CSC

ICT Solutions for Brilliant Minds

Introduction to quantum computing and FiQCI - QAOA

Olli Mukkula – Application specialist at CSC

Quantum Approximate Optimization Algorithm Introduction

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  1. Resistance to noise: QAOA can be implemented with low-depth circuits and the algorithm seems to be robust against noise beyond what would be expected for low-depth circuits.
  2. General purpose: QAOA has potentially a wide range of applications, because it solves combinatorial optimization problems; which includes a huge amount of different problems.

Quantum Approximate Optimization Algorithm Introduction

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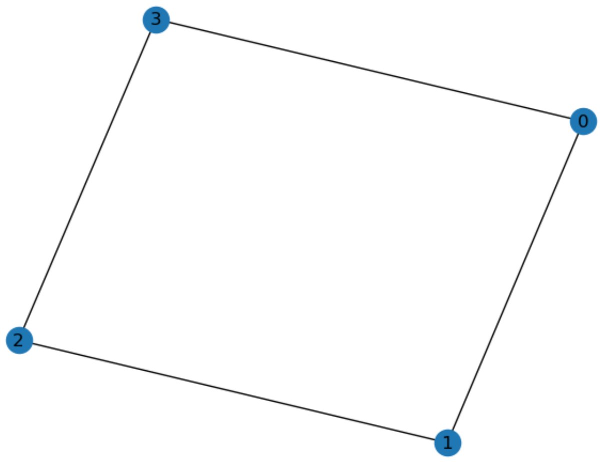
    4. Use classical optimizer to find quantum ansatz that minimizes the energy of the system.

Quantum Approximate Optimization Algorithm Introduction

- Example application: The MaxCut problem
  - *Divide vertices of a graph into two complementary sets, such that the number of edges between the partitions is as high as possible*



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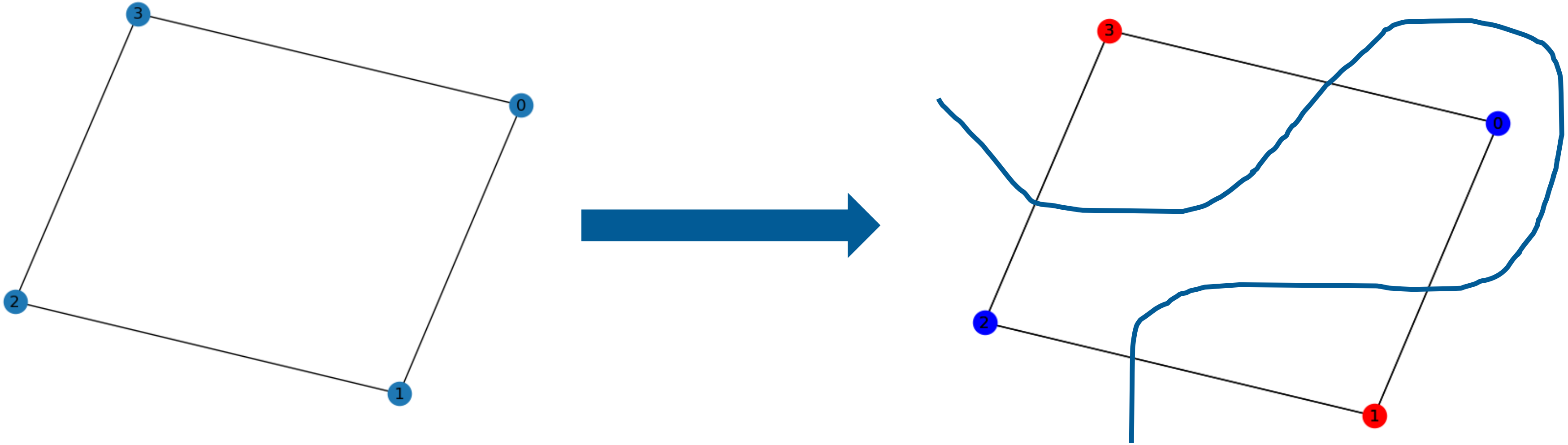
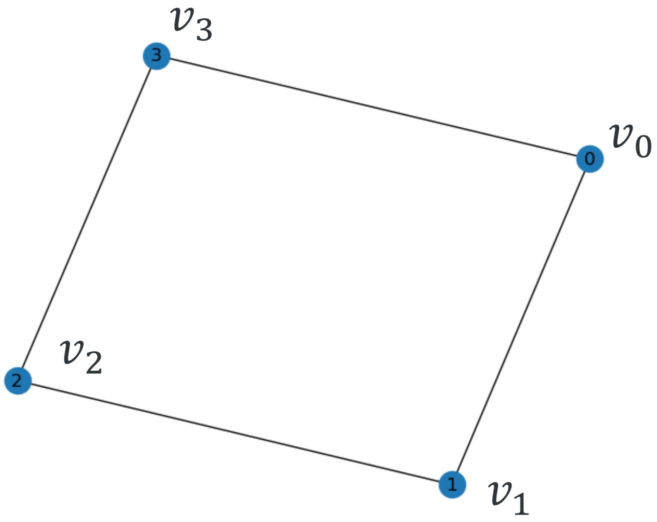


