Computational Linear Algebra, Module 7

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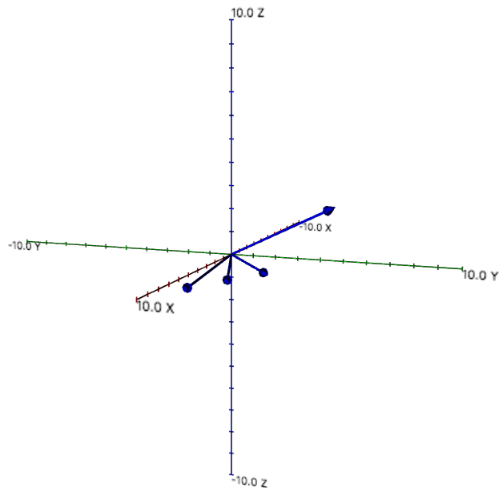
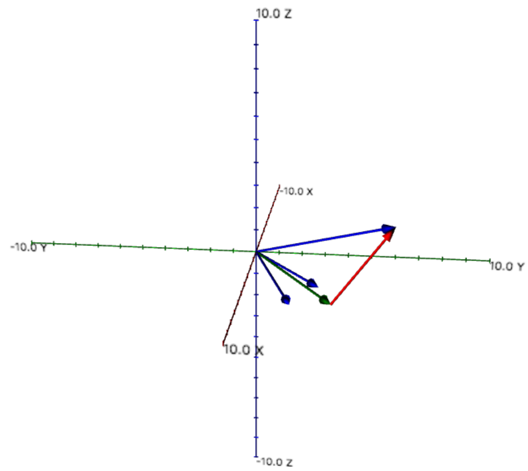
Due: April 11th, 2018

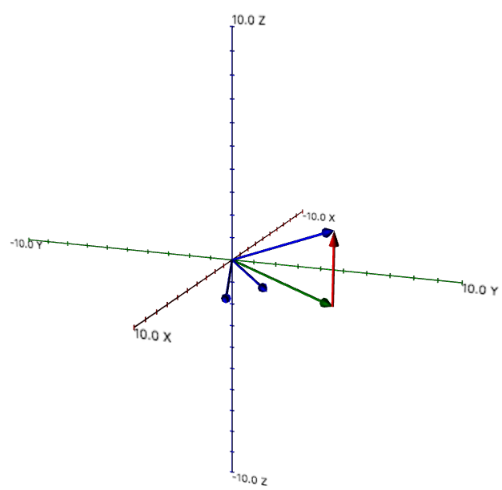
1. output:



2. c_3 is not included in the linear combination because it is linearly dependent on the other two columns.



If I increase the range, I get a y vector that is a bit closer, in the sense that it is the same as b, but without the z component.



need to insert drawing!

3. We need to show that $AB < AC$ for all AC. We know that

$$\begin{aligned} AB^2 + BC^2 &= AC^2 \\ AB^2 &= AC^2 - BC^2 \\ AB^2 < AC^2 & \\ AB &< AC \end{aligned}$$

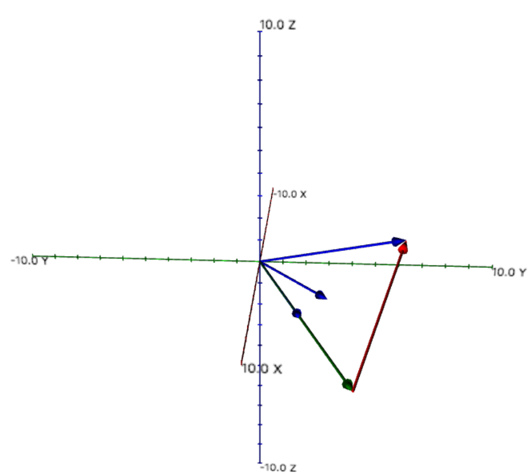
   So, the shortest distance from the point to the plane is on the perpendicular to the plane.

4. Both c_1 and c_2 are in the xy-plane, with z components of 0. So, the closest linear combination of them to b will be the projection of b on the xy-plane. Therefore, if we look at $\mathbf{z}$, the vector form y to b, since it is orthogonal to the projection, it is orthogonal to both c_1 and c_2.

5. Using the data in the example above and taking the dot product, the equations for α and β are

$$\begin{aligned} 42 - 40\alpha - 30\beta &= 0 \\ 38 - 30\alpha - 25\beta &= 0 \end{aligned}$$

   and by solving these equations, we get $\alpha = \frac{-27}{30}$ and $\beta = \frac{13}{5}$.



6. This works because of how matrix multiplication works. By have the c's as rows, we are going to multiply each c_i with z_i.

