Applied Cryptography and Network Security

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Lecture #5: AES
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University of Pittsburgh

Lecture Goals

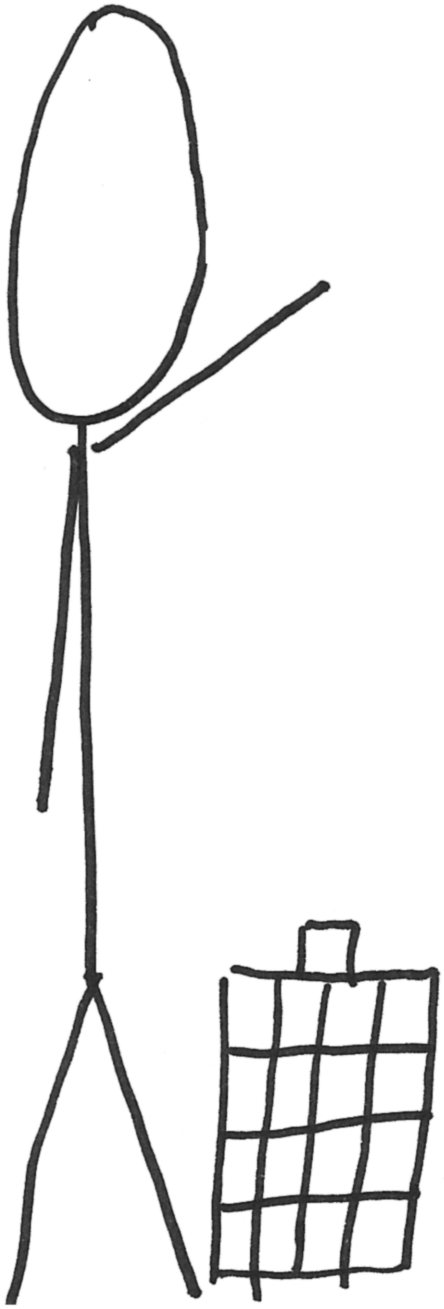
Develop a (brief) history of modern cryptography

Learn about the AES standardization process

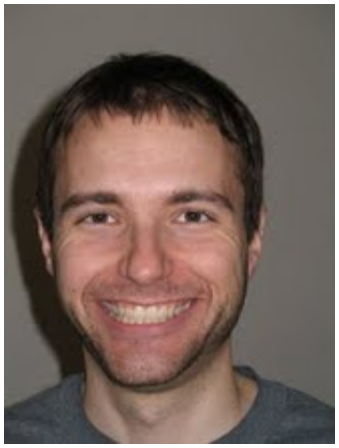
Understand AES:

- High level: Appreciate how AES utilizes confusion/diffusion
- Low level: Gain exposure to the mathematics behind AES

Appreciate that implementing something as complex as AES is a very non-trivial task



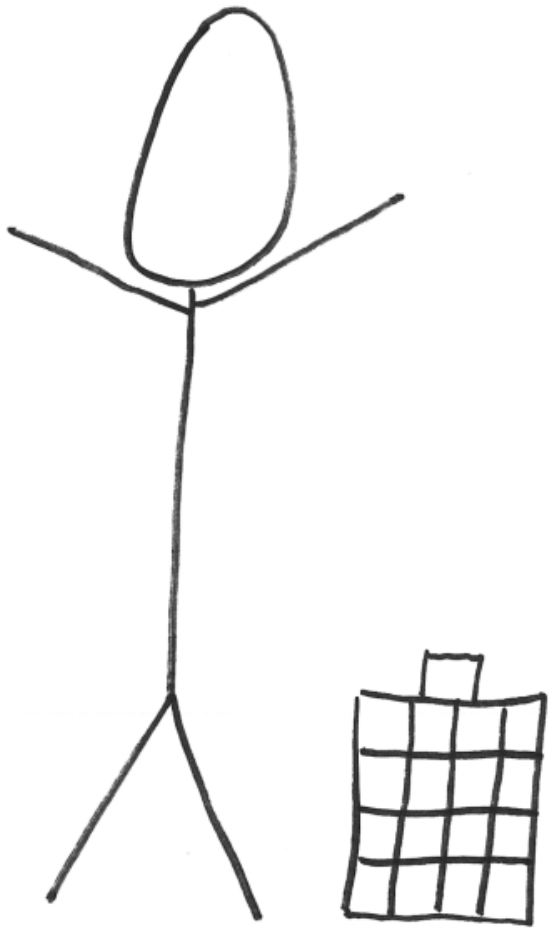
A Stick Figure Guide to the Advanced Encryption Standard (AES)



http://www.moserware.com/

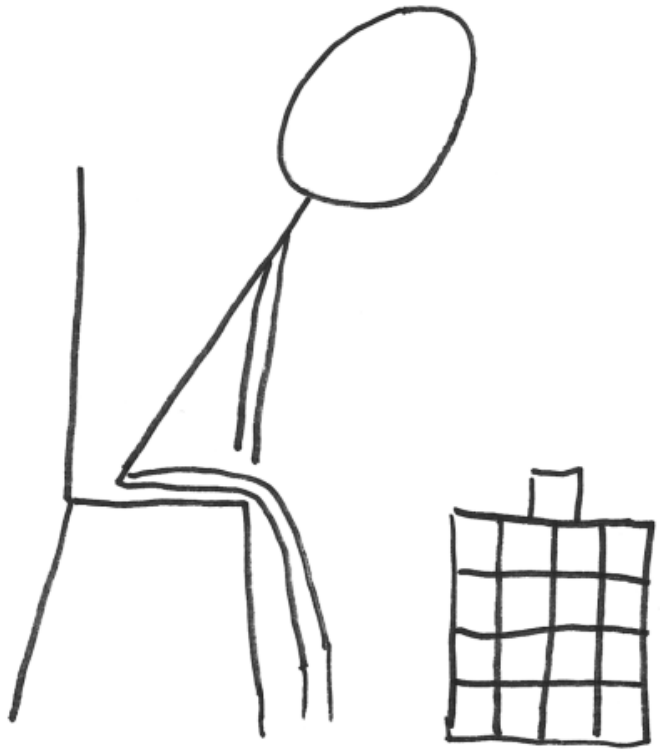
Act 1: Once Upon a Time…

I handle petabytes* of data every day. From encrypting juicy Top Secret intelligence to boring packets bound for your WiFi router, I do it all!

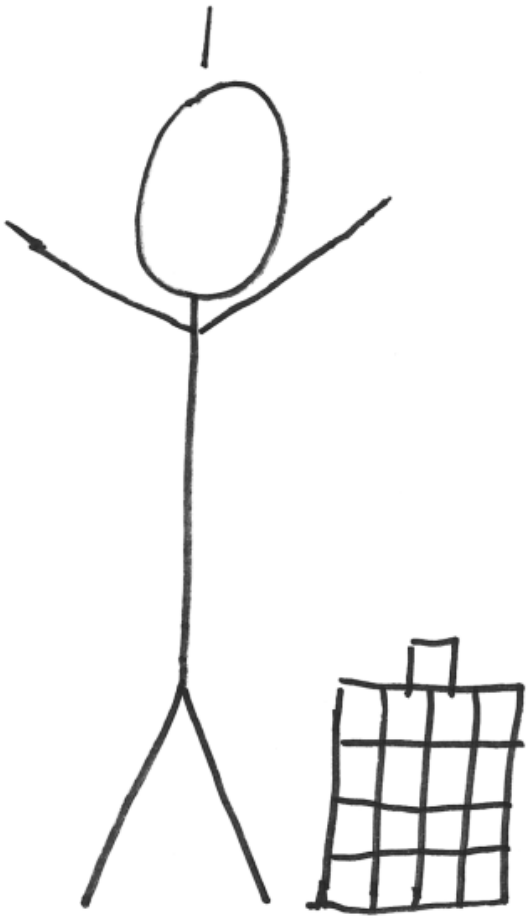


* 1 petabyte ≈ a lot

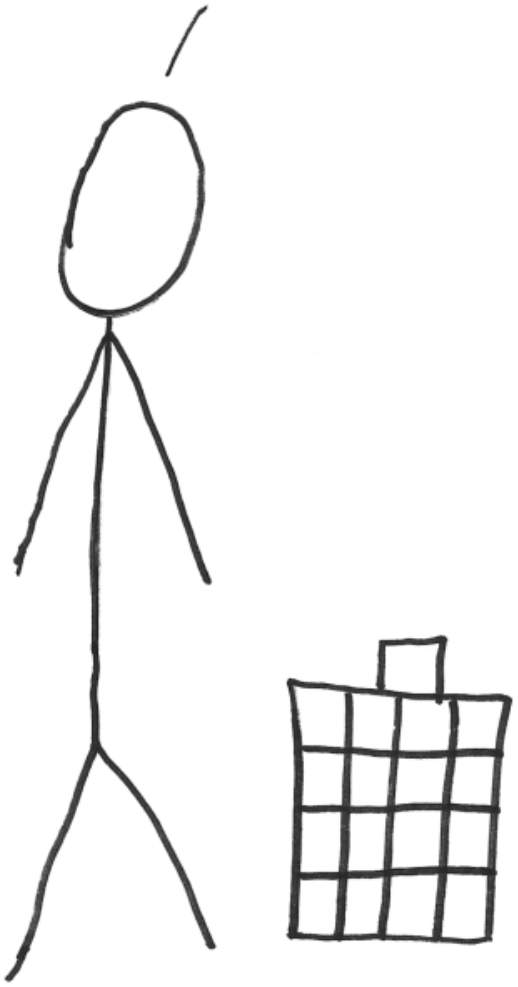
... and still no one seems to care about me or my story.



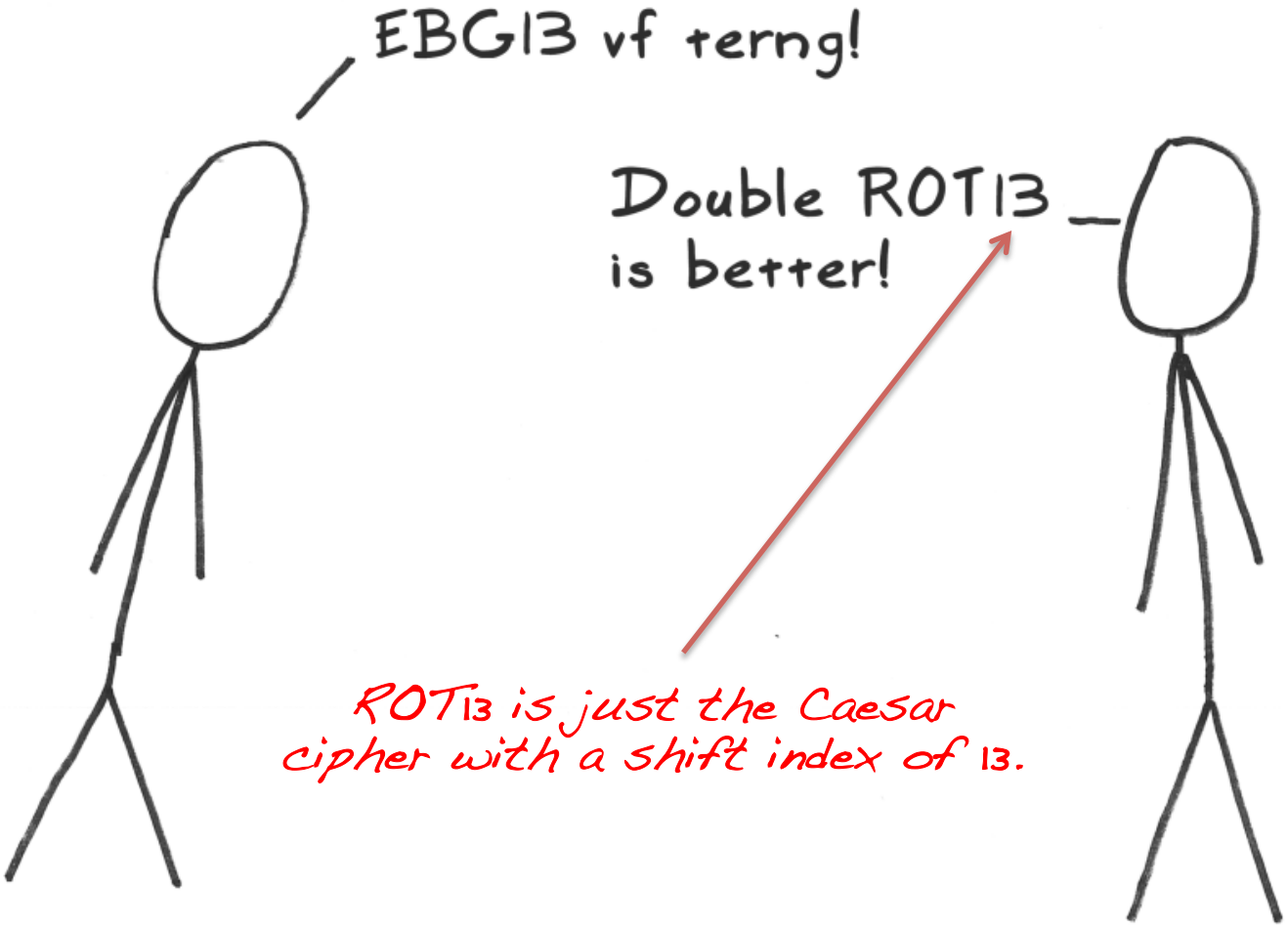
I've got a better-than-Cinderella story as I made my way to become king of the block cipher world.



Whoa! You're still there. You want to hear it? Well let's get started...



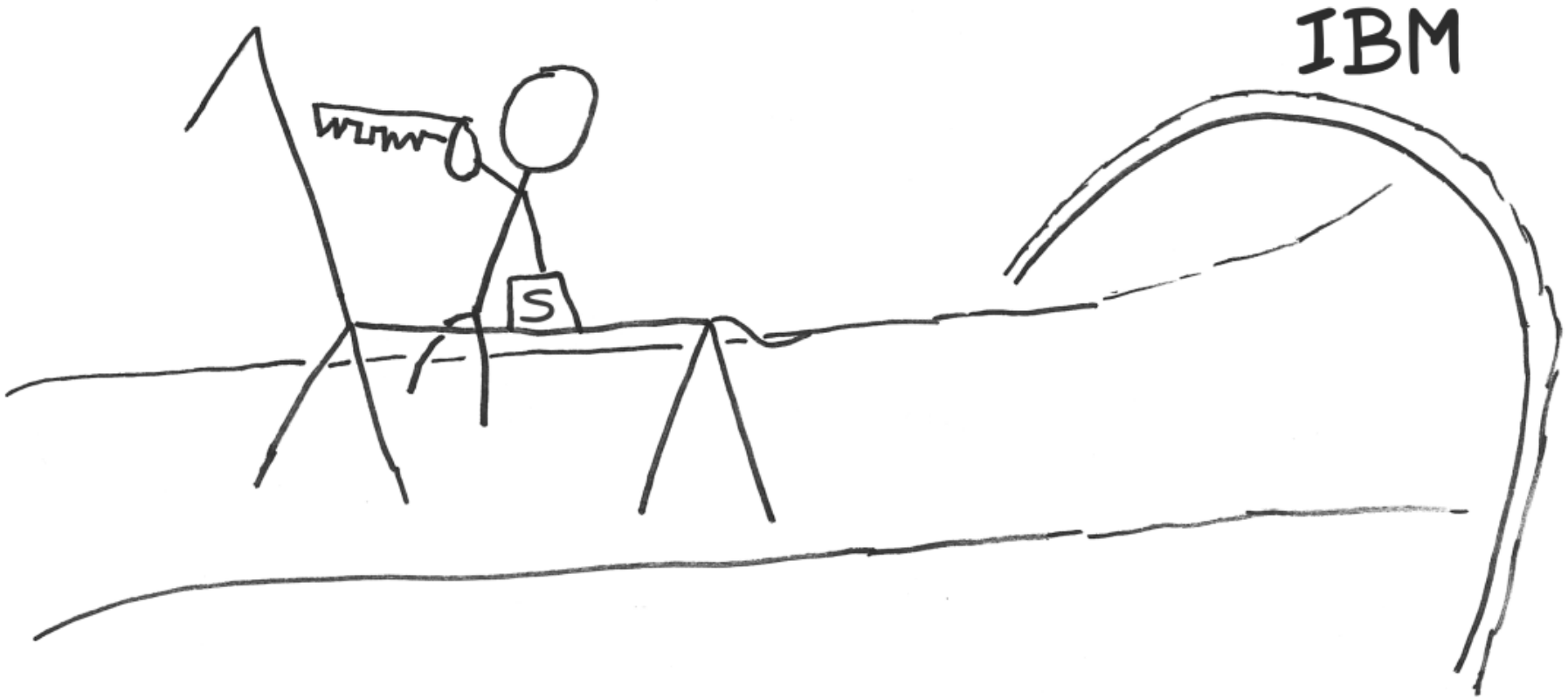
Once upon a time,* there was no good way for people outside secret agencies to judge good crypto.



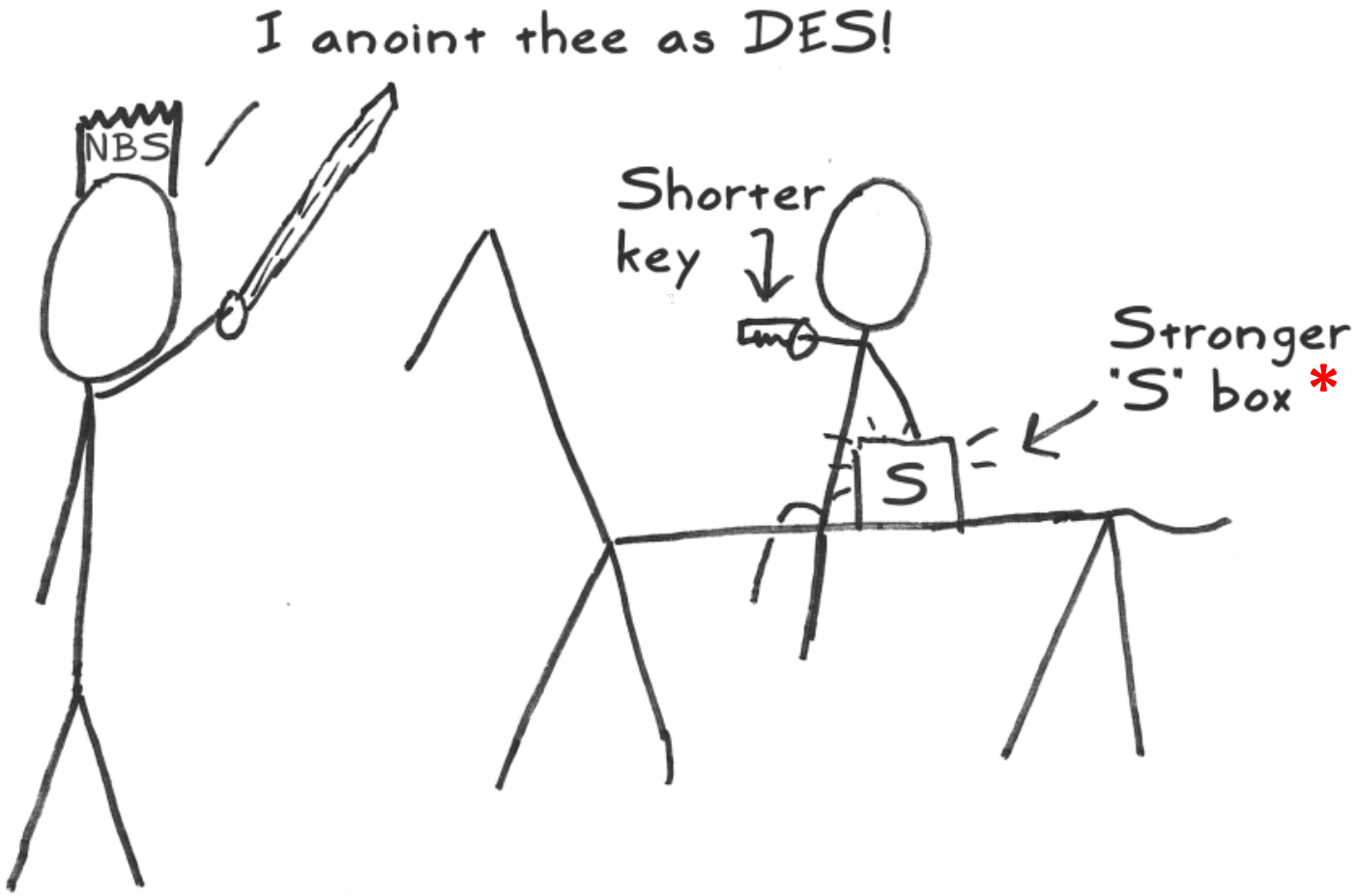
* ~ pre-1975 for the general public

A decree went throughout the land to find a good, secure, algorithm.

One worthy competitor named Lucifer came forward.



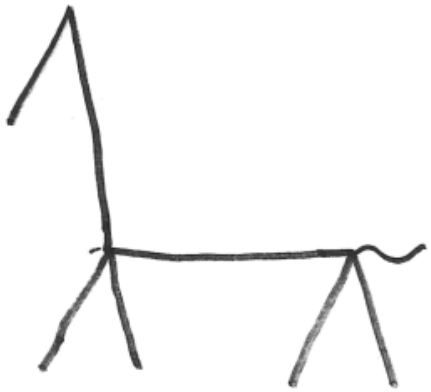
After being modified by the National Security Agency (NSA), he was anointed as the Data Encryption Standard (DES).



DES ruled in the land for over 20 years. Academics studied him intently. For the first time, there was something specific to look at. The modern field of cryptography was born.

"... to the best of our knowledge, DES is free from any statistical or mathematical weakness."

NSA

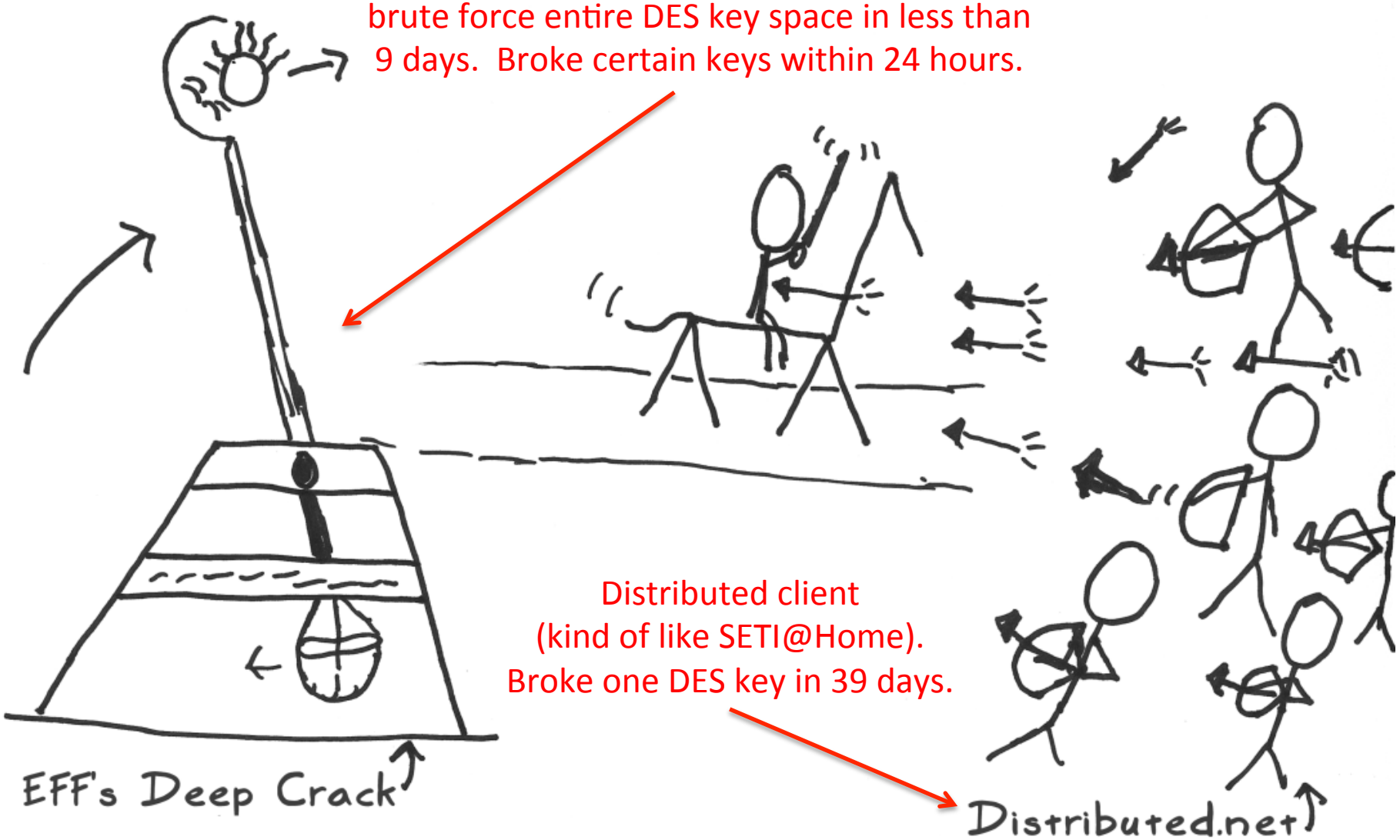


Check out that Feistel network!

Zzzz

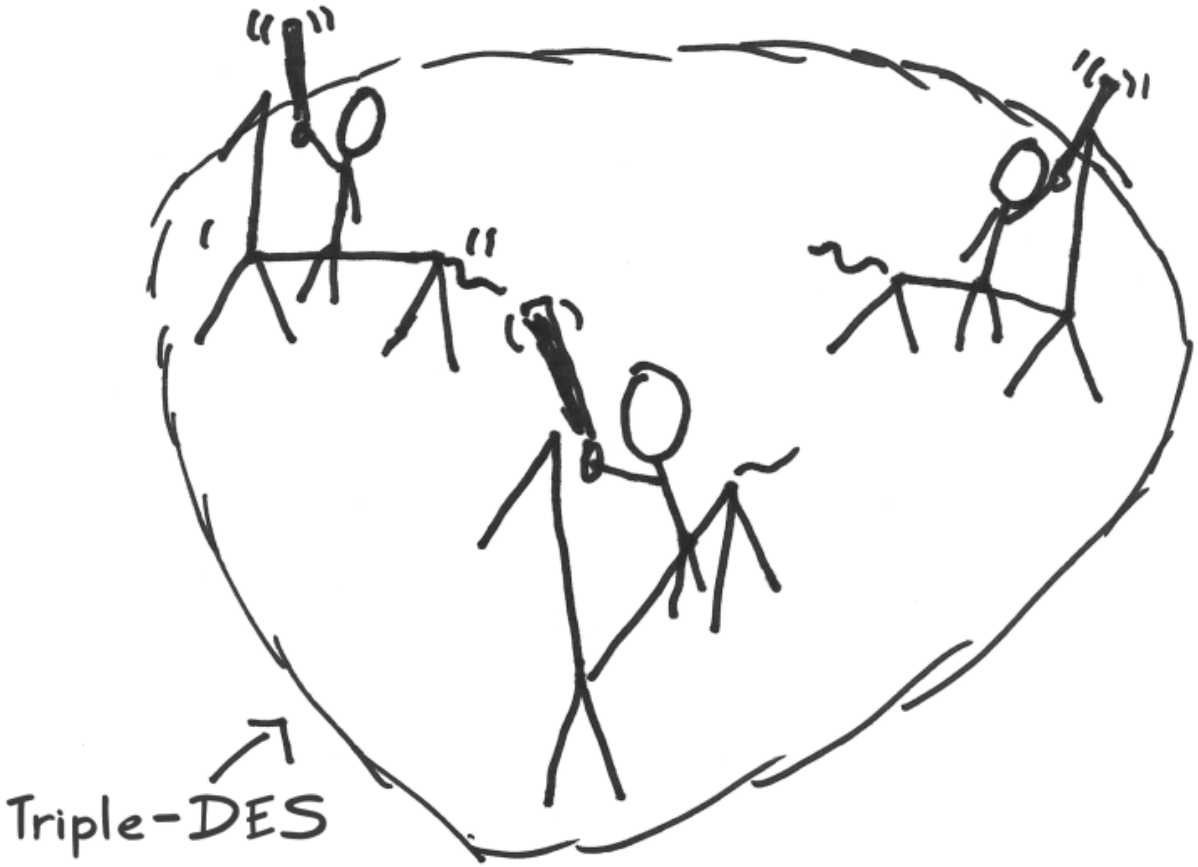
Over the years, many attackers challenged DES. He was defeated in several battles.

