Applied Cryptography and Network Security

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Lecture #7: Public Key Cryptography
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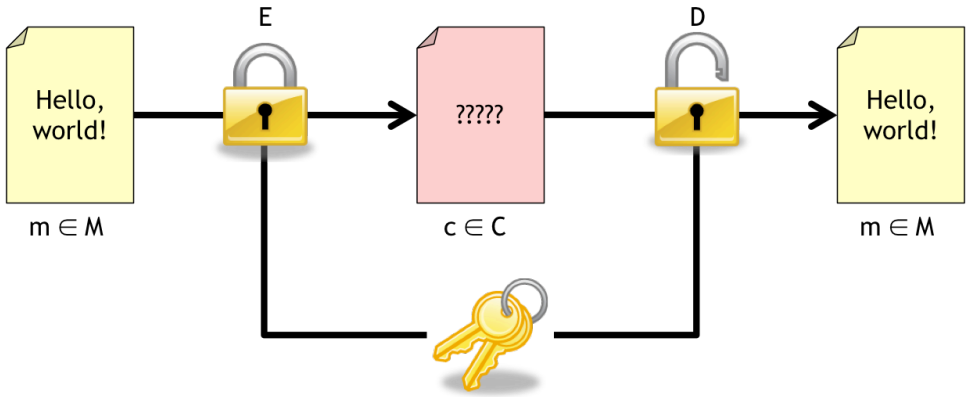
University of Pittsburgh

Motivation

Recall: In a symmetric key cryptosystem, the same key is used for both encryption and decryption



(Obvious) Note: the sender and recipient need a shared secret key

The good news is that symmetric key algorithms

- Have been well-studied by the cryptography community
- Are extremely fast, and thus good for encrypting bulk data
- Provide good security guarantees based on very small secrets

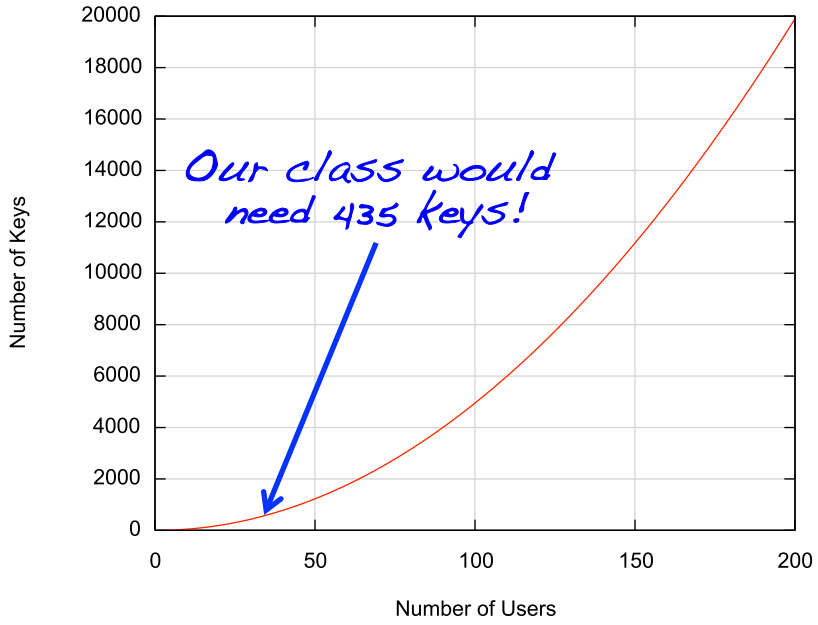
Unfortunately…

Symmetric key cryptography is not a panacea

Question: *What are some ways in which the need for a shared secret key might cause a problem?*

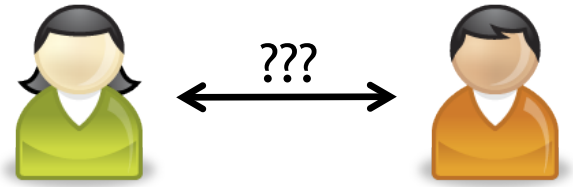
Problem 1: Key management

- In a network with *n* participants, $\binom{n}{2}$ = n(n-1)/2 keys are needed!
- This number grows very rapidly!



Problem 2: Key distribution

- How do Alice and Bob share keys in the first place?



- What if Alice and Bob have never met in person?
- What happens if they suspect that their shared key K_{AB} has been compromised?

Wouldn't it be great if we could securely communicate without needed pre-shared secrets?

Thought Experiment

Forget about bits, bytes, ciphers, keys, and math…

The Scenario: Assume that Alice and Bob have never met in person. Alice has a top secret widget that she needs to send to Bob using an untrusted courier service. Alice and Bob can talk over the phone if needed, but are unable to meet in person. Due to the high-security nature of their work, the phones used by Alice and Bob may by wiretapped by other secret agents.

