

SHARIF UNIVERSITY OF TECHNOLOGY

CAUSAL INFERENCE

**Nonlinear causal discovery with additive noise
model Based on simulated and real data**

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Question 1

a

linear with Gaussian noise

in this part we visualize that if we have linear relationship between Y and X and Gaussian noise, $P(X|y)$ is independent of y and also $P(Y|x)$ is independent of x .

as we can observe both $P(X|y)$ and $P(Y|x)$ are Gaussian for $y = -1, 0, 1$ and $x = -1, 0, 1$.

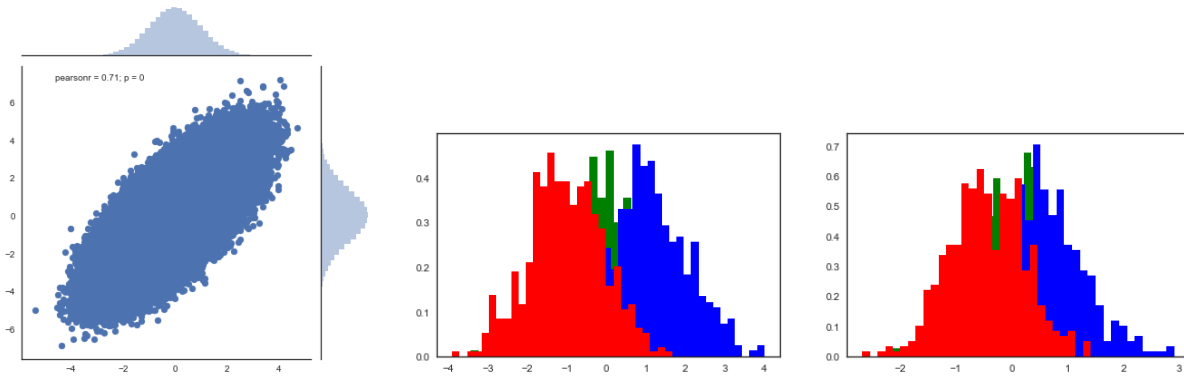


Figure 1: left picture is joint dist. of X and Y ; middle picture illustrate $P(Y|x)$; right picture illustrate $P(X|y)$

nonlinear with Gaussian noise

from previous we knew that if relationship between X and Y is nonlinear the probability of having causal model in both direction is zero. and here because X is the parent of Y therefore $P(Y|x)$ is independent of x , both $P(X|y)$ isn't independent of y .

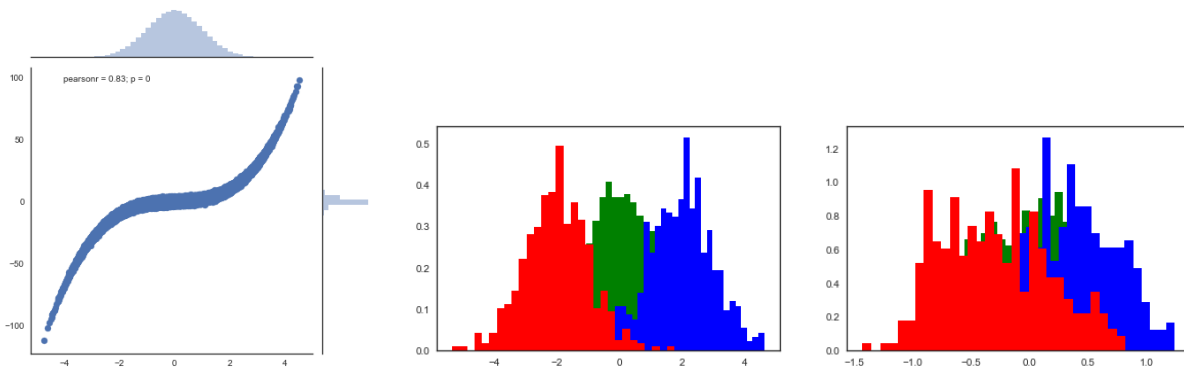
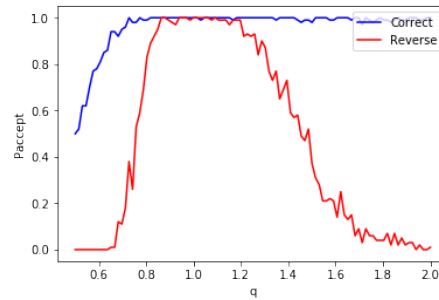


Figure 2: left picture is joint dist. of X and Y ; middle picture illustrate $P(Y|x)$; right picture illustrate $P(X|y)$

b

linear with range of q

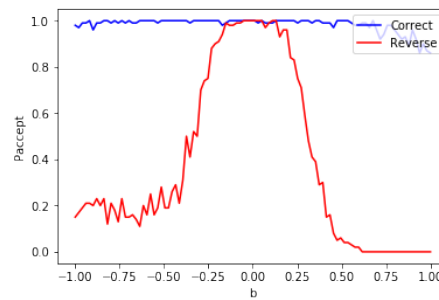
here is the plot.



this result adopt with our intuition because in linear model, the more Gaussian the noise become, the more probability to occur mistake to distinguish the right direction.

nonlinear with Gaussian noise

here is the plot.



this result adopt with our intuition, because with Gaussian noise, the more linear the model become, the more probability to occur mistake to distinguish the right direction.