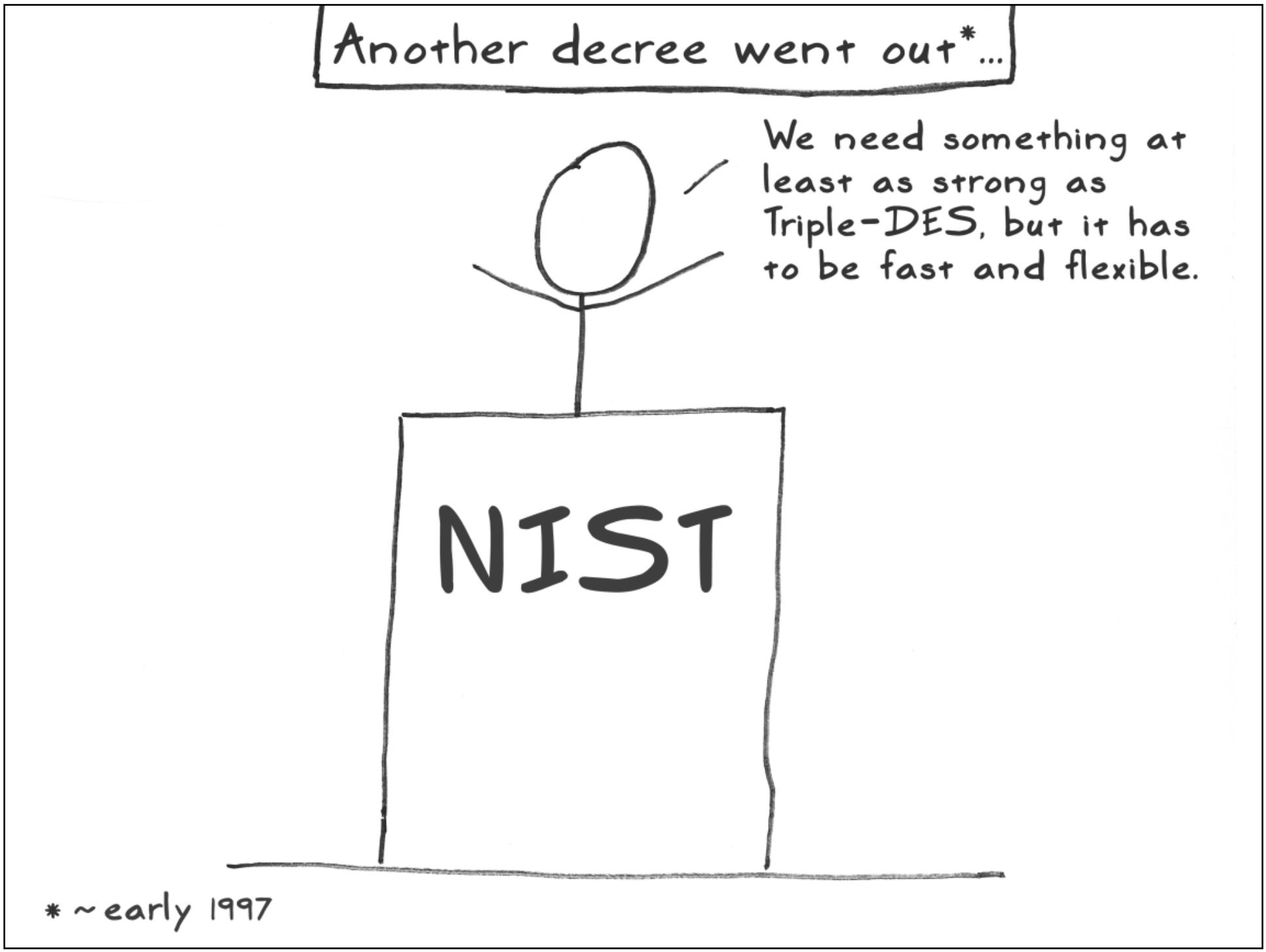
Built by the EFF for less than $250,000. 29 circuit boards with 64 chips in each. Could brute force entire DES key space in less than 9 days. Broke certain keys within 24 hours.



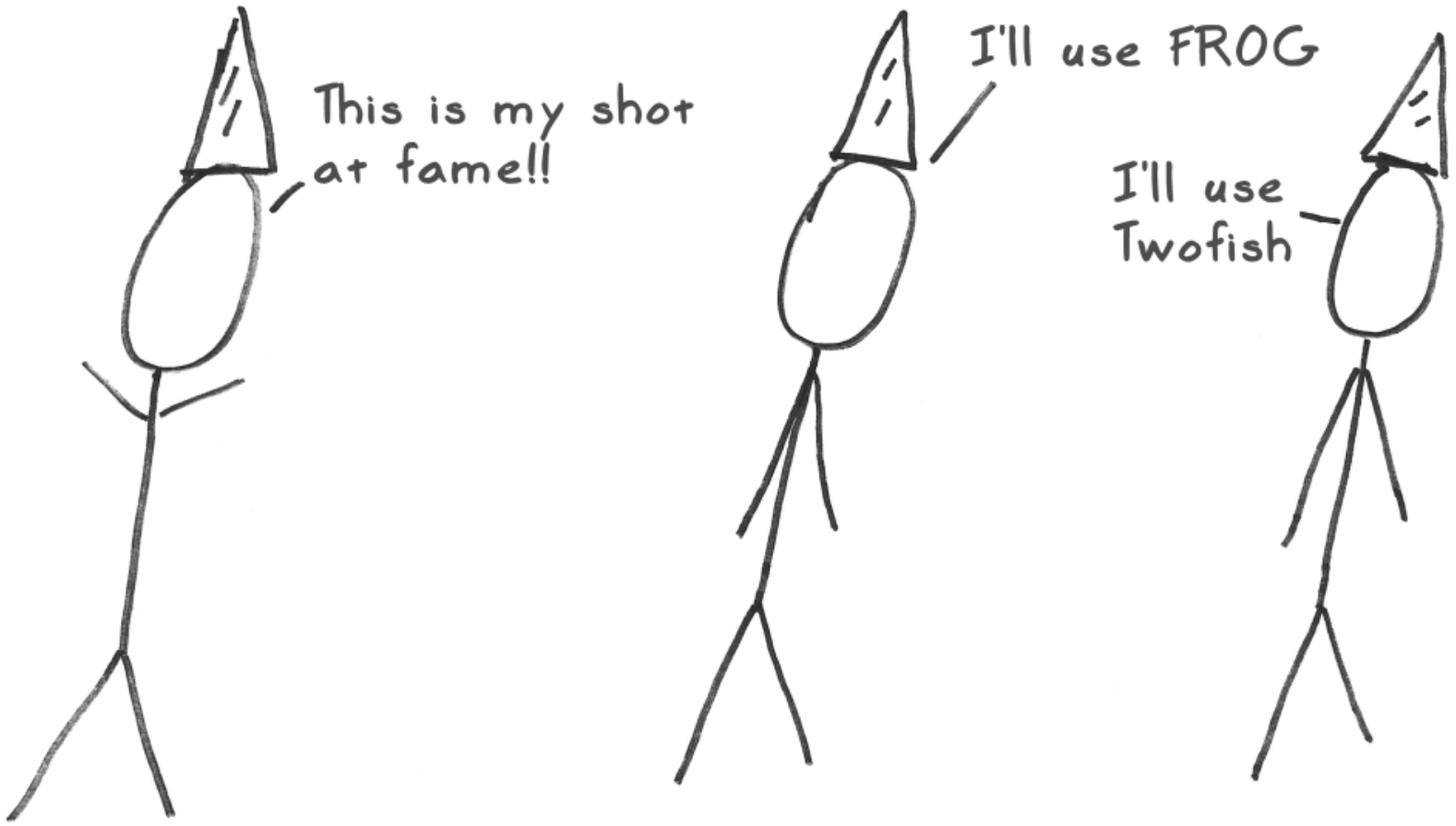
Distributed client (kind of like SETI@Home). Broke one DES key in 39 days.

The only way to stop the attacks was to use DES 3 times in row to form "Triple-DES." This worked. but it was awfully slow.

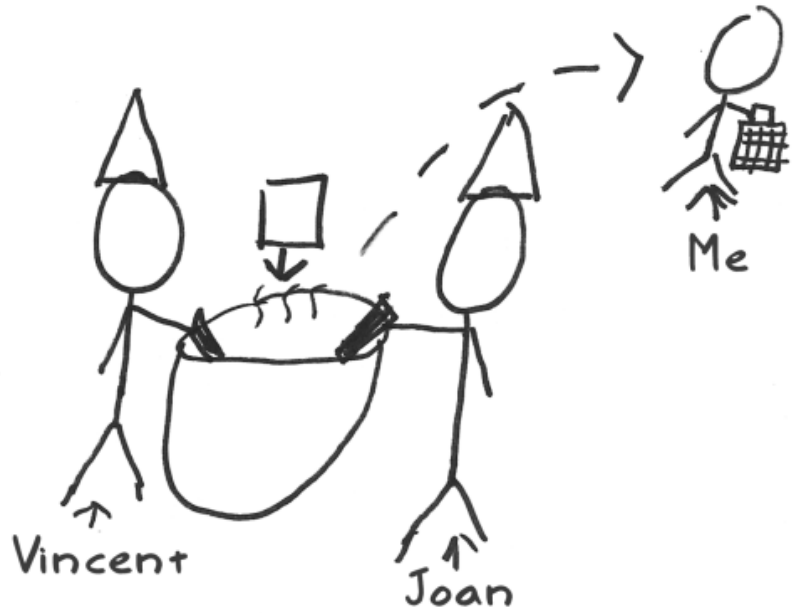



Another decree went out*...
We need something at least as strong as Triple-DES, but it has to be fast and flexible.
NIST
* ~early 1997

This call rallied the crypto wizards to develop something better.

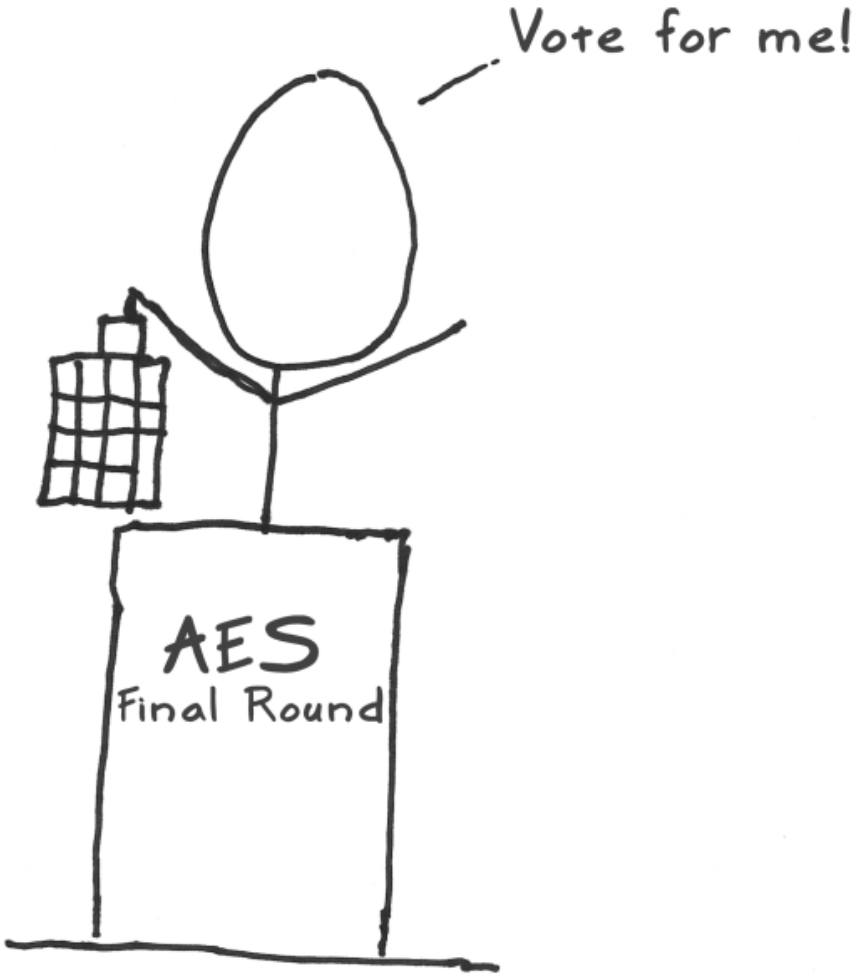


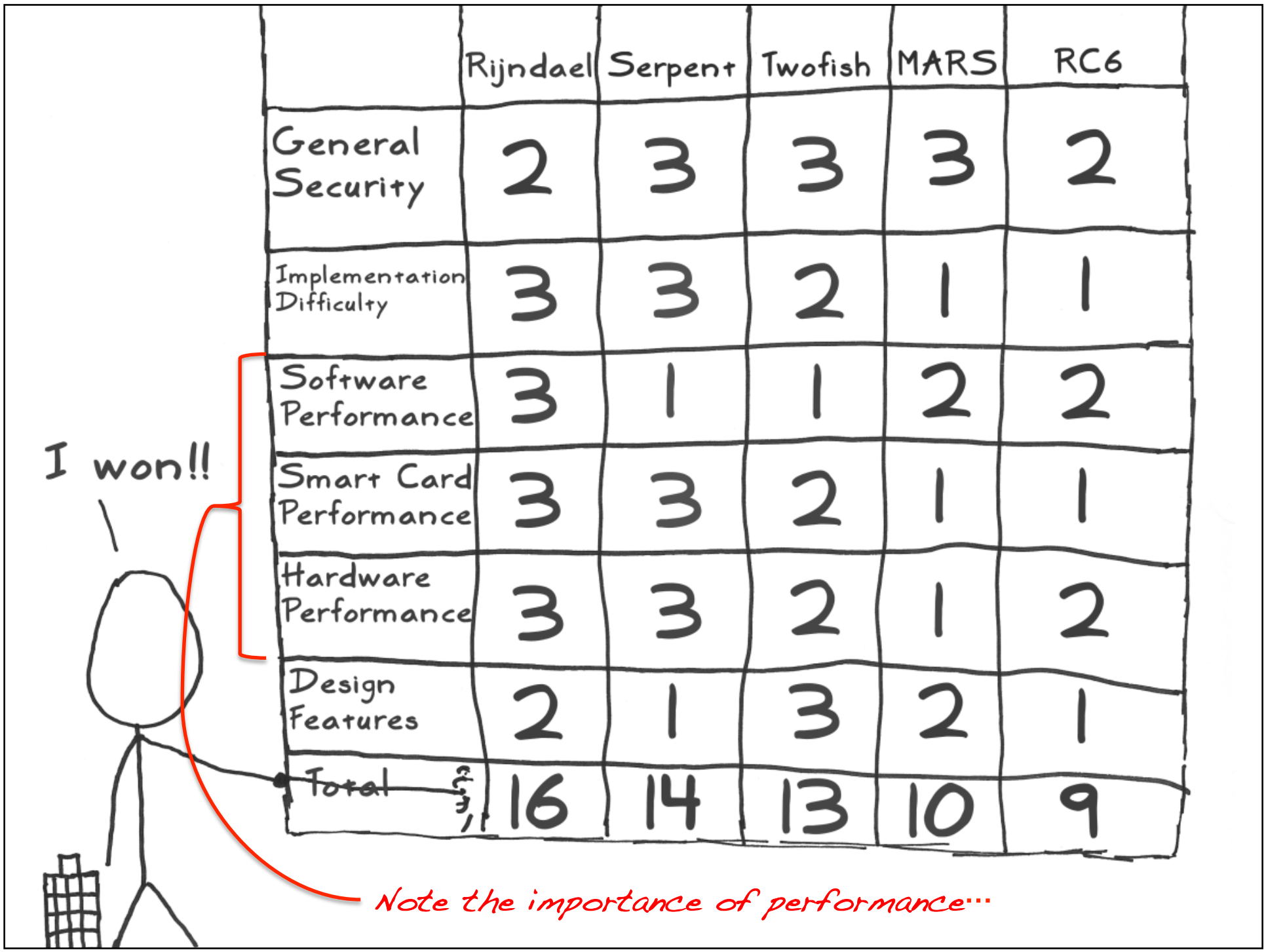
My creators, Vincent Rijmen and Joan Daemen, were among these crypto wizards. They combined their last names to give me my birth name: Rijndael.*

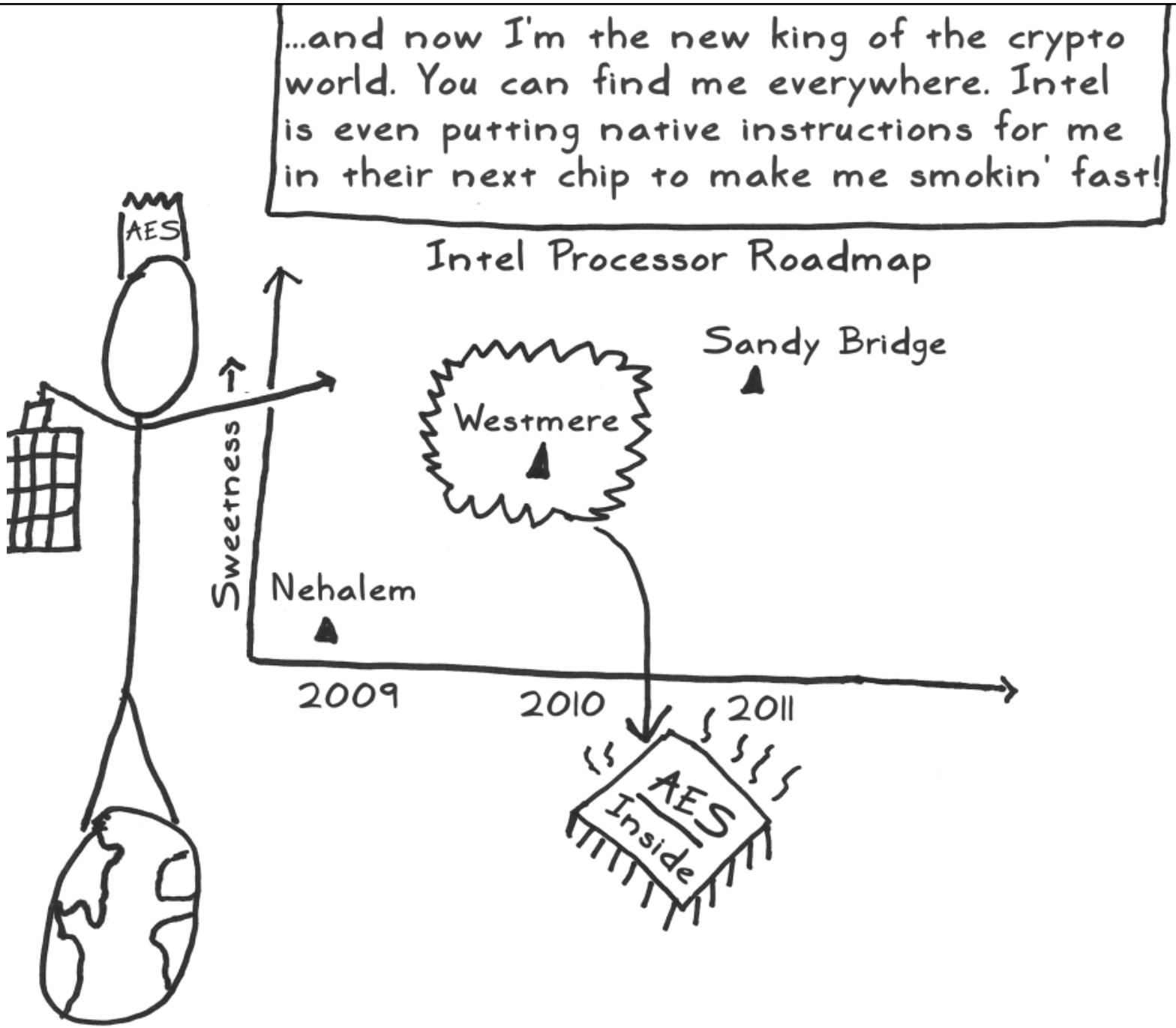


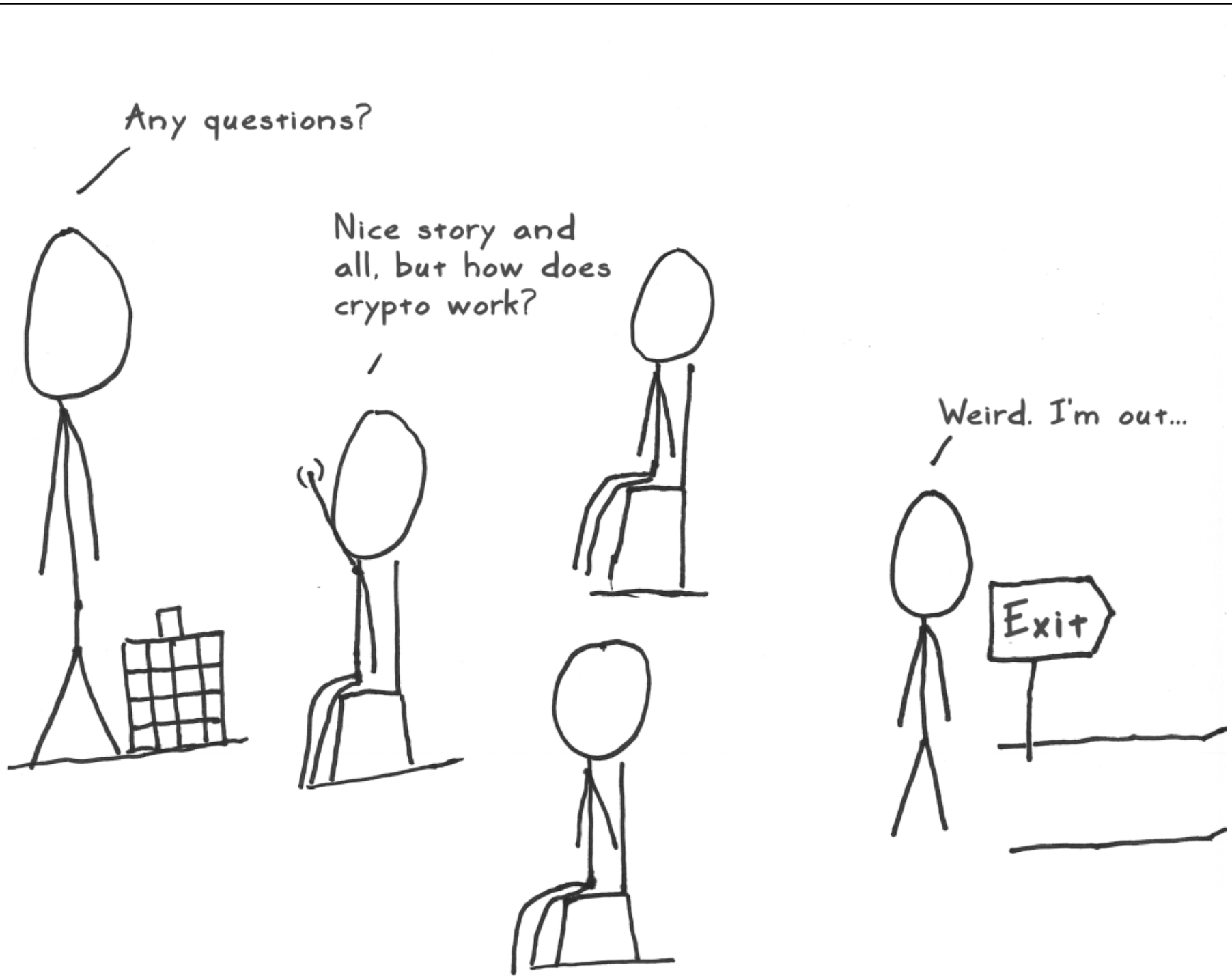
* That's pronounced "Rhine Dahl" for the non-Belgians out there.

Everyone got together to vote and...

