell} + \tilde{y}_t^1, \quad \forall t \in [t_1, t_1+1, \ldots, t_2]$
  (ii) $\mathbb{E}_{t^*}[\tilde{y}_{t^*+h}] = \tilde{\Theta}_h \varepsilon_{t^*} + \tilde{y}^*_{t^*+h}$

back