

# Public Debt and the r-star

*chatGPT explores how Public Debt could affect r-star*

Generated by ChatGPT

2024-10-20

## Introduction

This document discusses how public debt dynamics influence r-star, the natural rate of interest. We explore the key transmission channels through which public debt affects r-star and highlight the critical role of the sustainability condition in stabilizing r-star and long-term interest rates.

## 1. Debt Sustainability and Interest Rates

The idea behind debt sustainability is to ensure that a government's debt does not grow faster than its ability to pay it back, which is essentially its economic output (GDP). If debt grows too fast relative to GDP, it could become unsustainable, leading to difficulties in servicing the debt, potentially causing higher interest rates or even default.

### 1.1 Key Terms

- **Debt-to-Output Ratio:** The ratio of the government's debt to the country's GDP, often denoted as:

$$d_t = \frac{D_t}{Y_t}$$

where:

- $d_t$  = debt-to-output ratio at time  $t$ ,
- $D_t$  = nominal debt at time  $t$ ,
- $Y_t$  = nominal GDP at time  $t$ .

- **Real Interest Rate (r):** The interest rate on debt adjusted for inflation, representing the true cost of borrowing.
- **GDP Growth Rate (g):** The rate at which the economy is growing in real terms (adjusted for inflation).

## 1.2 The Sustainability Condition Formula

The sustainability condition says that for the debt-to-GDP ratio to remain stable or decrease, the growth rate of the debt must be lower than the difference between the real interest rate (r) and the growth rate of the economy (g). Mathematically, this can be expressed as:

$$\Delta d_t \leq (r - g)d_t - 1$$

If ( $r > g$ ), the debt is growing faster than the economy, and for sustainability, the government needs to run **primary surpluses** (where revenue exceeds non-interest expenditures) to offset this growth.

## 2. Link to Long-Term Interest Rates

The sustainability condition places a limit on how much debt a government can issue without triggering a rise in interest rates. If the condition is met, debt grows at a manageable pace relative to GDP, keeping investor confidence high and risk premiums low, thereby preventing a spike in long-term interest rates.

### 2.1 Market Perception of Default Risk and Confidence

Financial markets closely watch the debt-to-GDP ratio and the trajectory of debt growth. If debt is perceived as sustainable ( $r - g < 0$ ), markets remain confident in the government's ability to pay back debt, keeping long-term interest rates low.

### 2.2 Crowding Out and Private Investment

High public debt can crowd out private investment by pushing up interest rates, as the government competes for funds in financial markets. Conversely, if debt is sustainable, this crowding-out effect is minimized, and r-star remains closer to its fundamental drivers.

## 3. How Debt Sustainability Affects r-star

Public debt dynamics influence r-star through several channels. Here's how the sustainability condition, public debt levels, and their impact on long-term interest rates link back to r-star:

### **3.1 Risk Premium and r-star**

When a government's debt is perceived as sustainable, risk premiums remain low, preventing long-term interest rates from rising. If public debt becomes unsustainable, risk premiums increase, pushing up both long-term interest rates and r-star.

### **3.2 Crowding Out Effect on r-star**

If high debt crowds out private investment, the equilibrium interest rate (r-star) rises. If debt is perceived as sustainable, the crowding-out effect is minimized.

### **3.3 Expectations of Fiscal Policy and Savings Behavior**

When debt is sustainable, confidence in fiscal stability remains intact, minimizing precautionary savings. This balance between savings and investment helps keep r-star aligned with economic fundamentals.

### **3.4 Fiscal Policy and r-star**

Debt sustainability allows active fiscal policy to stabilize the economy, which can smooth fluctuations in r-star over time. Unsustainable debt limits fiscal policy options, potentially lowering r-star.

### **3.5 Global Capital Flows and r-star**

High public debt in a major economy can influence global capital flows. If perceived as risky, this raises global interest rates, influencing r-star in other economies.

## **Conclusion**

Debt sustainability is crucial for stabilizing r-star. When the sustainability condition ( $r - g < 0$ ) is met, debt levels are managed relative to GDP, preventing an undue rise in long-term interest rates. This ensures that r-star remains aligned with economic fundamentals, supporting balanced growth and economic stability.