PRINCIPAL COMPONENT ANALYSIS IN IMAGE PROCESSING

Article

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Abstract

Principal component analysis (PCA) is one of the statistical techniques frequently used in signal processing to the data dimension reduction or to the data decorrelation. Presented paper deals with two distinct applications of PCA in image processing. The first application consists in the image colour reduction while the three colour components are reduced into one containing a major part of information. The second use of PCA takes advantage of eigenvectors properties for determination of selected object orientation. Various methods can be used for previous object detection. Quality of image segmentation implies to results of the following process of object orientation evaluation based on PCA as well. Presented paper briefly introduces the PCA theory at first and continues with its applications mentioned above. Results are documented for the selected real pictures.

1 Introduction

Principal component analysis (Karhunen-Loeve or Hotelling transform) - PCA belongs to linear transforms based on the statistical techniques. This method provides a powerful tool for data analysis and pattern recognition which is often used in signal and image processing [1, 2] as a technique for data compression, data dimension reduction or their decorrelation as well. There are various algorithms based on multivariate analysis or neural networks [3, 4] that can perform PCA on a given data set. Presented paper introduces PCA as a possible tool in image enhancement and analysis.

2 The PCA Theory

Principal component analysis in signal processing can be described as a transform of a given set of n input vectors (variables) with the same length K formed in the n-dimensional vector $\mathbf{x} = [\mathbf{x_1}, \mathbf{x_2}, ...\mathbf{x_n}]^\mathbf{T}$ into a vector $\mathbf{y}$ according to

$$\mathbf{y} = \mathbf{A}\ (\mathbf{x} - \mathbf{m_x}) \tag{1}$$

This point of view enables to form a simple formula (1) but it is necessary to keep in the mind that each row of the vector $\mathbf{x}$ consists of K values belonging to one input. The vector $\mathbf{m_x}$ in Eq. (1) is the vector of mean values of all input variables defined by relation

$$\mathbf{m_x} = E\{\mathbf{x}\} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{x_k} \tag{2}$$

Matrix $\mathbf{A}$ in Eq. (1) is determined by the covariance matrix $\mathbf{C_x}$. Rows in the $\mathbf{A}$ matrix are formed from the eigenvectors $\mathbf{e}$ of $\mathbf{C_x}$ ordered according to corresponding eigenvalues in descending order. The evaluation of the $\mathbf{C_x}$ matrix is possible according to relation

$$\mathbf{C_x} = E\{(\mathbf{x} - \mathbf{m_x})(\mathbf{x} - \mathbf{m_x})^\mathbf{T}\} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{x_k}\mathbf{x_k^T} - \mathbf{m_x}\mathbf{m_x^T} \tag{3}$$

As the vector $\mathbf{x}$ of input variables is n-dimensional it is obvious that the size of $\mathbf{C_x}$ is n x n. The elements $\mathbf{C_x}(i,i)$ lying in its main diagonal are the variances

$$\mathbf{C_x}(i,i) = E\{(\mathbf{x_i} - m_i)^2\} \tag{4}$$

of $\mathbf{x}$ and the other values $\mathbf{C_x}(i,j)$ determine the covariance between input variables $\mathbf{x_i}$, $\mathbf{x_j}$.

$$\mathbf{C_x}(i,j) = E\{(\mathbf{x_i} - m_i)(\mathbf{x_j} - m_j)\} \tag{5}$$

between input variables x_i, x_j. The rows of $\mathbf{A}$ in Eq. (1) are orthonormal so the inversion of PCA is possible according to relation

$$\mathbf{x} = \mathbf{A^T y} + \mathbf{m_x} \tag{6}$$

The kernel of PCA defined by Eq. (1) has some other interesting properties resulting from the matrix theory which can be used in the signal and image processing to fulfil various goals as mentioned below.