Quantum Approximate Optimization Algorithm Introduction

- MaxCut cost function
  1. Assign variable $v_i \in \{0,1\}$ to each vertex, that defines whether the vertex belongs to 0-partition or 1-partition



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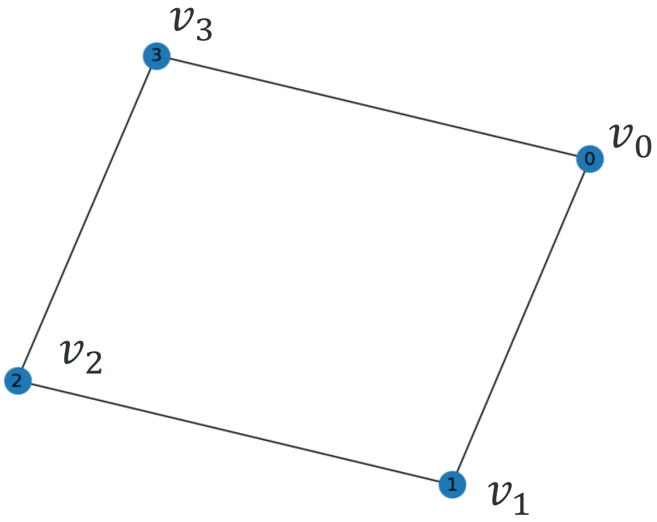
$$C = \underbrace{v_0 \wedge v_1}_{\substack{0 \text{ if } v_0 = v_1 \\ 1 \text{ if } v_0 \neq v_1}} + v_1 \wedge v_2 + v_2 \wedge v_3 + v_0 \wedge v_3$$



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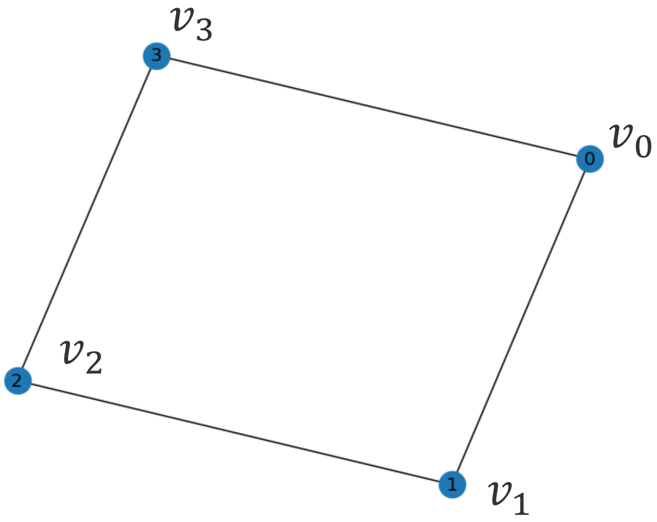
  3. Next, we move the cost function to energy landscape



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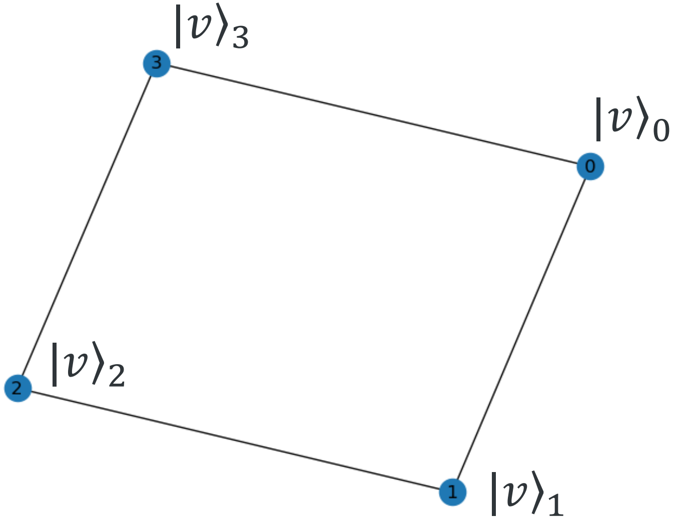

