# Causal Inferences for Data in Single-Case Design Research Across Multilevel Contextss

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# Agenda

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- Visual analysis in single-case design
- Multilevel modeling for single-case data
- Case study: Technology-assisted instruction for math
- Visual analysis results as functional relation
- Multilevel modeling results

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# Visual Analysis in single-case design

- A single-case design focuses on individual or group performance and demonstrates causal relationships between variables through experimental control (Kazdin, 2019).
- Behavior changes are evaluated through visual analysis of graphical data, examining changes in level, trend, and variability across phases.
- Interventions are provided to individual students (*N*=1) or small groups.

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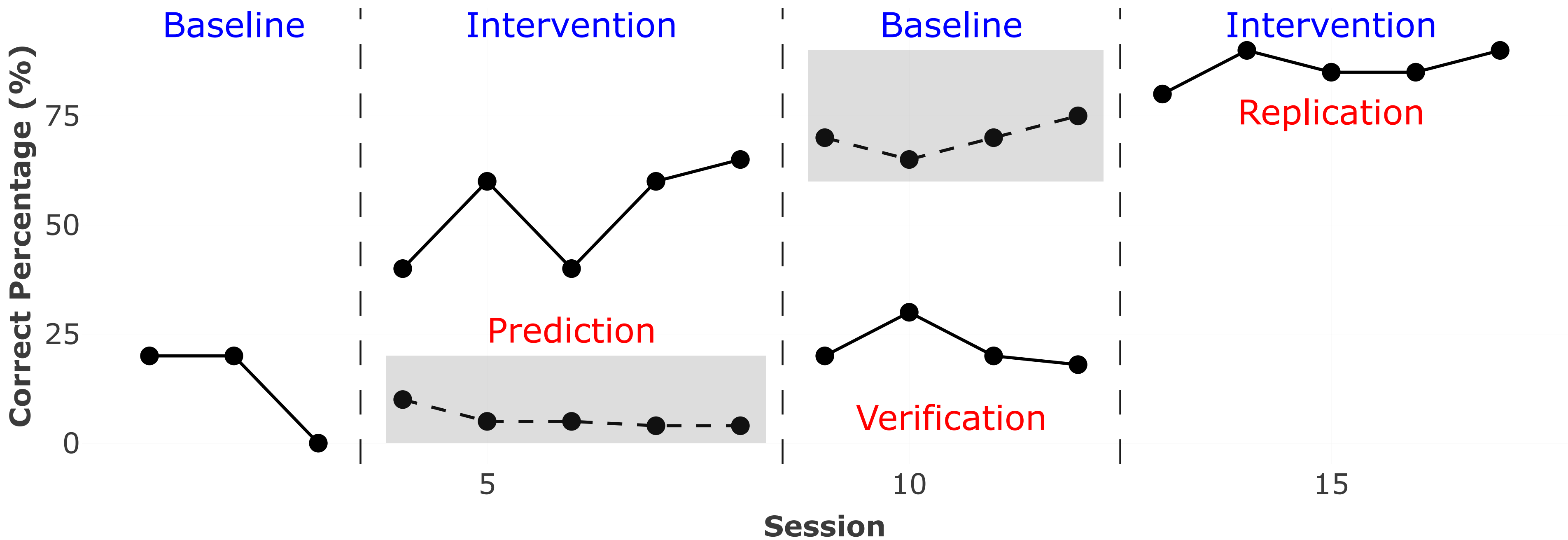


- Internal validity is established through baseline logic involving three elements: (a) prediction (what behavior would look like without intervention), (b) verification (demonstrating baseline would continue unchanged), and (c) replication (demonstrating effects multiple times).

# Multilevel Modeling for Single-Case Data

- Individual behavior is repeatedly measured over time.
- Behavioral changes between time periods are measured using a piecewise growth model.
- Observations at time *t* are related to observations at previous time points (autocorrelation coefficient).
- Unlike longitudinal studies such as panel surveys, data are measured intensively and frequently.
- In repeated measurements, correlation coefficients between nearby time points are higher than those between distant time points.