3 PCA Use for Image Compression

Data volume reduction is a common task in image processing. There is a huge amount of algorithms [1, 2, 4] based on various principles leading to the image compression. Algorithms based on the image colour reduction are mostly lossy but their results are still acceptable for some applications. The image transformation from colour to the gray-level (intensity) image $\mathbf{I}$ belongs to the most common algorithms. Its implementation is usually based on the weighted sum of three colour components **R, G, B** according to relation

$$\mathbf{I} = \mathbf{w_1 R} + \mathbf{w_2 G} + \mathbf{w_3 B} \tag{7}$$

The **R**, **G** and **B** matrices contain image colour components, the weights w_i were determined with regards to the possibilities of human perception [2]. The PCA method provides an alternative way to this method. The idea is based on Eq. (6) where the matrix $\mathbf{A}$ is replaced by matrix $\mathbf{A_l}$ in which only l largest (instead of n) eigenvalues are used for its forming. The vector $\hat{\mathbf{x}}$ of reconstructed variables is then given by relation

$$\hat{\mathbf{x}} = \mathbf{A_k^T y} + \mathbf{m_x} \tag{8}$$

True-colour images of size MxN are usually saved in the three-dimensional matrix $\mathbf{P}$ with size M x N x 3 which means that the information about intensity of colour components is stored in the 3 given planes . The vector of input variables $\mathbf{x}$ in Eq. (1) can be formed as the n=3-dimensional vector of each colour. Forming three 1-dimensional vectors $\mathbf{x_{1,2,3}}$ from each plane $\mathbf{P}(M,N,i)$ with the length of $M.N$ can be advantageous for better understanding and programming. The covariance matrix $\mathbf{C_x}$ and corresponding matrix $\mathbf{A}$ are then evaluated and the 3-dimensional reconstructed vector $\hat{\mathbf{x}}$ according to Eq. (8) can be called as the first, the second and the third component of the given image. The matrix theory implies that the image obtained by reconstruction with the matrix $\mathbf{A_1}$ (only the first - largest eigenvalue was used for its definition) contains the majority of information so this image should have the maximum contrast. This properties could be significant in the following image processing.

There is a selected real picture $\mathbf{P}$ and its R, G, B components in the Fig 1. Its three reconstructed components obtained according to Eq. (8) for each eigenvalues are presented in Fig. 2. The comparison of intensity images obtained from the original image as weighted colour sum evaluated by Eq. (7) and as the first principal component is presented in Fig. 3. The eigenvalues sorted in descending order belonging to the selected image are presented in Table 1.