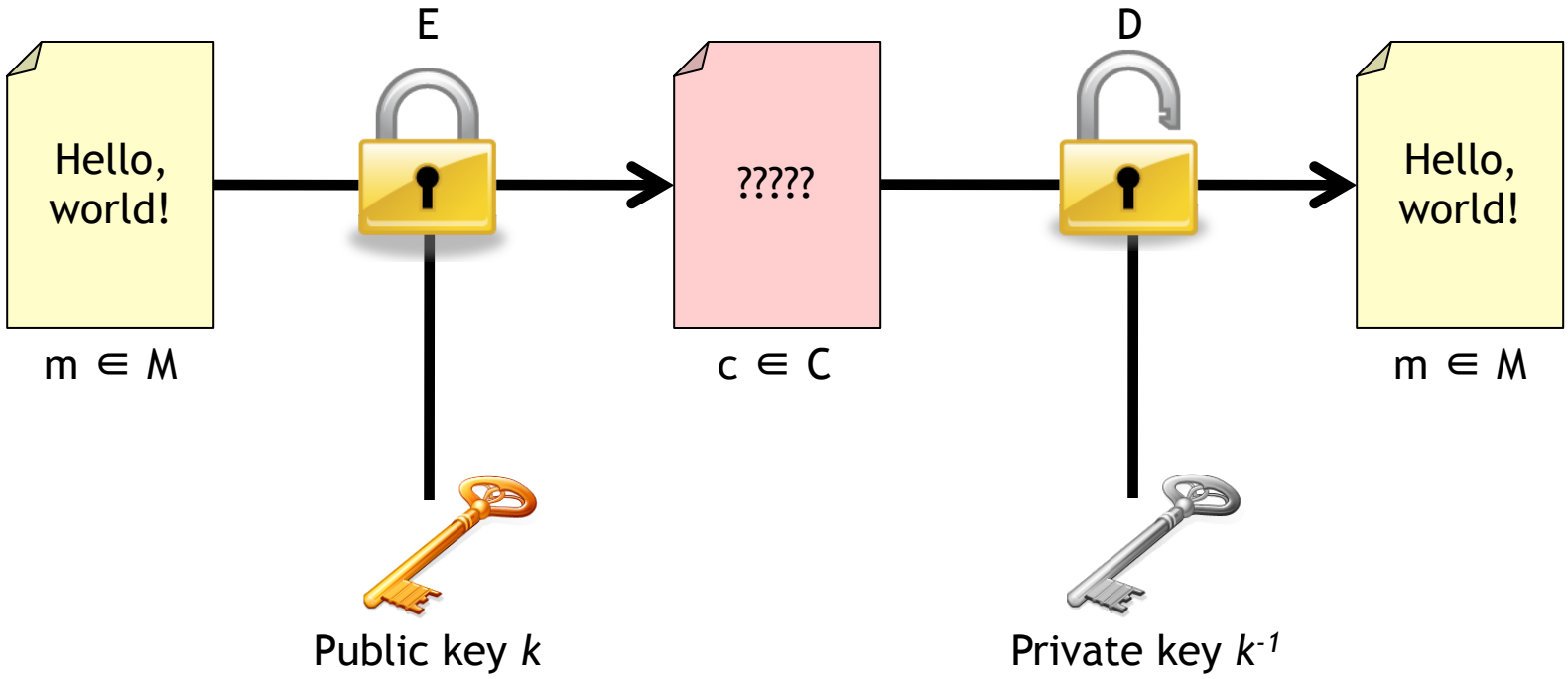
Problem: How can Alice send her widget to Bob while having very high assurance that Bob is the only person who will be able to access the widget if it is properly delivered?

Public key cryptosystems are a digital counterpart to our "strong-box" example

Formally, a cryptosystem can be represented as the 5-tuple (E, D, M, C, K)

- M is a message space
- K is a key space
- E : M × K → C is an encryption function
- C is a ciphertext space
- D : C × K → M is a decryption function

Note: Each "key" in K is actually a pair of keys, (k, k^{-1})



What can we do with public key cryptography?

First, we need some way of finding a user's public key

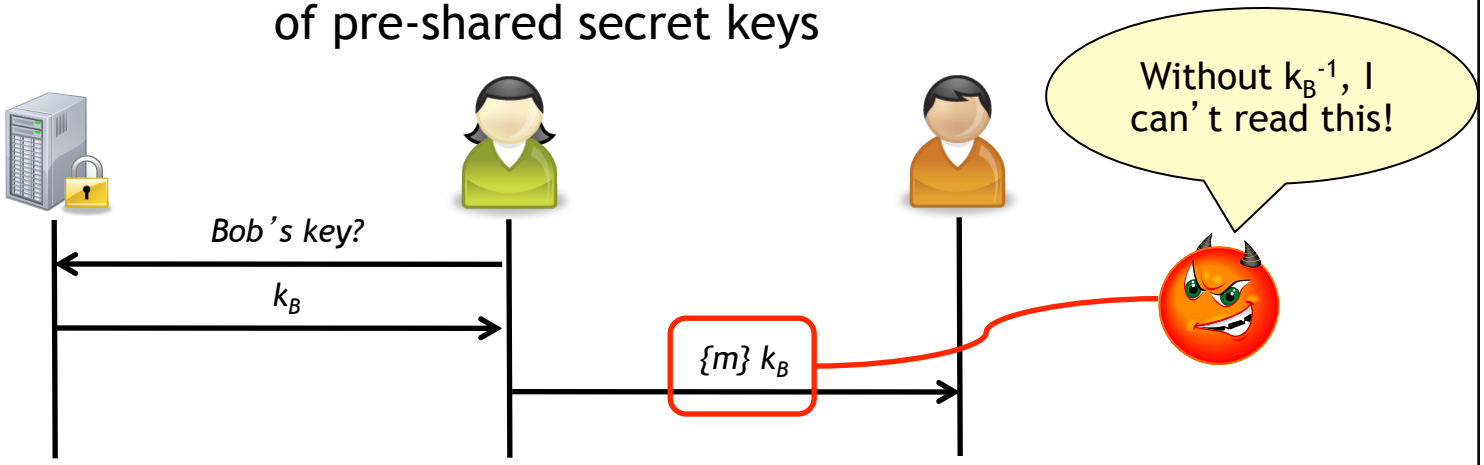
Print it in the newspaper

Post it on your webpage

A trusted keyserver (PKI)

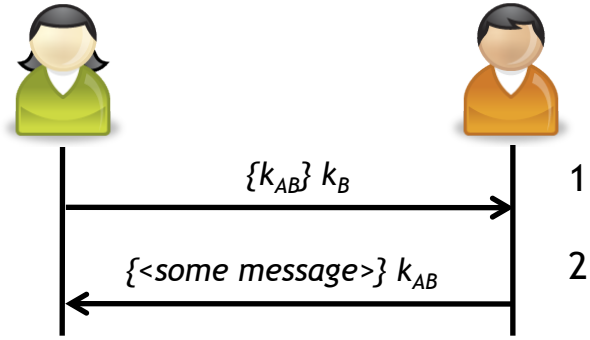
Important: It is critical to verify the authenticity of any public key! (How?)