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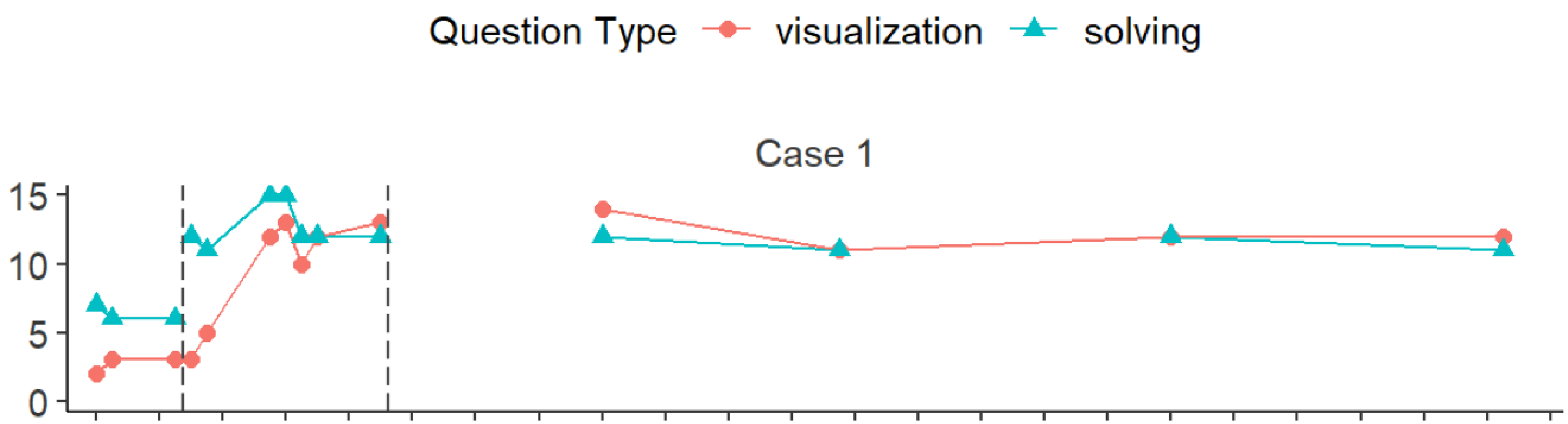
Shin, M., Hart, S. L., & Simmons, M. (2024). Meta-analysis of single-case design research: Application of multilevel modeling. *School Psychology, 39*(6), 625-635. https://doi.org/10.1037/spq0000637

# Piecewise Regression Models




- Distinct behavioral variations usually occur when experimental conditions (e.g., the provision or the withdrawal of interventions) change over time (Ledford & Gast, 2018).
- To accurately model this discontinuity between phases (i.e., changes in levels and trends), researchers can employ a piecewise regression model approach.
- After providing intervention, students' performance levels are expected to change immediately, and behaviors are expected to increase or decrease in the targeted direction during the intervention phase (Center et al, 1985)

# Case Study on Technology-Assisted Instruction (TAI) for Math

1. What are the effects of `TAI with teacher prompts` on `visualizing` and `solving fraction multiplication` word problems in middle school students with learning disabilities?
2. What are the moderating effects of `word problem question types` (visualization vs. problem-solving) on `changes between adjacent phases`?

Shin, M., & Park, J. (2024). Technology-assisted instruction with teacher prompts on fraction multiplication word problems: A single-case design with visual analysis and Bayesian multilevel modeling. *Assistive Technology*. Advance online publication. https://doi.org/10.1080/10400435.2024.2415366

# Method

- Four middle school students with mathematics learning disabilities
- Middle school in the Southeastern United States
- Resource math class (50 minutes of daily math instruction)
- One-on-one intervention by a special education teacher
- Inclusion criteria: Students in grades 6-8, mathematics achievement (state test), below school level, with individualized math program goals, and screening test scores below 30%
- Multiple-probe design across subjects with three phases: baseline, intervention, and maintenance

# Tau-U Effect Size Calculation

**Baseline to Intervention (A→B):**

$$\tau_{A \to B} = \frac{\sum_{i=1}^{n_A} \sum_{j=1}^{n_B} \operatorname{sign}(B_j - A_i)}{n_A \times n_B}$$

**Intervention to Maintenance (B→M):**

$$\tau_{B \to M} = \frac{\sum_{i=1}^{n_B} \sum_{j=1}^{n_M} \operatorname{sign}(M_j - B_i)}{n_B \times n_M}$$