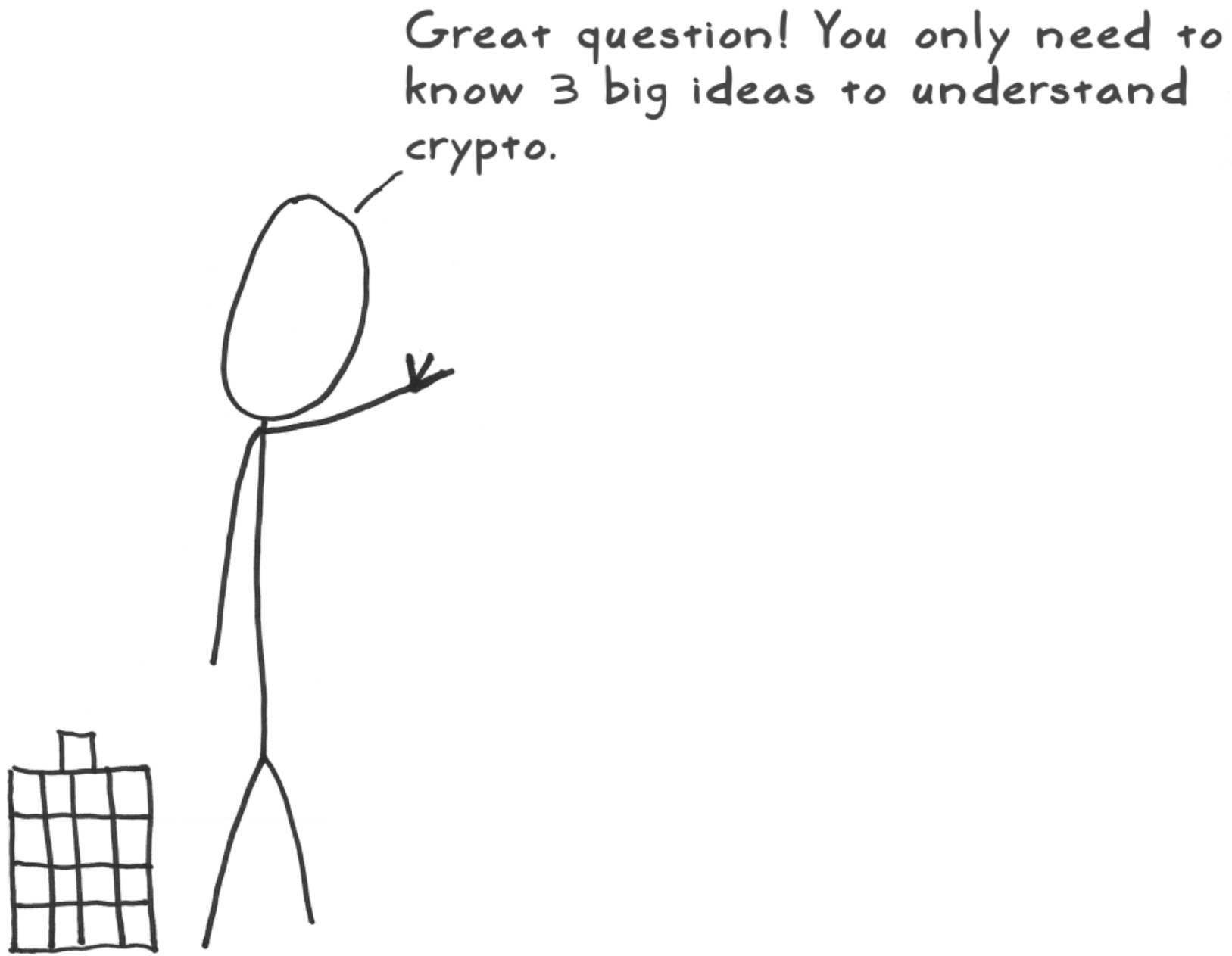



I won!!
Rijndael
Serpent
Twofish
MARS
RC6
General Security
2
3
3
3
2
Implementation Difficulty
3
3
2
1
1
Software Performance
3
1
1
2
2
Smart Card Performance
3
3
2
1
1
Hardware Performance
3
3
2
1
2
Design Features
2
1
3
2
1
Total
16
14
13
10
9
Note the importance of performance…


...and now I'm the new king of the crypto world. You can find me everywhere. Intel is even putting native instructions for me in their next chip to make me smokin' fast!
AES
Intel Processor Roadmap
Sandy Bridge
Westmere
Sweetness
Nehalem
2009
2010
2011
AES Inside

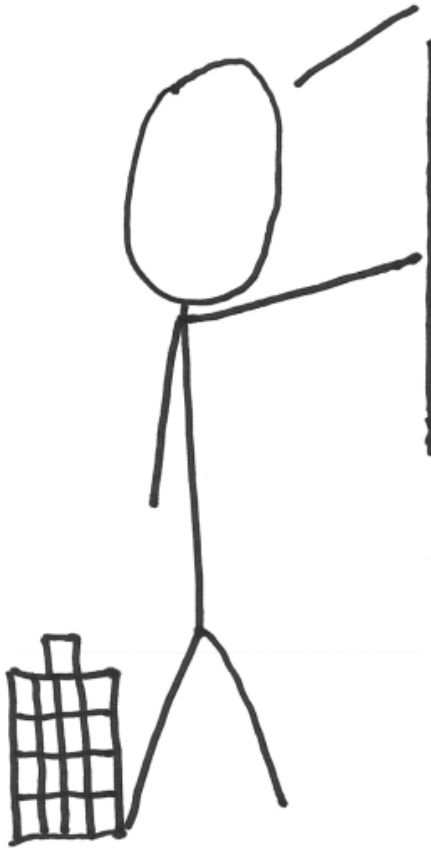

Any questions?
Nice story and all, but how does crypto work?
Weird. I'm out...
Exit

Act 2: Crypto Basics


Great question! You only need to know 3 big ideas to understand crypto.

Big Idea #1: Confusion

It's a good idea to obscure the relationship between your real message and your "encrypted" message. An example of this "confusion" is the trusty ol' Caesar Cipher:



Plaintext: ATTACK AT DAWN

↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓ ↓

Ciphertext: DWWDFN DW GDZQ

A + 3 letters = D

I told you that this would eventually be useful…

Big Idea #2: Diffusion

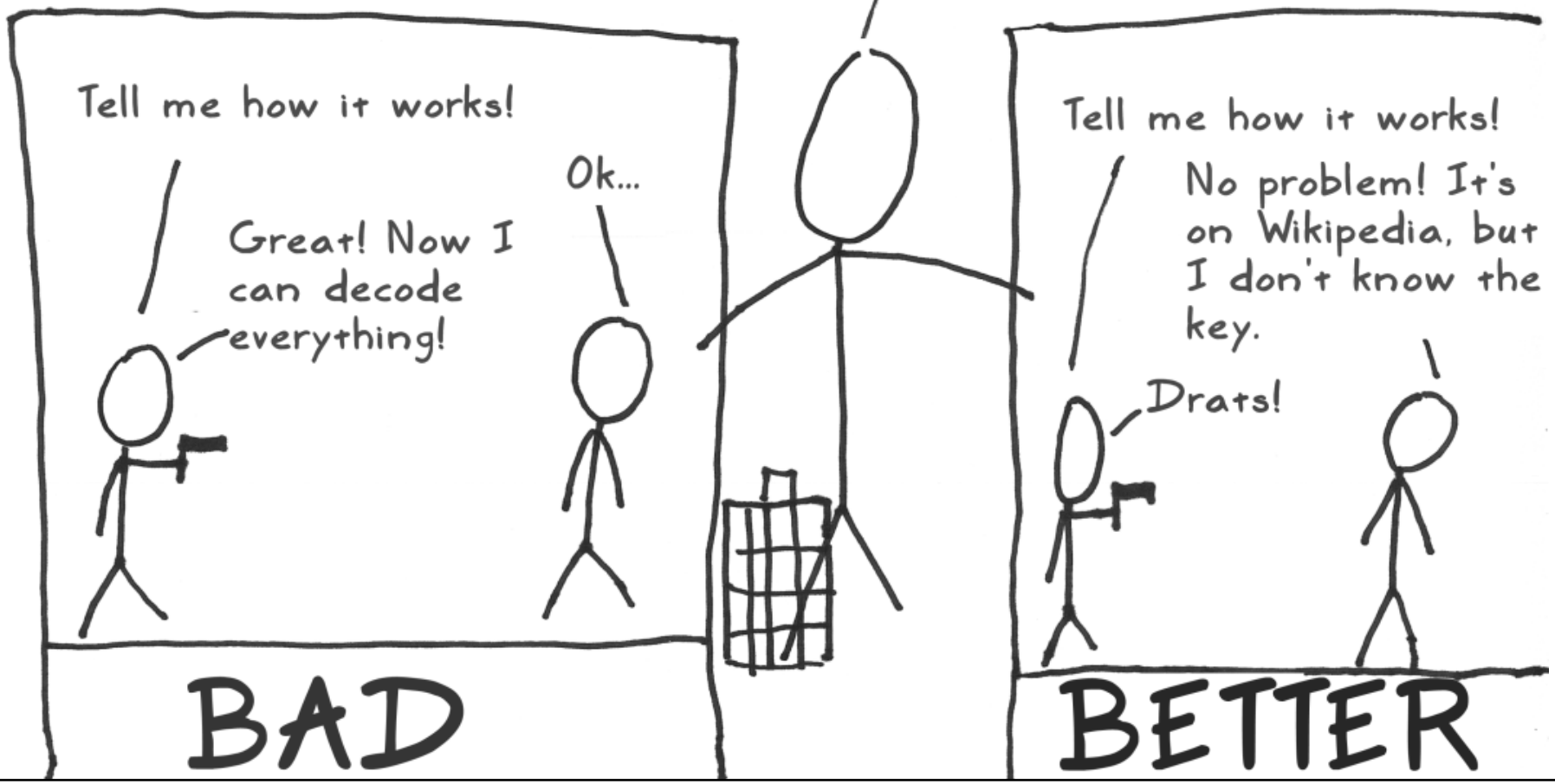
It's also a good idea to spread out the message. An example of this "diffusion" is a simple column transposition:

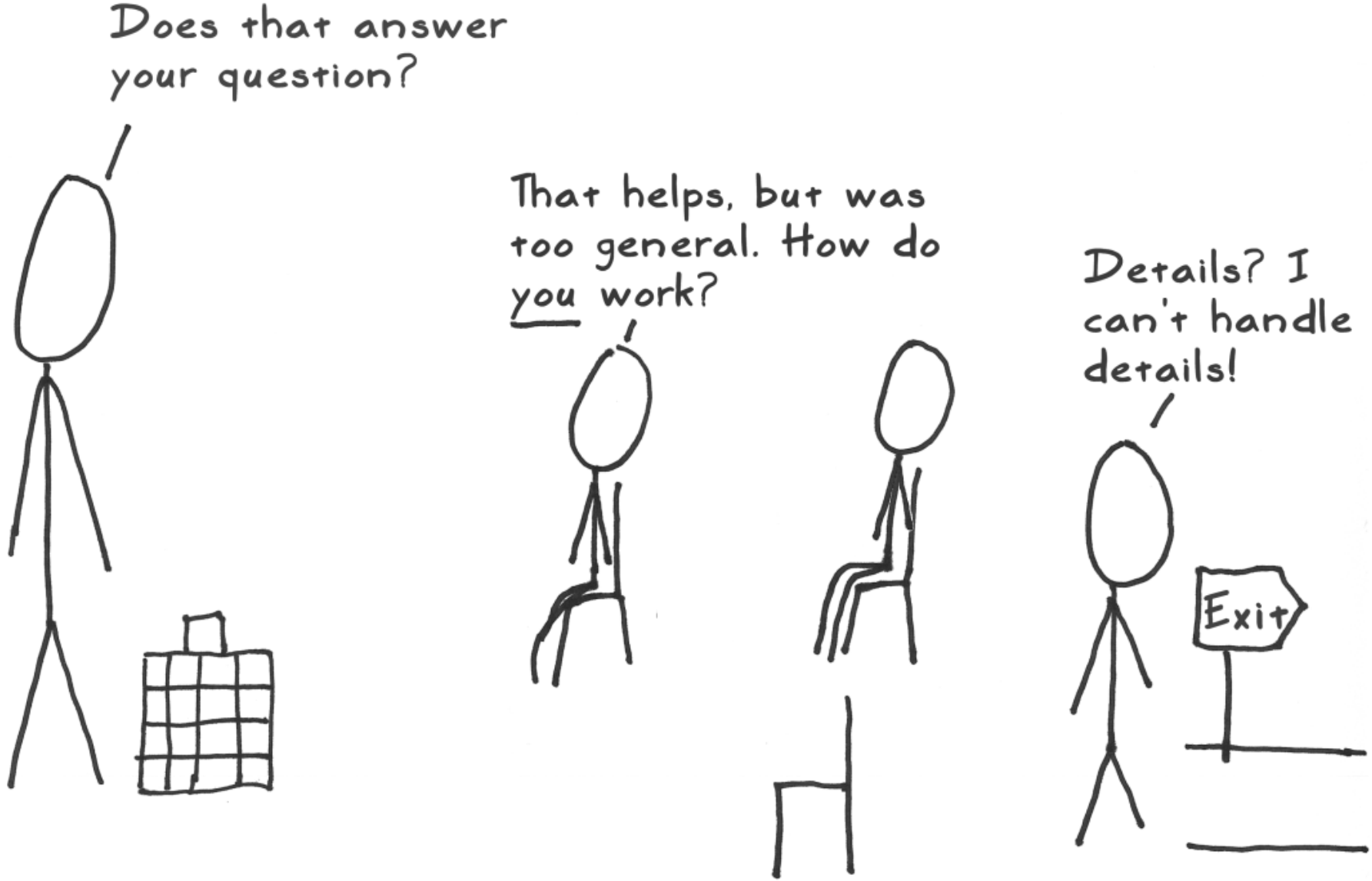
This is the "rail fence" cipher that we talked about last week

Big Idea #3: Secrecy Only in the Key

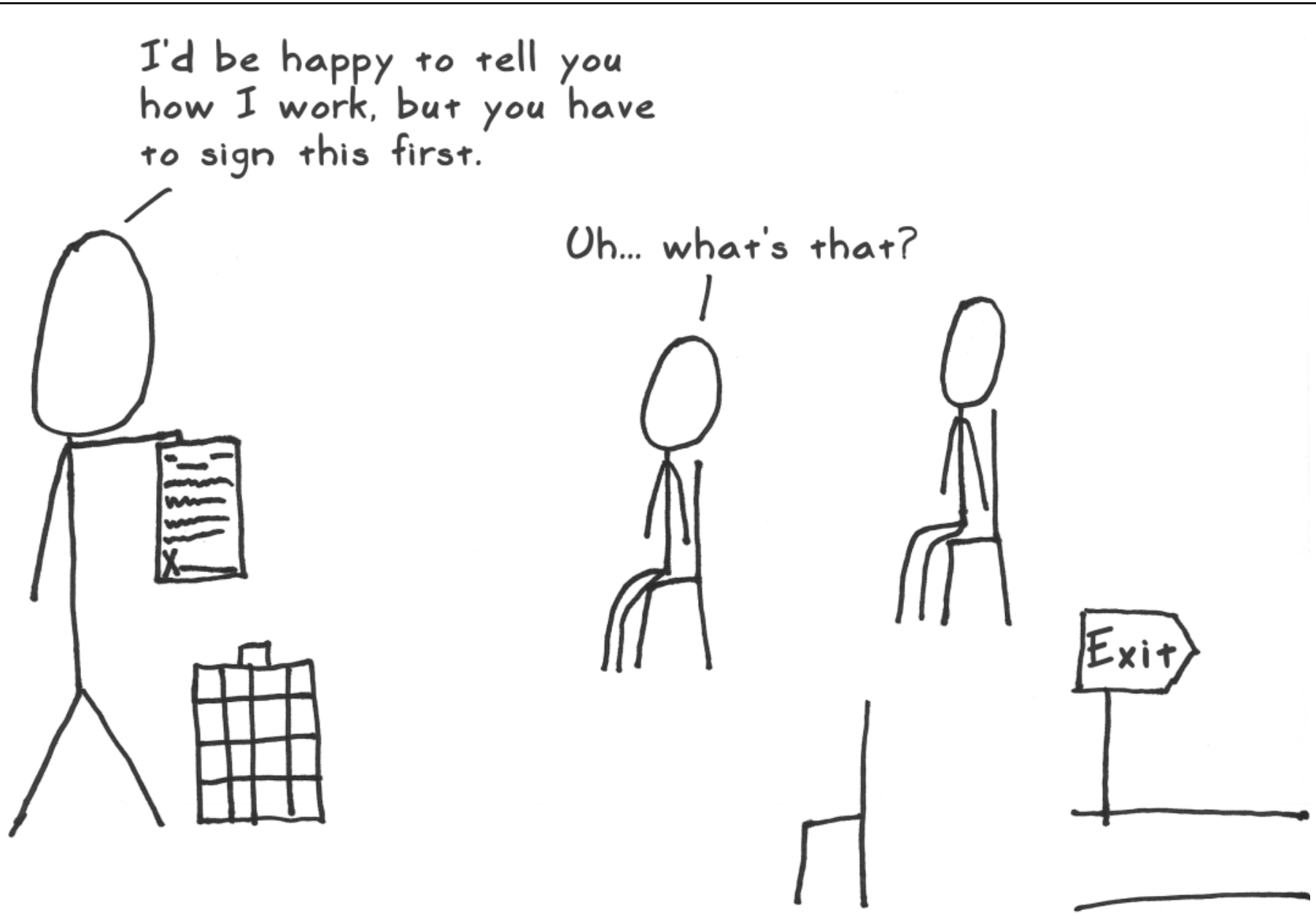
Kerchoffs' principle!

After thousands of years, we learned that it's a bad idea to assume that no one knows how your method works. Someone will eventually find that out.




Does that answer your question?
That helps, but was too general. How do you work?
Details? I can't handle details!
Exit

Act 3: Details

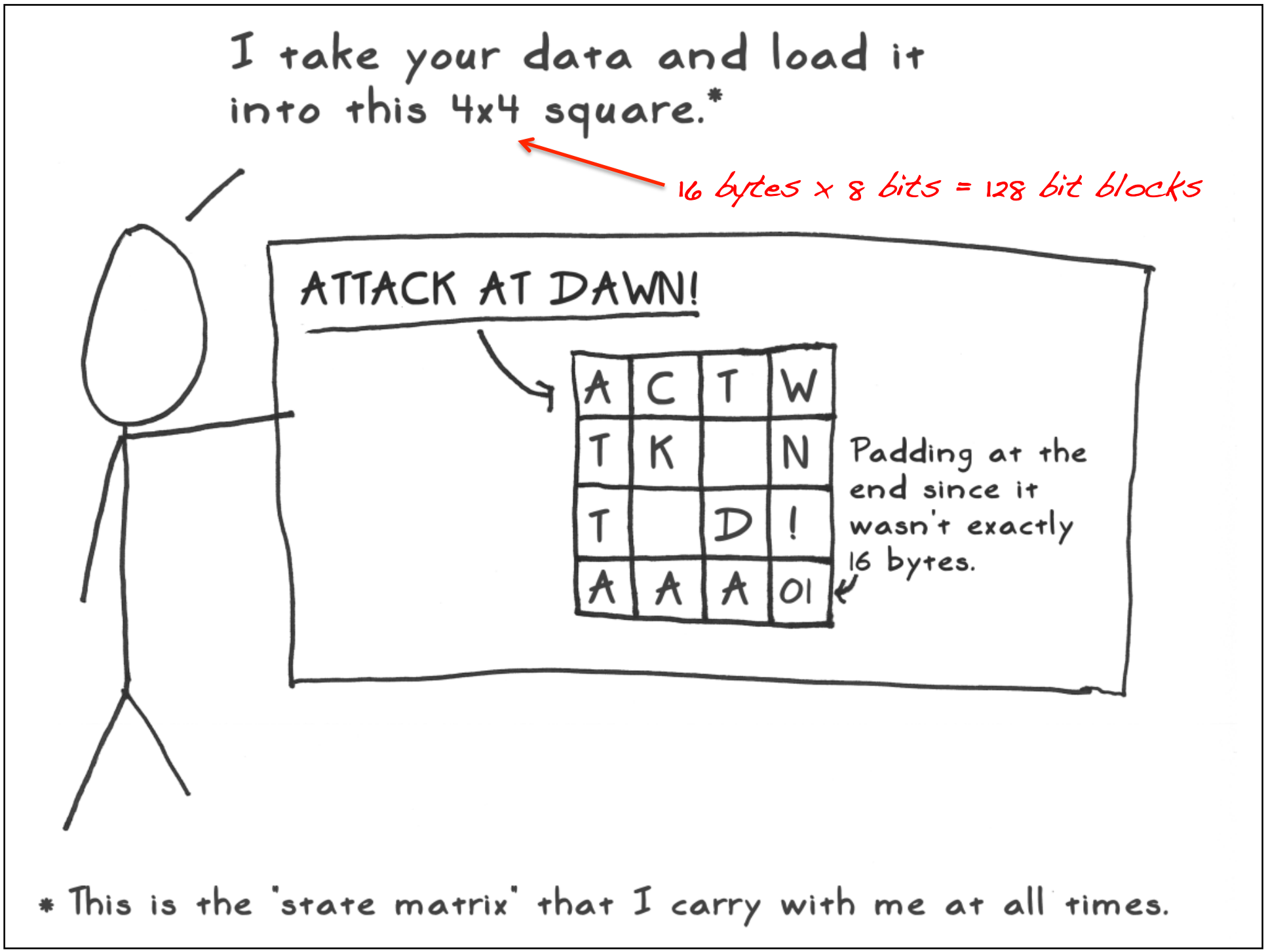

I'd be happy to tell you how I work, but you have to sign this first.
Uh... what's that?
Exit

Foot-Shooting Prevention Agreement

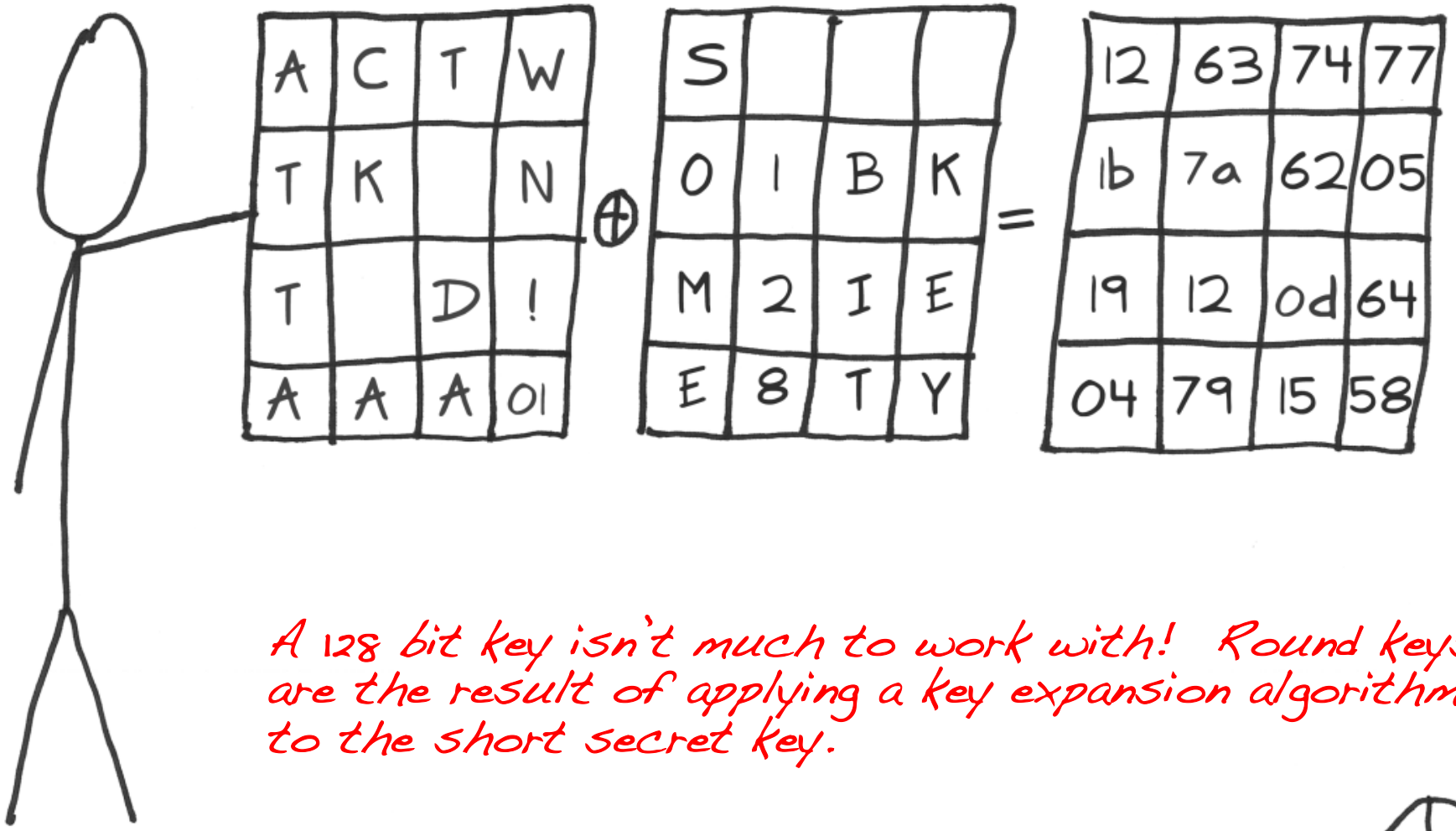
I, _____ , promise that once
Your Name
I see how simple AES really is, I will not implement it in production code even though it would be really fun.

This agreement shall be in effect until the undersigned creates a meaningful interpretive dance that compares and contrasts cache-based, timing, and other side channel attacks and their countermeasures.

X _____ _____
Signature Date


I take your data and load it into this 4x4 square.*
16 bytes x 8 bits = 128 bit blocks
ATTACK AT DAWN!
A C T W
T K N
T D !
A A A 01
Padding at the end since it wasn't exactly 16 bytes.
* This is the 'state matrix' that I carry with me at all times.

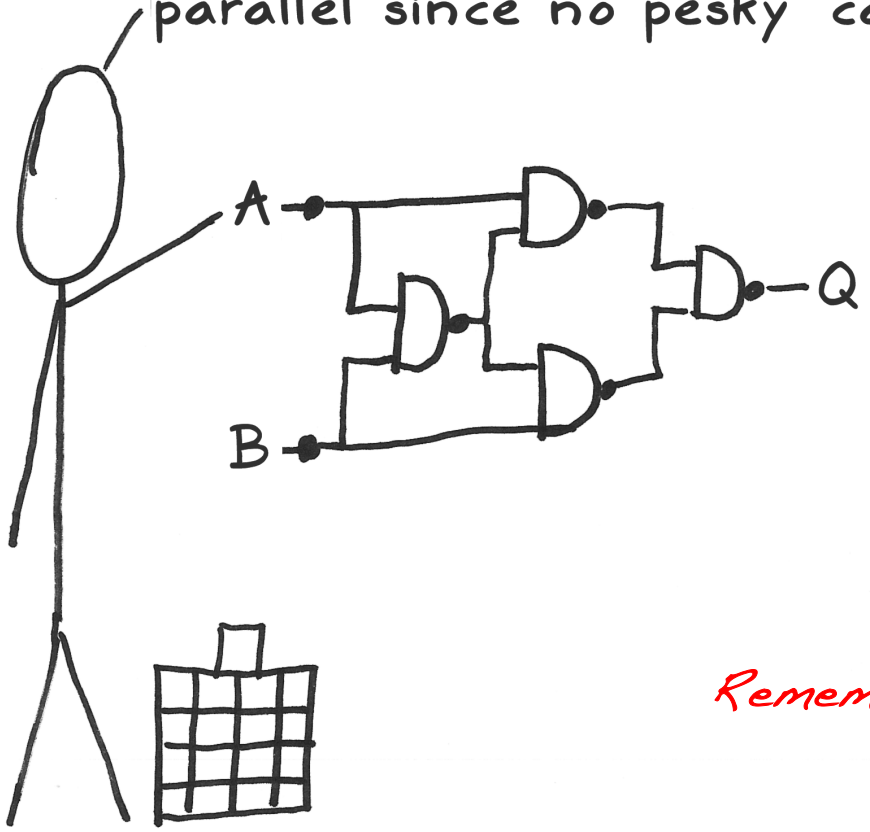
The initial round has me xor each input byte with the corresponding byte of the first round key.



A 128 bit key isn't much to work with! Round keys are the result of applying a key expansion algorithm to the short secret key.

A Tribute to XOR

There's a simple reason why I use xor to apply the key and in other spots: it's fast and cheap – a quick bit flipper. It uses minimal hardware and can be done in parallel since no pesky "carry" bits are needed.

