

# Computational Linear Algebra, Assignment 2, Part I

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Due: March 7th, 2018

- 1.
2. We can conclude that  $\text{nullspace}(A) = \text{nullspace}(A^T A)$  by proposition 8.5. We can further conclude that  $\dim(\text{nullspace}(A)) = \dim(\text{nullspace}(A^T A))$  by Theorem 8.3. ■
3. Suppose  $A$  is an  $m \times n$  matrix with rank  $r$ . By Theorem 8.3, we know  $\dim(\text{nullspace}(A)) = n - r$ . We also know that  $A^T A$  is an  $n \times n$  matrix. By proposition 8.5,  $\text{nullspace}(A) = \text{nullspace}(A^T A)$ , and by Theorem 8.3,  $\dim(\text{nullspace}(A)) = \dim(\text{nullspace}(A^T A)) = n - r$ . Now, since both  $A$  and  $A^T A$  have  $n$  columns, we can conclude that  $A^T A$  also has rank  $r$ . Thus,  $\text{rank}(A) = \text{rank}(A^T A)$ . ■