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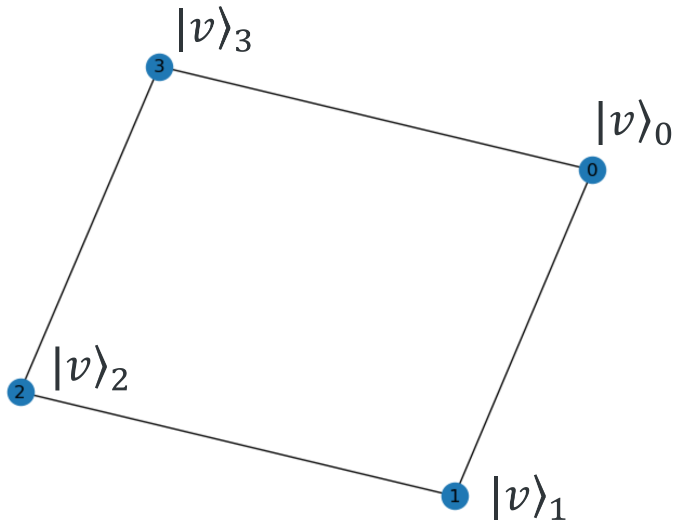

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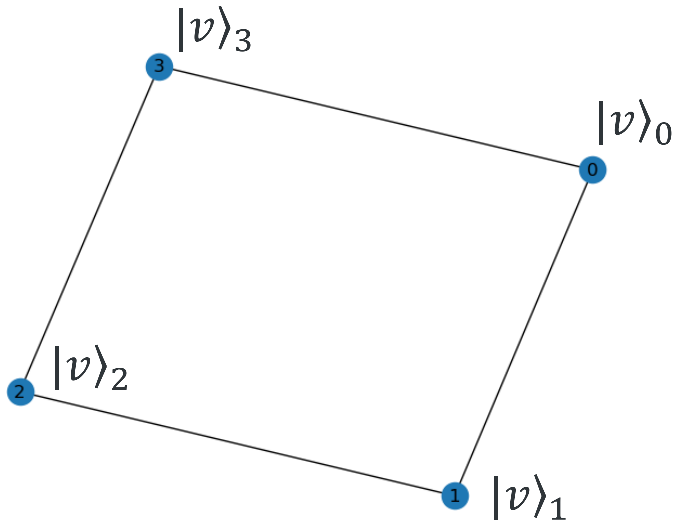
  3. By using Pauli Z-operations: $Z_i|0\rangle_i = |0\rangle_i$ and $Z_i|1\rangle_i = -|1\rangle_i$, we can find the matching expression:

$$H = \frac{1 - Z_0 Z_1}{2} + \frac{1 - Z_1 Z_2}{2} + \frac{1 - Z_2 Z_3}{2} + \frac{1 - Z_0 Z_3}{2}$$



Quantum Approximate Optimization Algorithm
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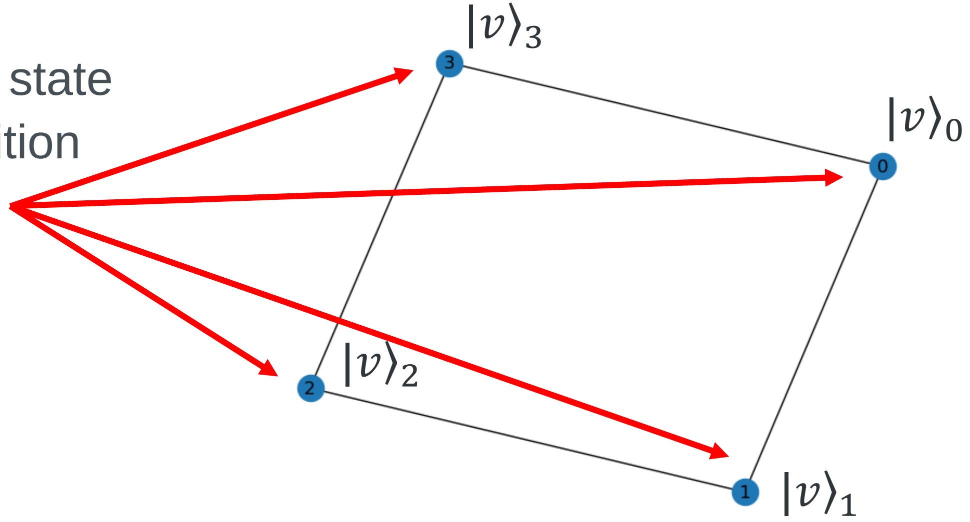
- When the ansatz that minimizes the energy is found, the most measured state for that ansatz holds tells which partition gives the MaxCut: $|v\rangle = |v_0 v_1 v_2 v_3\rangle$