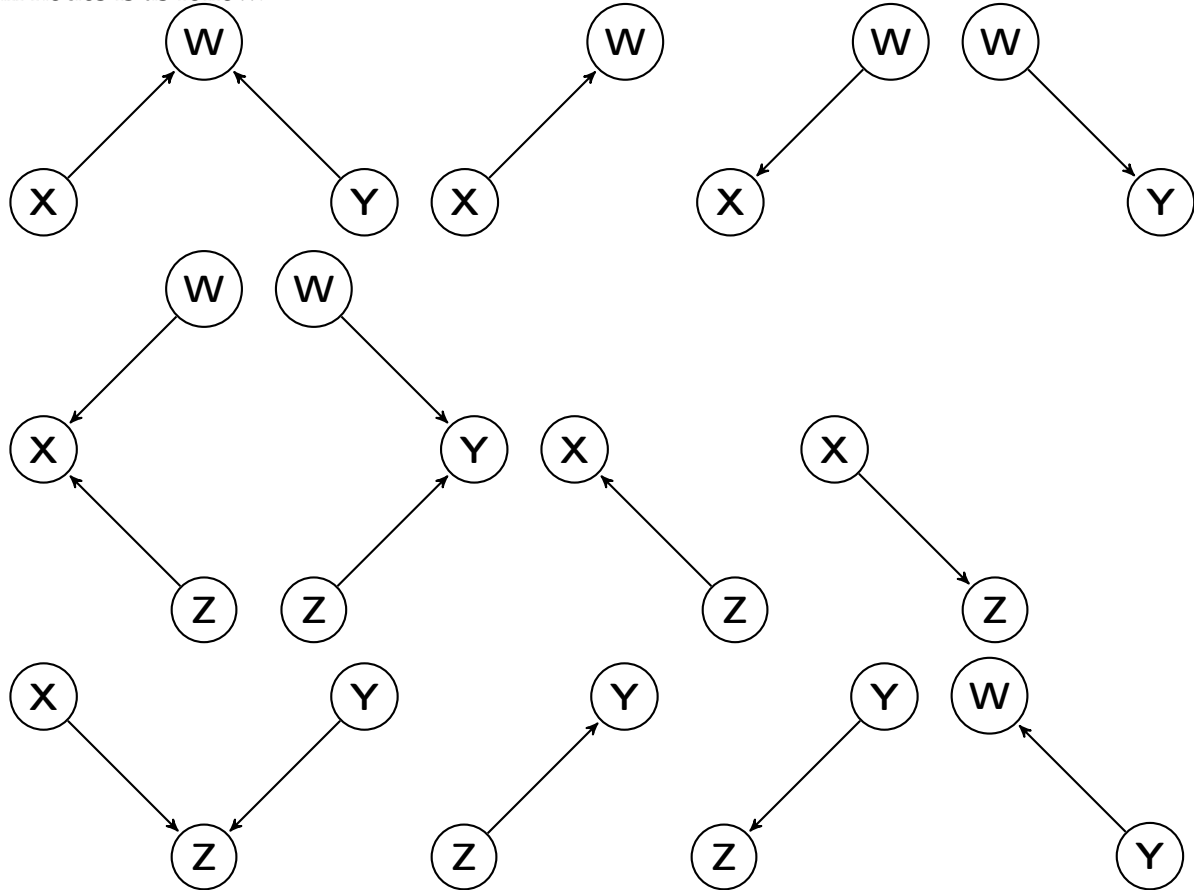
c

| | test stat | threshold |
|--|-----------|-----------|
| <i>eruption</i> - - - - > <i>waiting</i> | 0.52 | 0.78 |
| <i>waiting</i> - - - - > <i>eruption</i> | 1.60 | 0.67 |
| <i>Lengths</i> - - - - > <i>Rings</i> | 3.75 | 0.63 |
| <i>Rings</i> - - - - > <i>Lengths</i> | 5.05 | 0.62 |

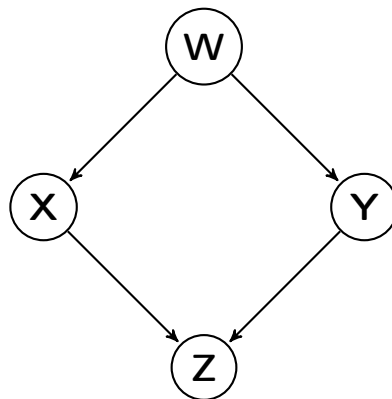
in faithful database we can infer *eruption* - - - - > *waiting* is the right direction, and from abalone database with SVR and Hsic we couldn't infer right direction, because hypothesis testing is rejected in both directions. but if we have to choose one of them, we will choose *Lengths* - - - - > *Rings*, because it is rejected easier than opposite direction in hypothesis testing.

