Computational Linear Algebra, Module 8

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Due: April 11th, 2018

1. "fundamental theorem of a" yields the following completions:
   (a) fundamental theorem of algebra
   (b) fundamental theorem of arithmetic

   "fundamental theorem of b" yields the following completions:
   (a) fundamental theorem of boolean algebra
   (b) fundamental theorem of biomedical informatics

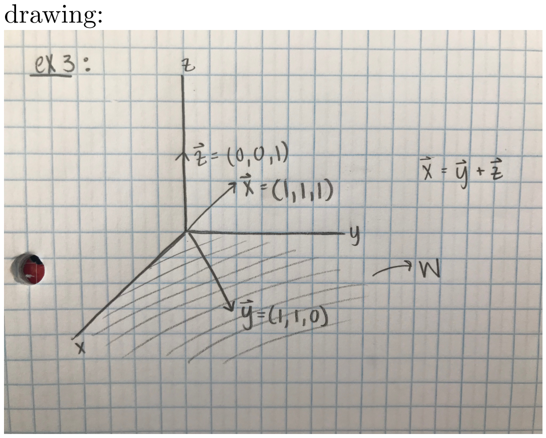
   "fundamental theorem of j" is the first search term that seems to fail.

2.

$$\begin{aligned} x_1 &= -x_3 - x_4 - 2x_5 \\ x_2 &= -x_3 + x_4 + x_5 \end{aligned}$$

   When $x_3 = 1, x_4 = 0, x_5 = 0$, $x_1 = -1$ and $x_2 = -1$.
   When $x_3 = 0, x_4 = 0, x_5 = 1$, $x_1 = -2$ and $x_2 = 1$.

3. drawing:



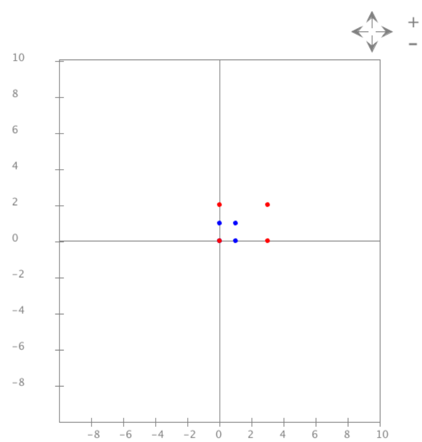
4. We know that the dim(W) = r, and we also know that the dimension of n-D space is n. So, if $W^{\perp}$ is the set of all vectors orthogonal to W, then it must have dimension $n - r$.

5. The nullspace of A is the set of all vectors that are orthogonal to A. Since z is the nullspace(A), $Az = 0$ by definition.

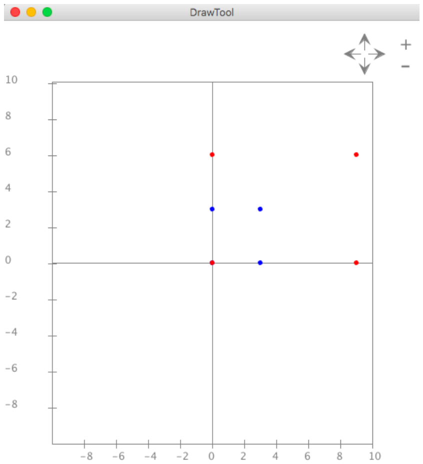
6. By multiplying A and the rowspace of A, you will only get the columns of A that have a pivot in them, therefore b is the columnspace of A.

7. Suppose A is an $m \times n$ matrix with rank r. By Theorem 8.3, we know dim(nullspace(A)) = n-r. We also know that $A^T A$ is an $n \times n$ matrix. By proposition 8.5, nullspace(A) = nullspace($A^T A$)), and by Theorem 8.3, dim(nullspace(A))=dim(nullspace($A^T A$))=n-r. Now, since both A and $A^T A$ have n columns, we can conclude that $A^T A$ also has rank r. Thus, rank(A) = rank($A^T A$). ■

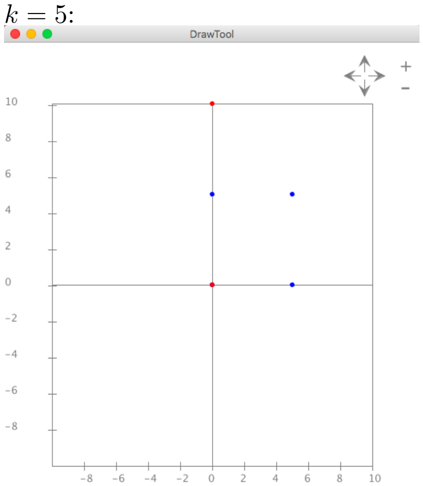
8. $k = 1$:



9. $k = 3$:


DrawTool
10
8
6
4
2
0
−2
−4
−6
−8
−8 −6 −4 −2 0 2 4 6 8 10

10. $k = 5$:


DrawTool
10
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−8 −6 −4 −2 0 2 4 6 8 10