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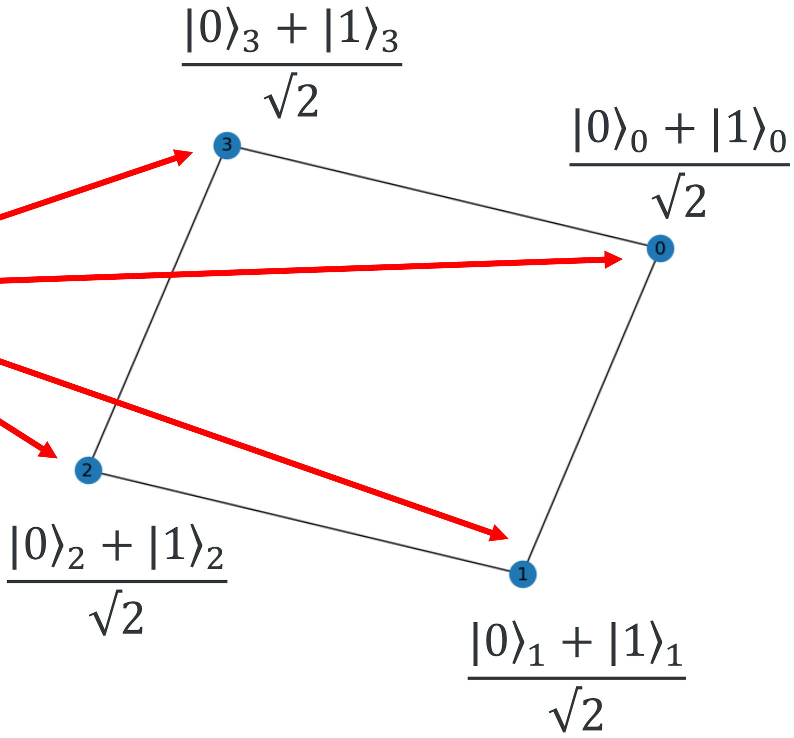
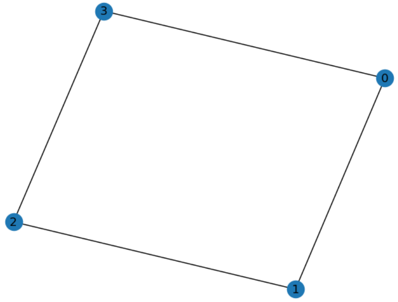
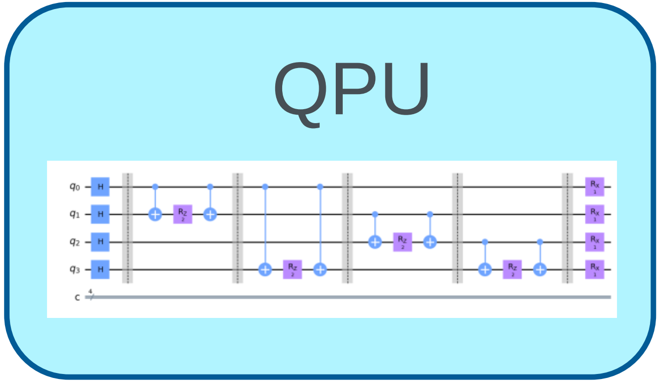
- Quantum computers can bring qubits to superposition to simultaneously evaluate every possible combination:
  - $|0000\rangle, |0001\rangle, \ldots, |1110\rangle, |1111\rangle$

