ICT Solutions for Brilliant Minds

Introduction to quantum computing and FiQCI – Deutsch Algorithm

Olli Mukkula – Application specialist at CSC

Deutsch Algorithm

- Deutsch algorithm was invented by David Deutch in 1985. It was the first algorithm developed for quantum computers

Prof. David Deutsch - Creator of the first quantum algorithm

Source: [https://www.daviddeutsch.org.uk/]

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- Deterministic algorithm with 100% success rate on noiseless quantum computer
- Solves the specific problem (Deutsch problem) exponentially faster than any classical algorithm

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Is it true that $f(0) = f(1)$

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NOT-function

$f(0) = 1$

$f(1) = 0$

Deutsch Algorithm Introduction

- The Deutsch problem:

Consider a function $f : \{0,1\} \to \{0,1\}$

Is it true that $f(0) = f(1)$

- There are four possible functions:

Is the function constant or varied?

Constant functions

Always zero

$$f(0) = 0$$
$$f(1) = 0$$

Always one

$$f(0) = 1$$
$$f(1) = 1$$

Varied functions

Identity-function

$$f(0) = 0$$
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NOT-function

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Deutsch Algorithm
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- Deutsch problem is a black box problem
- Black box oracle:
  - Has an input and an output
  - Internal workings are unknown

Deutsch Algorithm

- If function is $f(x) = \{0,1\}$ is a black box, it is non-reversible
  - Can't solve x based on the output

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- Quantum computer only needs <u>one evaluation!</u>

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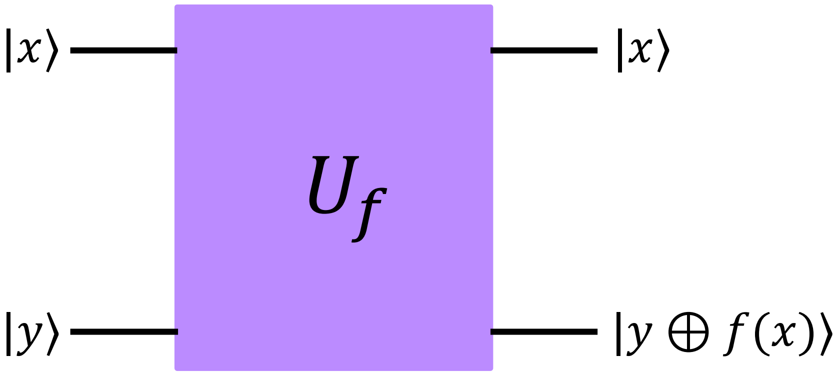
- Using two qubits, we can build Quantum oracle U_f ('black box' quantum gate) that implements $f(x)$



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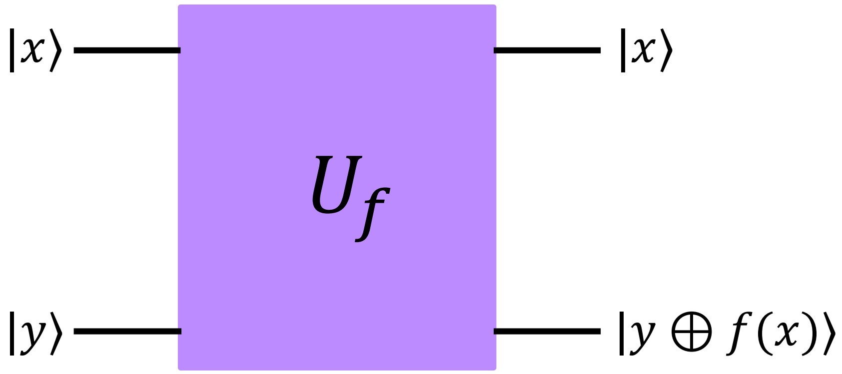
- Let's analyze the output state $|x\ \ y \oplus f(x)\rangle$ to see what it can tell us about the oracle



Deutsch Algorithm

- Operation $\oplus$ is called exclusive-OR (XOR

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
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Table of XOR operations

Deutsch Algorithm

- Operation $\oplus$ is called exclusive-OR (XOR)
- We can calculate all the possible output states

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Table of XOR operations

	Output state: $\mathrm{U_f}\lvert\mathrm{x\ y}\rangle = \lvert x\ \ y \oplus f(x)\rangle$			
Initial state	$f(0)=0$ $f(1)=0$	$f(0)=1$ $f(1)=1$	$f(0)=0$ $f(1)=1$	$f(0)=1$ $f(1)=0$
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  - If $f(x)$ is varied, the quantum oracle is two-qubit operation (CNOT or NOT-CNOT). Phase kickback will reverse the phase of the 1st qubit.
  - Reversing the superposition of the 1st qubit causes it flip if its phase was reversed
  - Whether the 1st qubit was flipped reveals if $f(x)$ is constant or varied

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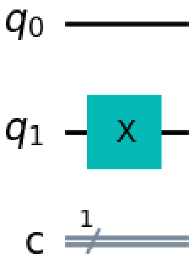
- Deutsch algorithm implementation:

1. Allocate qubits (Flip the 2nd qubit with X-gate): $|0\rangle|1\rangle$



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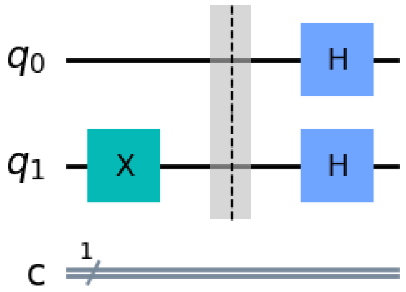

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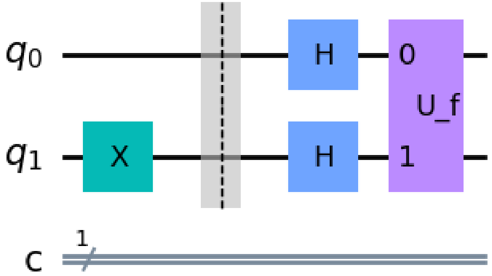

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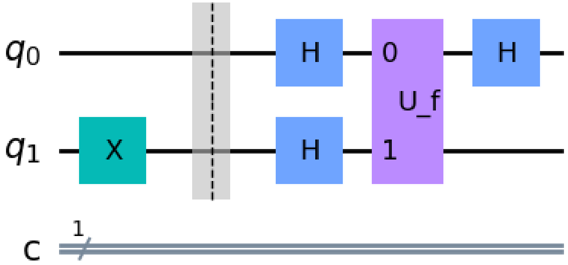

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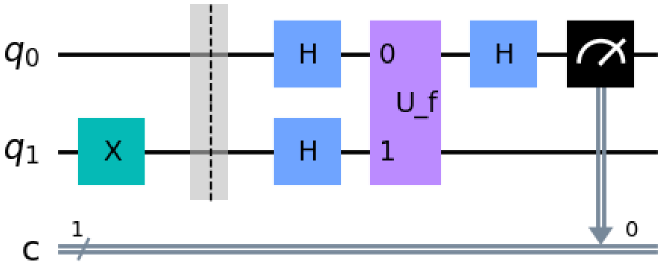

5. Measure:
   - $|0\rangle$ if $f(0) = f(1)$
   - $|1\rangle$ if $f(0) \neq f(1)$

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- Generalization for n-qubits is called *Deutsch-Jozsa algorithm*

- It has no practical use, but it inspired the development of quantum algorithms for practically relevant problems