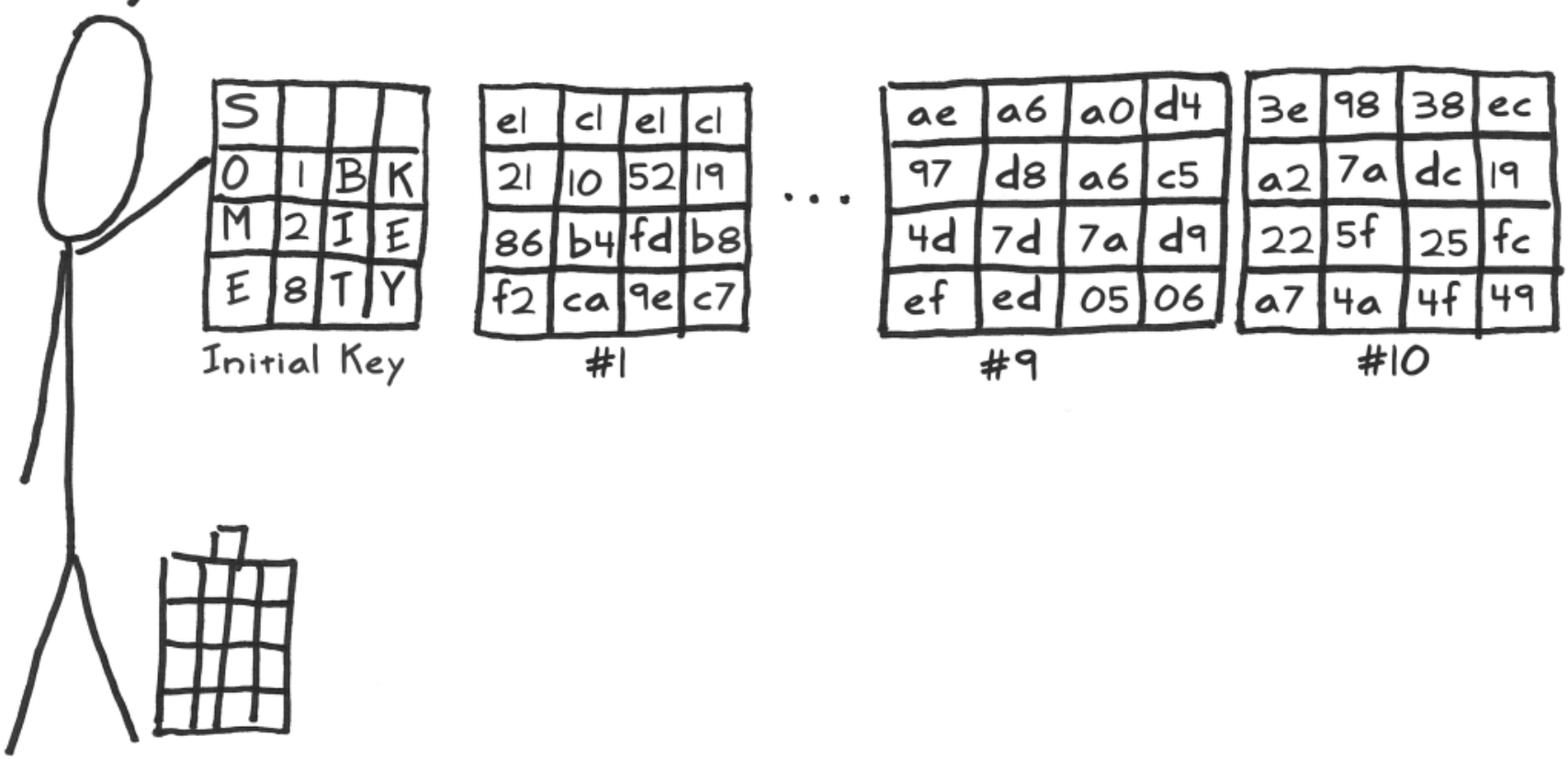


AES ♡ ⊕

Remember the performance focus of AES?

Key Expansion: Part 1

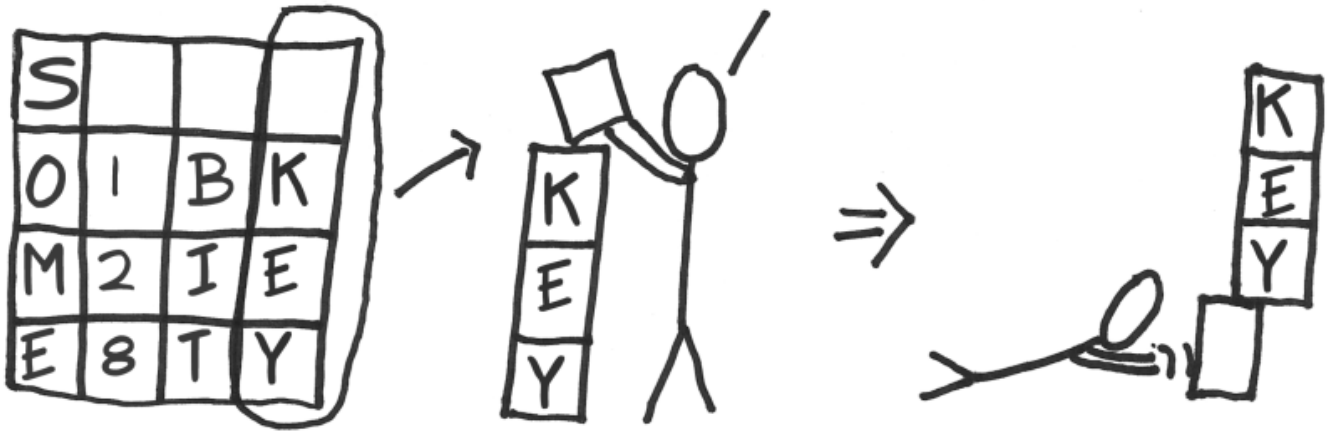
I need lots of keys for use in later rounds. I derive all of them from the initial key using a simple mixing technique that's really fast. Despite its critics,* it's good enough.



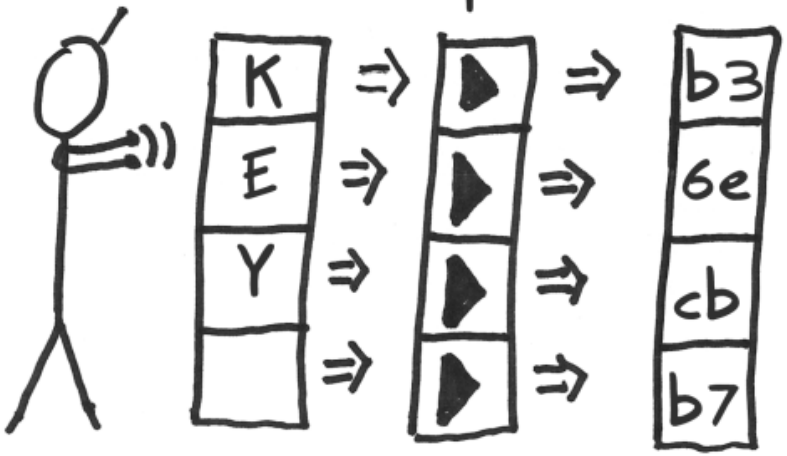
* By far, most complaints against AES's design focus on this simplicity.

Key Expansion: Part 2a

(1) I take the last column of the previous round key and move the top byte to the bottom:

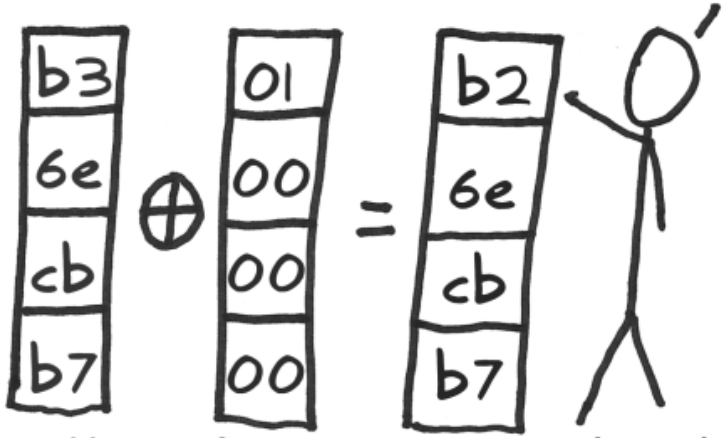


(2) Next, I run each byte through a substitution box that will map it to something else:

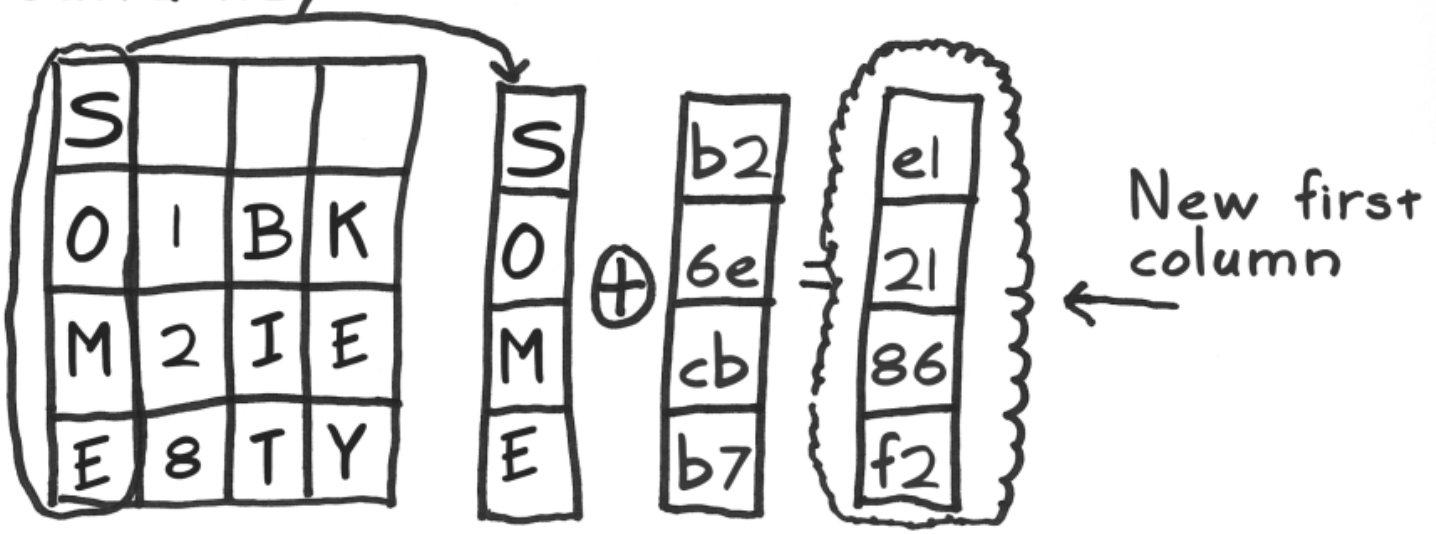


Key Expansion: Part 2b

(3) I then xor the column with a "round constant" that is different for each round.

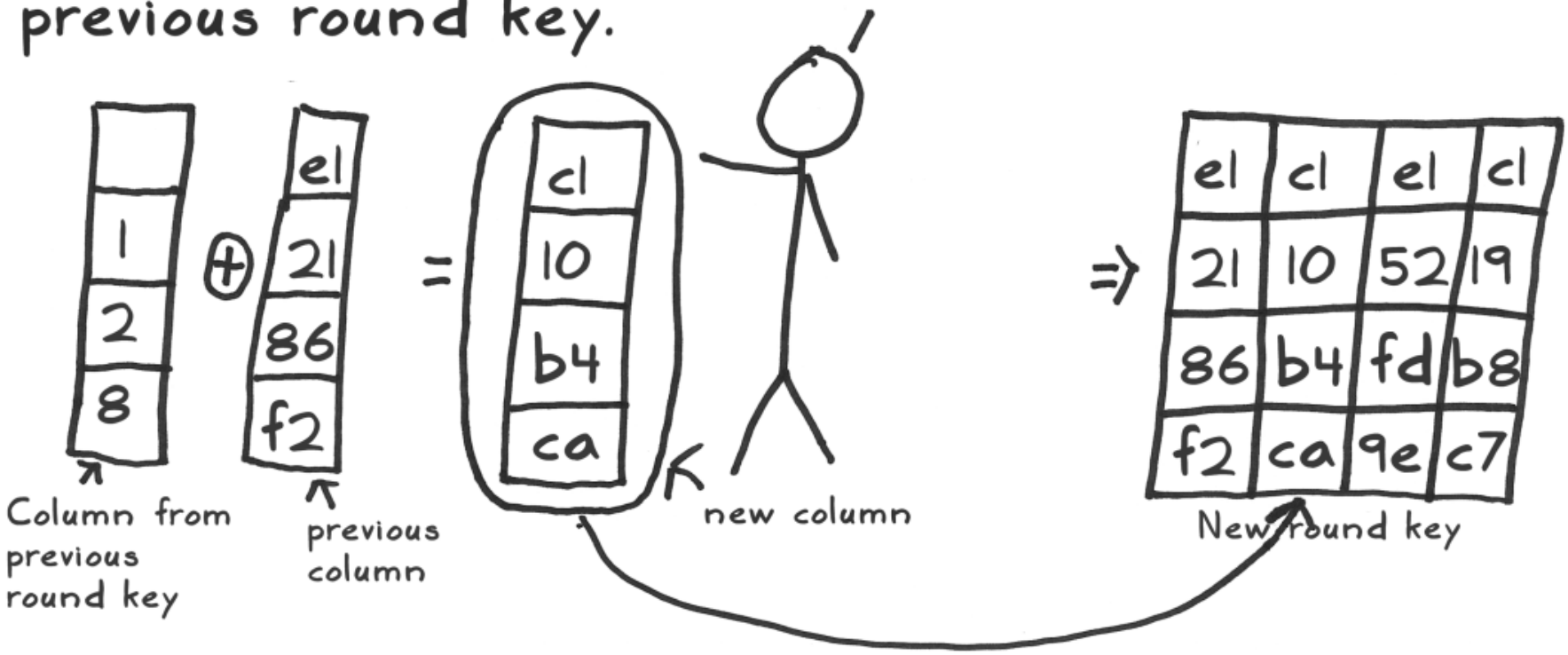


(4) Finally, I xor it with the first column of the previous round key:



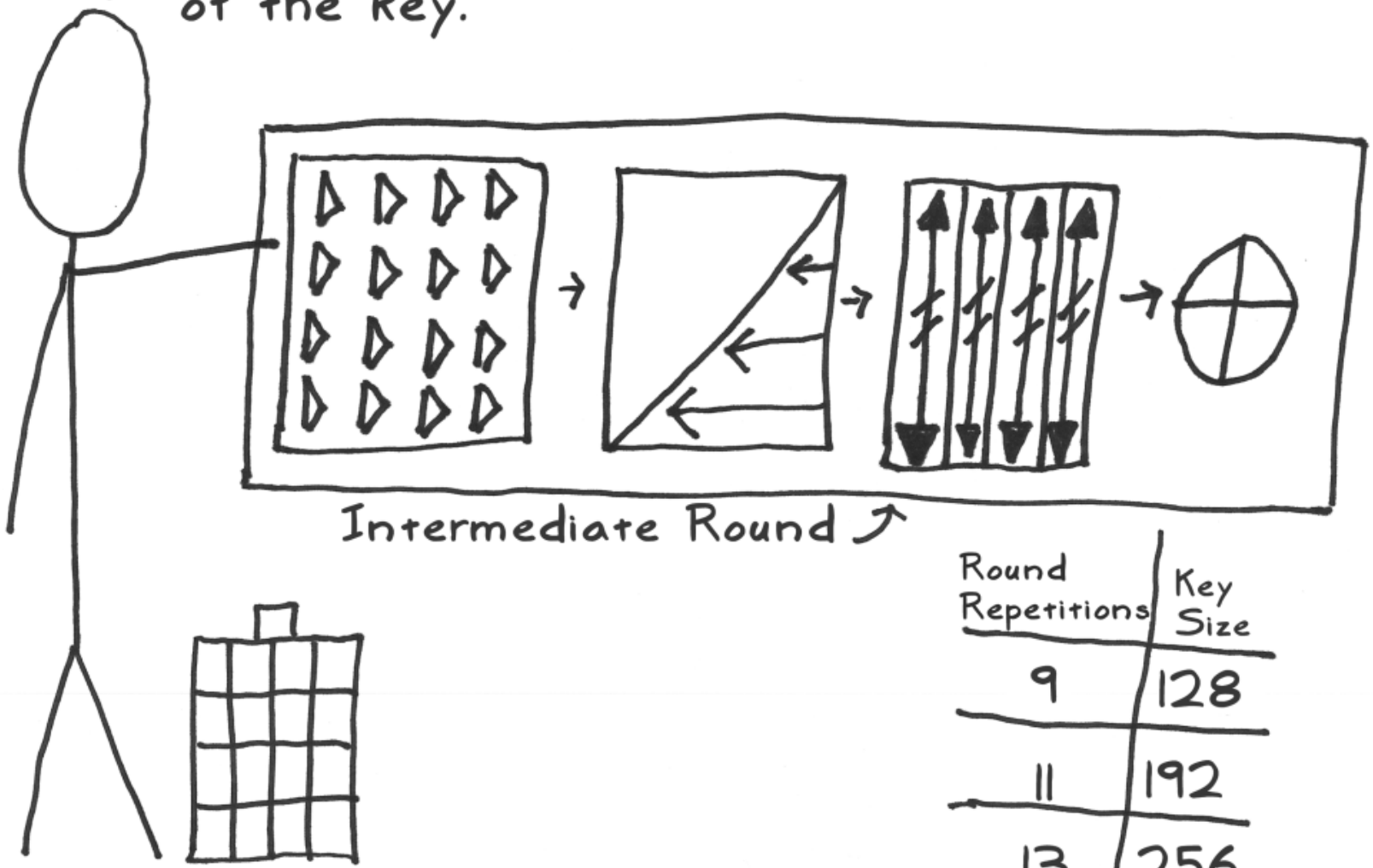
Key Expansion: Part 3

The other columns are super-easy,* I just xor the previous column with the same column of the previous round key.



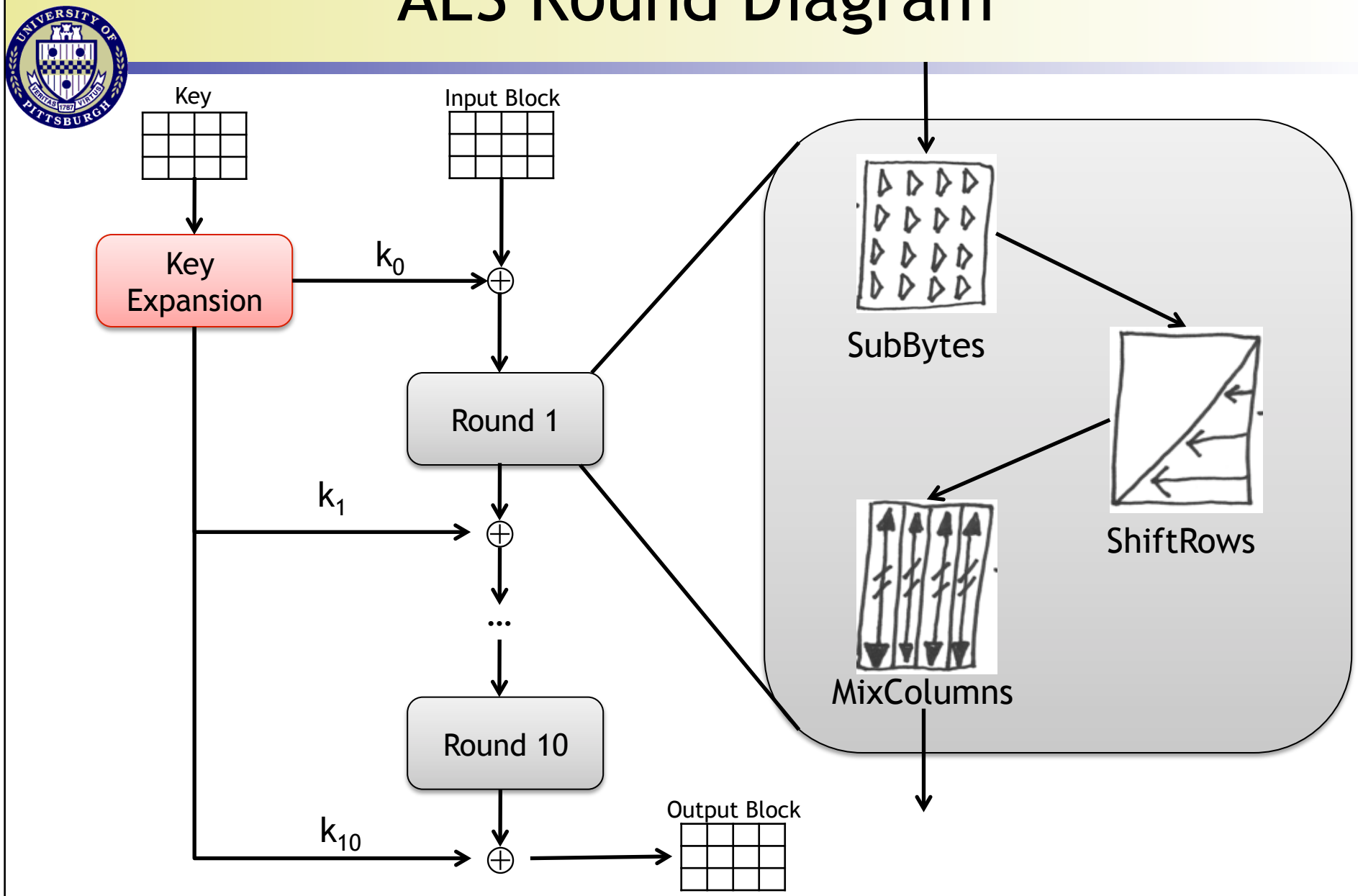
* Note that 256 bit keys are slightly more complicated.

Next, I start the intermediate rounds. A round is just a series of steps I repeat several times. The number of repetitions depends on the size of the key.



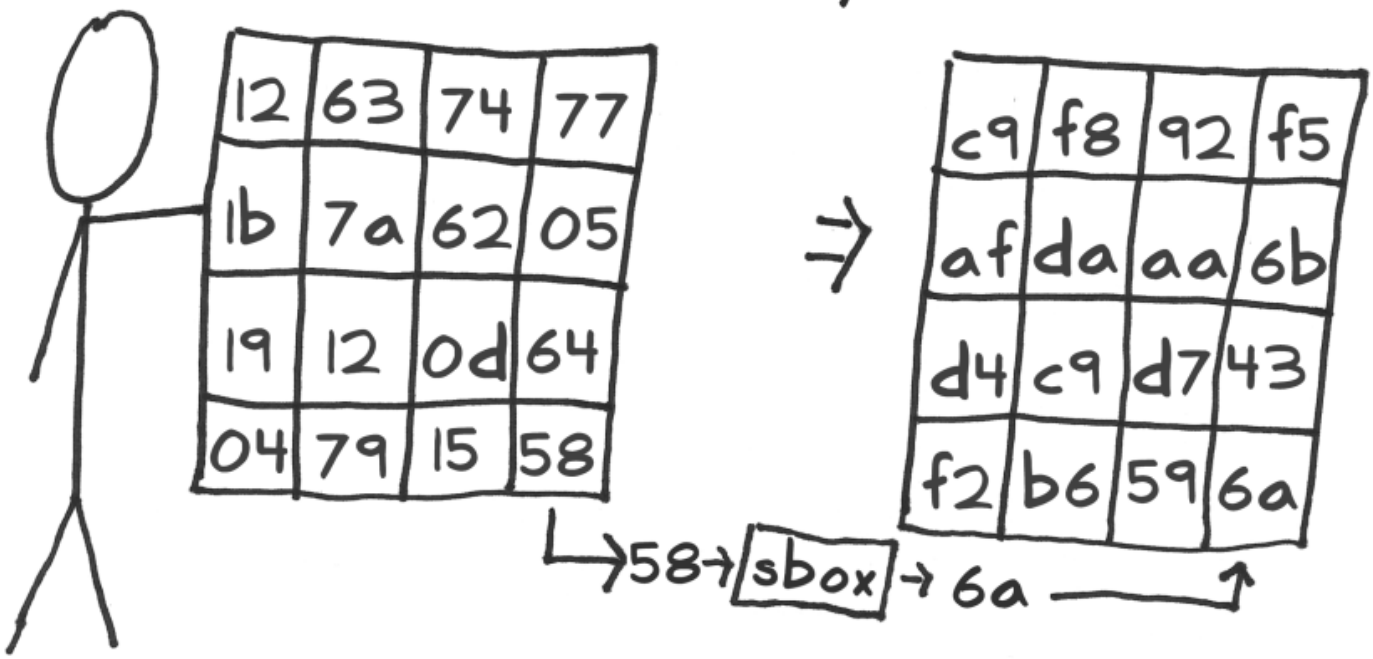
Round Repetitions	Key Size
9	128
11	192
13	256

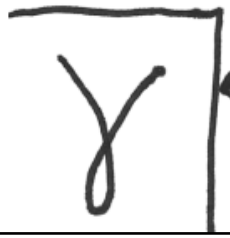
AES Round Diagram



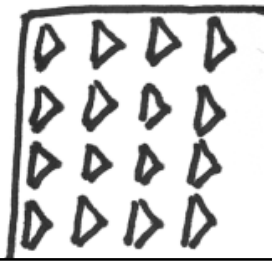
Applying Confusion: Substitute Bytes

I use confusion (Big Idea #1) to obscure the relationship of each byte. I put each byte into a substitution box (sbox), which will map it to a different byte:





Denotes "confusion"



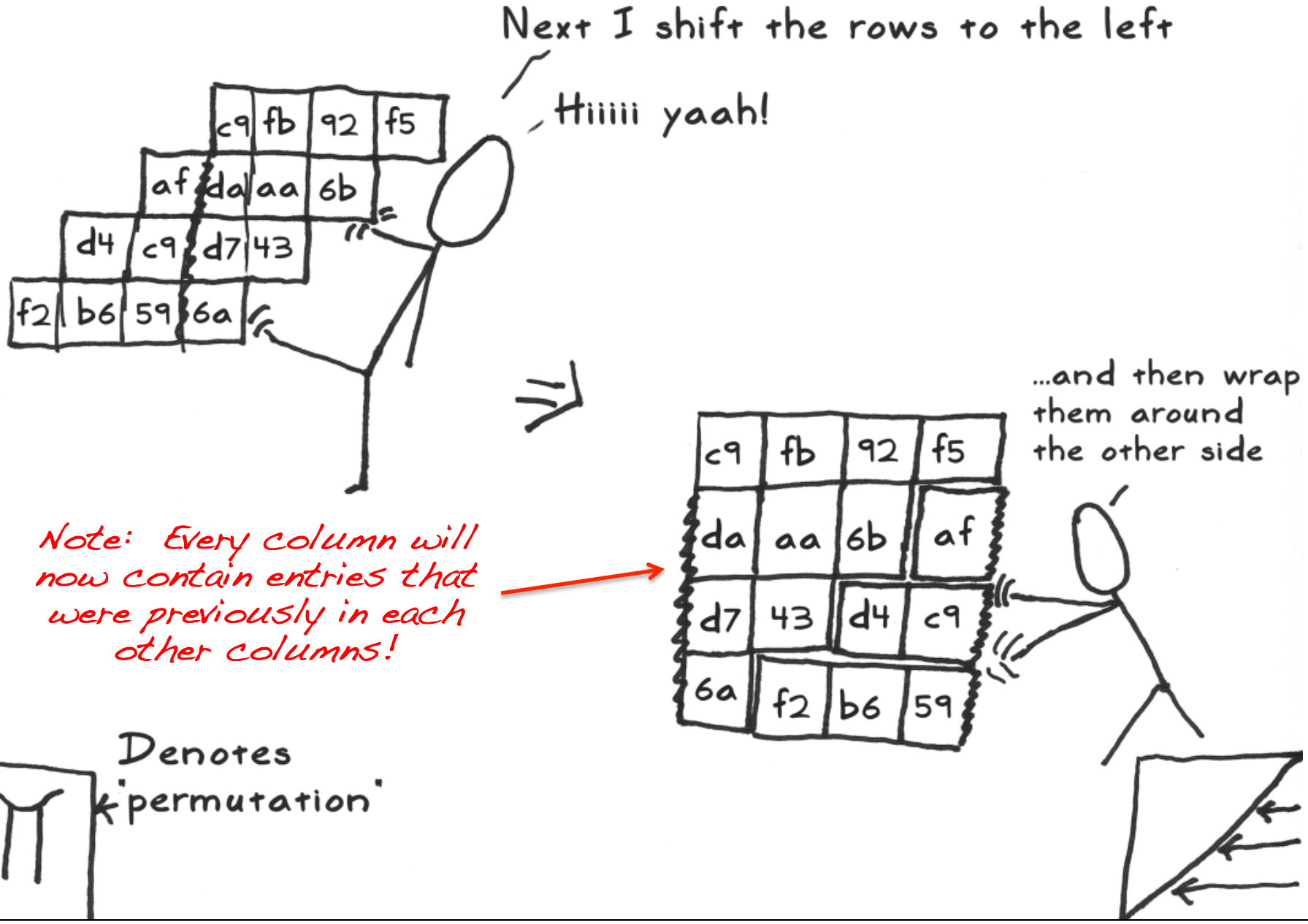
The S-Box is just a big lookup table!