$\operatorname{sign}(x)$ = +1 if $x > 0$, -1 if $x < 0$, and 0 if $x = 0$

$n_A$ = number of baseline observations

$n_B$ = number of intervention observations

$n_M$ = number of maintenance observations

$\tau \in [-1, 1]$: positive values indicate improvement or sustained gains

# Visual Analysis Results as Functional Relation

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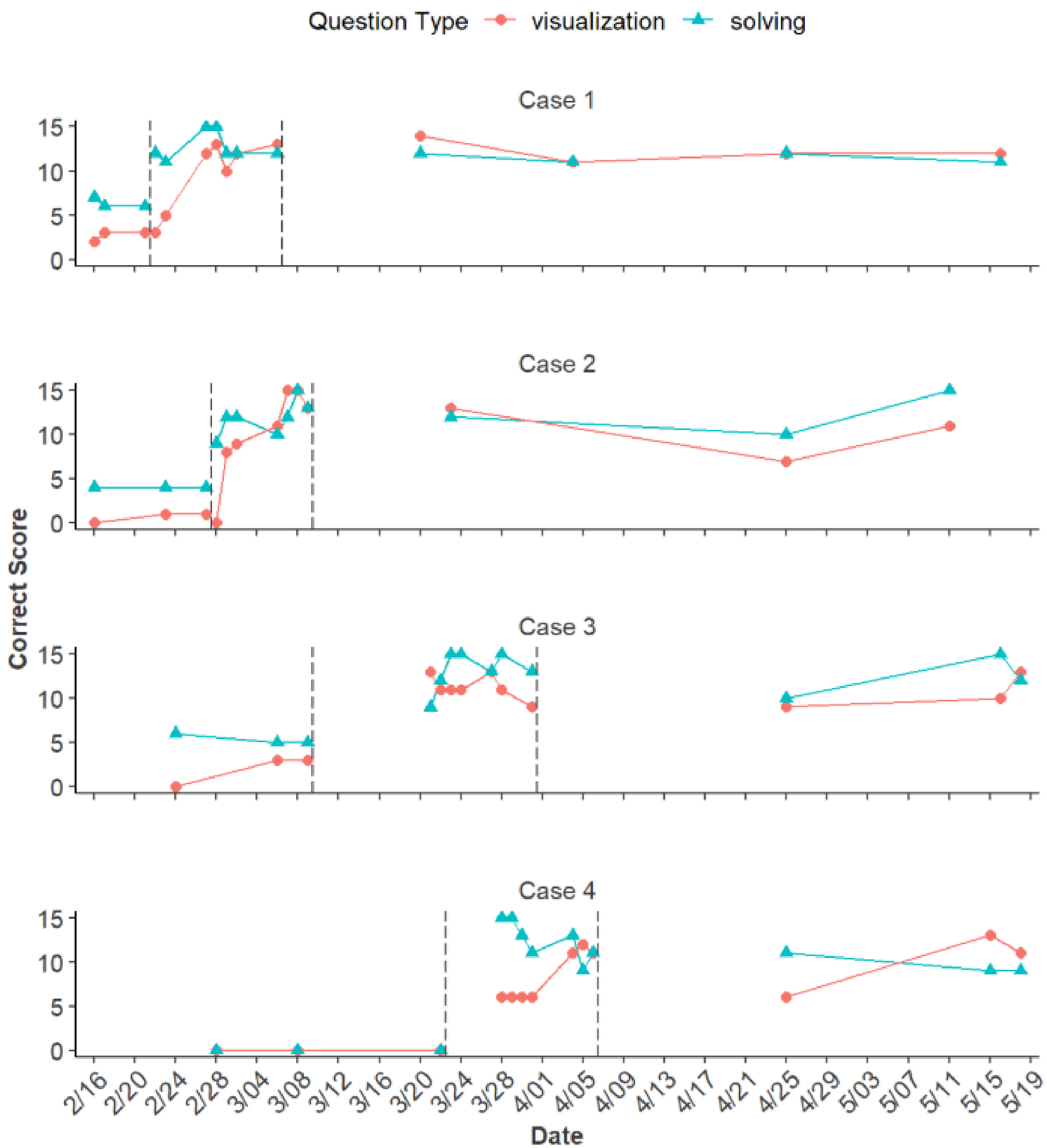
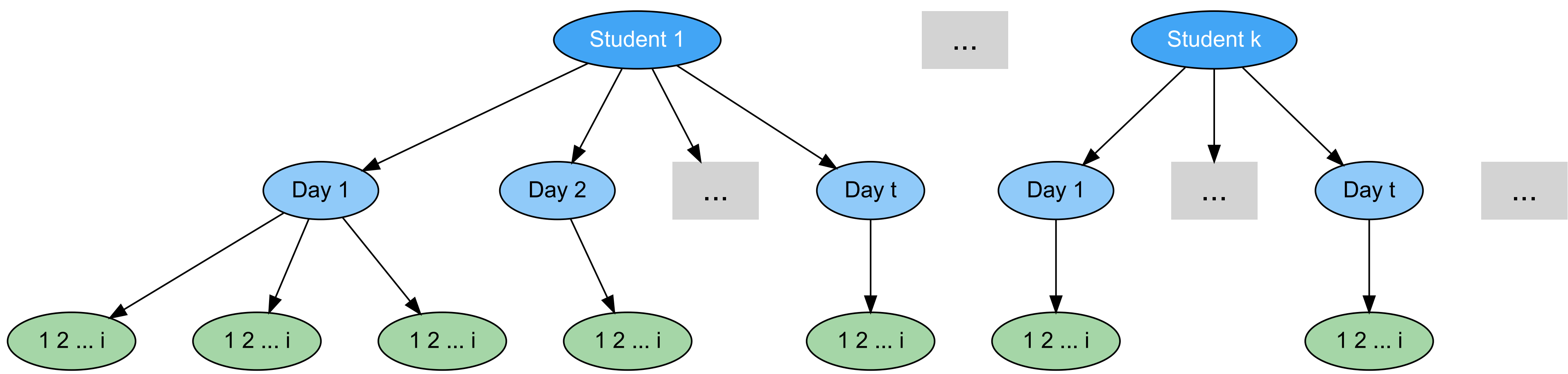