Public key cryptography allows us to send private messages without the use of pre-shared secret keys



Public key cryptography can also help us exchange symmetric keys

1. Generate a key k_{AB}
2. Encrypt k_{AB} using Bob's public key k_B
3. Transmit



1. Decrypt k_{AB} using private key k_B^{-1}
2. Encrypt message using k_{AB}
3. Transmit

Note: Only Bob can decode k_{AB}, since only he knows k_B^{-1}

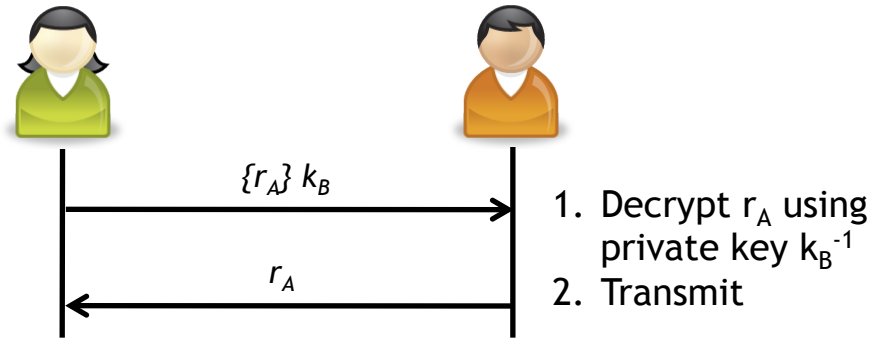
- Unfortunately, Bob doesn't know who this key is from ☺
- Key exchange is not quite this easy in practice, but it isn't *much* harder

Question: Why on earth do we want to exchange symmetric keys?!

- Public key cryptography is usually pretty slow…
  - Based on fancy math, not bit shifting
  - Symmetric key algorithms are orders of magnitude faster
- It's always a good idea to change keys periodically

Public key cryptography can also be used to authenticate users

1. Pick r_A at random
2. Encrypt r_A using Bob's public key k_B
3. Transmit



1. Decrypt r_A using private key k_B^{-1}
2. Transmit

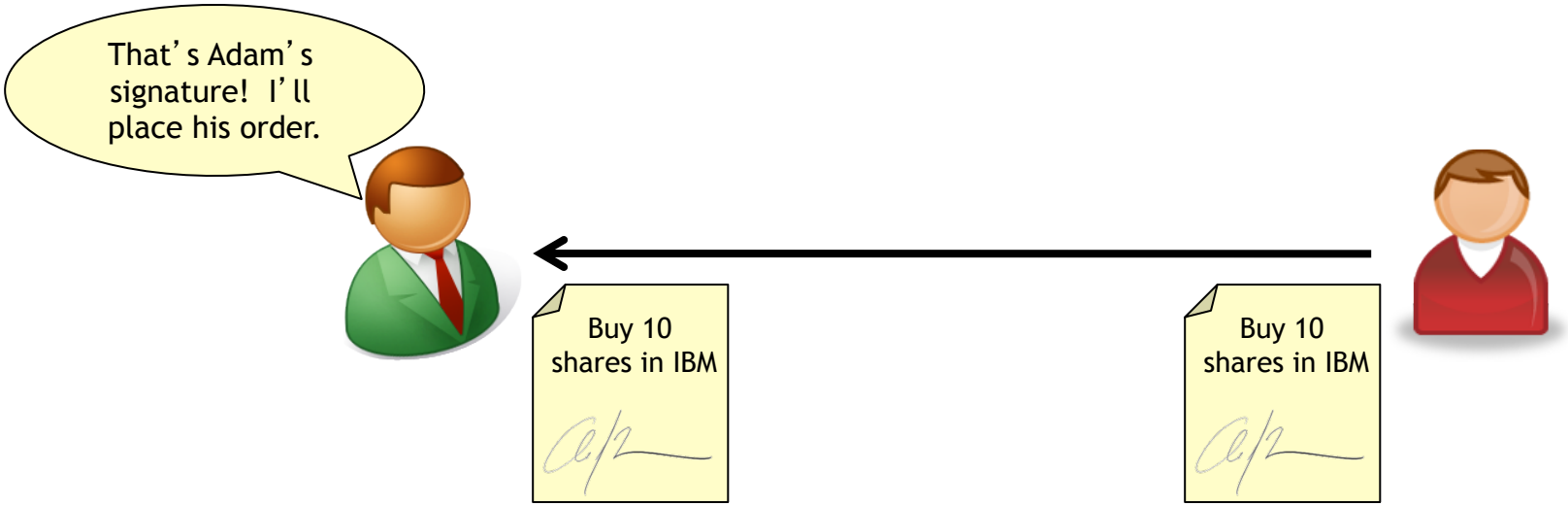
Note: As in the previous key exchange, only Bob can decrypt r_A, since only Bob knows k_B^{-1}.

It is of absolute importance that the random numbers used during this type of protocol are not predictable and are never reused (Why?)

- Unpredictable:
  - The security of this protocol is a proof of possession of k_B^{-1}
  - If predictable, an adversary can guess the "challenge" without decrypting!
  - Clearly, this is bad
- Reusing challenges may* lead to replay attacks (When?)

