Computational Linear Algebra, Module 9 (1-10)

Maya Shende

Due: March 28st, 2017

1. Yes, this does draw the function $g(x) = x - \frac{5}{3}x^3$
2. When more terms are added to the approximation function, we see that it looks like the sine function
3. Errors for interval $[0, 2\pi]$:

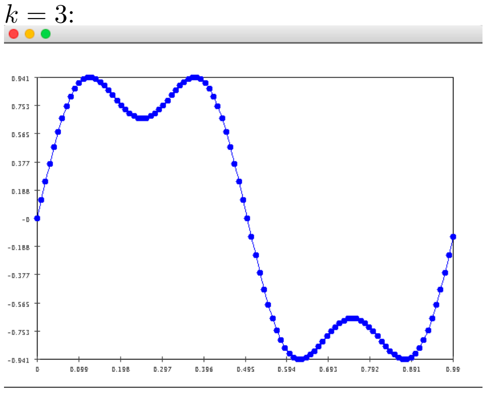
n=3	44.03945
n=5	38.71829
n=7	18.99292
n=13	0.22859

The difference between $n = 3$ and $n = 13$ is $44.03945 - 0.22859 = 43.81086$. The errors for $n = 3$ and $n = 13$ for the interval $[0, \pi]$ are: $n = 3 \rightarrow 1.13981$ and $n = 13 \rightarrow 4.30594 \times 10^{-6}$

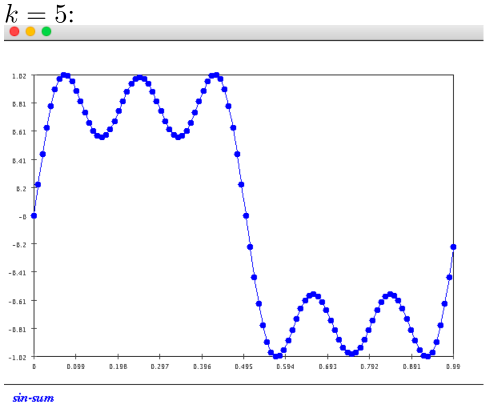
For the interval $[0, \pi/2]$, the errors are $n = 3 \rightarrow 0.01836$ and $n = 13 \rightarrow 4.30594 \times 10^{-11}$

The error at each point is multiplied by deltaX because we are calculating the area between the two curves to find the error.

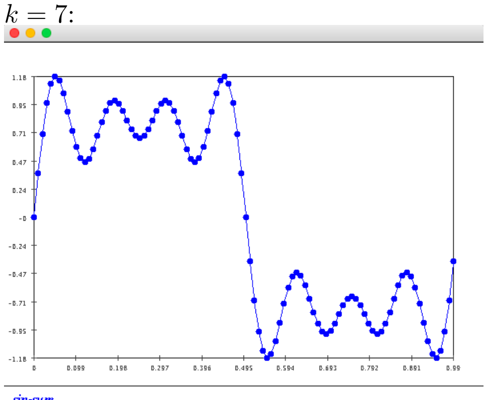
4. Taylor series expansion: $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \ldots$. The coefficients printed in the alpha array are the same as those we saw in the earlier exercise.
5. $k = 3$:



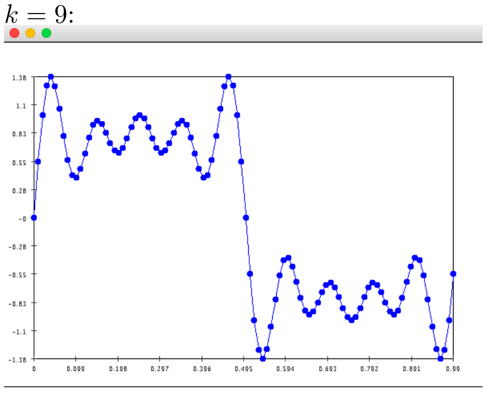
$k = 5$:


1.02
0.81
0.61
0.41
0.2
-0
-0.2
-0.41
-0.61
-0.81
-1.02
0
0.099
0.198
0.297
0.396
0.495
0.594
0.693
0.792
0.891
0.99
sin-sum

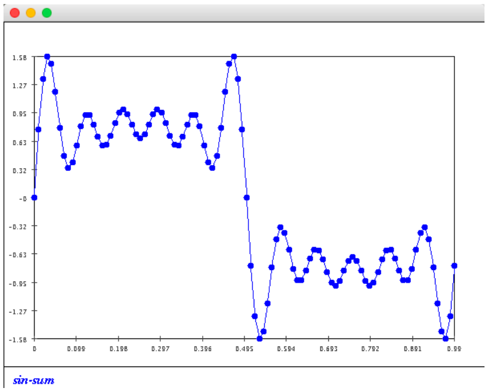
$k = 7$:


1.18
0.95
0.71
0.47
0.24
-0
-0.24
-0.47
-0.71
-0.95
-1.18
0
0.099
0.198
0.297
0.396
0.495
0.594
0.693
0.792
0.891
0.99
sin-sum

$k = 9$:


1.38
1.1
0.83
0.55
0.28
-0
-0.28
-0.55
-0.83
-1.1
-1.38
0
0.099
0.198
0.297
0.396
0.495
0.594
0.693
0.792
0.891
0.99
sin-sum

$k = 11$:



6. Any piecewise function cannot be represented by a linear combination of $\sin(2\pi kx)$. For example,

$$h(x) = \begin{cases} 1 & x \leq 0 \\ 3 & 0 \leq x \leq 20 \\ 5 & x > 20 \end{cases}$$

7. The second graph that is output by this program is a subsection of the first graph, and this is a fractal so the subsection looks like the full graph.

8. $k = 0 \rightarrow$ evaluates to 1
$k = 1 \rightarrow$ evaluates to n
$k = n - 1 \rightarrow$ evaluates to n, which is the same as when $k = 1$
$k = n \rightarrow$ evaluates to 1, which is the same as when $k = 0$

9. $\binom{n}{k} = \frac{n!}{k!(n-k)!} \rightarrow$ this is derived by taking the total number of combinations possible, and dividing out the repeated combinations.

k	$\binom{n}{k}$
0	1
1	5
2	10
3	10
4	5
5	1

10. We know that $\binom{n}{k} = \frac{n!}{k!(n-k)!}$, $\binom{n-1}{k} = \frac{(n-1)!}{k!(n-1-k)!}$ and $\binom{n-1}{k-1} = \frac{(n-1)!}{(k-1)!(n-k)!}$.

So now we have:

$$\begin{aligned}
\binom{n-1}{k} + \binom{n-1}{k-1} &= \frac{(n-1)!}{k!(n-1-k)!} + \frac{(n-1)!}{(k-1)!(n-k)!} \\
&= \frac{(n-1)!(n-k) + k(n-1)!}{k!(n-k)!} \\
&= \frac{n! - k(n-1)! + k(n-1)!}{k!(n-k)!} \\
&= \frac{n!}{k!(n-k)!} \\
&= \binom{n}{k}
\end{aligned}$$

Therefore, $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$ ■

$\binom{n}{k}$ is a symmetric function because of how choosing combinations works.
Pascal's Triangle:

				1				
			1		1			
		1		2		1		
	1		3		3		1	
1		4		6		4		1

Each row of Pascal's Triangle gives the values of $\binom{n}{k}$, where k is the column of the row and n is the row number.