

Computational Linear Algebra: Module 1

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Due: January 31st, 2018

Exercise 1

- We ran the demos and they worked.

Exercise 2

$$\begin{aligned}x_1 + 3x_2 &= 7.5 \\ 4x_2 + 2x_2 &= 10\end{aligned}$$

- I solved this equation by hand, and found that $x_1 = 1.5$ and $x_2 = 2$
- We verified this solution by running the code
- Algorithm: For 2 equations, solve the first equation for x_1 , then plug this into the second equation to solve for x_2 . Then plug x_2 back into the first equation to solve for x_1 .
- General Algorithm: For n equations, solve the first equation for x_1 , plug this into the remaining equation to get $n-1$ new equations. Then, solve the second equation for x_2 and plug this into the remaining $n-2$ new equations. Repeat this process until you have $x_n = \text{some number}$. Then plug each variable back into the above equation to solve for the remaining $n-1$ variables.

Exercise 3

- For these equation, I solved the first equation for x_1 and plugged this into the second and third equations. Then, I solved the second equation for x_2 and plugged this into the third equation. By doing this, I found that $x_3 = \text{plug in value there}$.
- I verified this result by running the code.

Exercise 4

- variableCoeffs: 2D array
- rightHandSide: 1D array containing the b vector from $Ax = b$

Exercise 5

$$\begin{aligned}x_1 + 3x_2 &= 7.5 \\ 2x_1 + 6x_2 &= 10\end{aligned}$$

- When I apply my algorithm to this set of equations, I find that $15=10$, which is not true. So, I can conclude that there is no solution to this system of equations.

Exercise 6

$$\begin{aligned}x_1 - 2.5x_3 + 3x_5 &= 5 \\ x_2 - 4x_3 + 3x_5 &= 10\end{aligned}$$

- There are more unknowns than equations, so there are multiple solutions. In these problems, the equations act as constraints on the unknowns, so since there are less constraints, there are more possibilities for the values of the unknowns.

Exercise 7

$$x_1 + 3x_2 = 7.5$$

$$4x_1 + 2x_2 = 10$$

$$x_1 + x_2 = 3.5$$

- In this case, there is a solution: $x_1 = 1.5$ and $x_2 = 2$. However, for such systems where there are more equations than unknowns, we can guarantee a unique solution.

Exercise 8

$$x_1 + 3x_2 = 7.5000$$

$$4x_1 + 2x_2 = 10.000$$

$$x_1 + x_2 = 3.4999$$

- Here we see that there is not a solution. This is because the last equation is written with more precision, so the solution we got in Exercise 7 is not exact here.

Exercise 9

- Want to handle any number of equations
- Want to handle any number of unknowns
- Want to handle situations where there can be multiple solutions, or situations where there is no solution.

Exercise 10

- There is one parameter for the ellipse
- The minimum number of points needed to draw an ellipse is 5 points.

Exercise 11

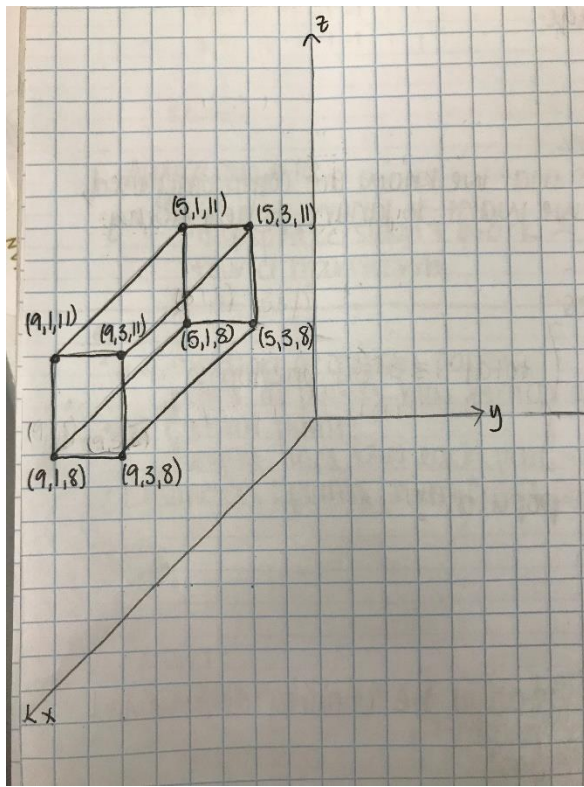
	Advantages	Disadvantages
A	<ul style="list-style-type: none">- Easy to store- Easily generalizable	<ul style="list-style-type: none">- Computationally expensive- Memory expensive
B	<ul style="list-style-type: none">- Exact- Real-time- Less computationally expensive	<ul style="list-style-type: none">- Can't generalize; have to write equations for each curve explicitly- Maybe there isn't an easy mathematical formula