In addition to encryption, public key systems also let us create digital signatures

Goal: If Bob is given a message *m* and a signature S(*m*) supposedly computed by Adam, he can determine whether or not Adam actually wrote the message m



In order for this to occur, we require that

- The signature S(*m*) must be unforgeable
- The signature S(*m*) must be verifiable

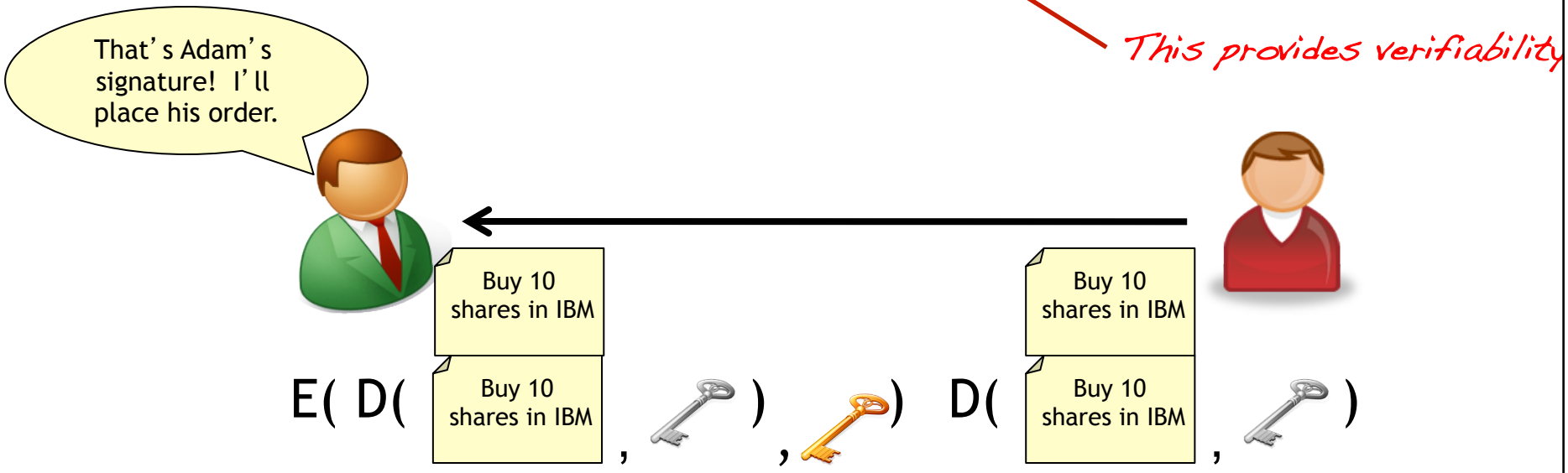
Question: *How can we do this?*

In many public key cryptosystems, the encryption and decryption operations are commutative

That is, $D(E(m, k), k^{-1}) = E(D(m, k^{-1}), k) = m$

In such a system, we can use digital signatures as follows:

- To sign a message, compute $D(m, k^{-1})$ ← This is unforgeable
- Transmit m and $D(m, k^{-1})$ to the recipient
- The recipient uses the sender's public key to verify that $E(D(m, k^{-1}), k) = m$ ← This provides verifiability



Question: Does encryption with a shared key have the same properties?

Features and Requirements

These features all require that for a given key pair (k, k^{-1}), k can be made public and k^{-1} must remain secret

So, in a public key cryptosystem it must be

1. Computationally easy to encipher or decipher a message
2. Computationally infeasible to derive the private key from the public key
3. Computationally infeasible to determine the private key using a chosen plaintext attack

Informally, easy means "polynomial complexity", while infeasible means "no easier than a brute force search"

How do public key cryptosystems work?

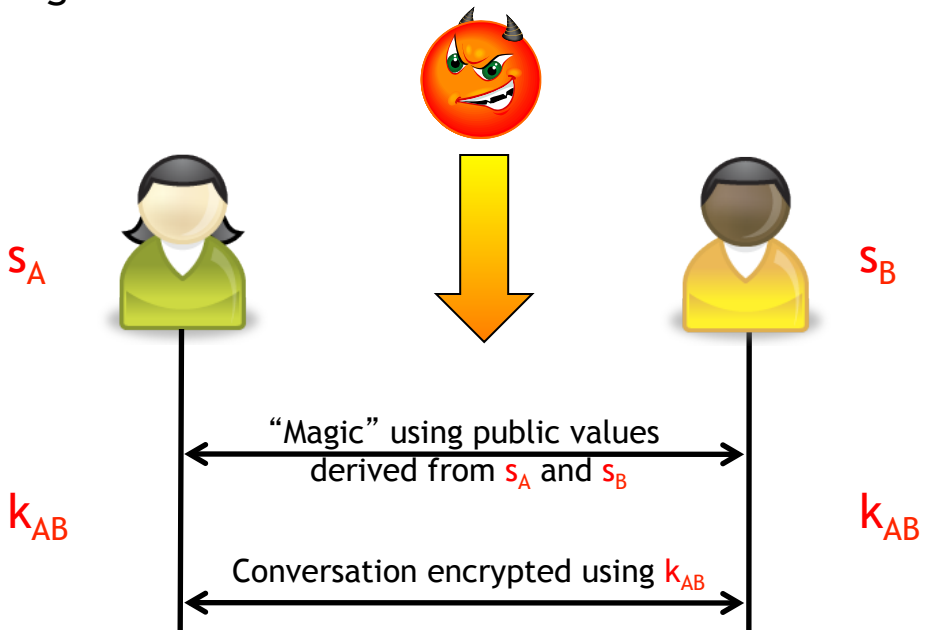
Diffie and Hellman proposed* the notion of public key cryptography

Diffie and Hellman did not succeed in developing a full-fledged public key cryptosystem

- I.e., their system cannot be used to encrypt/decrypt document directly
- Rather, it allows two parties to agree on a shared secret using an entirely public channel

Question: Why is this an interesting problem to solve?

- Key exchange!



Diffie and Hellman proposed their system in 1976

Seminal paper: Whitfield Diffie and Martin E. Hellman, "New Directions in Cryptography," IEEE Transactions on Information Theory (22)6 : 644 – 654, Nov. 1976

Problem: The widening use of telecommunications coupled with the key distribution problems inherent with secret key cryptography point to the fact that current solutions are not scalable!

