Numerical Solutions of Lorenz Equations

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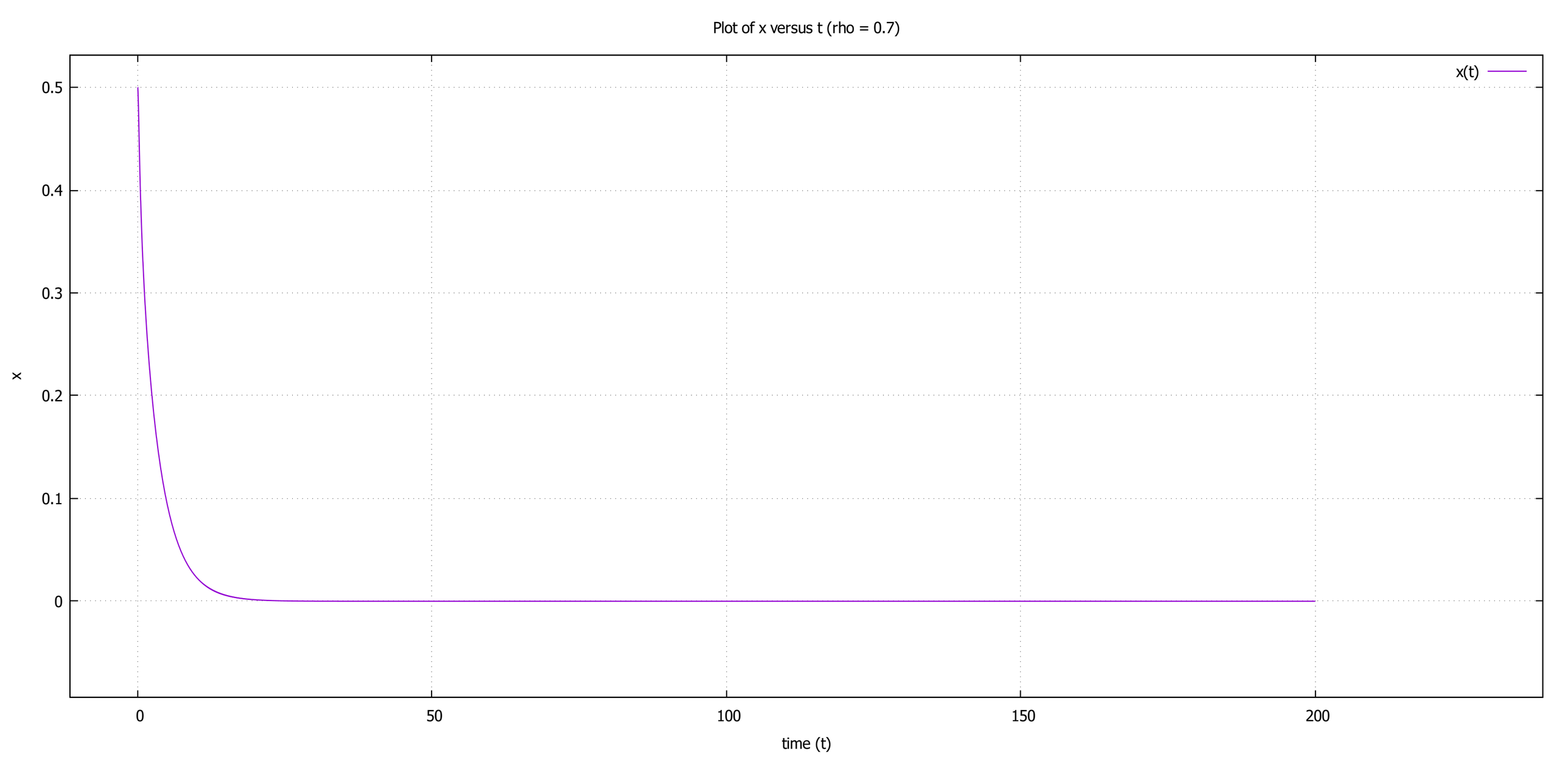