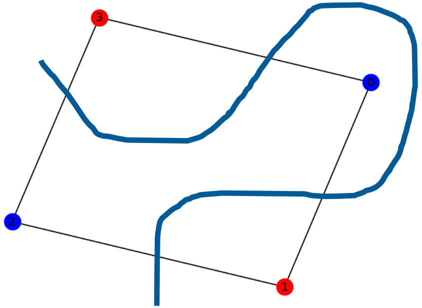


Quantum Approximate Optimization Algorithm Introduction

- Implementing MaxCut QAOA on a quantum computer







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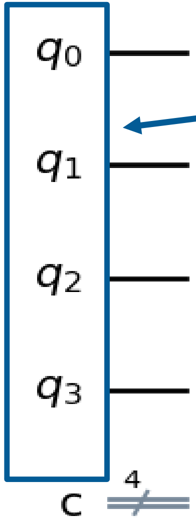
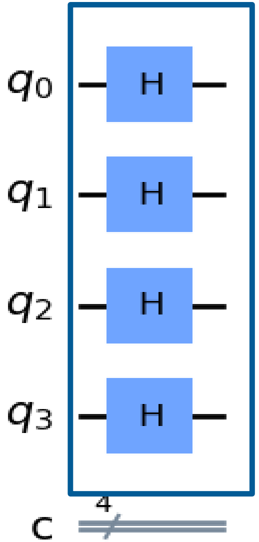


- Allocate one qubit for each vertex in the graph
- Every qubit starts at the ground state

$$|\psi\rangle = |0\rangle_0|0\rangle_1|0\rangle_2|0\rangle_3$$

$$= |0000\rangle$$

Quantum Approximate Optimization Algorithm Introduction



- Apply Hadamard gate to each qubit to create superposition of all possible solutions

$$|\psi\rangle = \frac{|0\rangle_0 + |1\rangle_0}{\sqrt{2}} \frac{|0\rangle_1 + |1\rangle_1}{\sqrt{2}} \frac{|0\rangle_2 + |1\rangle_2}{\sqrt{2}} \frac{|0\rangle_3 + |1\rangle_3}{\sqrt{2}}$$

$$= \frac{|0000\rangle + |0001\rangle + \ldots + |1110\rangle + |1111\rangle}{4}$$

Quantum Approximate Optimization Algorithm
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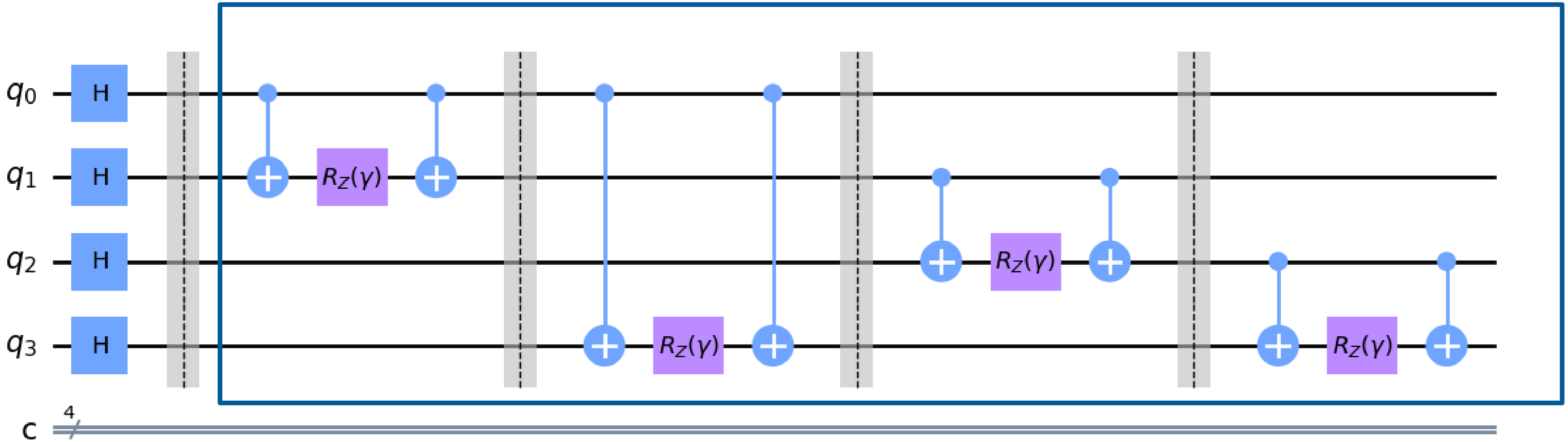
Operate the system with our Hamiltonian

$$U(\gamma)_C = \prod_{(j,k)\in \text{edges}} e^{-i\gamma C_{jk}}, \qquad \text{where } C_{j,k} = \frac{1 - Z_j Z_k}{2}$$