This paper accomplishes many things:

- Clearly articulates why the key distribution problem must be solved
- Motivates the need for digital signatures
- Presents the first public key cryptographic algorithm
- Opened the "challenge" of designing a general purpose public key cryptosystem

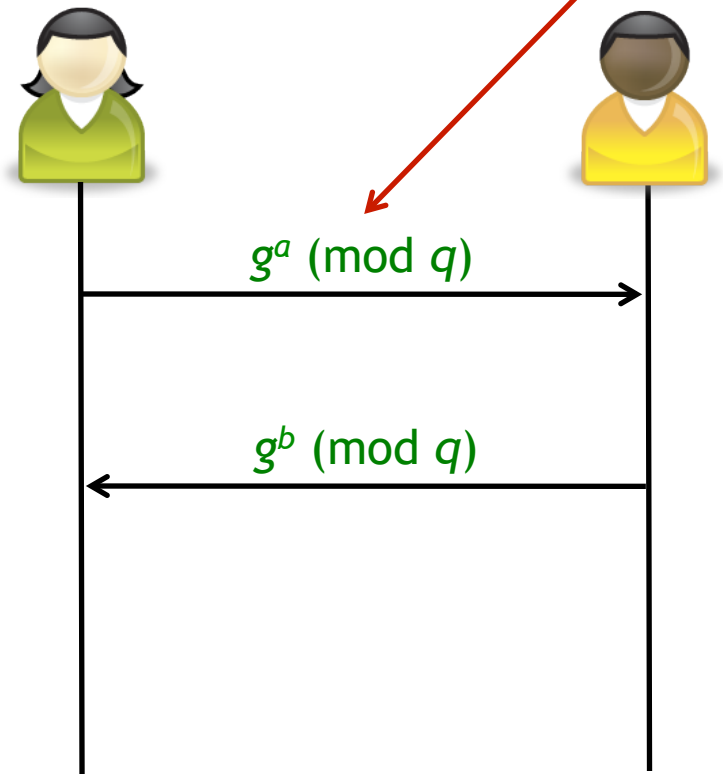
Variants of the Diffie-Hellman key exchange algorithm are still used today!

How does the Diffie-Hellman protocol work?

Step 0: Alice and Bob agree on a finite cyclic group G of (large) prime order q, and a generator g for this group. This information is all public.

a is Alice's private key

$g^a \pmod q$ is Alice's public key



Step 1:

- Randomly choose $a \in \{1, 2, \ldots, q\text{-}1\}$
- Compute $g^a \pmod q$
- Send $g^a \pmod q$

Step 2:

- Randomly choose $b \in \{1, 2, \ldots, q\text{-}1\}$
- Compute $g^b \pmod q$
- Send $g^b \pmod q$

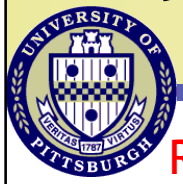
Step 3:

- Compute $(g^b \pmod q)^a \pmod q) = g^{ba} \pmod q = K_{ab}$

Step 3':

- Compute $(g^a \pmod q)^b \pmod q) = g^{ab} \pmod q = K_{ab}$

Why is the Diffie-Hellman key exchange protocol safe?



Recall: We need to show that it is hard for a "bad guy" to learn any of the secret information generated by this protocol, assuming that they know all public information

Public information: G, g, q, $g^a \pmod q$, $g^b \pmod q$

Private information: a, b, $K_{ab} = g^{ab} \pmod q$

Tactic 1: Can we get $g^{ab} \pmod q$ from $g^a \pmod q$ and $g^b \pmod q$?

- We can get $g^{am+bn} \pmod q$ for arbitrary m and n, but this is no help…

Tactic 2: Can we get a from $g^a \pmod q$?

- This called taking the discrete logarithm of $g^a \pmod q$
- The discrete logarithm problem is widely believed to be very hard to solve in certain types of cyclic groups

Conclusion: If solving the discrete logarithm problem is hard, then the Diffie-Hellman key exchange is secure!

Hm, interesting…

Recall from last week that:

- Block ciphers secure data through confusion and diffusion
- Designing block cipher mechanisms is equal parts art and science
- The security of a block cipher is typically accepted over time (Assurance!)
  - Recall the initial skepticsim over DES
  - The NIST competitions promote this as well

In public key cryptography, the relationship between k and k^{-1} is intrinsically mathematical

Result: The security of these systems is also rooted in mathematical relationships, and proofs of security involve reductions to mathematically "hard" problems

- E.g., Diffie-Hellman safe if the discrete logarithm is hard

The RSA cryptosystem picks up where Diffie and Hellman left off

RSA was proposed* by Ron Rivest, Adi Shamir, and Leonard Adelman in 1978. It can be used to encrypt/decrypt and digitally sign arbitrary data!

Key generation:

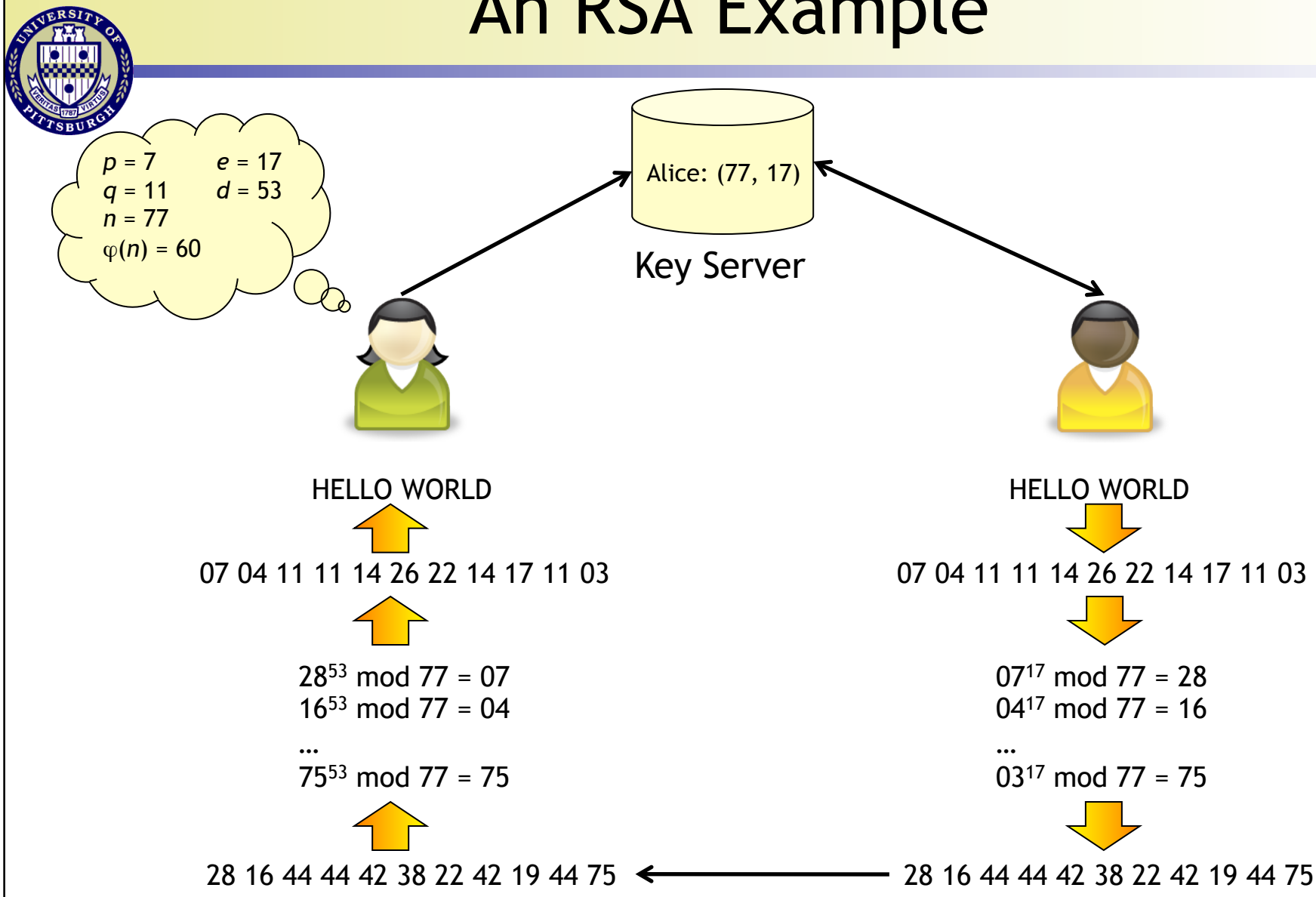
- Choose two large prime numbers p and q, compute $n = pq$
- Compute $\varphi(n) = (p-1)(q-1)$
- Choose an integer d such that $\gcd(d, \varphi(n)) = 1$
- Calculate e such that $ed \equiv 1 \pmod{\varphi(n)}$
- Public key: n, e
- Private key: p, q, d

We'll discuss how to do these steps an why they work in the next lecture

Usage:

- Encryption: $M^e \pmod{n}$
- Decryption: $C^d \pmod{n} = M^{ed} \pmod{n} = M^{k\varphi(n)+1} \pmod{n} = M^1 \pmod{n} = M$

An RSA Example



What is involved in breaking RSA?

To break RSA, an attacker would need to derive the decryption exponent *d* from the public key (*n*, *e*)

Mathematicians think that this is a hard problem

This is conjectured to be as hard as factoring *n* into *p* and *q*. Why?

- Given *p*, *q*, and *e*, we can compute $\varphi(n)$
- This allows us to compute *d* easily!

But what if there is some entirely unrelated way to derive *d* from the public key (*n*, *e*)?

Question: Should this make you uneasy? Why or why not?

My Answer: Probably not, since this wacky new attack would also have applications to factoring large numbers.

As always, nothing is really that easy…

The bad news: Naive implementations of RSA are vulnerable to chosen ciphertext attacks

The good news: These attacks can be prevented by using a padding scheme like OAEP prior to encryption

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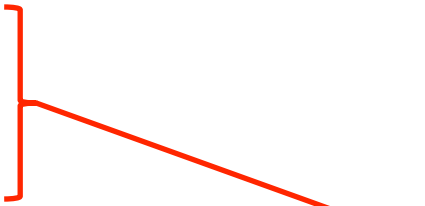
Don't implement cryptography yourself! Use a standardized implementation, and verify that it is standards compliant.

Lastly, don't forget that implementations can be subjected to attacks

- Timing attacks
- Power consumption attacks
- Etc…



More on this in Lecture 15

Unfortunately, RSA is slow when compared to symmetric key algorithms like AES or HMAC-X

Using RSA as part of a hybrid cryptosystem can speed up encryption

- Generate a symmetric key k_s
- Encrypt m with k_s
- Use RSA to encrypt k_s using public key k
- Transmit $E_{ks}(m)$, $E_k(k_s)$

Using hash functions can help speed up signing operations

- Intuition: $H(m) << m$, so signing $H(m)$ takes far less time than signing m
- Why is this safe? H's preimage resistance property!

Some public-key systems have an interesting property known as malleability

Informally, a malleable cryptosystem allows meaningful modifications to be made to ciphertexts without revealing the underlying plaintext

- E.g., $E(x) \otimes E(y) = E(x + y)$

See: Pascal Paillier, Public-Key Cryptosystems Based on Composite Degree Residuosity Classes, EUROCRYPT 1999, pages 223-238.