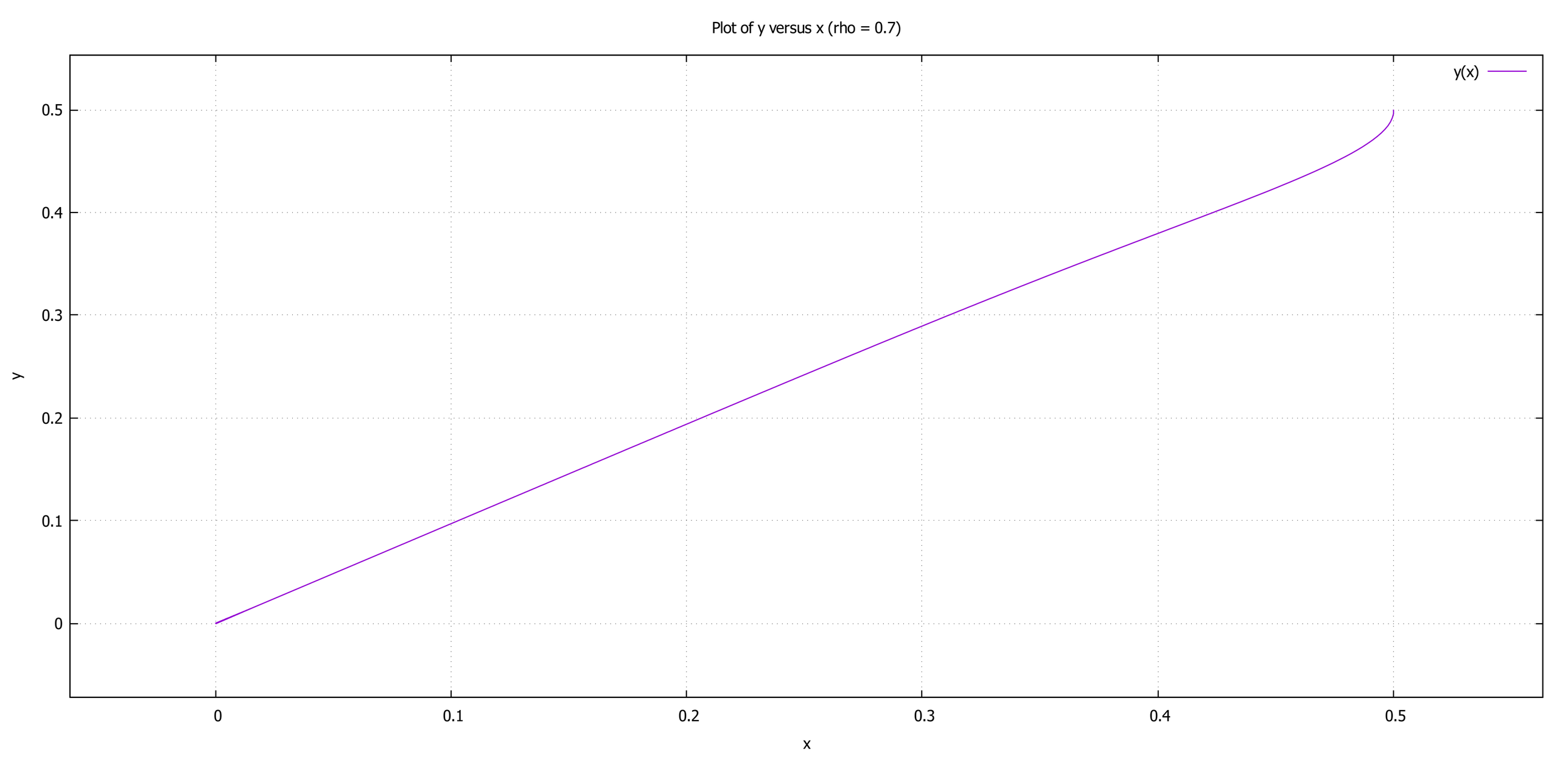
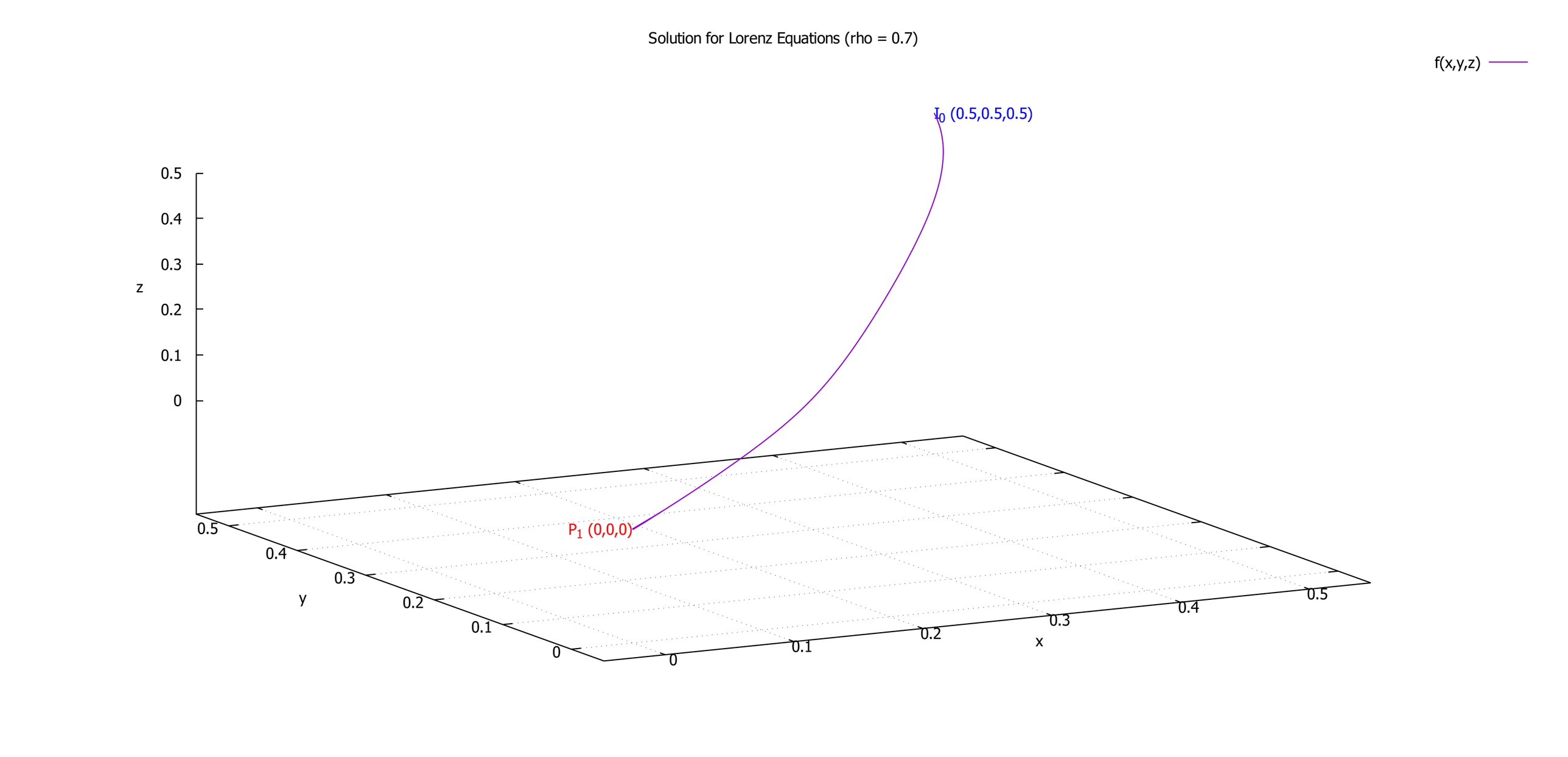
1 For a point near P_1 ($\rho < 1$)