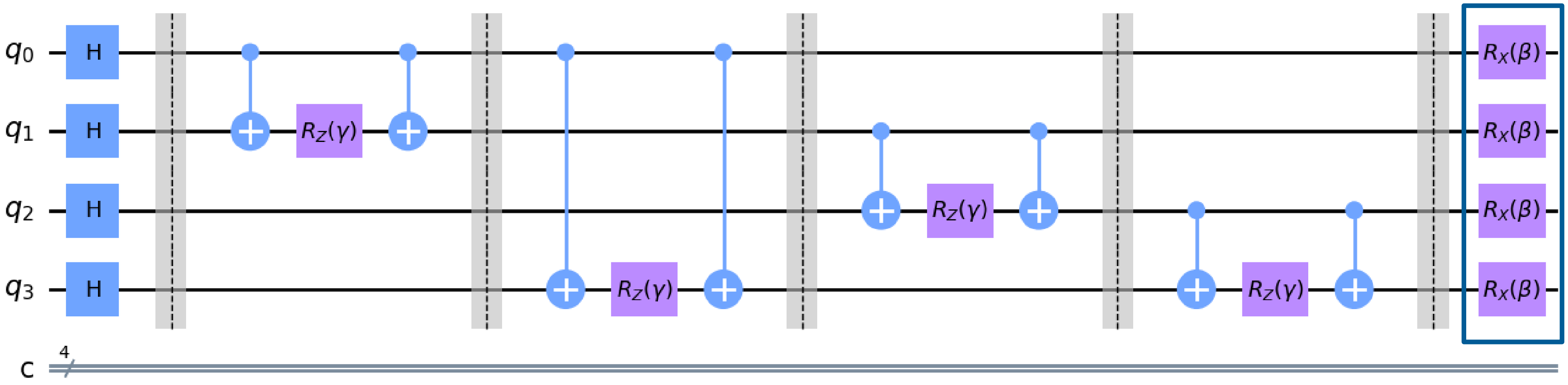


Quantum Approximate Optimization Algorithm Introduction

Apply mixer Hamiltonian

(in case the system gets 'stuck 'in its eigenstate)

$$U_B(\beta) = \prod_{j \in \text{vertices}} R_x(\beta)_j$$



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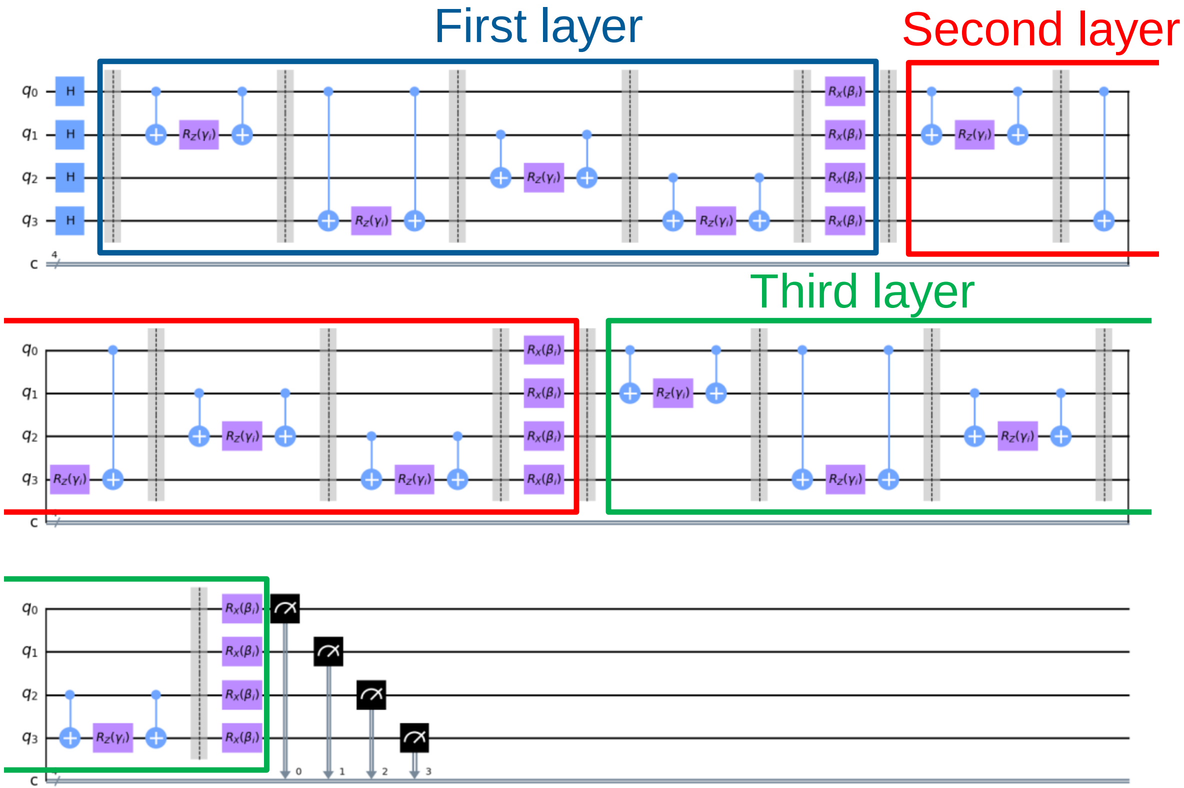
- Apply multiple layers of $U_C(\gamma)$ and $U_B(\beta)$ with the lists of parameters $[\gamma_0, \gamma_1, \ldots]$ and $[\beta_0, \beta_1, \ldots]$.
- Then measure all qubits