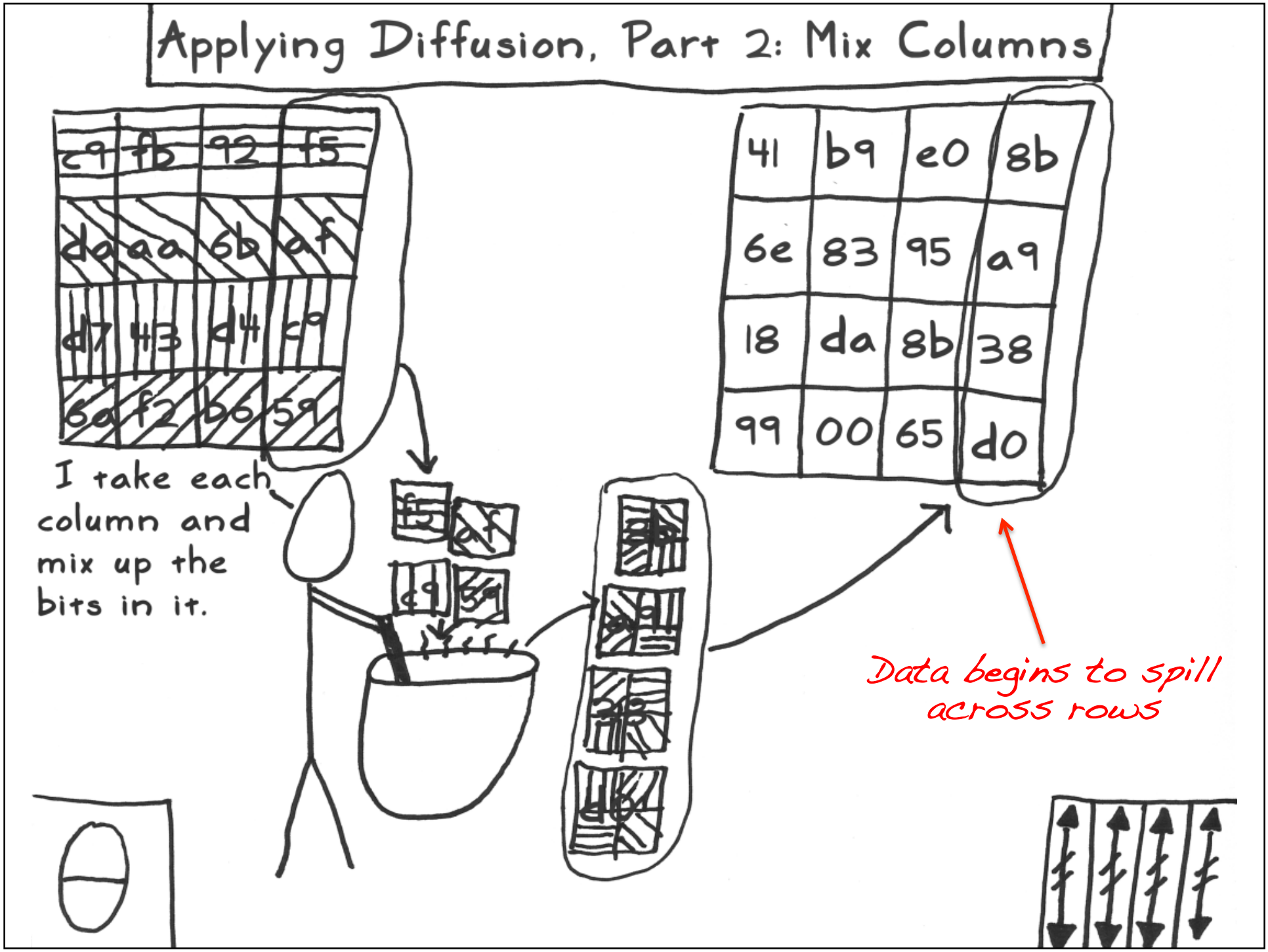
x \ y	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
1	ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
2	b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
3	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
4	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
5	53	d1	00	ed	20	fc	b1	5b	6a	cb	be	39	4a	4c	58	cf
6	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
7	51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
8	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
9	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
a	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
b	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
c	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
d	70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
e	e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
f	8c	a1	89	0d	bf	e6	42	68	41	99	2d	0f	b0	54	bb	16

Figure 7. S-box: substitution values for the byte xy (in hexadecimal format).

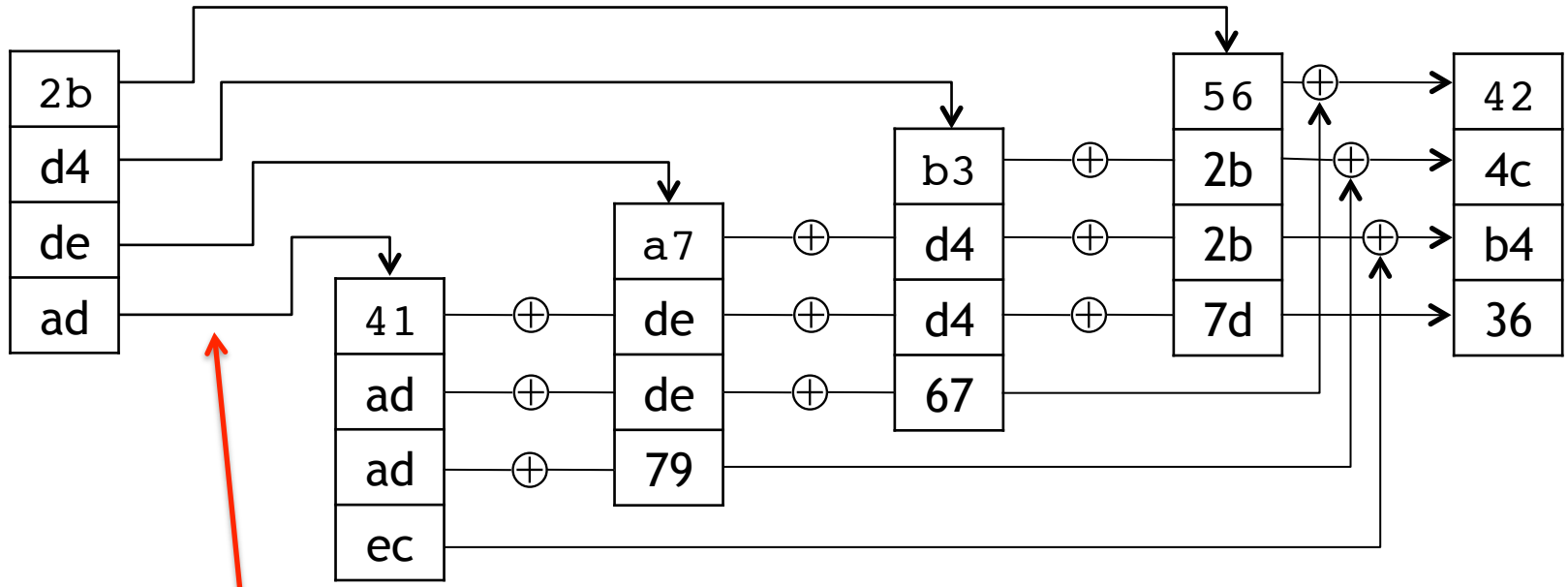
Figure from FIPS PUB 197: The Official AES Specification

Applying Diffusion, Part 1: Shift Rows




Applying Diffusion, Part 2: Mix Columns
c9 fb 92 f5
da aa 6b af
d7 43 d4 c9
6a f2 b6 59
I take each column and mix up the bits in it.
41 b9 e0 8b
6e 83 95 a9
18 da 8b 38
99 00 65 d0
Data begins to spill across rows

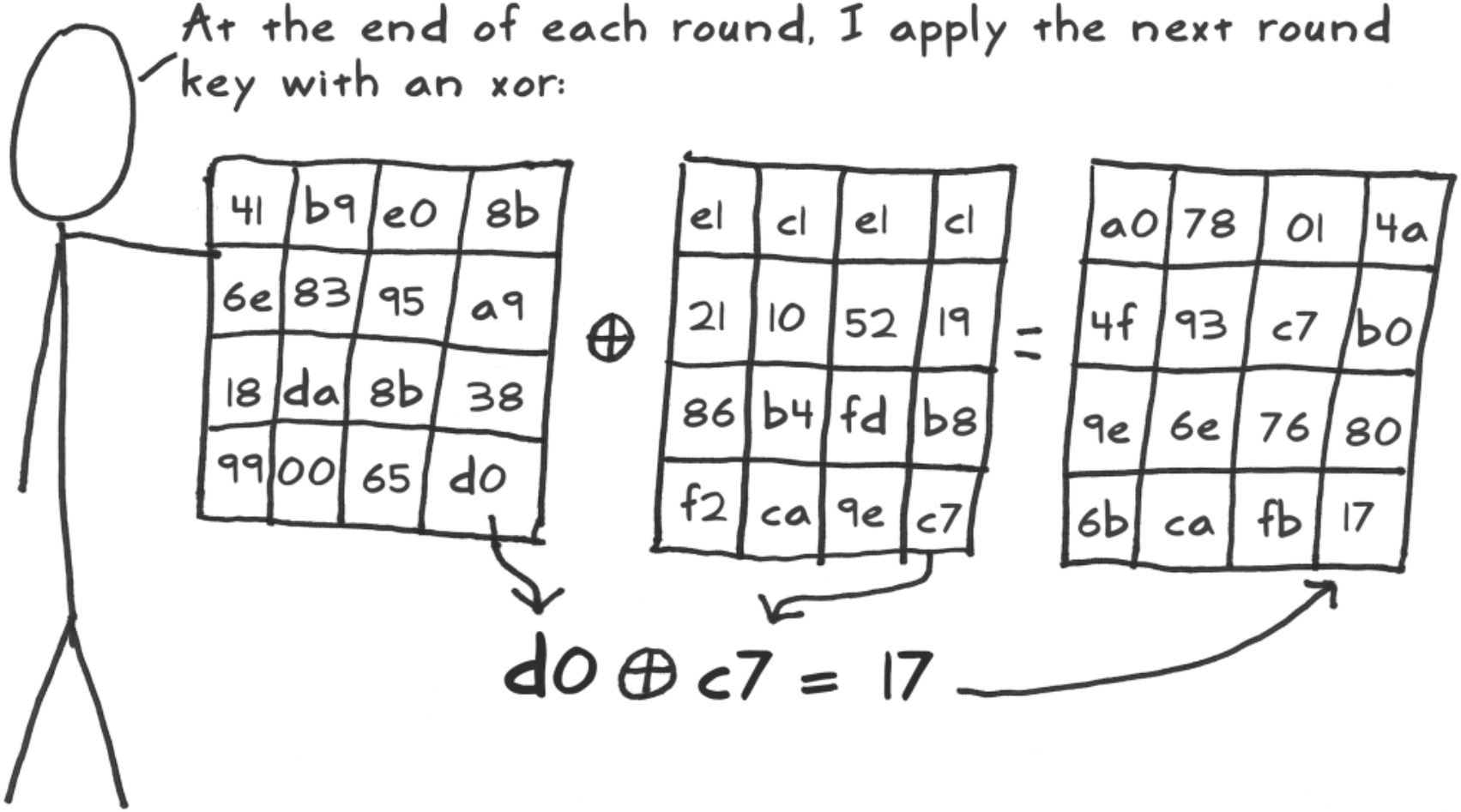
MixColumns is essentially table lookups and XORs



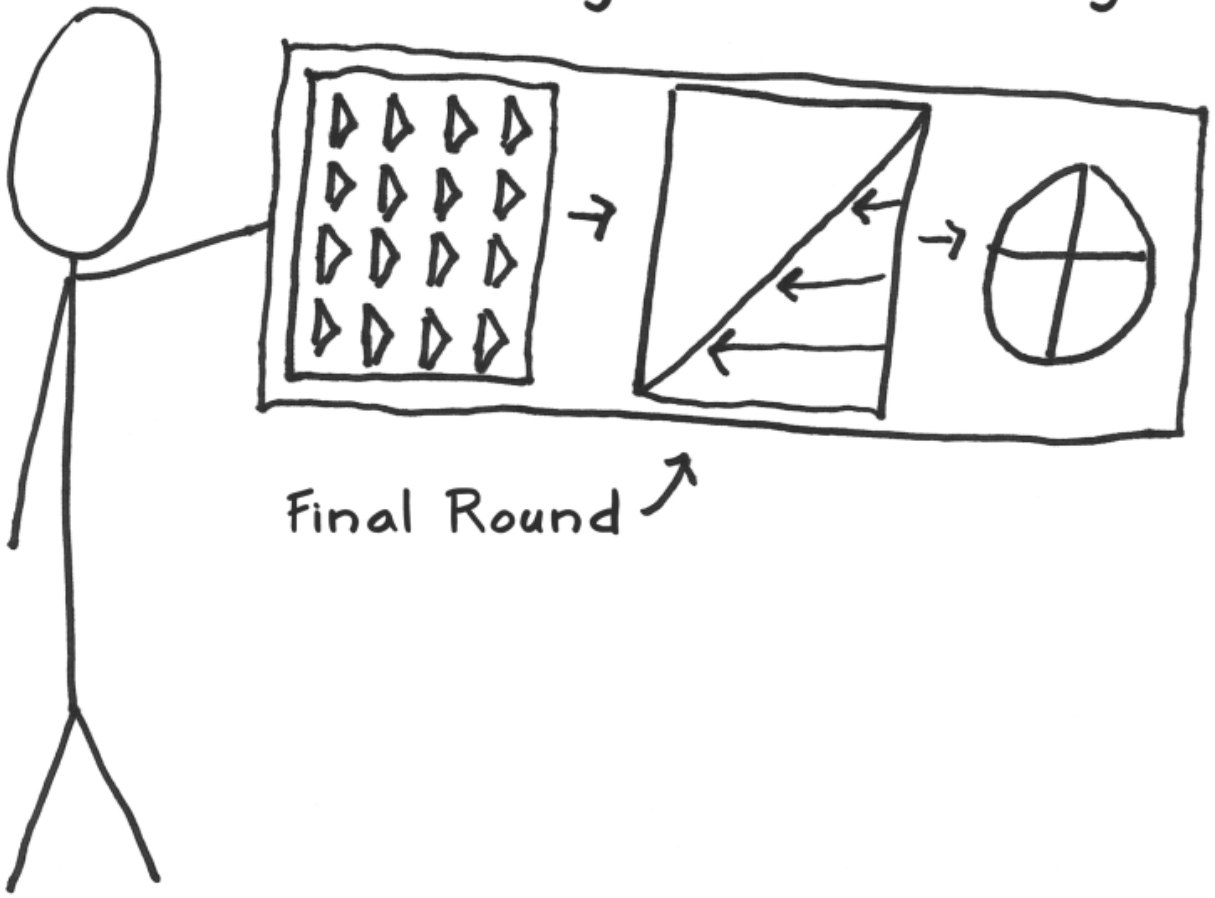
Each line involves a table lookup

See also Figures 3-25 and 3-26 in KPS

Applying Key Secrecy: Add Round Key

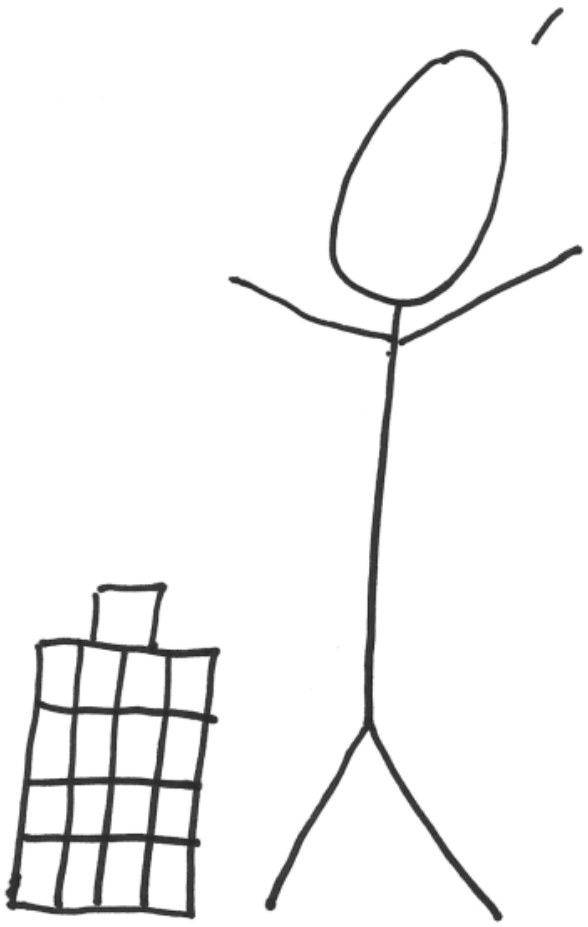


In the final round, I skip the "Mix Columns" step since it wouldn't increase security* and would just slow things down:

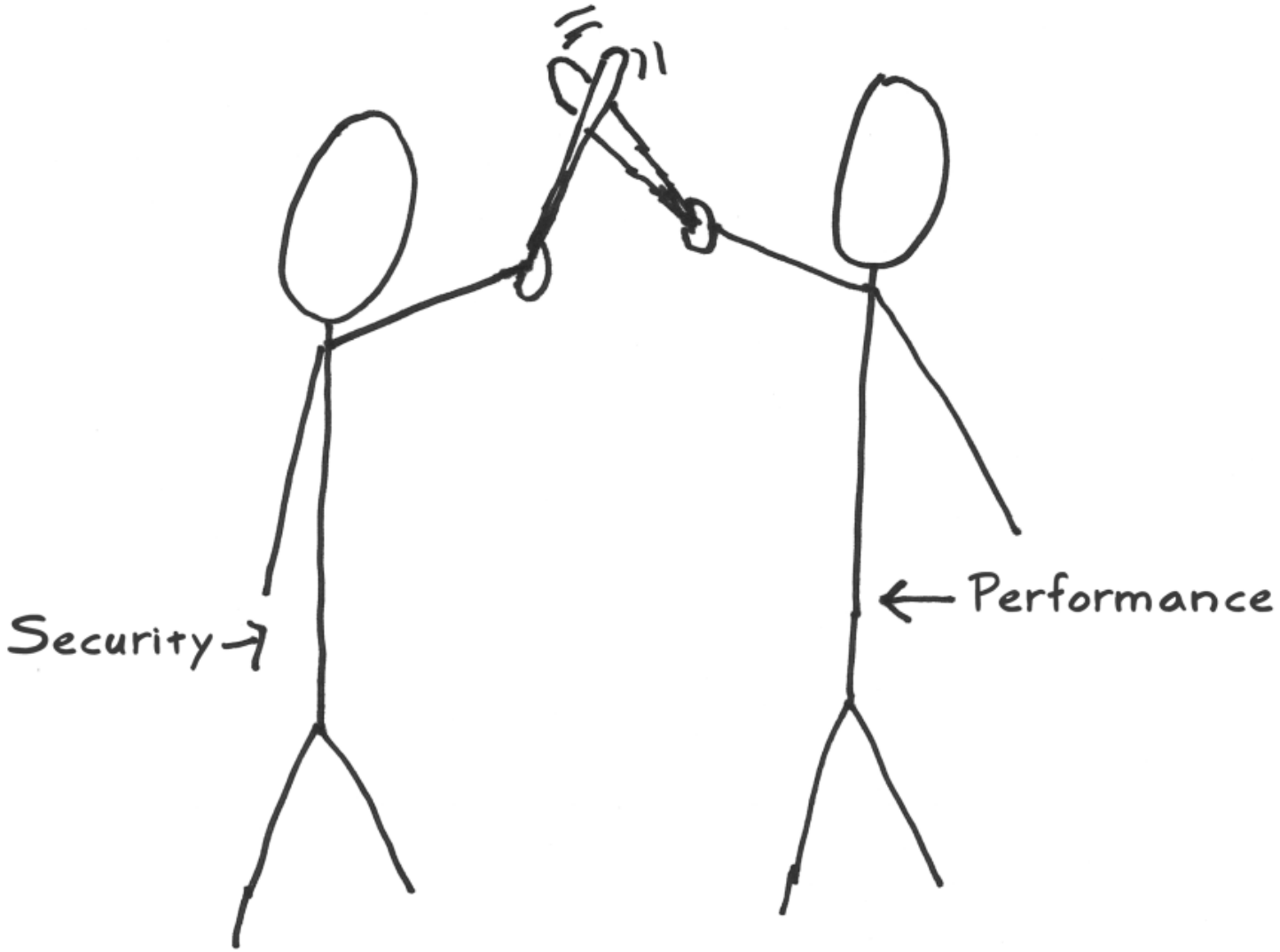


*The diffusion it would provide wouldn't go to the next round.

...and that's it. Each round I do makes the bits more confused and diffused. It also has the key impact them. The more rounds, the merrier!

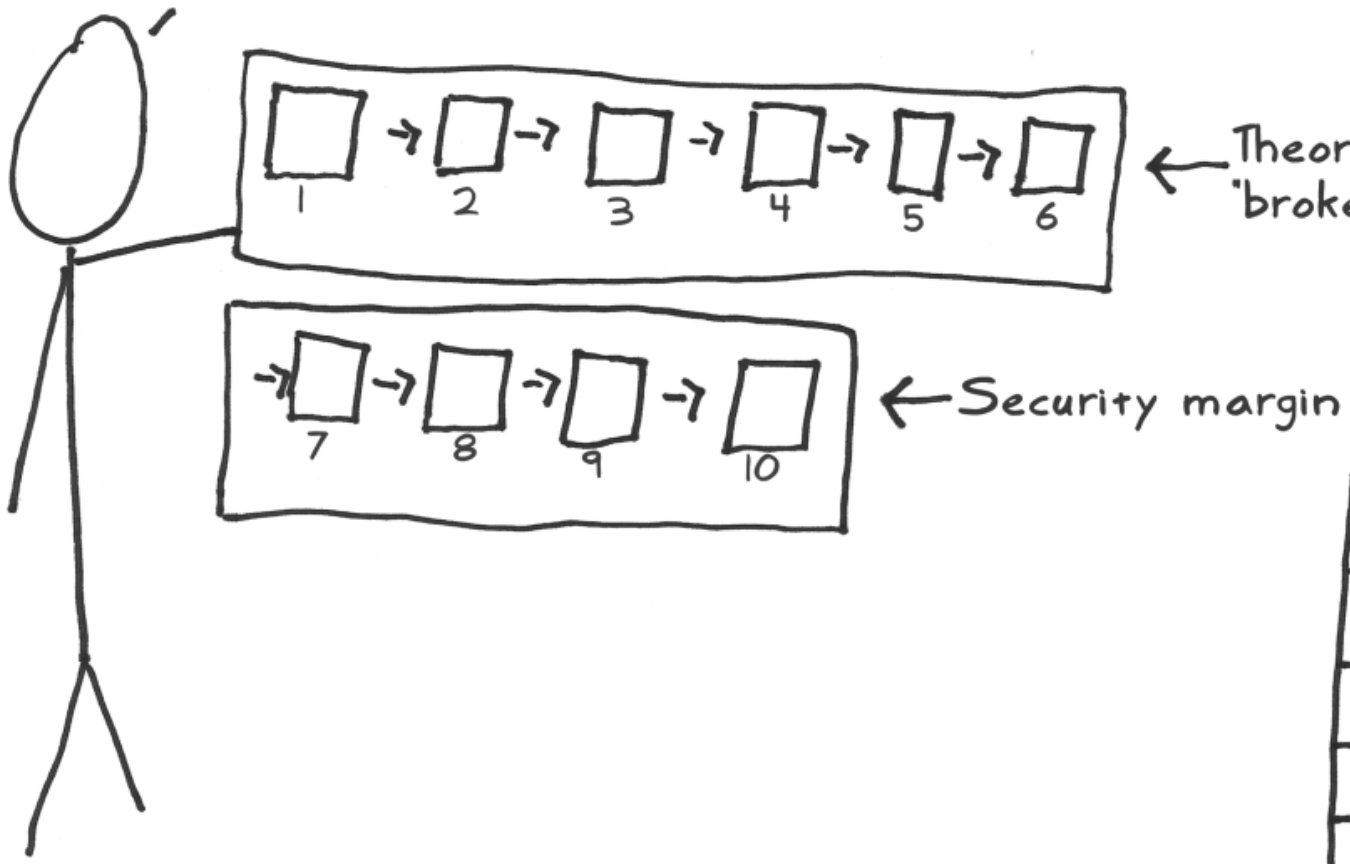


Determining the number of rounds always involves several tradeoffs.



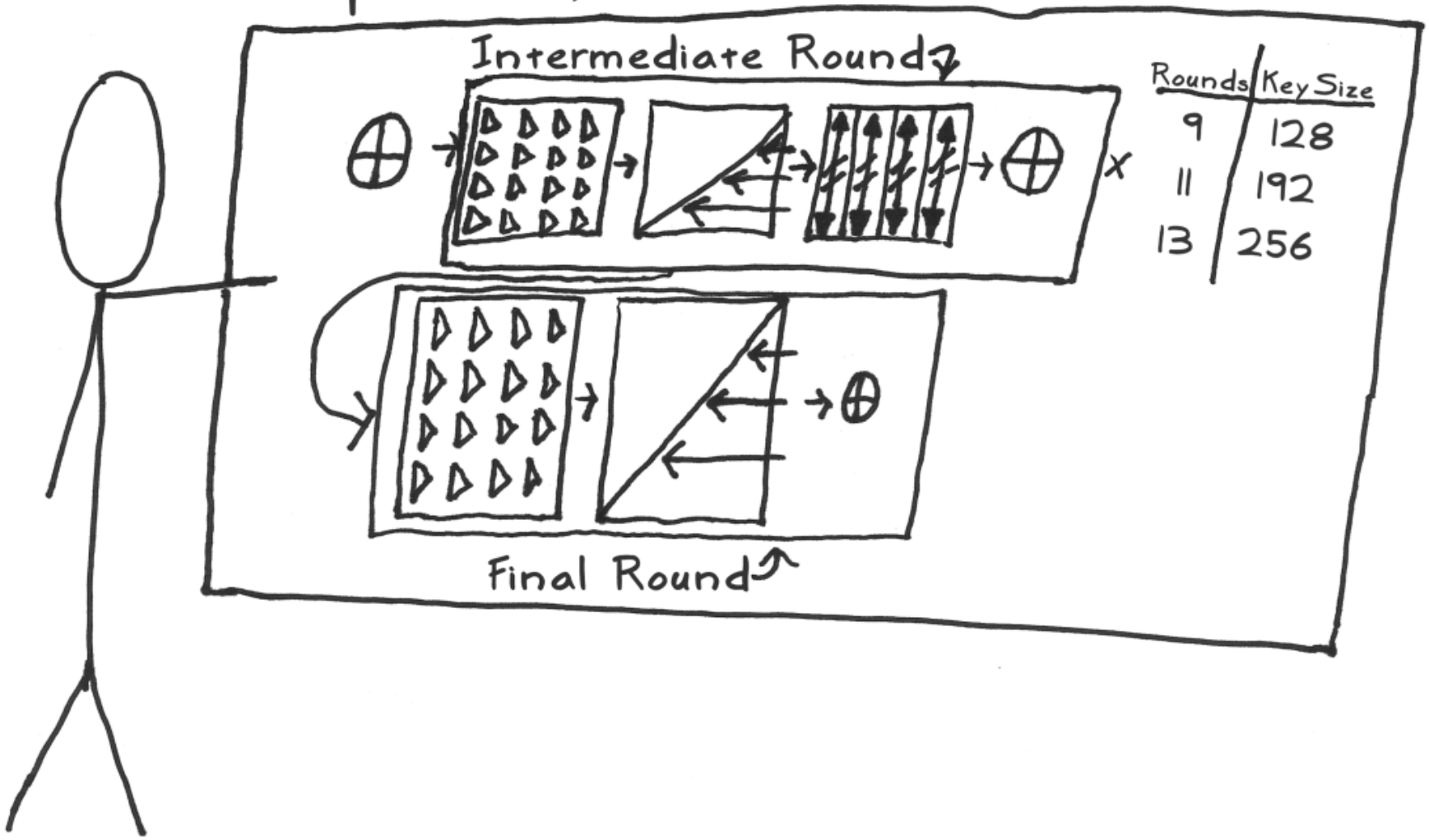
"Security always comes at a cost to performance" - Vincent Rijmen

When I was being developed, a clever guy was able to find a shortcut path through 6 rounds. That's not good! If you look carefully, you'll see that each bit of a round's output depends on every bit from two rounds ago. To increase this diffusion "avalanche," I added 4 extra rounds. This is my "security margin."