Exercise 12

- A: location of control points, pairs of points between which there are lines
- B: control points, tangent lines between each pair of control points, points to use for the curve

- Parameters: points, numCurvePoints

Exercise 13



This cube looks like it is lying in the negative-y side of the xz-plane, when in reality it is in the positive xz-plane. This is where eye-coordinate matters. In the 2D world, it looks like the mapping is as follows:

- $(9,1,8) \rightarrow (0,0)$
- $(9,3,8) \rightarrow (2,0)$
- $(9,1,11) \rightarrow (0,3)$
- $(9,3,11) \rightarrow (2,3)$
- $(5,1,8) \rightarrow (4,4)$
- $(5,3,8) \rightarrow (6,4)$
- $(5,1,11) \rightarrow (4,7)$
- $(5,3,11) \rightarrow (6,7)$

Exercise 14

- The eye coordinate determines where it “looks” like the cube is positioned

Exercise 15

- The size of the image is 20x20 (400) pixels

- The size of the png file is 90 bytes
- I don't know if this is lossy or lossless compressions, but I can see that the png file is compressed because the number of bytes is less than it should be given the size of the image (in pixels)

Exercise 16

- Demo1:
 - Original data: Vector, length=8: (0.000 1.000 2.000 3.000 4.000 5.000 6.000 7.000)
 - Inverted Data: Vector, length=8: (-0.052 1.037 2.059 2.820 4.281 4.680 6.279 6.838)

- Demo2:

original image: Matrix, numRows=8 numCols=8:

```
0.505 1.237 2.630 3.728 4.415 5.843 6.703 7.402
8.863 9.316 10.930 11.609 12.326 13.220 14.732 15.405
16.878 17.411 18.228 19.638 20.566 21.329 22.844 23.014
24.004 25.152 26.833 27.416 28.003 29.160 30.971 31.453
32.486 33.013 34.448 35.521 36.758 37.753 38.988 39.745
40.907 41.913 42.774 43.089 44.964 45.588 46.815 47.878
48.008 49.765 50.175 51.522 52.139 53.467 54.950 55.634
56.877 57.280 58.844 59.738 60.540 61.257 62.416 63.626
```

Decompressed image: Matrix, numRows=8 numCols=8:

```
0.453 1.298 2.604 3.709 4.474 5.763 6.778 7.357
8.897 9.279 10.934 11.647 12.253 13.310 14.652 15.452
16.948 17.317 18.309 19.575 20.610 21.302 22.857 23.008
23.803 25.411 26.645 27.511 27.997 29.107 31.043 31.405
32.795 32.619 34.723 35.402 36.730 37.877 38.844 39.839
40.555 42.359 42.467 43.212 45.012 45.429 46.994 47.762
48.314 49.378 50.440 51.419 52.091 53.611 54.788 55.738
56.700 57.504 58.691 59.796 60.570 61.171 62.511 63.565
```

- Demo 3:

original image: Matrix, numRows=8 numCols=8:

```
0.505 1.237 2.630 3.728 4.415 5.843 6.703 7.402
8.863 9.316 10.930 11.609 12.326 13.220 14.732 15.405
16.878 17.411 18.228 19.638 20.566 21.329 22.844 23.014
24.004 25.152 26.833 27.416 28.003 29.160 30.971 31.453
32.486 33.013 34.448 35.521 36.758 37.753 38.988 39.745
40.907 41.913 42.774 43.089 44.964 45.588 46.815 47.878
48.008 49.765 50.175 51.522 52.139 53.467 54.950 55.634
56.877 57.280 58.844 59.738 60.540 61.257 62.416 63.626
```