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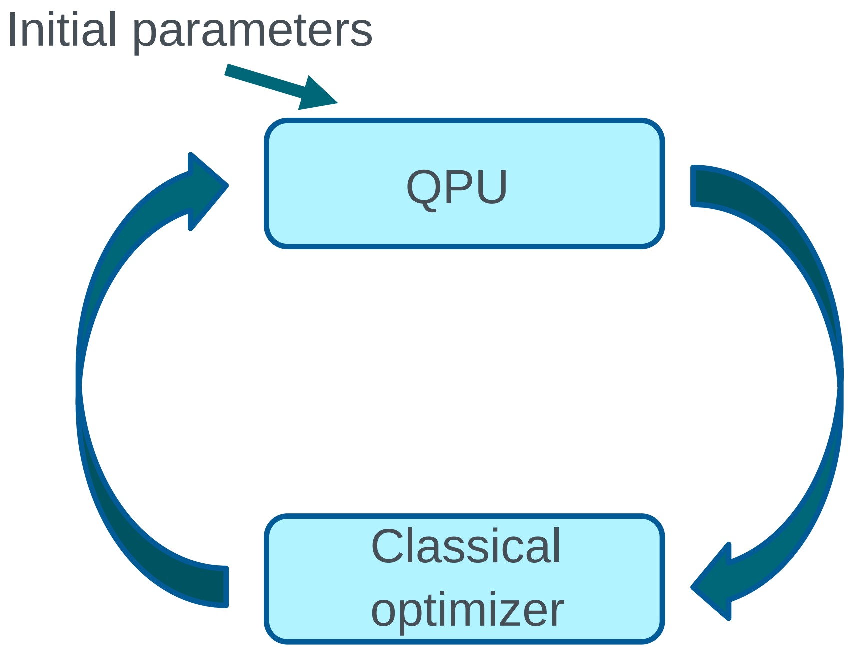
    1. Guess the quantum ansatz (initial parameters γ and β)



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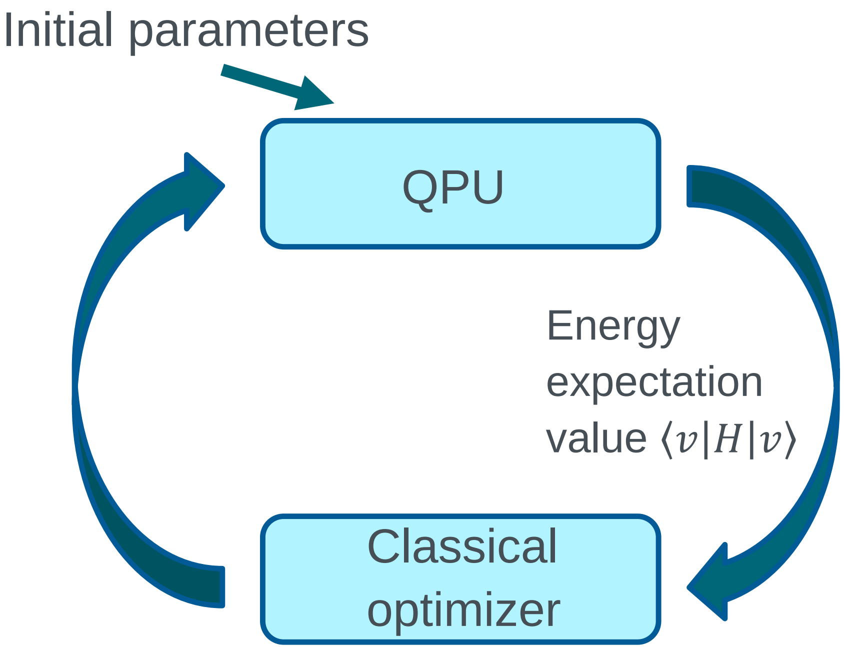