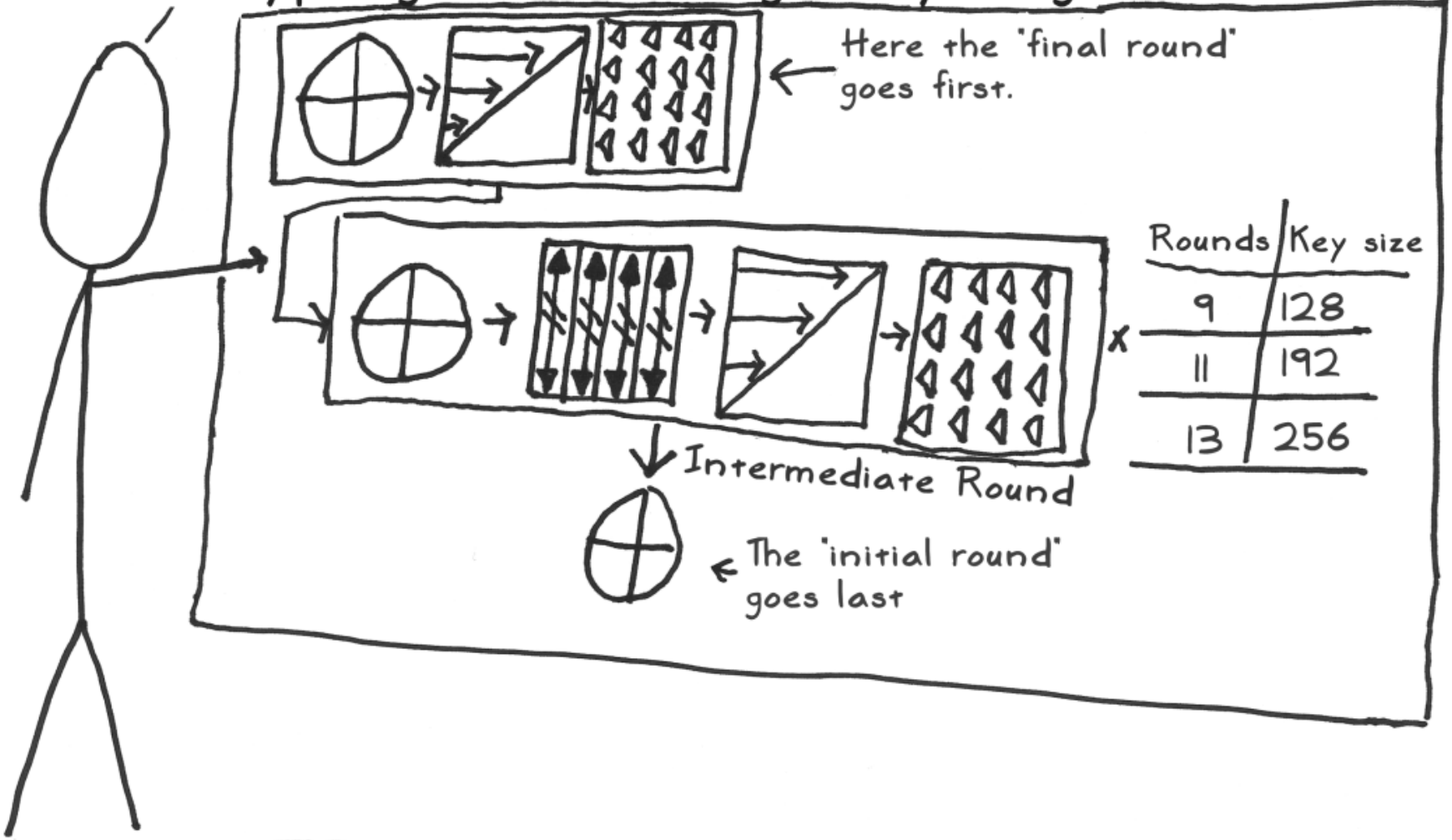


FIPS 197 Spec	
Key Size	Rounds
128	10
192	12
256	14

So in pictures, we have this:



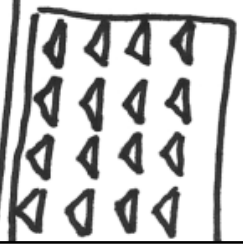
Decrypting means doing everything in reverse



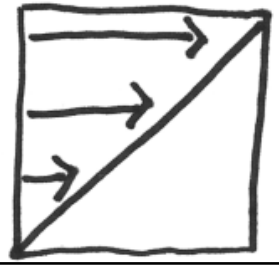
Rounds	Key size
9	128
11	192
13	256

Add Round Key Inverse

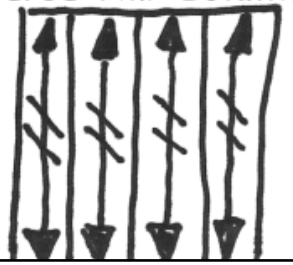
Inverse Substitute Bytes



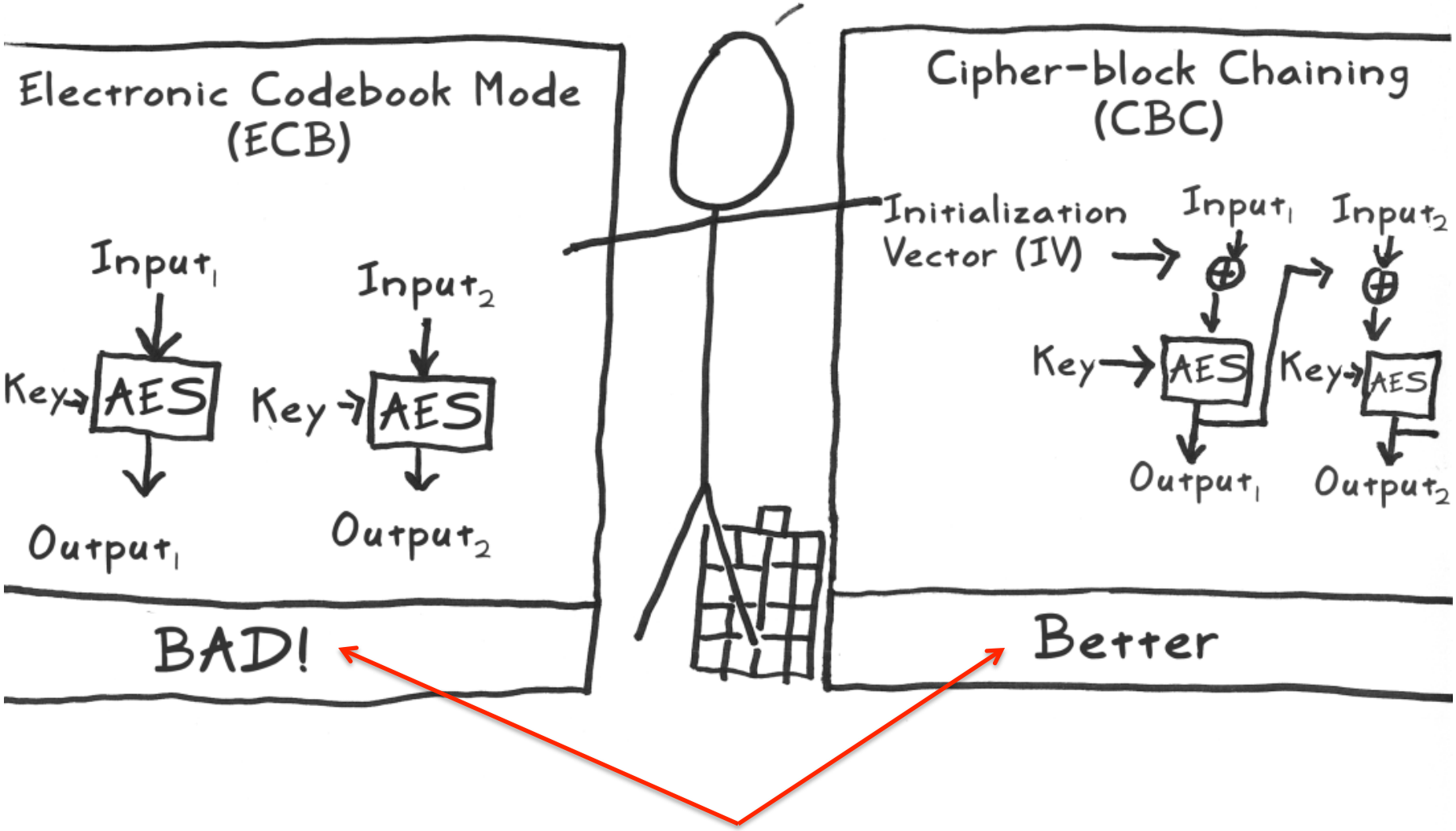
Inverse Shift Rows



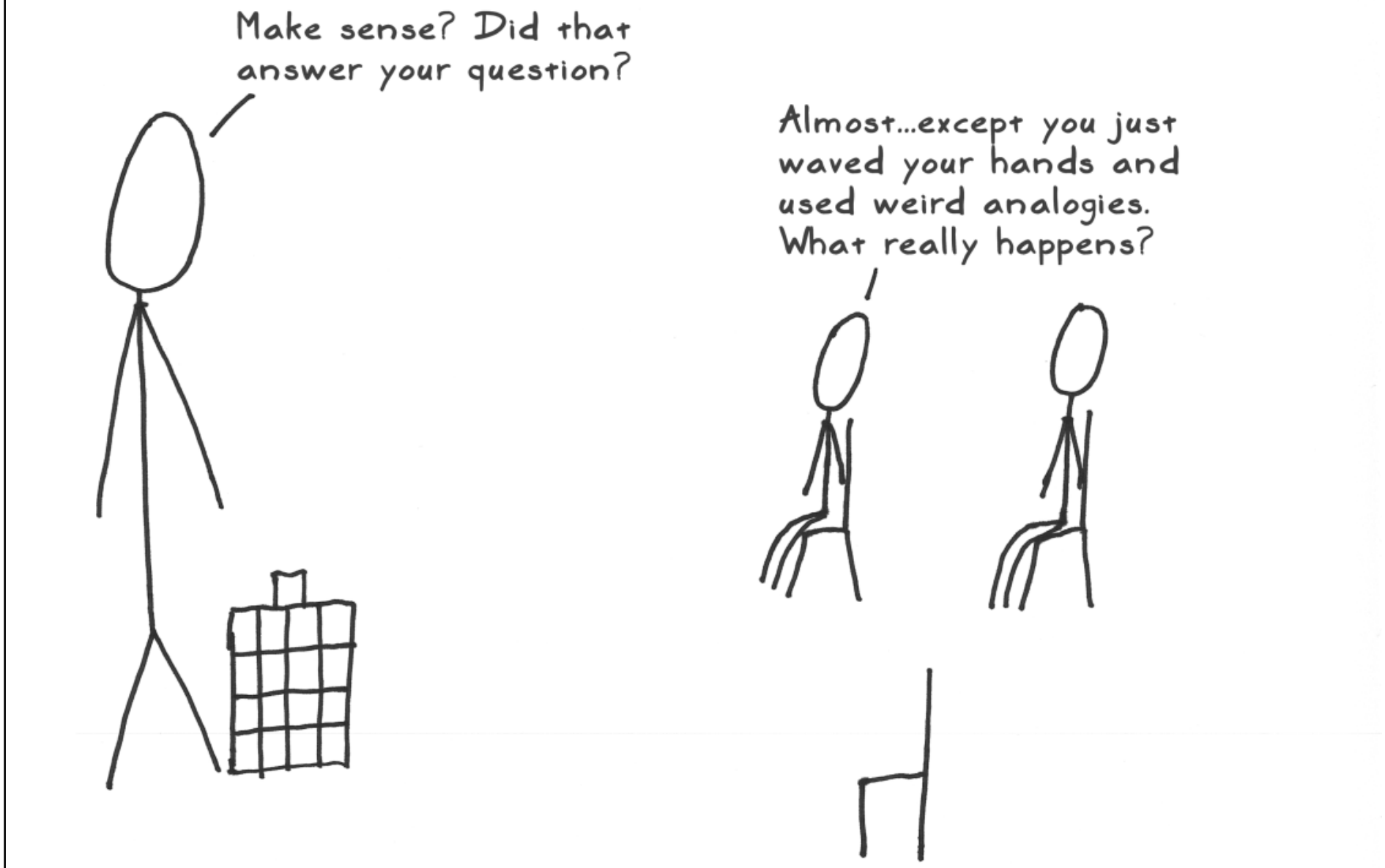
Inverse Mix Columns

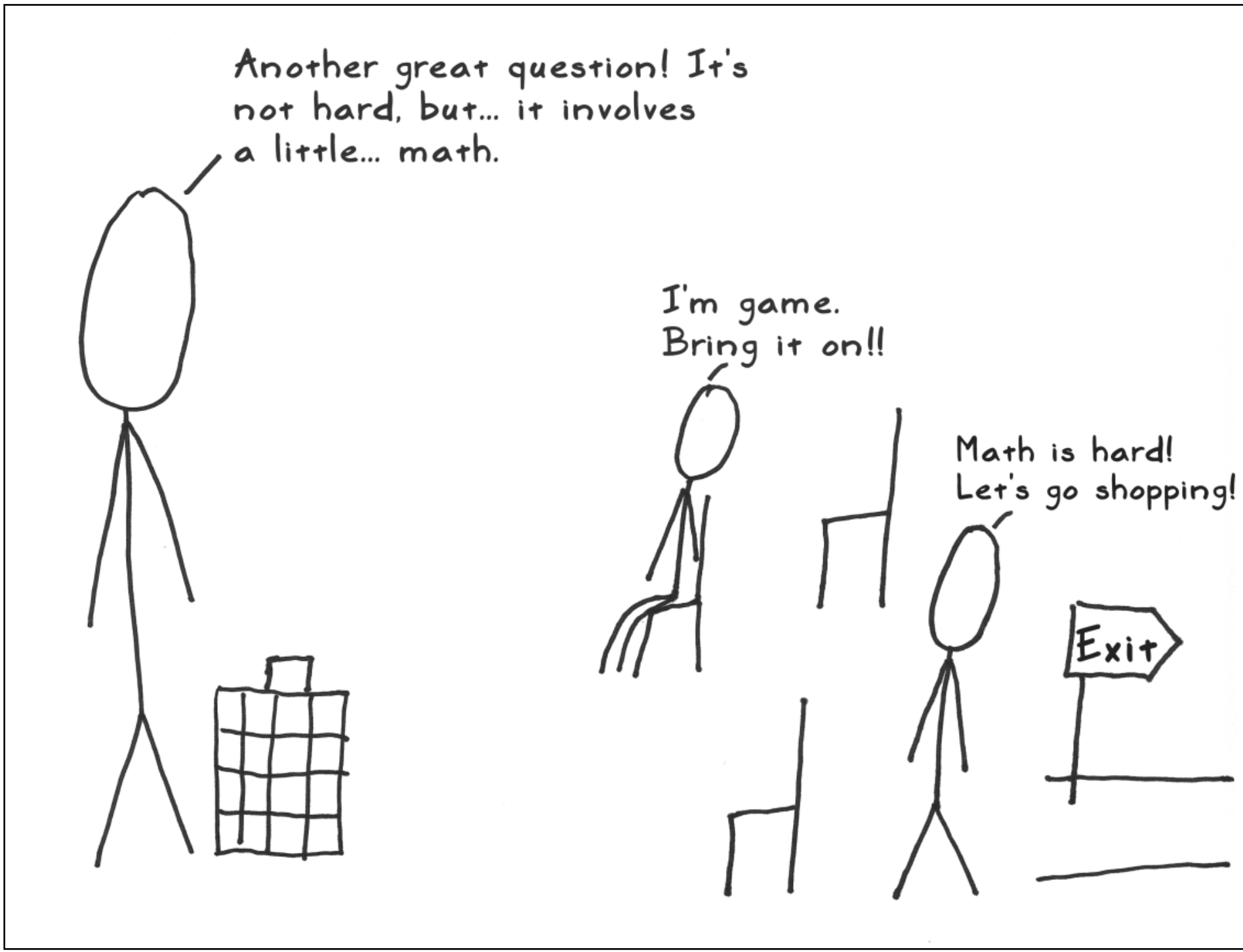


One last tidbit: I shouldn't be used as-is, but rather as a building block to a decent "mode."

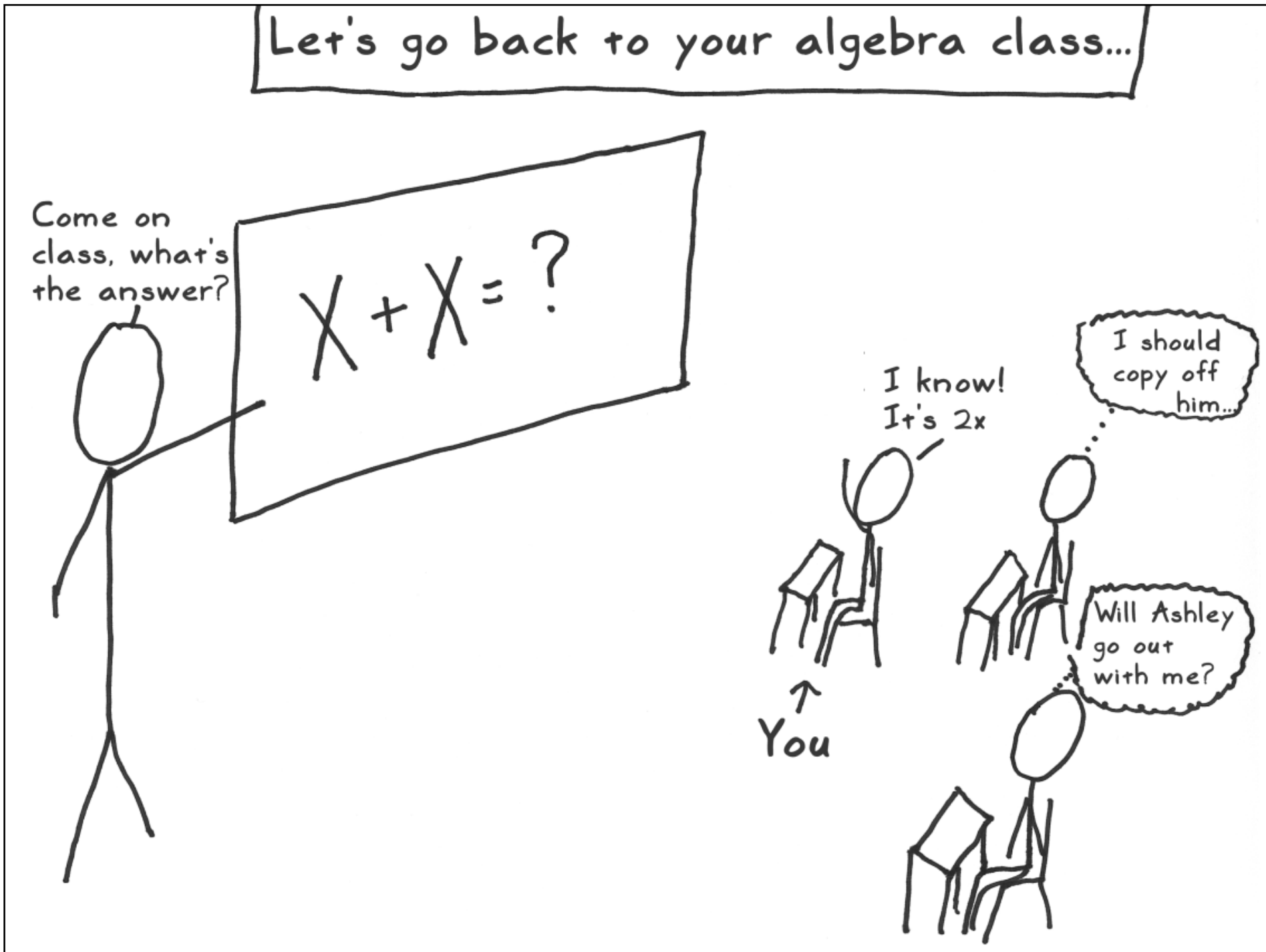


Question: Why? Isn't AES supposed to be safe to use?

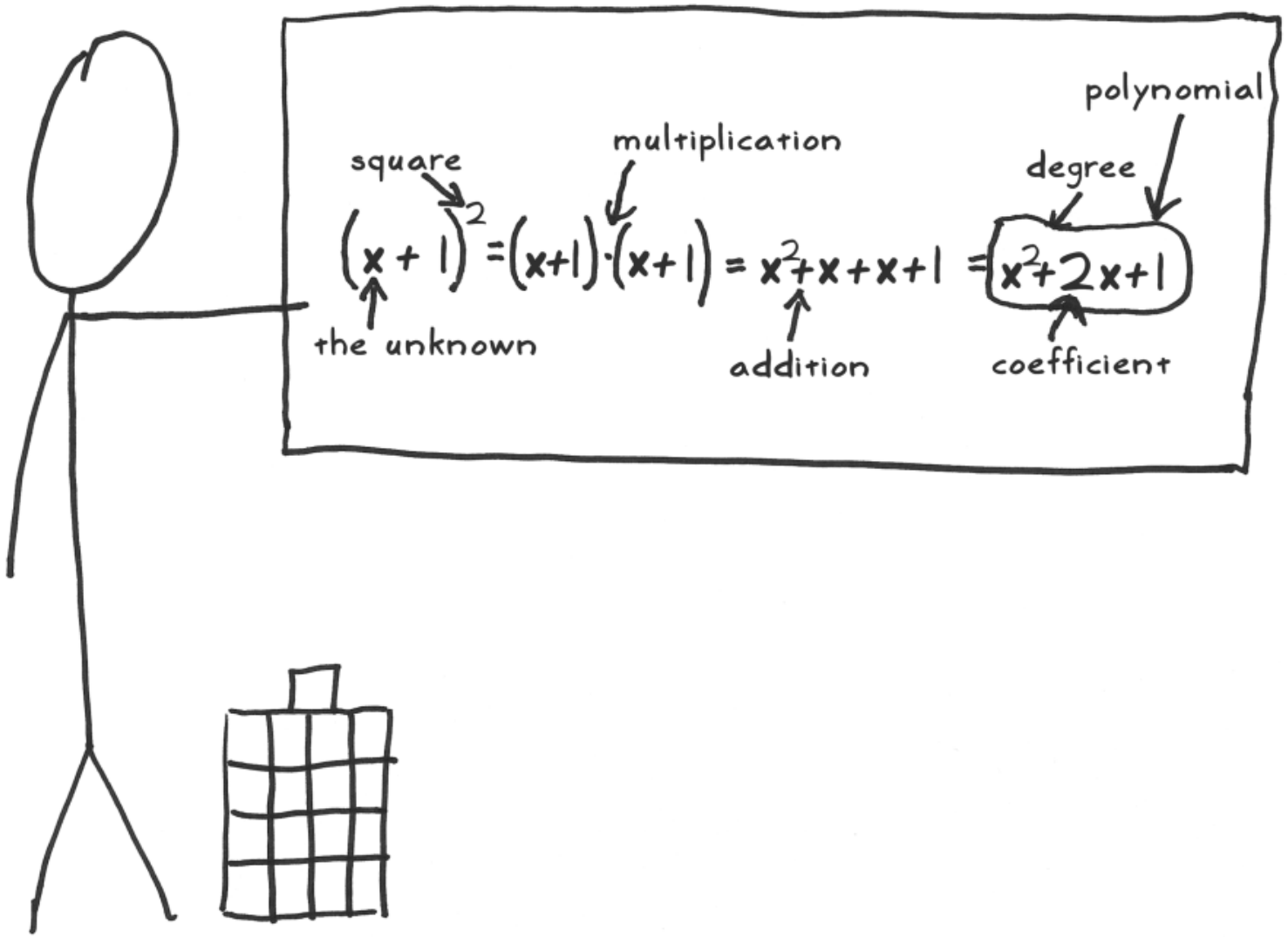

Make sense? Did that answer your question?
Almost...except you just waved your hands and used weird analogies. What really happens?


Another great question! It's not hard, but... it involves a little... math.
I'm game. Bring it on!!
Math is hard! Let's go shopping!
Exit

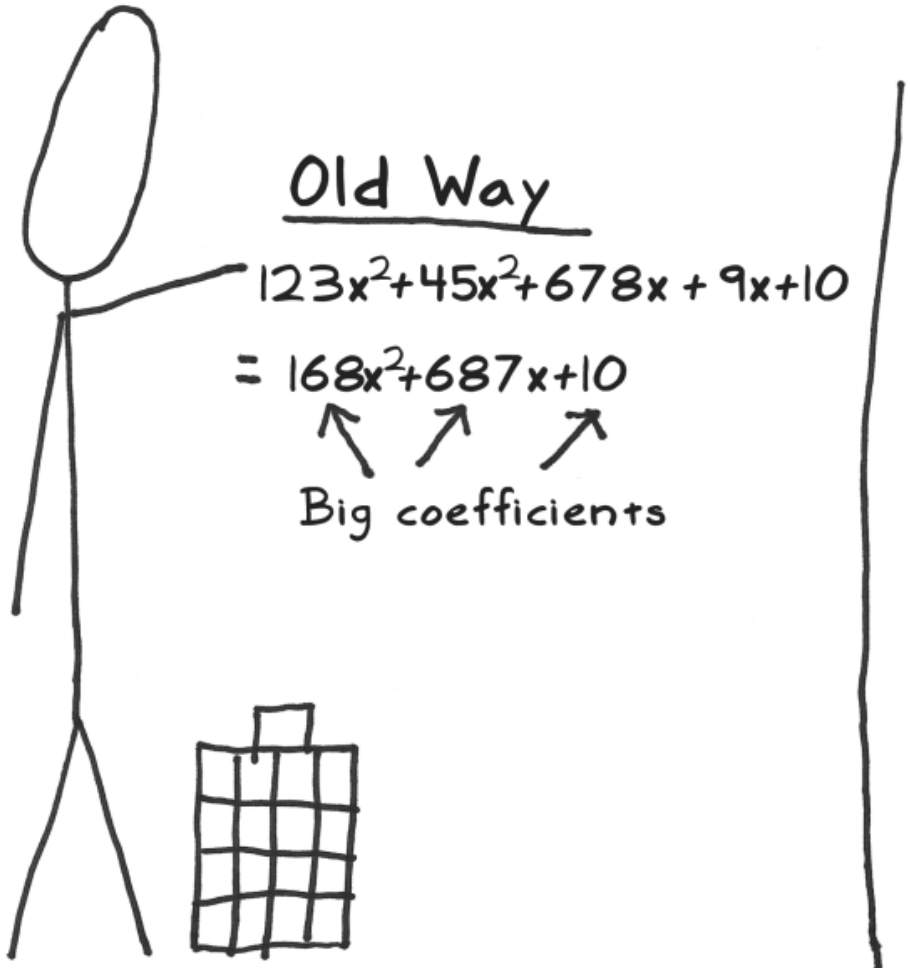
Act 4: Math!