7. B is the transpose of A.

8.

9. $A \to (m \times n)$, $A^T \to (n \times m)$, $A^T A \to (n \times n)$. So,

$$\begin{aligned}(A^T A)^{-1} A^T b &\to (n \times n)(n \times m)(m \times 1) \\ &\to (n \times m)(m \times 1) \\ &\to (n \times 1)\end{aligned}$$

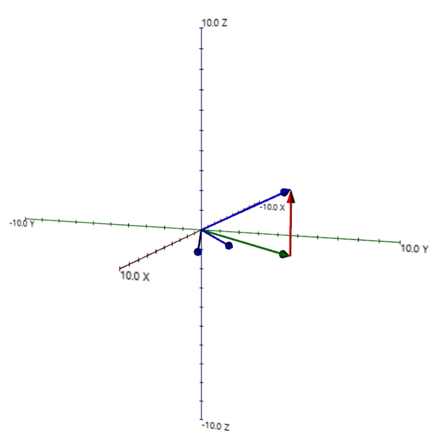
10.

$$\begin{aligned}(A^T A)^{-1} &= A^{-1}(A^T)^{-1} \\ (A^T A)^{-1} A^T &= A^{-1}(A^T)^{-1} A^T \\ &= A^{-1} \\ (A^T A)^{-1} A^T b &= A^{-1} b\end{aligned}$$

11. If $\mathbf{A}^{-1}$ exists, the dimension and the column rank are equal. Since those two are equal, the row rank is also equal to the other two. This means that is $\mathbf{A}^{-1}$ exists, then $\left(\mathbf{A}^T\right)^{-1}$ must also exist.

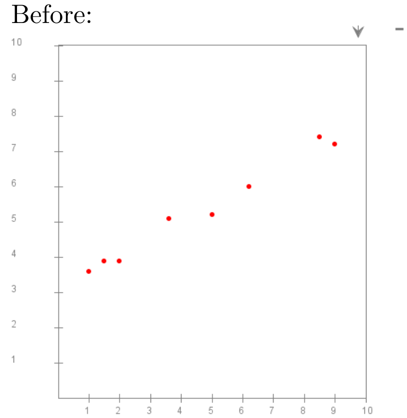
12. We have $AB^{-1} = B^{-1}A^{-1}$. Now, in exercise 10, if we replace A^T with A and A with B, then we can use the same proof to prove Theorem 7.1.

13. Matrix (2x2):
0.250 -0.300
-0.300 0.400
x[0]=-0.9000000000000004 x[1]=2.6000000000000014

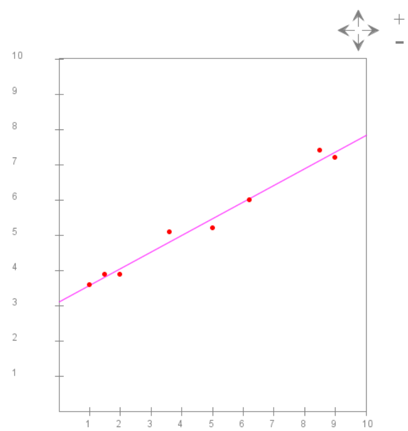


14. The columns is not linearly independent because no inverse exists.

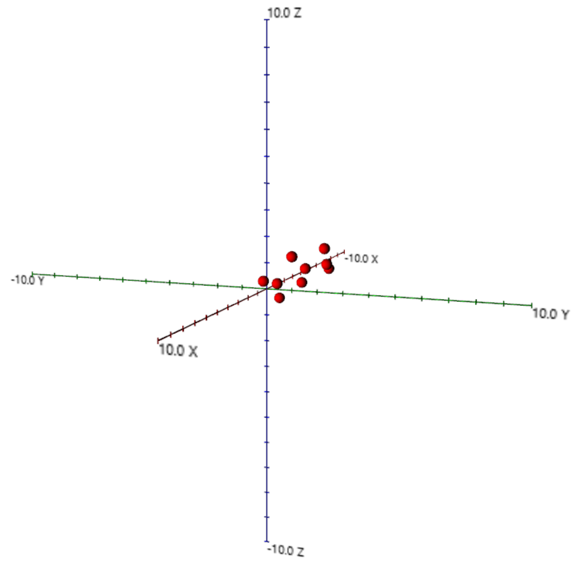
15. Before:



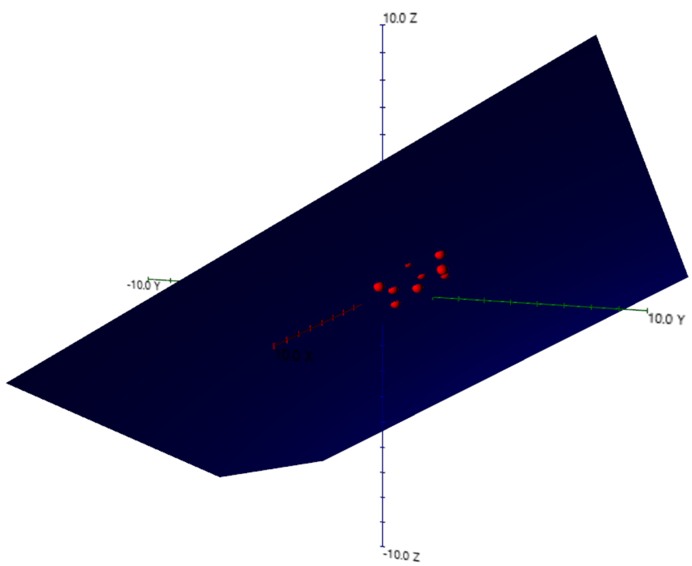
After:



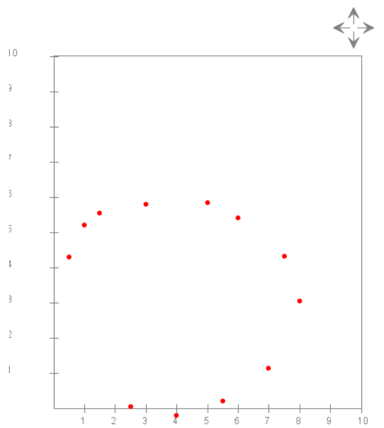
16. Before:



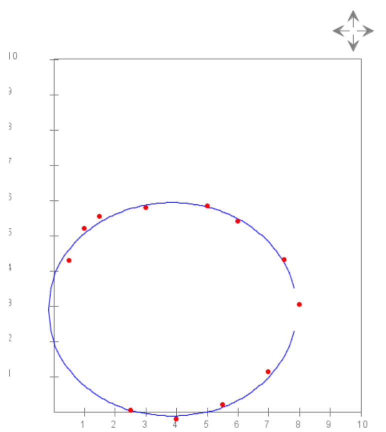
After:



17. Before :

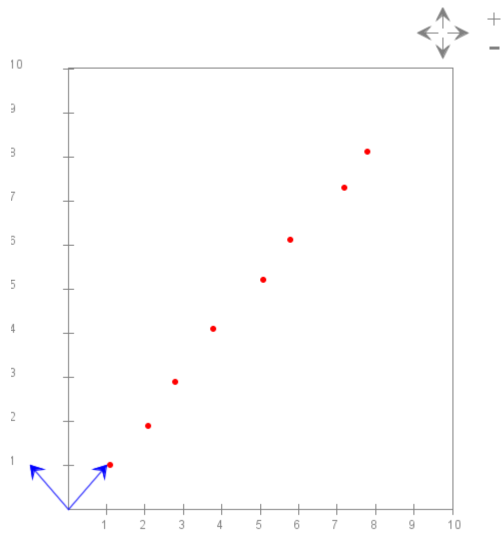


After:



18. Before change of basis:
meanX= 4.462, meanY= 4.575 varX= 5.846, varY= 6.505 covariance=49.260

After change of basis:
meanX= 4.519, meanY= 0.056 varX= 6.166, varY= 0.009 covariance= 1.319



19.

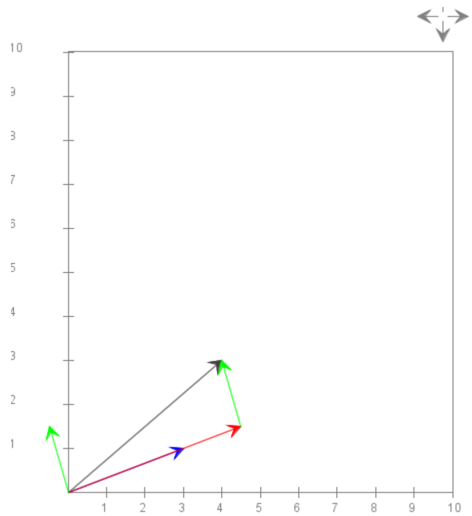
Coordinates after change of basis:
(1.050, -0.050)
(2.000, -0.100)
(2.850, 0.050)
(3.950, 0.150)
(5.150, 0.050)
(5.950, 0.150)
(7.250, 0.050)
(7.950, 0.150)

20.