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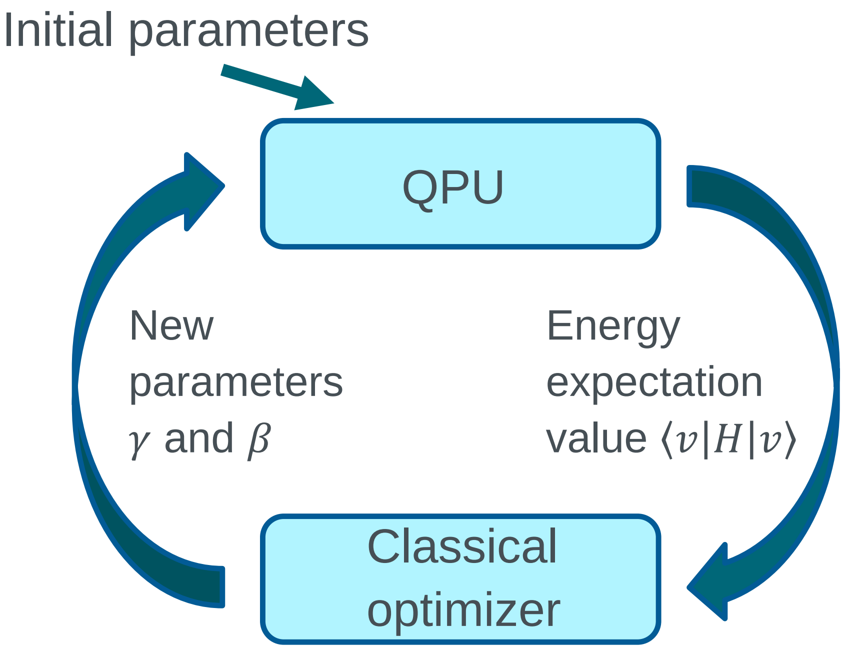

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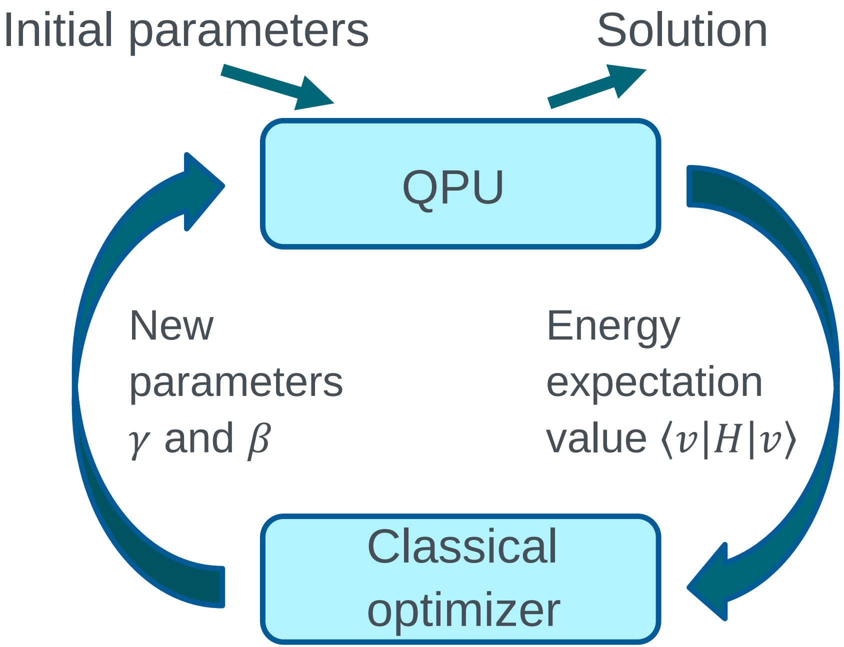
  3. Use classical optimizer to find the parameters that minimize the energy
  4. Run the QAOA circuit with the optimal parameters; The most probable measured state gives the solution to MaxCut



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- MaxCut itself has use cases in physics (Ising model) and machine learning (binary classification)
- Potentially faster than classical algorithms
  - Performs well in specific problems
  - Sometimes slowed down by parameters optimization and bad ansatzes