Let's go back to your algebra class...
Come on class, what's the answer?
X + X = ?
I know! It's 2x
I should copy off him...
Will Ashley go out with me?
You

Reviewing the Basics...



We'll change things slightly. In the old way, coefficients could get as big as we wanted. In the new way, they can only be 0 or 1:



Old Way

$123x^2+45x^2+678x+9x+10$

$= 168x^2+687x+10$

Big coefficients

New Way

$x^2 \oplus x^2 \oplus x^2 \oplus x \oplus x \oplus 1$

$= x^2 \oplus 1$ — The "new" add*

Small coefficients

$x^2 \oplus x^2 \oplus x^2 = (x^2 \oplus x^2) \oplus x^2$

$= 0 \oplus x^2$

$= x^2$

I.e., Coefficients drawn from Z_2. I told you that you'd need material from CS0441 again!

*Nifty Fact: In the new way, addition is the same as subtraction (e.g. $x \oplus x = x - x = 0$)

Remember how multiplication could make things grow fast?

$$(x^7+x^5+x^3+x)\cdot(x^6+x^4+x^2+1)$$

$$= x^{7+6}+x^{7+4}+x^{7+2}+x^{7+0}+x^{5+6}+x^{5+4}+x^{5+2}+x^{5+0}$$

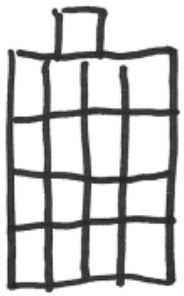
$$+x^{3+6}+x^{3+4}+x^{3+2}+x^{3+0}+x^{1+6}+x^{1+4}+x^{1+2}+x^{1+0}$$

$$= x^{13}+x^{11}+x^9+x^7+x^{11}+x^9+x^7+x^5+x^9+x^7+x^5+x^3+x^7+x^5+x^3+x$$

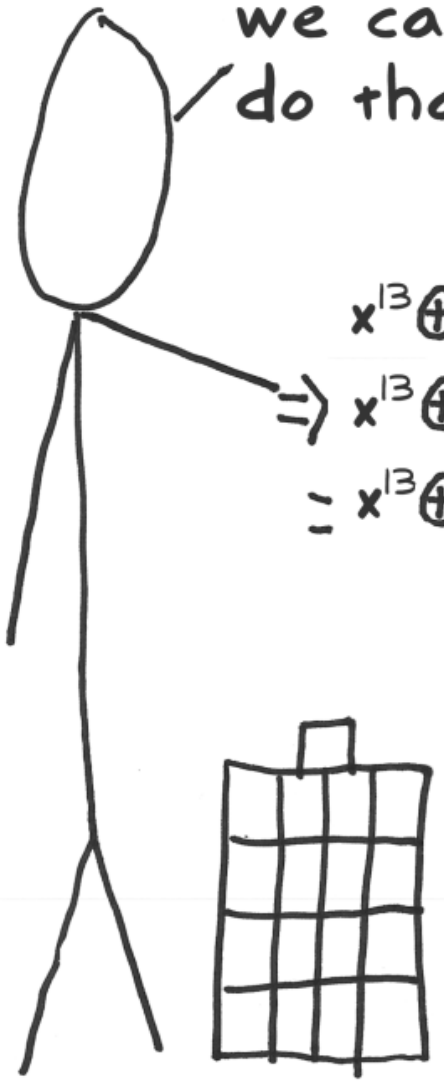
$$= x^{13}+x^{11}+x^{11}+x^9+x^9+x^9+x^7+x^7+x^7+x^7+x^5+x^5+x^5+x^3+x^3+x$$

$$= x^{13}+2x^{11}+3x^9+4x^7+3x^5+2x^3+x$$

Big and yucky!



With the "new" addition, things are simpler, but the x^{13} is still too big. Let's make it so we can't go bigger than x^7. How can we do that?



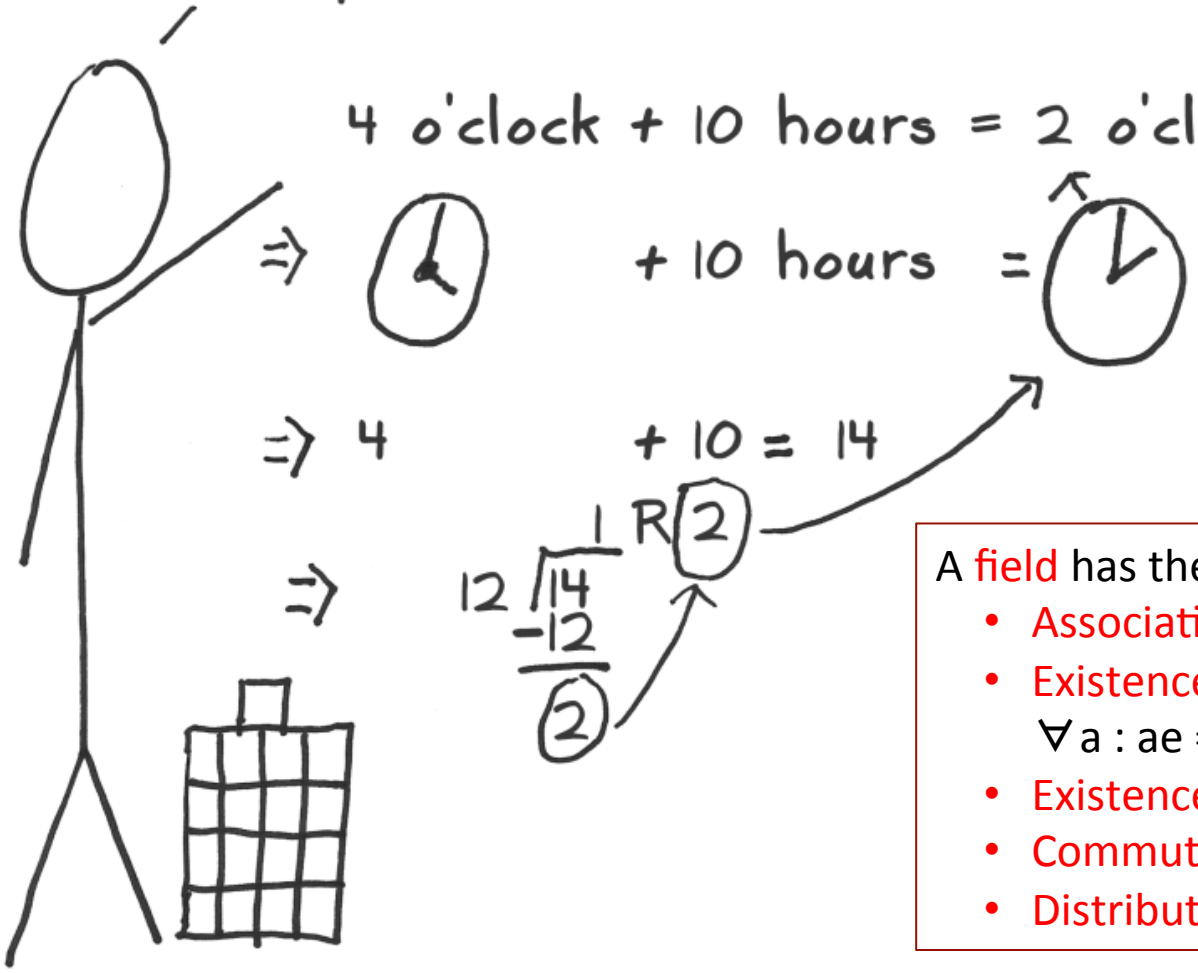
$$x^{13} \oplus 2x^{11} \oplus 3x^9 \oplus 4x^7 \oplus 3x^5 \oplus 2x^3 \oplus x$$

$$\Rightarrow x^{13} \oplus 0x^{11} \oplus x^9 \oplus 0x^7 \oplus x^5 \oplus 0x^3 \oplus x$$

$$= x^{13} \oplus x^9 \oplus x^5 \oplus x$$

Question: How might we accomplish this?

We use our friend, "clock math*," to do this. Just add things up and do long division. Keep a close watch on the remainder:

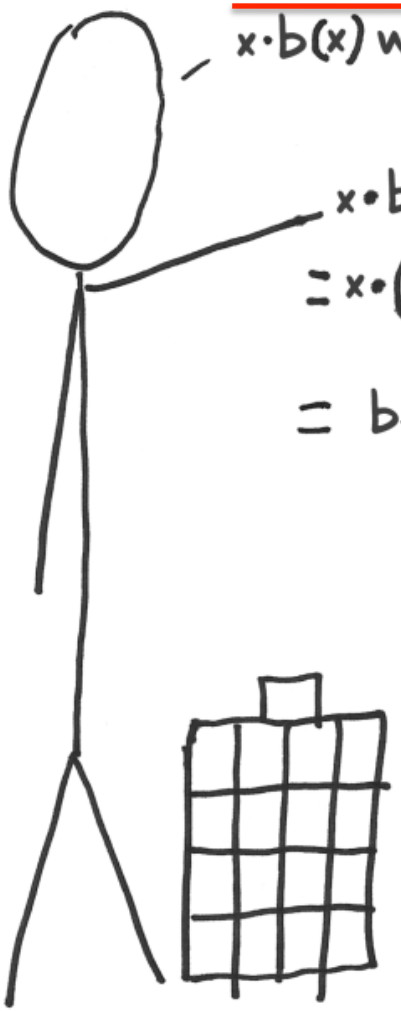


A field has the following properties:

- Associativity: (ab)c = a(bc)
- Existence of an identity element e: $\forall a : ae = a$
- Existence of inverses: $\forall a : aa^{-1} = e$
- Commutativity: ab = ba
- Distributivity: a(b+c) = ab + bc

* This is also known as "modular addition." Math geeks call this a "group." AES uses a special group called a "finite field."

We can do "clock" math with polynomials. Instead of dividing by 12, my creators told me to use $m(x) = x^8 \oplus x^4 \oplus x^3 \oplus x \oplus 1$. Let's say we wanted to multiply $x \cdot b(x)$ where $b(x)$ has coefficients $b_7 \ldots b_0$:



$$x \cdot b(x)$$

$$= x \cdot (b_7x^7 \oplus b_6x^6 \oplus b_5x^5 \oplus b_4x^4 \oplus b_3x^3 \oplus b_2x^2 \oplus b_1x \oplus b_0)$$

$$= b_7x^8 \oplus b_6x^7 \oplus b_5x^6 \oplus b_4x^5 \oplus b_3x^4 \oplus b_2x^3 \oplus b_1x^2 \oplus b_0x)$$

Eeek! x^8 is too big. We must make it smaller.

Note: Rather than reducing elements of our field modulo an integer, we will reduce them modulo another polynomial. This polynomial is irreducible, which means it's kind of like a prime number (i.e., it has no other factors).

* Remember that each b_n (e.g. b_7) is either 0 or 1.

We divide it by $m(x) = x^8 \oplus x^4 \oplus x^3 \oplus x \oplus 1$ and take the remainder:

$m(x)$

$$
\begin{array}{r|l}
 & b_7 \\
\hline
x^8 \oplus x^4 \oplus x^3 \oplus x \oplus 1 & b_7x^8 \oplus b_6x^7 \oplus b_5x^6 \oplus b_4x^5 \oplus b_3x^4 \oplus b_2x^3 \oplus b_1x^2 \oplus b_0x \\
 & \quad b_7x^8 \qquad\qquad\qquad\qquad \oplus b_7x^4 \oplus b_7x^3 \oplus b_7x \oplus b_7 \\
\hline
 & b_6x^7 \oplus b_5x^6 \oplus b_4x^5 \oplus (b_3 \oplus b_7)x^4 \oplus (b_2 \oplus b_7)x^3 \\
 & \qquad\qquad\qquad\qquad \oplus b_1x^2 \oplus (b_0 \oplus b_7)x \oplus b_7
\end{array}
$$

Remainder

$$b_6x^7 \oplus b_5x^6 \oplus b_4x^5 \oplus b_3x^4 \oplus b_2x^3 \oplus b_1x^2 \oplus b_0x$$

$$\oplus b_7 \cdot (x^4 \oplus x^3 \oplus x \oplus 1)$$

Note how the b's are shifted left by 1 spot.

This is just b_7 multiplied by a small polynomial.

Now we're ready for the hardest blast from the past: logarithms. After logarithms, everything else is cake! Logarithms let us turn multiplication into addition:

$$\log(x \cdot y) = \log(x) + \log(y)$$

So... $\log(10 \cdot 100) = \log(10^1) + \log(10^2)$

$= 2 + 1 = 3$

In reverse:

$\log^{-1}(1) = 10^1 = 10$

$\log^{-1}(2) = 10^2 = 100$

$\log^{-1}(3) = 10^3 = 1{,}000$

$\Rightarrow 10 \cdot 100 = 1{,}000$

We can use logarithms in our new world. Instead of using 10 as the base, we can use the simple polynomial of $x \oplus 1$ and watch the magic unravel.*

$$(x \oplus 1)^1 = x \oplus 1$$

$$(x \oplus 1)^2 = (x \oplus 1)(x \oplus 1) = x^2 \oplus x \oplus x \oplus 1 = x^2 \oplus 1$$

$$(x \oplus 1)^3 = (x \oplus 1)^2 \cdot (x \oplus 1) = x^3 \oplus x^2 \oplus x \oplus 1$$

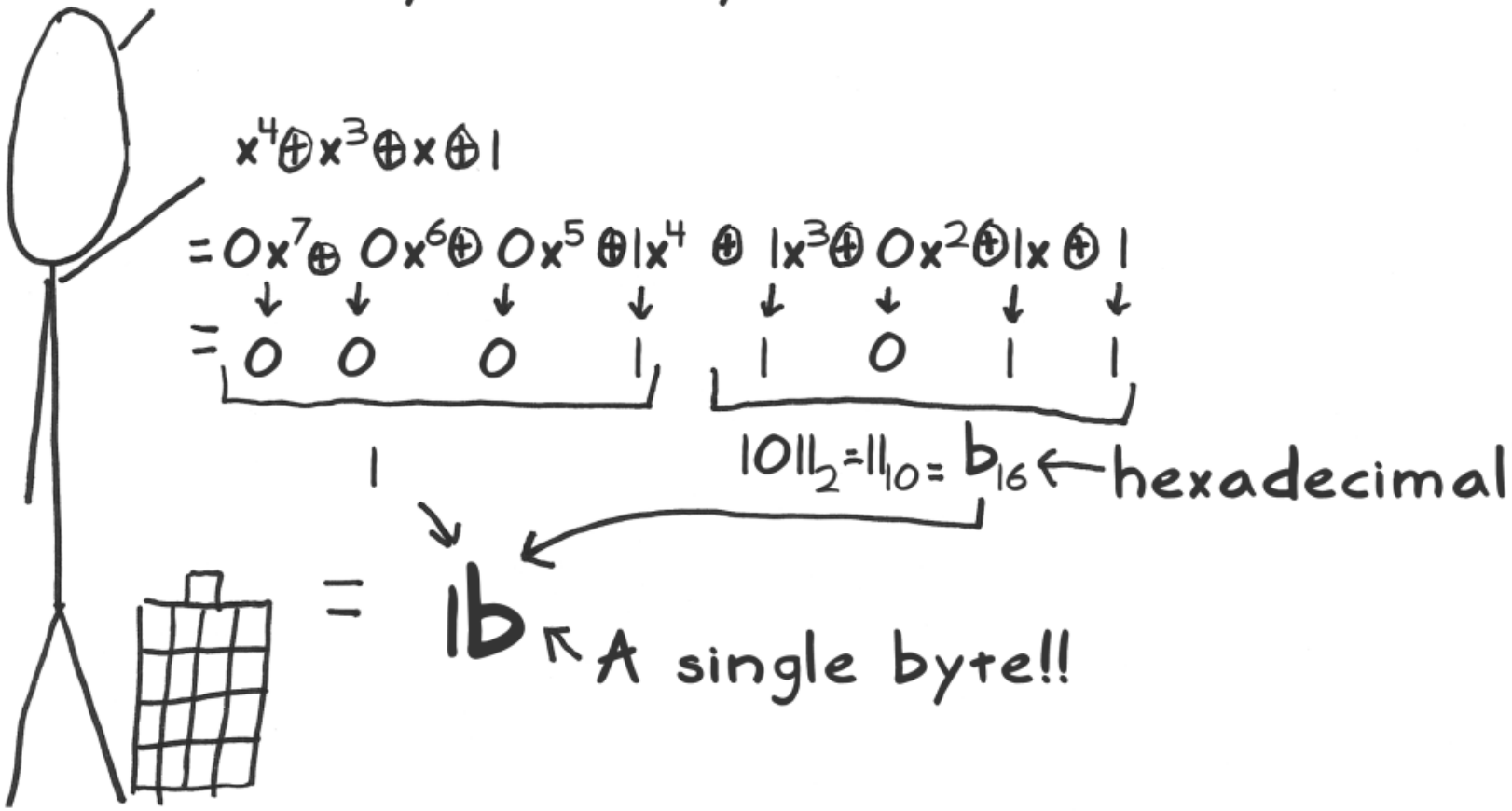
So...

$$\log_{x \oplus 1}(x \oplus 1) = 1, \log_{x \oplus 1}(x^2 \oplus 1) = 2, \log_{x \oplus 1}(x^3 \oplus x^2 \oplus x \oplus 1) = 3$$

The polynomial **(x ⊕ 1)** *is called a* <u>generator</u> *of our field* $GF(2^8)$

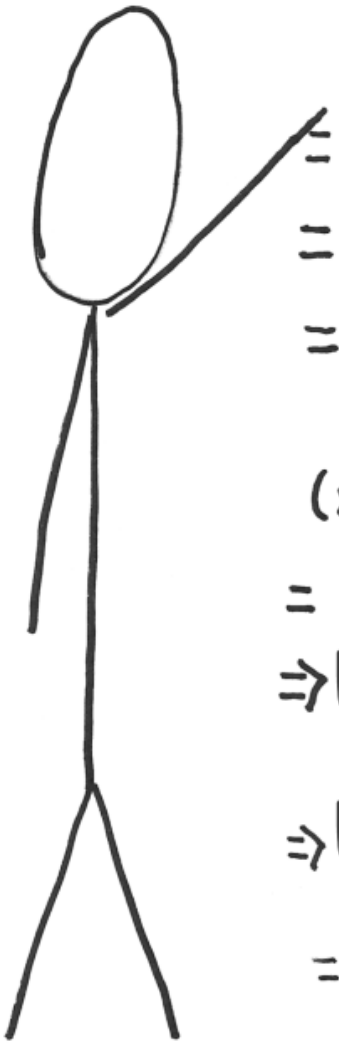
*If you keep multiplying by $(x \oplus 1)$ and then take the remainder after dividing by $m(x)$, you'll see that you generate all possible polynomial below x^8. This is very important!

Why bother with all of this math?* Encryption deals with bits and bytes, right? Well, there's one last connection: a 7^{th} degree polynomial can be represented in exactly 1 byte since the new way uses only 0 or 1 for coefficients:



*Although we'll work with bytes from now on, the math makes sure everything works out.

With bytes, polynomial addition becomes a simple xor. We can use our logarithm skills to make a table for speedy multiplication.*



$$(x^4 \oplus x^3 \oplus x \oplus 1) \oplus (x^7 \oplus x^5 \oplus x^3 \oplus x)$$

$$= 1b \oplus aa \quad \leftarrow \text{byte xor}$$

$$= b1$$

$$= x^7 \oplus x^5 \oplus x^4 \oplus 1$$

$$(x^4 \oplus x^3 \oplus x \oplus 1) \cdot (x^7 \oplus x^5 \oplus x^3 \oplus x)$$

$$= 1b \cdot aa \quad \text{logarithm table lookup}$$

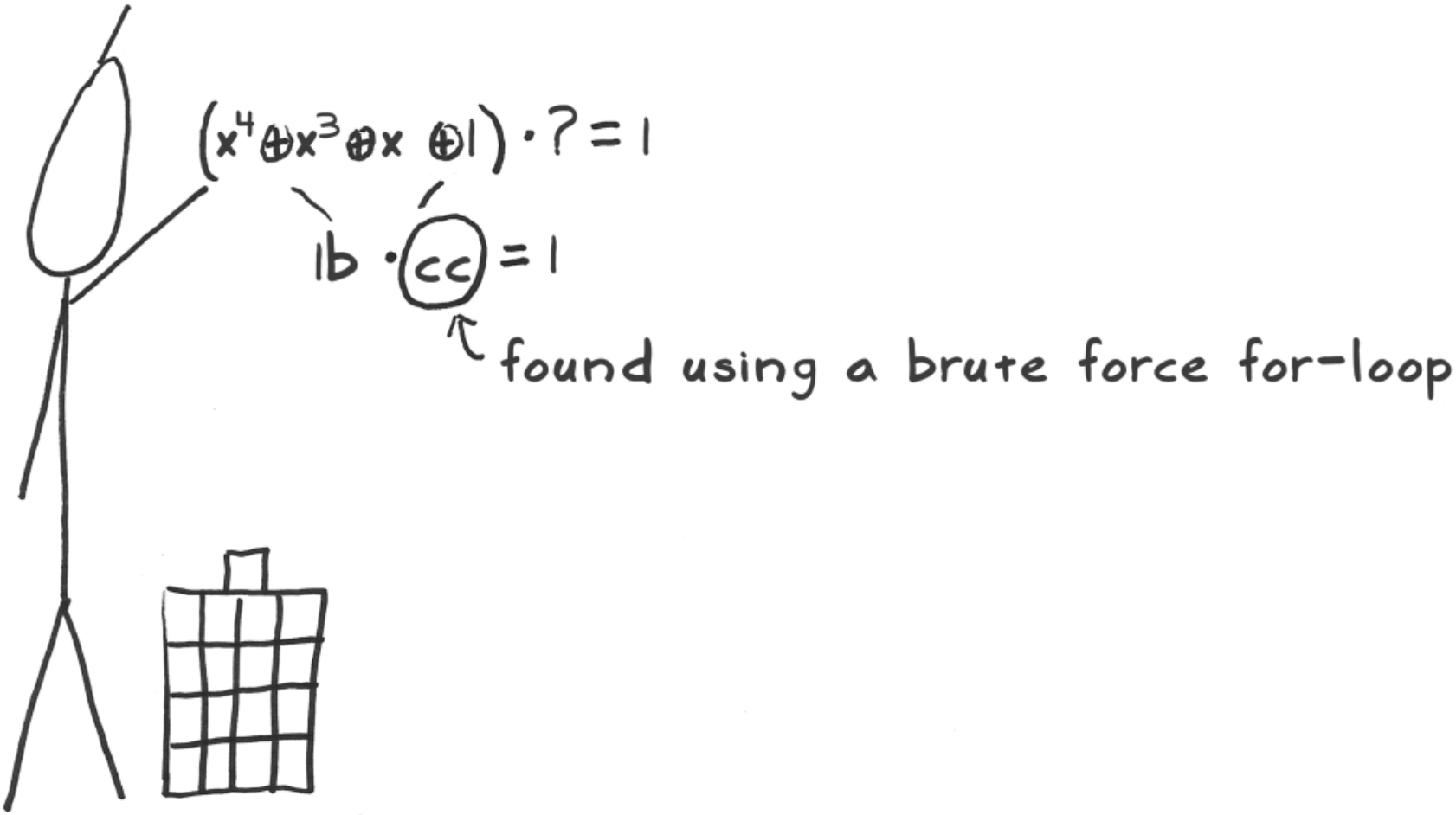
$$\Rightarrow \log(1b) + \log(aa) = c8 + 1f = e7$$

$$\Rightarrow \log^{-1}(e7) = 8c \Rightarrow 1b \cdot aa \quad \text{inverse table lookup}$$

$$= x^7 \oplus x^3 \oplus x^2$$

* We can create the table as we keep multiplying by $(x \oplus 1)$.

Since we know how to multiply, we can find the "inverse" polynomial byte for each byte. This is the byte that will undo/invert the polynomial back to 1. There are only 255* of them, so we can use brute force to find them:



* There are only 255 instead of 256 because 0 has no inverse.

Now we can understand the mysterious s-box. It takes a byte "a" and applies two functions. The first is "g" which just finds the byte inverse. The second is "f" which intentionally makes the math uglier to foil attackers.

$$g(a) = a^{-1}$$

$$f(a) = \begin{bmatrix} 1&1&1&1&1&0&0&0 \\ 0&1&1&1&1&1&0&0 \\ 0&0&1&1&1&1&1&0 \\ 0&0&0&1&1&1&1&1 \\ 1&0&0&0&1&1&1&1 \\ 1&1&0&0&0&1&1&1 \\ 1&1&1&0&0&0&1&1 \\ 1&1&1&1&0&0&0&1 \end{bmatrix} \cdot \begin{bmatrix} a_7 \\ a_6 \\ a_5 \\ a_4 \\ a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

Why build our S-box using properties of the inverse over $GF(2^8)$? It is known to have good non-linearity properties. This makes cryptanalysis harder!

$$sbox[a] = f(g(a))$$

$$sbox[58] = f(g(58))$$

$$sbox[58] = f(18) = 6a$$

$$58 \cdot 18 = 01$$

Translation

Linear transformation

The S-Box, Redux

x \ y	0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f
0	63	7c	77	7b	f2	6b	6f	c5	30	01	67	2b	fe	d7	ab	76
1	ca	82	c9	7d	fa	59	47	f0	ad	d4	a2	af	9c	a4	72	c0
2	b7	fd	93	26	36	3f	f7	cc	34	a5	e5	f1	71	d8	31	15
3	04	c7	23	c3	18	96	05	9a	07	12	80	e2	eb	27	b2	75
4	09	83	2c	1a	1b	6e	5a	a0	52	3b	d6	b3	29	e3	2f	84
5	53	d1	00	ed	20	fc	b1	5b	6a	cb	be	39	4a	4c	58	cf
6	d0	ef	aa	fb	43	4d	33	85	45	f9	02	7f	50	3c	9f	a8
7	51	a3	40	8f	92	9d	38	f5	bc	b6	da	21	10	ff	f3	d2
8	cd	0c	13	ec	5f	97	44	17	c4	a7	7e	3d	64	5d	19	73
9	60	81	4f	dc	22	2a	90	88	46	ee	b8	14	de	5e	0b	db
a	e0	32	3a	0a	49	06	24	5c	c2	d3	ac	62	91	95	e4	79
b	e7	c8	37	6d	8d	d5	4e	a9	6c	56	f4	ea	65	7a	ae	08
c	ba	78	25	2e	1c	a6	b4	c6	e8	dd	74	1f	4b	bd	8b	8a
d	70	3e	b5	66	48	03	f6	0e	61	35	57	b9	86	c1	1d	9e
e	e1	f8	98	11	69	d9	8e	94	9b	1e	87	e9	ce	55	28	df
f	8c	a1	89	0d	bf	e6	42	68	41	99	2d	0f	b0	54	bb	16

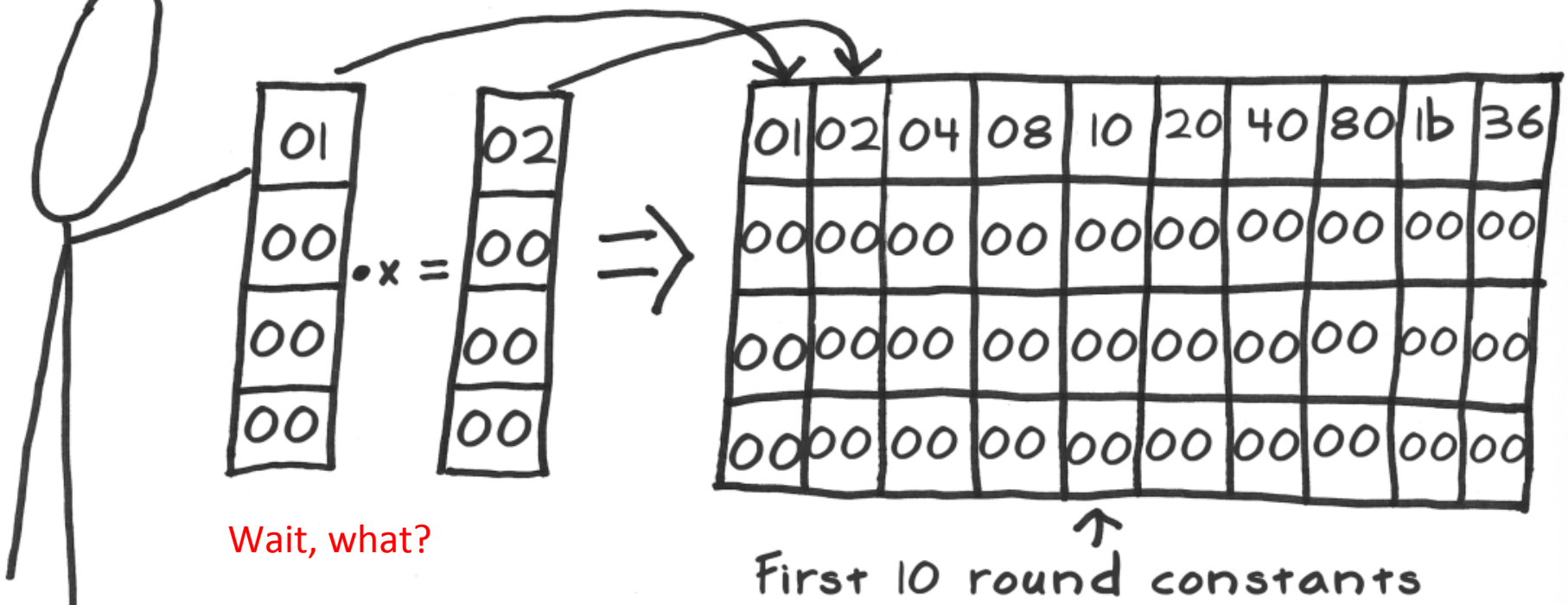
Recall: sbox[58] = f(g(58))

sbox[58] = f(18) = 6a

58·18=01

Doing the lookup is way faster than doing the actual math!