MacKenzie et al. define a tag-based cryptosystem

- Messages encrypted relative to a key and a tag
- Only messages with the same tag can be combined

For example:

- $E(m, t) \otimes E(m', t) = E(mm', t)$
- $E(m, t) \otimes E(m', t') =$ <garbage>

See: Philip MacKenzie, Michael K. Reiter, and Ke Yang, "Alternatives to Non-malleability: Definitions, Constructions, and Applications (Extended Abstract)", TCC 2004, pages 171-190.

Discussion

Question 1: Why might malleability be an interesting property for a cryptosystem to have?

- Tallying electronic votes
- Aggregating private values
- A primitive for privacy-preserving computation
- …

Question 2: Why might this be bad?

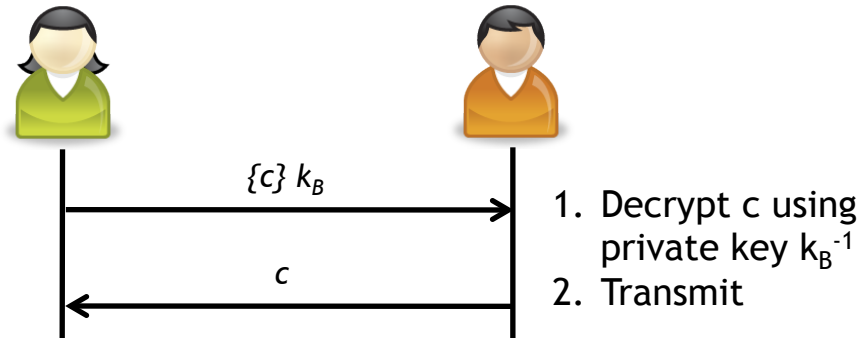
- Modifications by an active attacker!
- ***Example:*** Modifying an encrypted payment

In short, these types of cryptosystems have interesting properties, but require care to use properly.

Note that public key cryptography allows us to prove knowledge of a secret without revealing that secret

Example: Decrypting a challenge

1. Pick challenge c at random
2. Encrypt c using Bob's public key k_B
3. Transmit

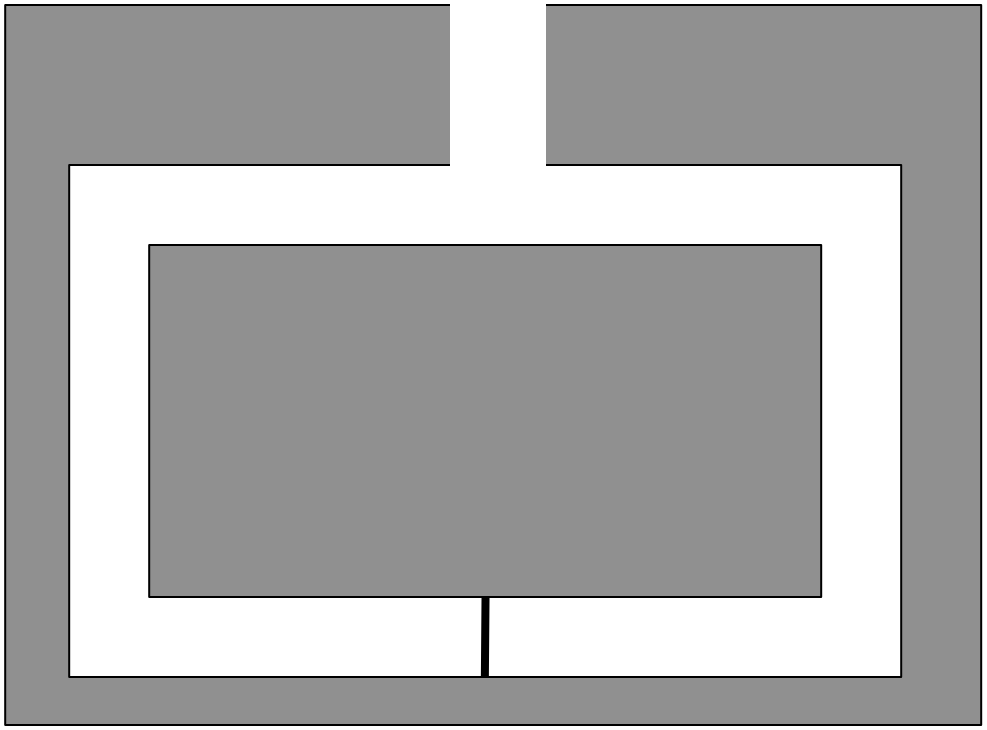


Note: Revealing the challenge, c, does not leak information about the private key k_B^{-1}, yet Alice is (correctly) convinced that Bob knows k_B^{-1}

This type of protocol is called a zero knowledge protocol

Zero-knowledge proofs are so easy to understand, even a child can figure it out!

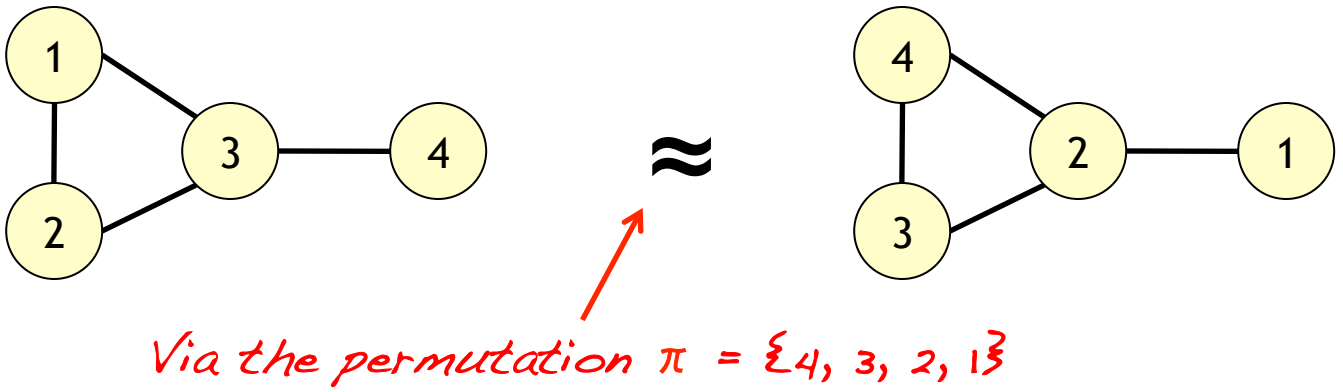
Example: The secret cave



Note: To ensure correctness, this "protocol" needs to be run multiple times (Why?)