- Improvements in both visualization and word problem solving questions compared to baseline.
- Visualization:
  A vs. B Tau = 0.76 ~ 1.00,
  B vs. M Tau = -0.29 ~ 0.33
- Word Problem Solving:
  A vs. B Tau = 1.00,
  B vs. M Tau = -0.71 ~ 0.10

# Bayesian Multilevel Modeling

- `Cumulative Link Mixed Effects`: Analyzed 6-point ordinal scale data for each item (Level 1) nested within repeated measurement days (Level 2) for each student (Level 3).




- Bayesian approach is recommended to avoid convergence issues that arise from excessive zeros in baseline phases (Li et al., 2024).
- By incorporating prior information, researchers can fit models that avoid extreme values (e.g., random effect variances close to zero) with small sample sizes (Meteyard & Davies, 2020).

$$
\begin{aligned}
\operatorname{logit}[\pi_{kijl}(Y > k)] &= \ln\left(\frac{\pi\left(Y_{ijl} > k\right)}{\pi\left(Y_{ijl} \leq k\right)}\right) \\
&= -\alpha_k + \left(\beta_{0jl} + \beta_{1jl}\left(t - T_{1jl}\right)\right. \\
&+ \beta_{2jl}\left(t > T_{2jl}\right) + \beta_{3jl}\left(t - T_{2jl}\right) \\
&\times \left(t > T_{2jl}\right) + \beta_{4jl}\left(t > T_{3jl}\right) \\
&\left.+\beta_{5jl}\left(t - T_{3jl}\right) \times \left(t > T_{3jl}\right)\right),
\end{aligned}
$$

- $\pi_{kijl}$ = cumulative probability of exceeding cut point $k$ (6-point rating)
- $T_{1jl}, T_{2jl}, T_{3jl}$ = baseline, intervention, maintenance start points
- $(t > T_{2jl})$, $(t > T_{3jl})$ = logical indicators (0 = false, 1 = true)
- $\beta_{0jl} \sim \beta_{5jl}$ = level and trend changes
- Demonstrated `higher maintenance effects` on `word problem-solving` questions compared to `visualization` questions.
- Intervention effects: level (logit = 2.6) and trend (logit = 0.22) changes

# Conclusions

- Although the small number of participants limits the generalizability of the findings, single-case design research is often aimed at understanding and refining interventions on an individual level, making it particularly useful in situations where large-scale trials may not be feasible or necessary.
- Integrating visual analysis (Tau-U) and Bayesian multilevel modeling supplements intervention evaluation.
- Visual analysis establishes functional relations while multilevel modeling quantifies phase changes in nested data.
- Future research should extend this intervention to establish causal inferences across diverse backgrounds and instructional group settings.