$$\alpha = \frac{\mathbf{w} \cdot \mathbf{v}}{\mathbf{v} \cdot \mathbf{v}} = \frac{(4)(6) + (3)(2)}{(6)(6) + (2)(2)} = \frac{30}{40} = \frac{3}{4}$$

$$\mathbf{z} = \begin{bmatrix} 4 \\ 3 \end{bmatrix} - \frac{3}{4} \begin{bmatrix} 6 \\ 2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 1.5 \end{bmatrix}$$

$$\mathbf{z} \cdot \mathbf{v} = (-0.5)(6) + (1.5)(2) = 0$$

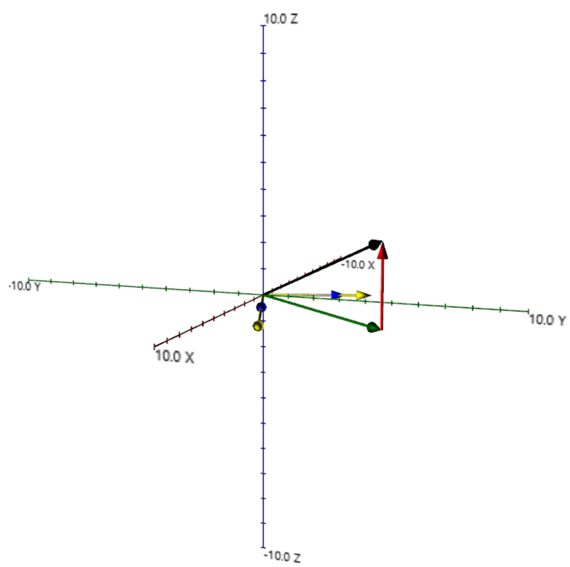


21.

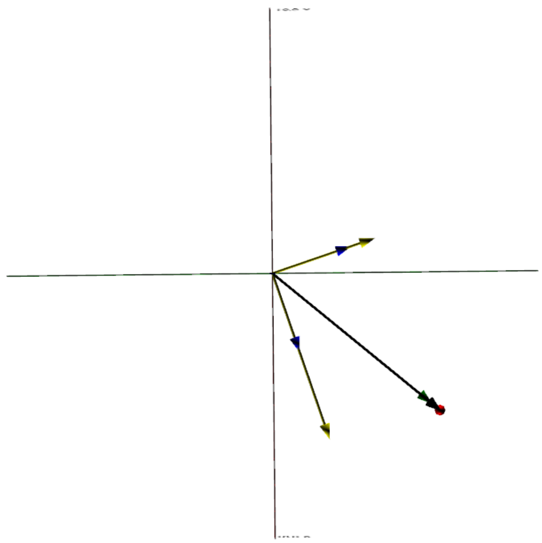
alpha = 1.5
z=(-0.5,1.5)
z dot v = 0.0

The additional arrow is **z**.

22. before:



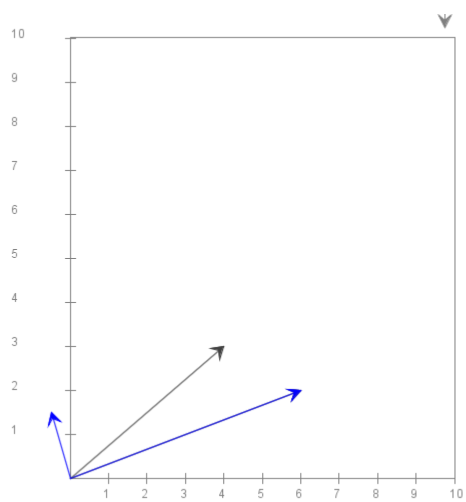
after:



z dot v1 = -2.6645352591003757E-15
z dot v2 = 8.881784197001252E-16

23.
- $\mathbf{v}_1 \cdot \mathbf{v}_2 = (6)(-1) + (2)(3) = 0$
- $\alpha_1 = \frac{(4)(6)+(2)(3)}{(6)(6)+(2)(2)} = .75$ and $\alpha_2 = \frac{(4)(-1)+(3)(3)}{(-1)(-1)+(3)(3)} = .5$
- $\text{proj}_{v_1} = .75 \begin{bmatrix} 6 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{9}{2} \\ \frac{3}{2} \end{bmatrix}$ and $\text{proj}_{v_2} = .5 \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ \frac{3}{2} \end{bmatrix}$
- $\mathbf{w} = \begin{bmatrix} \frac{9}{2} \\ \frac{3}{2} \end{bmatrix} + \begin{bmatrix} -\frac{1}{2} \\ \frac{3}{2} \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$

24.



25.

26. v1 dot v2 = 3.552713678800501E-15
    v1 dot v3 = 5.329070518200751E-15
    v2 dot v3 = -3.552713678800501E-15