We can also understand those crazy round constants in the key expansion. I get them by starting with "1" and then keep multiplying by "x":



Wait, what?

$1 = 0x^7 + 0x^6 + 0x^5 + 0x^4 + 0x^3 + 0x^2 + 0x + 1 = 0000\ 0001 = 01$

$1 \cdot x = x = 0x^7 + 0x^6 + 0x^5 + 0x^4 + 0x^3 + 0x^2 + 1x + 0 = 0000\ 0010 = 02$

$x \cdot x = x^2 = 0x^7 + 0x^6 + 0x^5 + 0x^4 + 0x^3 + 1x^2 + 0x + 0 = 0000\ 0100 = 04$

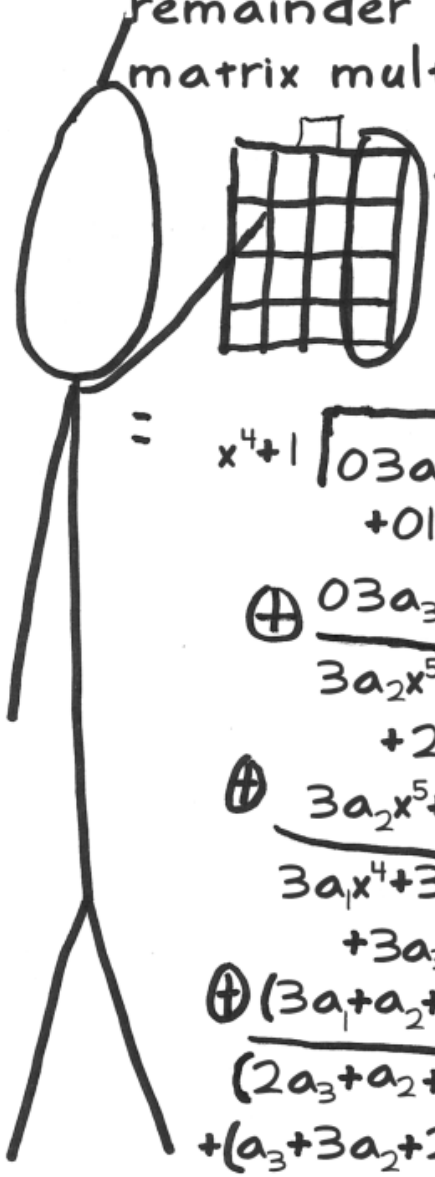
$x^2 \cdot x = x^3 = 0x^7 + 0x^6 + 0x^5 + 0x^4 + 1x^3 + 0x^2 + 0x + 0 = 0000\ 1000 = 08$

…

Observation: We're just left rotating here…

Mix Columns is the hardest. I treat each column as a polynomial. I then use our new multiply method to multiply it by a specially crafted polynomial and then take the remainder after dividing by x^4+1. This all simplifies to a matrix multiply:

i.e., Table lookups and XORs



$$\begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix}$$

$$b(x) = c(x) \cdot a(x) \bmod x^4+1$$

$$= (03x^3+01x^2+01x+02) \cdot (a_3x^3+a_2x^2+a_1x+a_0) \bmod x^4+1$$

special polynomial — the column

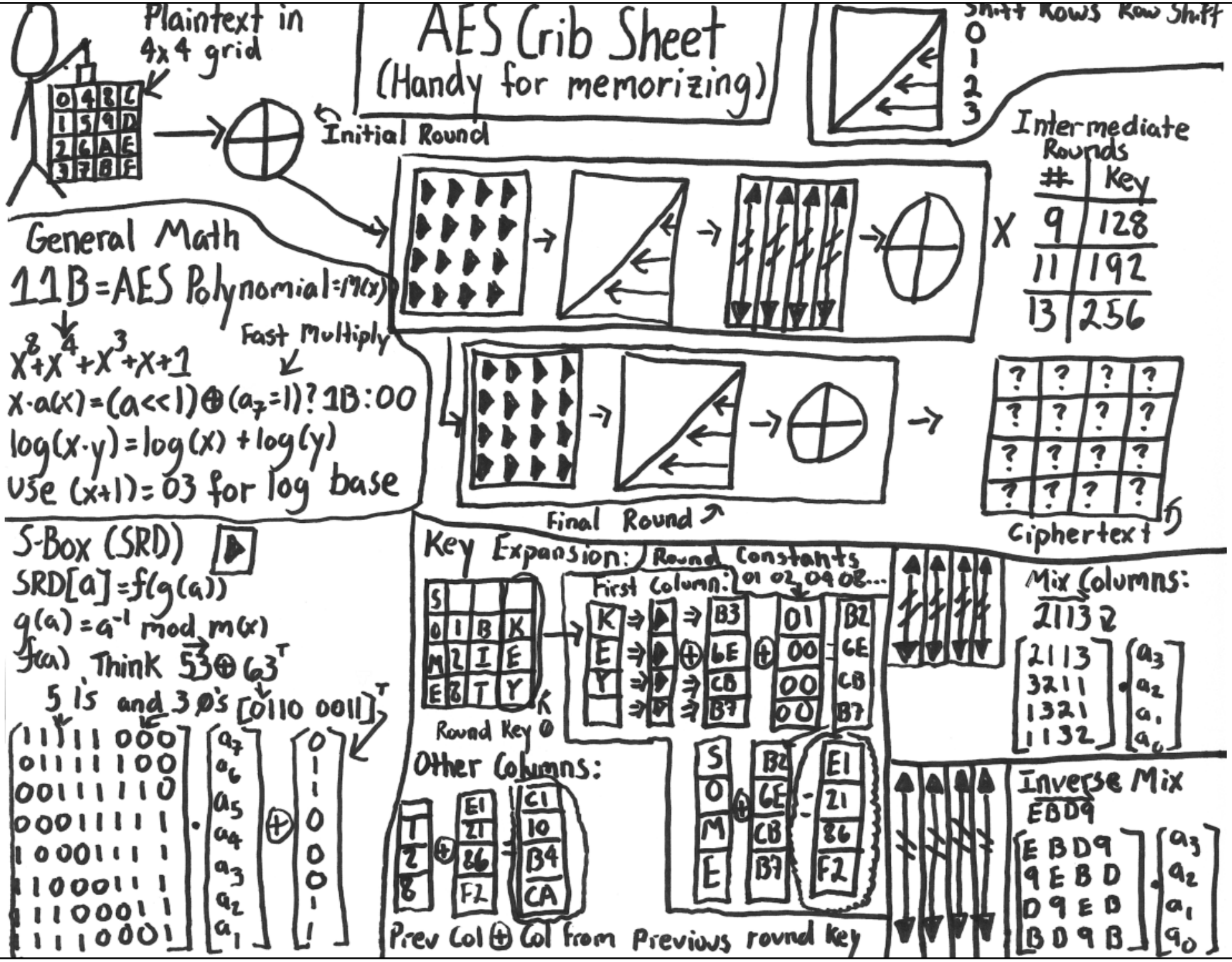
$$= x^4+1 \;\Big|\; \begin{array}{l} 03a_3 \cdot x^2+(3a_2+a_3)x+(3a_1+a_2+a_3) \\ \hline 03a_3x^6+03a_2x^5+03a_1x^4+03a_0x^3+01a_3x^5+01a_2x^4+01a_1x^3+01a_0x^2 \\ \quad +01a_3x^4+01a_2x^3+01a_1x^2+01a_0x+02a_3x^3+02a_2x^2+02a_1x+02a_0 \end{array}$$

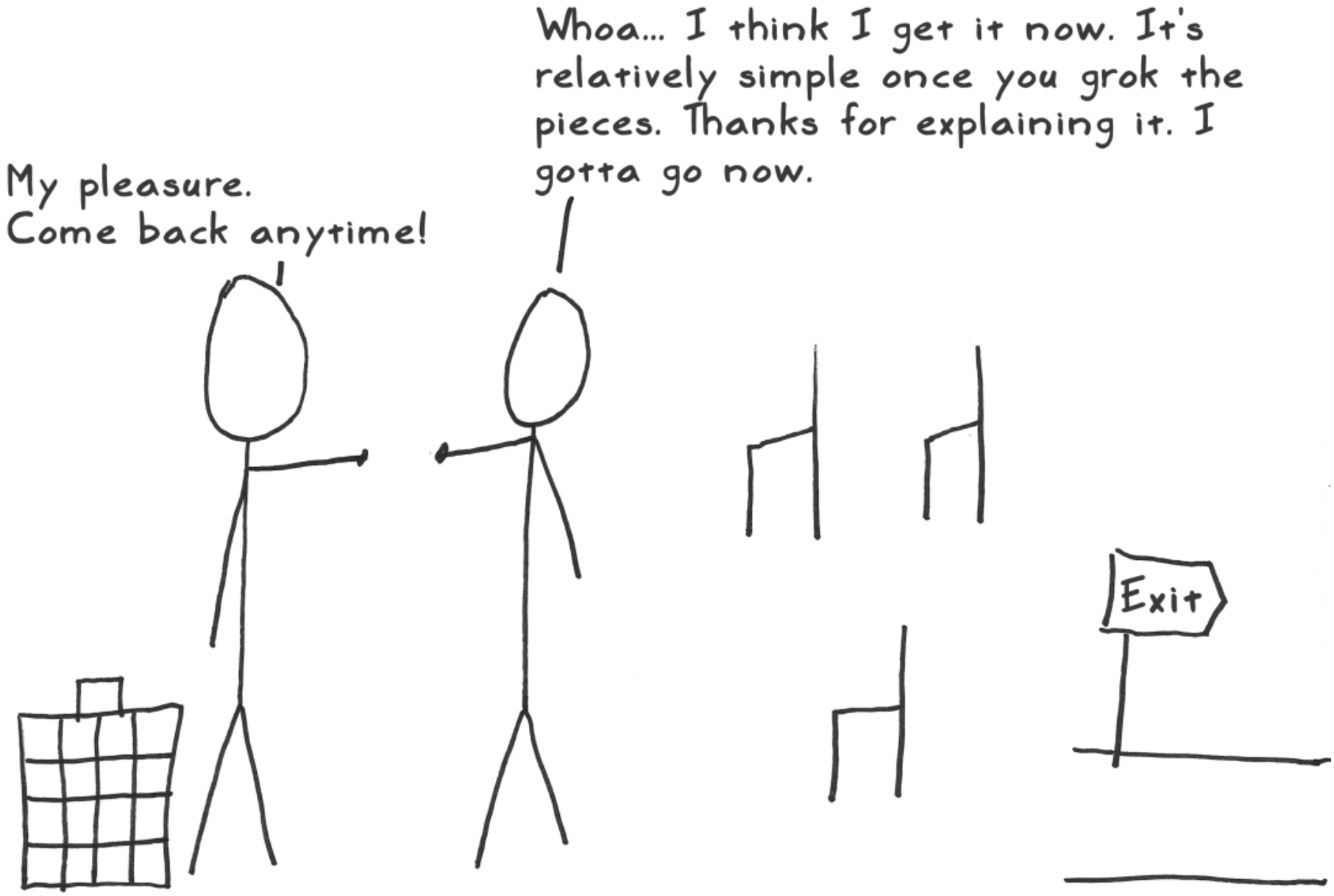
$$\oplus \; \begin{array}{l} 03a_3x^6+03a_3x^2 \\ \hline 3a_2x^5+3a_1x^4+3a_0x^3+a_3x^5+a_2x^4+a_1x^3+a_0x^2+a_3x^4+a_2x^3+a_1x^2+a_0x+2a_3x^3 \\ \quad +2a_2x^2+2a_1x+2a_0+3a_3x^2 \end{array}$$

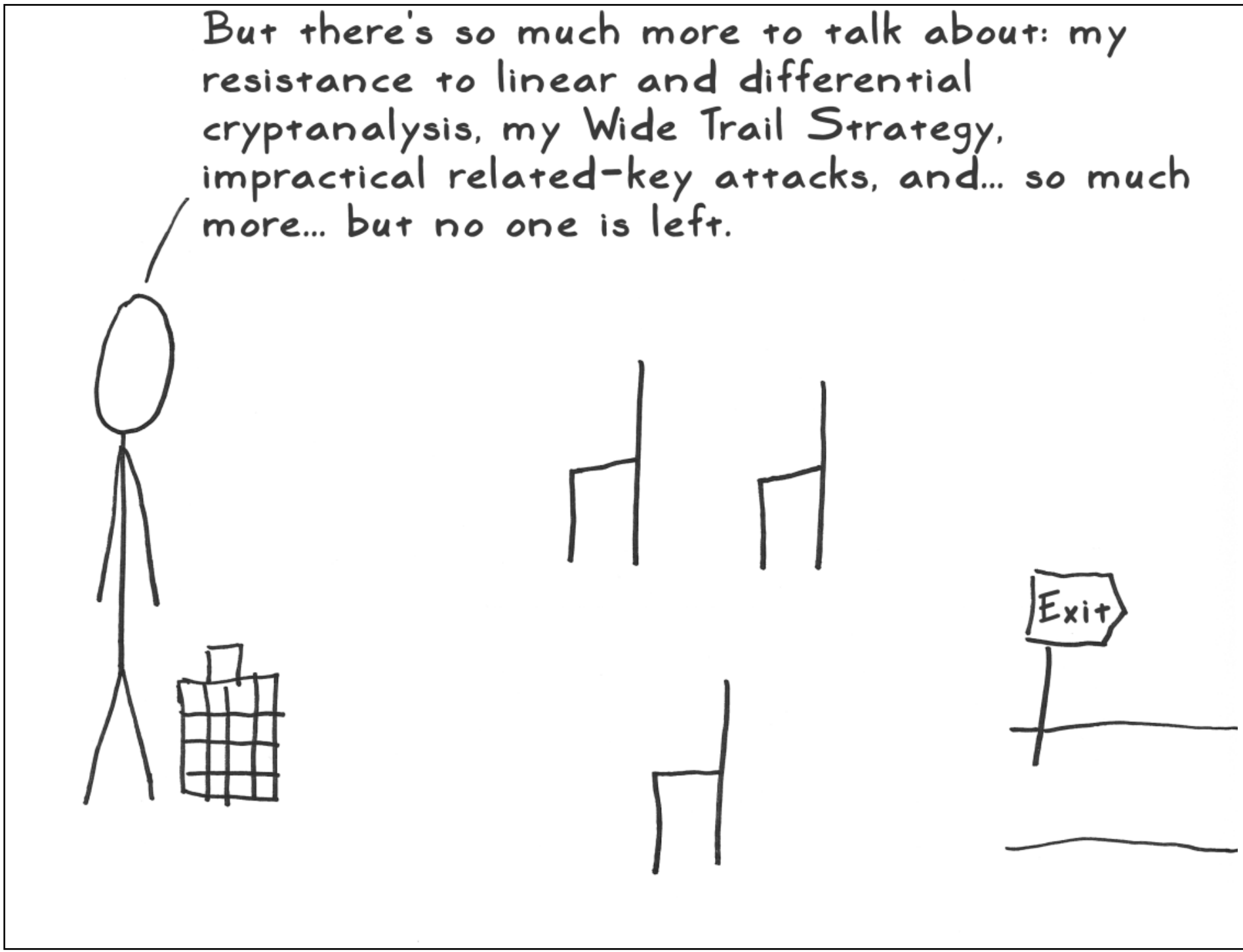
$$\oplus \; \begin{array}{l} 3a_2x^5+a_3x^5+3a_2x+a_3x \\ \hline 3a_1x^4+3a_0x^3+a_2x^4+a_1x^3+a_0x^2+a_3x^4+a_2x^3+a_1x^2+a_0x+2a_3x^3+2a_2x^2+2a_1x+2a_0 \\ \quad +3a_3x^2+3a_2x+a_3x \end{array}$$

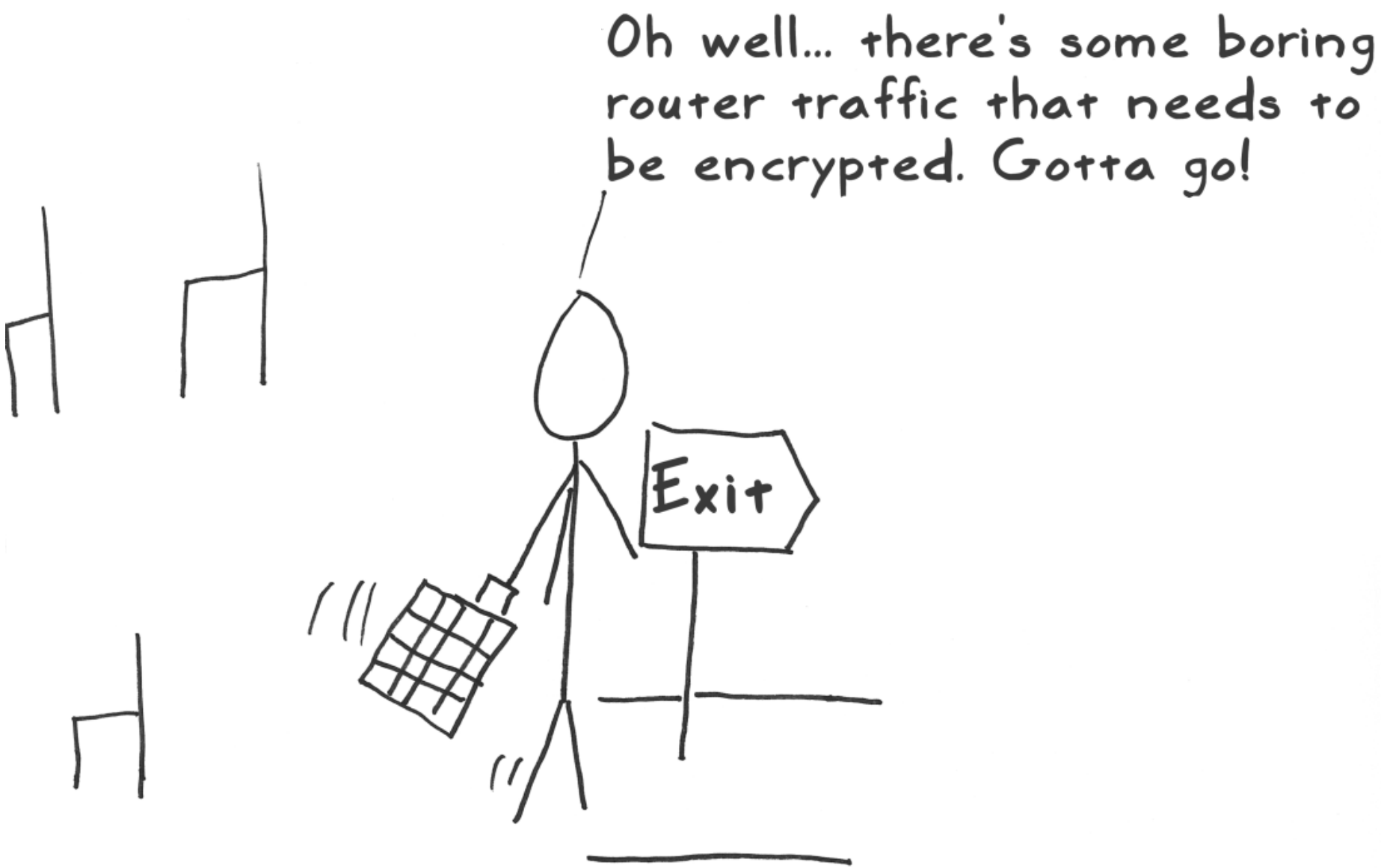
$$\oplus \; \begin{array}{l} (3a_1+a_2+a_3)x^4+(3a_1+a_2+a_3) \\ \hline (2a_3+a_2+a_1+3a_0)x^3+(3a_3+2a_2+a_1+a_0)x^2 \\ +(a_3+3a_2+2a_1+a_0)x+(a_3+a_2+3a_1+2a_0) \end{array} \Rightarrow \begin{bmatrix} 2 & 1 & 1 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 3 & 2 & 1 \\ 1 & 1 & 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix} = \begin{bmatrix} b_3 \\ b_2 \\ b_1 \\ b_0 \end{bmatrix}$$

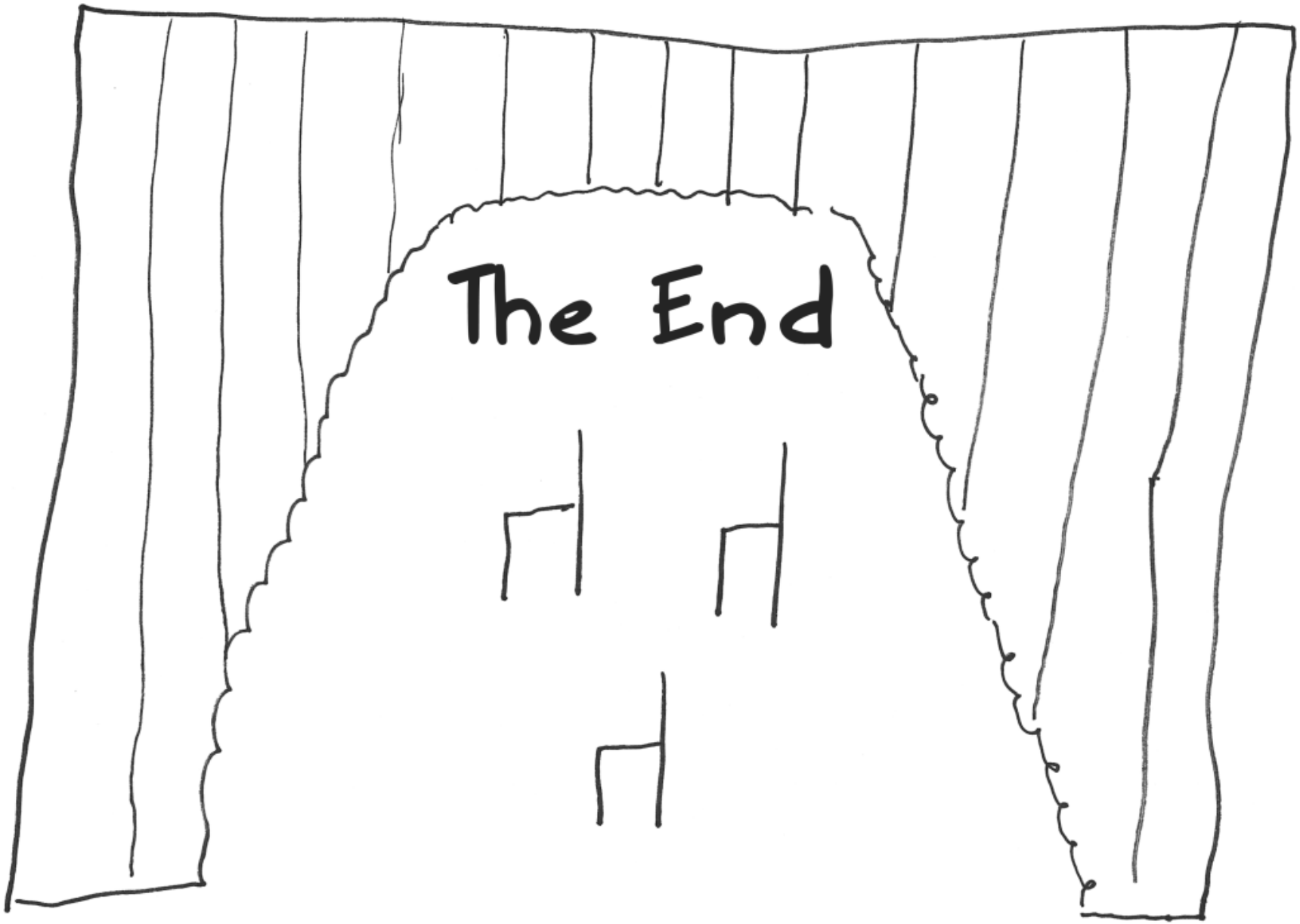
AES Crib Sheet
(Handy for memorizing)




Whoa... I think I get it now. It's relatively simple once you grok the pieces. Thanks for explaining it. I gotta go now.
My pleasure.
Come back anytime!
Exit


But there's so much more to talk about: my resistance to linear and differential cryptanalysis, my Wide Trail Strategy, impractical related-key attacks, and... so much more... but no one is left.
Exit


Oh well... there's some boring router traffic that needs to be encrypted. Gotta go!
Exit


The End

Final Thoughts…

In theory, you now understand how AES works

- High level: Apply confusion and diffusion to 128-bit blocks
- Medium level:
  - Key expansion to get 10 round keys
  - SubBytes via some crazy S-Box
  - ShiftRows to spread around the bytes
  - MixColumns to further spread around the bytes
  - XOR with round key, and repeat
- Low level:
  - S-Box is not random, it's based on inverses in GF(2^8)
  - Constants in MixColumns also derived from math in GF(2^8)
  - All of the insane shifting, XORing, and table lookups are really additions, multiplications, and inversion of polynomials in disguise!

Good non-linearity properties

Take away point: Crypto is difficult to implement properly, let alone design. Unless you plan to pursue higher education in mathematics, probably best to understand it as a tool, but leave the design to experts ☺