Decompressed image: Matrix, numRows=8 numCols=8:

```
0.591 1.207 2.665 3.644 4.442 5.599 6.932 7.382
```

8.794	9.416	10.726	11.629	12.433	13.465	14.716	15.224
16.778	17.344	18.639	19.514	20.445	21.378	22.593	23.218
24.030	25.326	26.436	27.470	28.048	29.337	30.911	31.435
32.499	33.039	34.502	35.454	36.741	37.637	38.962	39.876
40.964	41.578	42.789	43.635	44.828	45.590	46.811	47.736
47.994	49.625	50.483	51.500	52.088	53.318	54.974	55.678
56.870	57.556	58.619	59.412	60.691	61.322	62.513	63.592

It looks like as you go through the demos, the compression becomes less and less lossy.

Exercise 17

- Query image 0: image 3
- Query image 1: image 7

Exercise 18

- All eigen images have a little bit of everything from the collection of images; ghost-like images are eigenvectors

Exercise 19

- One advantage of this approach is that it means that webpages that are more “mentioned” elsewhere will have more incoming edges. However, a webpage that is considered very important as a stand-alone page may not have a lot of links to it, thus causing it to have very few incoming edges and thus lowering its “importance”.

Exercise 20

- Node 6 has 3 incoming edges, which is the most number of incoming edges
- Node 2 looks like it has the highest visit probability

Exercise 21

- This is different because node 4 doesn’t have an edge to 6 or 5, so 4 doesn’t go anywhere.

Exercise 22

- `linMagic.pageRankViaEigenValues(...)`
- `linMagic.pageRankViaPowerMethod(...)`

Exercise 23

- The **poodle** was developed as a water retriever, and the distinctive clipping of its heavy coat was initiated to increase the animal's efficiency in the water. ... An elegant-looking dog, often ranked as one of the most intelligent of all breeds, the **poodle** has been bred in three size varieties—standard, miniature, and toy.
- **Labrador retriever**, breed of sporting dog that originated in Newfoundland and was brought to England by fishermen about 1800. It is an outstanding gun dog, consistently dominating field trials. Standing 21.5 to 24.5 inches (55 to 62 cm) and weighing 55 to 80 pounds (25 to 36 kg), it is more solidly built than other retrievers and has shorter legs. Distinctive features include its otterlike tail, thick at the base and tapered toward the end, and its short, dense coat of black,

brown (“chocolate”), or yellow. The Labrador retriever is characteristically rugged, even-tempered, and gentle. In England it has been used in military and police work, as a rescue dog, and as a guide dog for the blind. An ideal family pet, the Labrador retriever became in the 1990s the most popular dog breed in the United States.

- These two pieces of text are both broadly about dogs, or breeds of dogs, however a simple keyword text on both would show that the first is about poodles and the second is about Labrador retrievers.

Exercise 24

	0	1	2	3	4	5	6
0	1	1	1	.5	-.5	-1	-1
1	1	1	1	.5	-.5	--1	-1
2	1	1	1	.5	-.5	-1	-1
3	.5	.5	.5	1	.5	-1	-1
4	-.5	-.5	-.5	.5	1	-1	-1
5	-1	-1	-1	-1	-1	1	1
6	-1	-1	-1	-1	-1	1	1

Exercise 25

- LSA demo doesn’t work
- Stop words are words that we know are commonly used, like “the”, which we want to ignore when doing text analysis.

Exercise 26

- 33/54 ratings are unknown
- Movie 1 is the most popular
- A movie with less ratings could be unfairly determined as the most popular movie if the few ratings that it has are really high.
- Users 4 and 5 may be similar, as well as users 7 and 8

Exercise 27

- The input is the known ratings and the output is the unknown ratings.

Exercise 28

- We compiled and executed this, but neither of us have working laptop speakers, so we listened to groups around us to hear the signals, filtered and unfiltered.

Exercise 29

- There are about 3,270,000 pdf documents with the word “matrix”. About 1/3 of these are scientific publications.