Question 2

in this part we have to distinguish right directions on graph. for this purpose we applied SVR (a nonlinear Regression) and Hsic (a hypothesis test to check that whether input variables and residual are independent or not). in this graph there are 4 undirected line. totally there are 16 states to survey, but with few attention we can understand in 2 states we have cycle in graph. 12 possible conditions for all nodes is as follow.



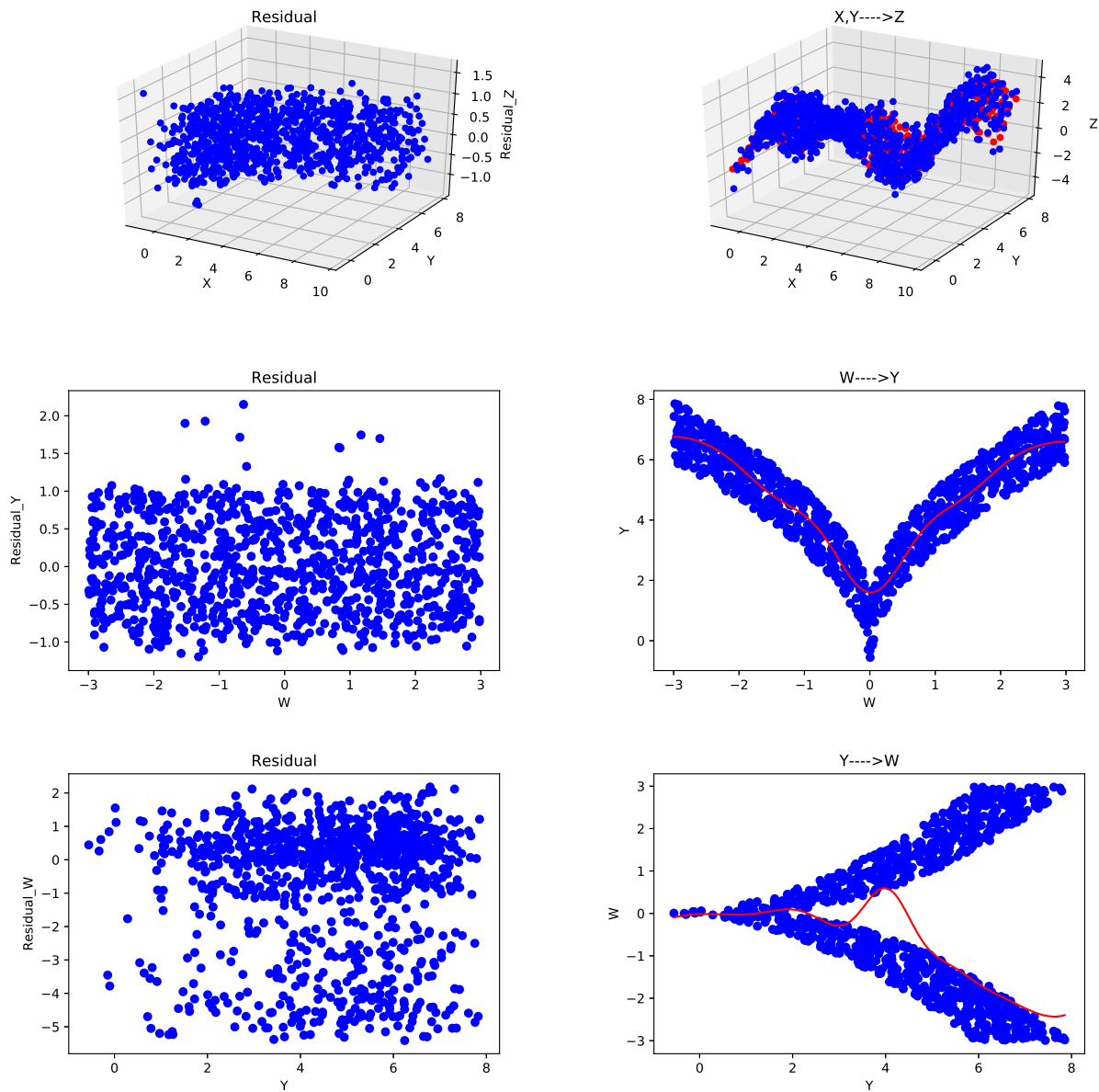
Hsic test couldn't reject null hypothesis in just three conditions(3,4 and 9). therefore we have :



| | test stat | threshold |
|-------------------------|-----------|-----------|
| $W \text{ --- } > Y$ | 0.45 | 0.76 |
| $W \text{ --- } > X$ | 0.11 | 0.77 |
| $X, Y \text{ --- } > Z$ | 0.19 | 0.70 |
| $W \text{ --- } > Z$ | 0.24 | 0.72 |

one more interesting thing here is, residual of Z is independent of W from the time we regress on W and also independent from X and Y when we regress on X and Y. this result adopt with our intuition.

few example plots for regression and residual.



Resources

- Nonlinear causal discovery with additive noise models, Patrik O. Hoyer et al.