We can construct a more realistic ZK system based on the (hard) problem of determining graph isomorphism

Informally, two graphs are isomorphic if the only difference between them is the names of their nodes



Determining whether two graphs are isomorphic is an NP-Complete problem. This means that if the graphs are large, solving this problem will take a long time, but checking a solution is very easy.

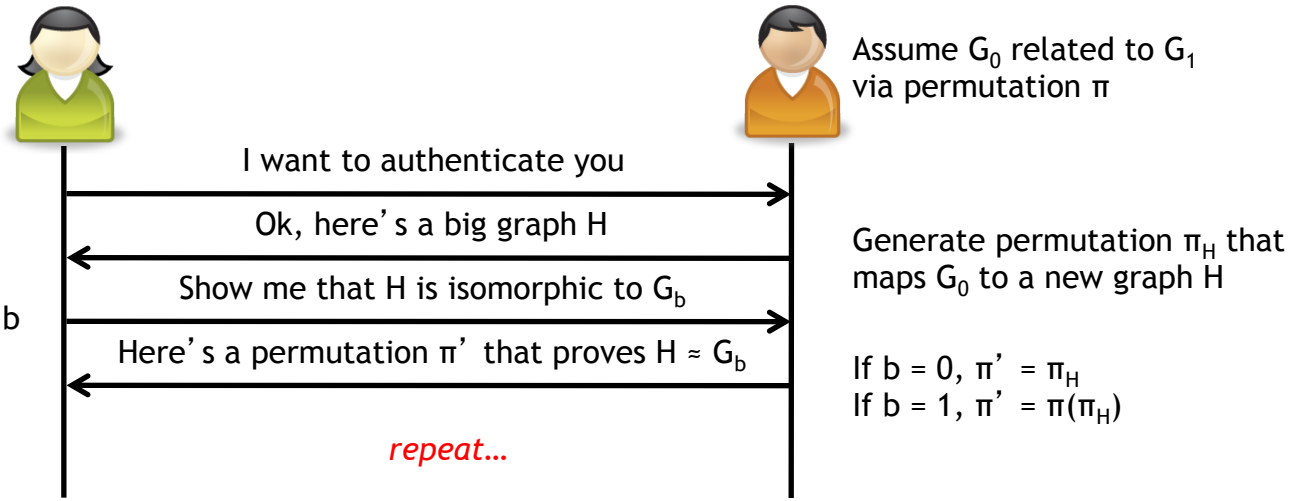
Authenticating via Graph Isomorphism

Our protocol has fairly simple parameters

- Public key: Two (big) isomorphic graphs G_0 and G_1
- Private key: The permutation mapping $G_0 \rightarrow G_1$

How do we find these efficiently?

The protocol:



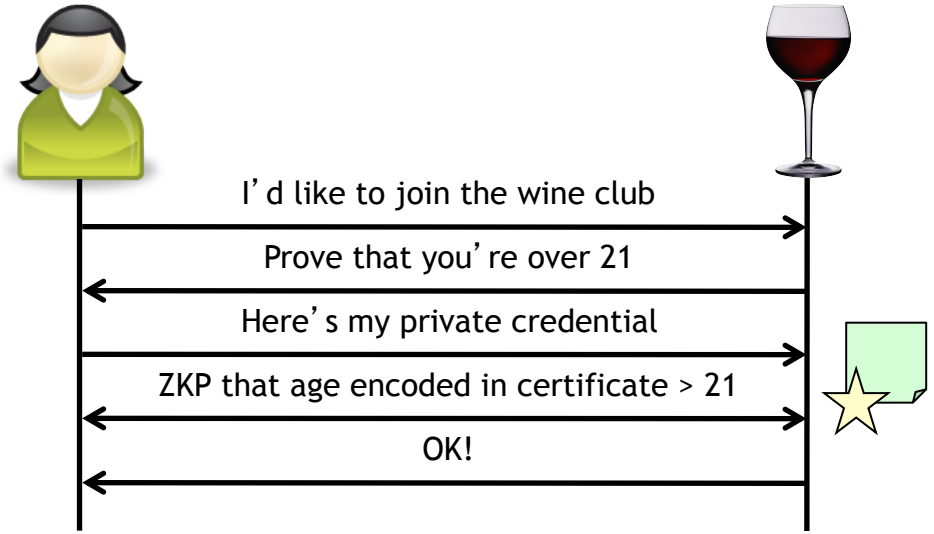
Why does this work?

- Answering this once means that Bob knows (at least) the permutation mapping from G_b to H.
- Doing this m times means that Bob knows the mapping between G_0 and G_1 with probability $1 - 0.5^m$
- Note that this leaks no information regarding the permutation π (Why?)

Zero knowledge proofs of knowledge can be used to solve a variety of interesting authorization problems

Private/Anonymous credential systems allow users to prove that they have certain attributes without actually revealing these attributes

Example: Purchasing wine over the Internet



The private credential scheme proposed by Stefan Brands enables many types of attribute properties to be checked in a zero-knowledge fashion

Conclusions

Secret key cryptography has a key distribution problem

Public key cryptography overcomes this problem!

- Public encryption key
- Private decryption key

Digital signatures provide both integrity protection and non-repudiation

Malleable cryptosystems are useful, but their usage entails certain risks

Zero knowledge proof systems have many interesting applications

Next time: *Really* understanding RSA