Table 1: EIGENVALUES OF SELECTED REAL IMAGE PRESENTED IN FIG. 1

λ_1	λ_2	λ_3
0.6103	0.3231	0.0418

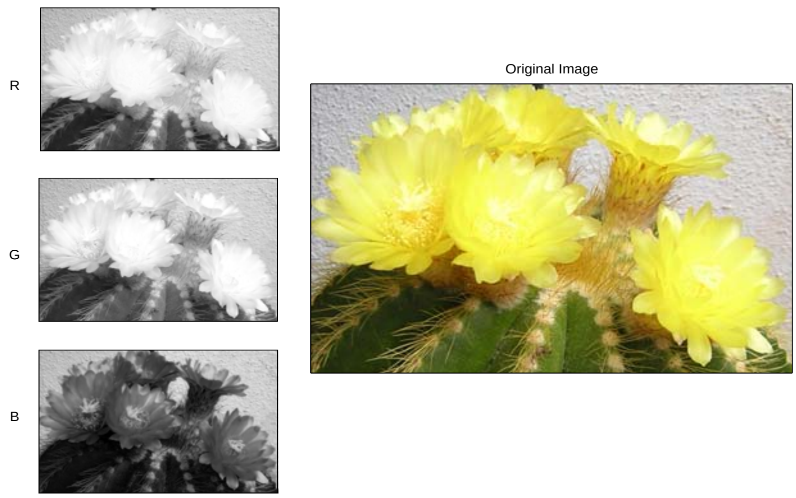

Figure 1: Original image and its three colour components

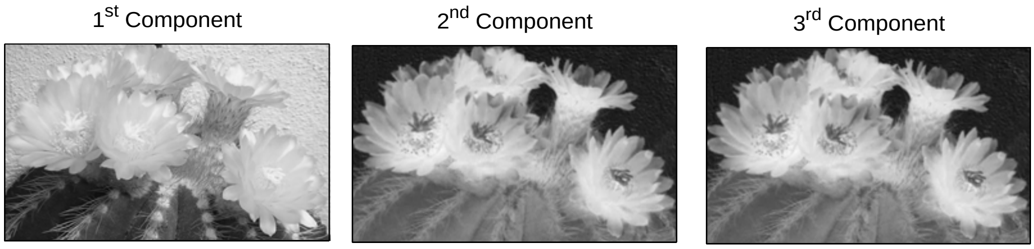

Figure 2: PCA of a selected image

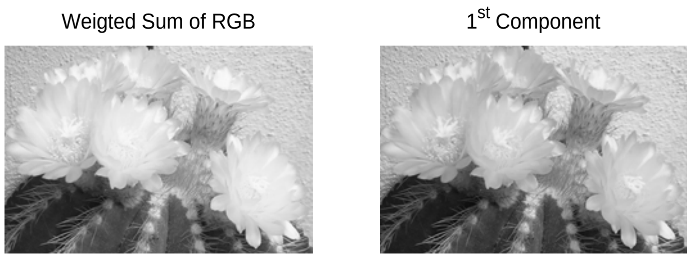

Figure 3: Comparison of various method of image colour reduction. The gray-level image evaluated as weighted sum of R,G,B colours (left) and the gray-level image counted on the base of the PCA method (right)

4 PCA Use for Determination of Object Rotation

Properties of PCA can be used for determination of selected object orientation or its rotation, too [2, 4]. Various method of image segmentation to object definition (like thresholding, edge detection or others) must be used at first. Binary image containing object boundary or its area in black (or white) pixels on the inverse background results from this process. After that two vectors **a** and **b** containing the cartesian x and y coordinates of object's pixels can be simply formed. The vector **x** in the Eq. (1) is in this case a 2-dimensional vector consisting of **a** and **b** respectively. The mean vector $\mathbf{m_x}$ and the covariance matrix $\mathbf{C_x}$ are computed as well as its eigenvector **e**. Its two elements - vectors e_1 and e_2 enable the evaluation of object rotation in the cartesian axis or object rotation around the center given by $\mathbf{m_x}$. Fig. 4 illustrates the PCA use for the determination of selected object orientation. The object boundary was detected at first by means of LoG filter in the original gray-level image. The original has been rotated by a given angle with the bilinear interpolation method use and the process of image segmentation and PCA has been applied again. Resulted eigenvectors e_1 and e_2 are drawn in each binary image, too and their orientation were compared with the rotation angle.

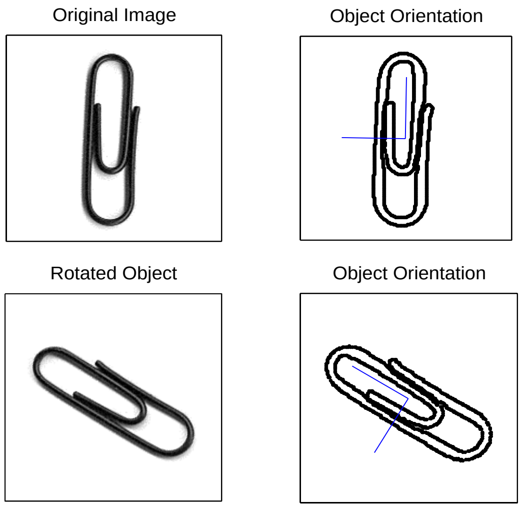

Figure 4: Illustration of object orientation determination in the selected image

5 Conclusion

The presented paper dealt with two possible application of PCA in image processing. Other application in this area can be studied as well. Our interest will be focussed on the PCA method use for processing of biomedical signals and images. Further attention will be payed to the method of Independent Component Analysis related to PCA, too.

References

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