Computational Linear Algebra, Assignment 1

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1. Suppose $z_1 = a_1 + ib_1$ and $z_2 = a_2 + ib_2$.

   (a) Prove $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$ Proof:

$$\begin{aligned}
\overline{z_1 + z_2} &= \overline{a_1 + ib_1 + a_2 + ib_2} \\
&= \overline{a_1 + a_2 + i(b_1 + b_2)} \\
&= (a_1 + a_2) - i(b_1 + b_2) \\
&= (a_1 - ib_1) + (a_2 - ib_2) \\
&= \overline{a_1 + ib_1} + \overline{a_2 + ib_2} \\
&= \overline{z_1} + \overline{z_2}
\end{aligned}$$

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   (b) Prove $\overline{z_1 - z_2} = \overline{z_1} - \overline{z_2}$ Proof:

$$\begin{aligned}
\overline{z_1 - z_2} &= \overline{(a_1 + ib_1) - (a_2 + ib_2)} \\
&= \overline{(a_1 - a_2) + i(b_1 - b_2)} \\
&= (a_1 - a_2) - i(b_1 - b_2) \\
&= (a_1 - ib_1) - (a_2 - ib_2) \\
&= \overline{a_1 + ib_1} - \overline{a_2 + ib_2} \\
&= \overline{z_1} - \overline{z_2}
\end{aligned}$$

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   (c) Prove $\overline{z_1 z_2} = (\overline{z_1})(\overline{z_2})$ Proof:

$$\begin{aligned}
\overline{z_1 z_2} &= \overline{(a_1 + ib_1)(a_2 + ib_2)} \\
&= \overline{a_1a_2 + ia_1b_2 + ia_2b_1 - b_1b_2)} \\
&= \overline{((a_1a_2 - b_1b_2) + i(a_1b_2 + a_2b_1))} \\
&= (a_1a_2 - b_1b_2) - i(a_1b_2 + a_2b_1) \\
&= a_1a_2 - b_1b_2 - ia_1b_2 - ia_2b_1 \\
&= (a_1 - ib_1)(a_2 - ib_2) \\
&= (\overline{z_1})(\overline{z_2})
\end{aligned}$$

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