Following are the plots when initial point is $I_0 \equiv (0.5, 0.5, 0.5)$ and $\sigma = 10, \beta = 2.667, \rho = 0.7$. Here, $P_1 \equiv (0, 0, 0)$

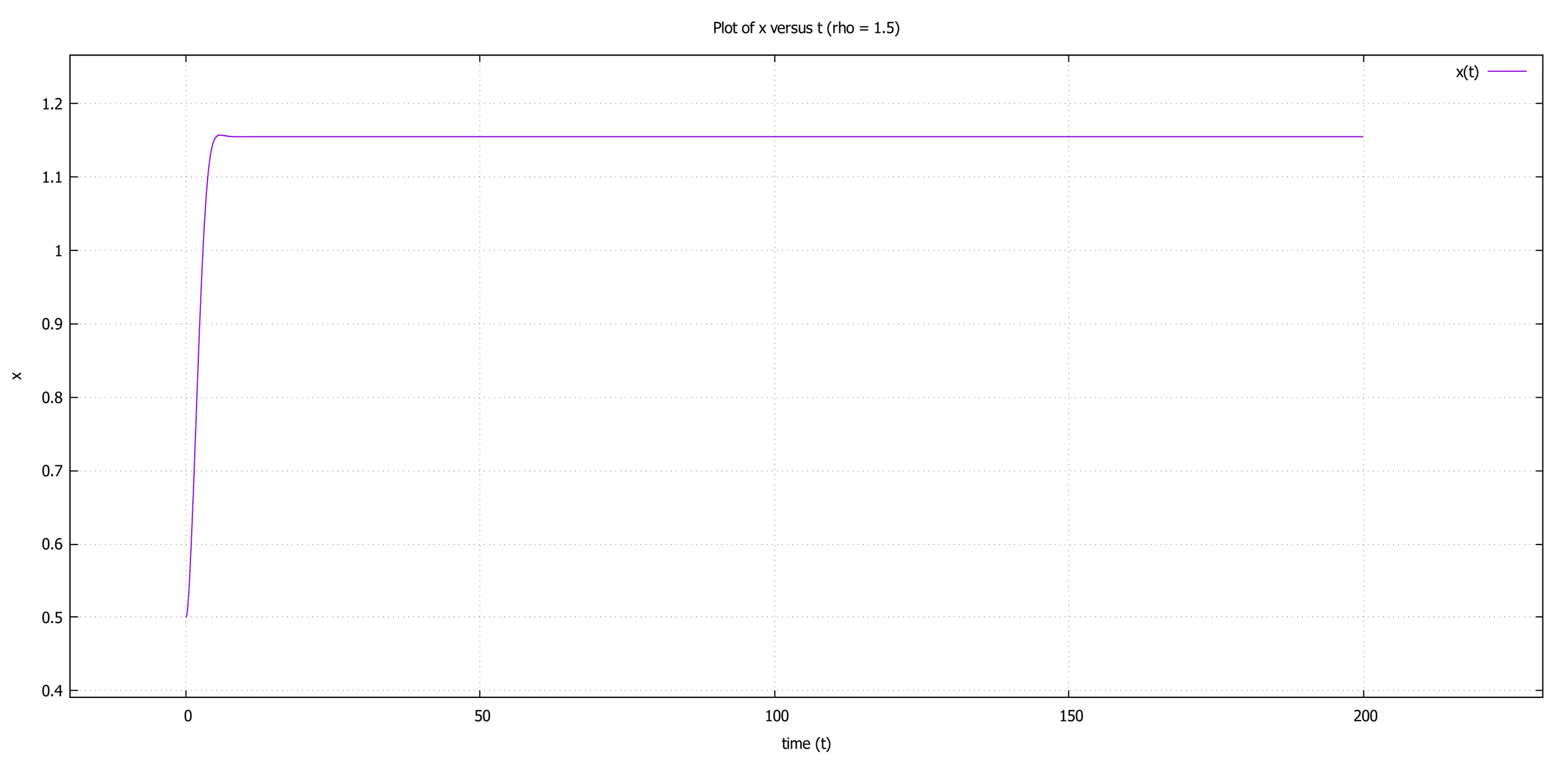

Plot of x versus t (rho = 0.7)
x(t)
0.5
0.4
0.3
0.2
0.1
0
x
0
50
100
150
200
time (t)

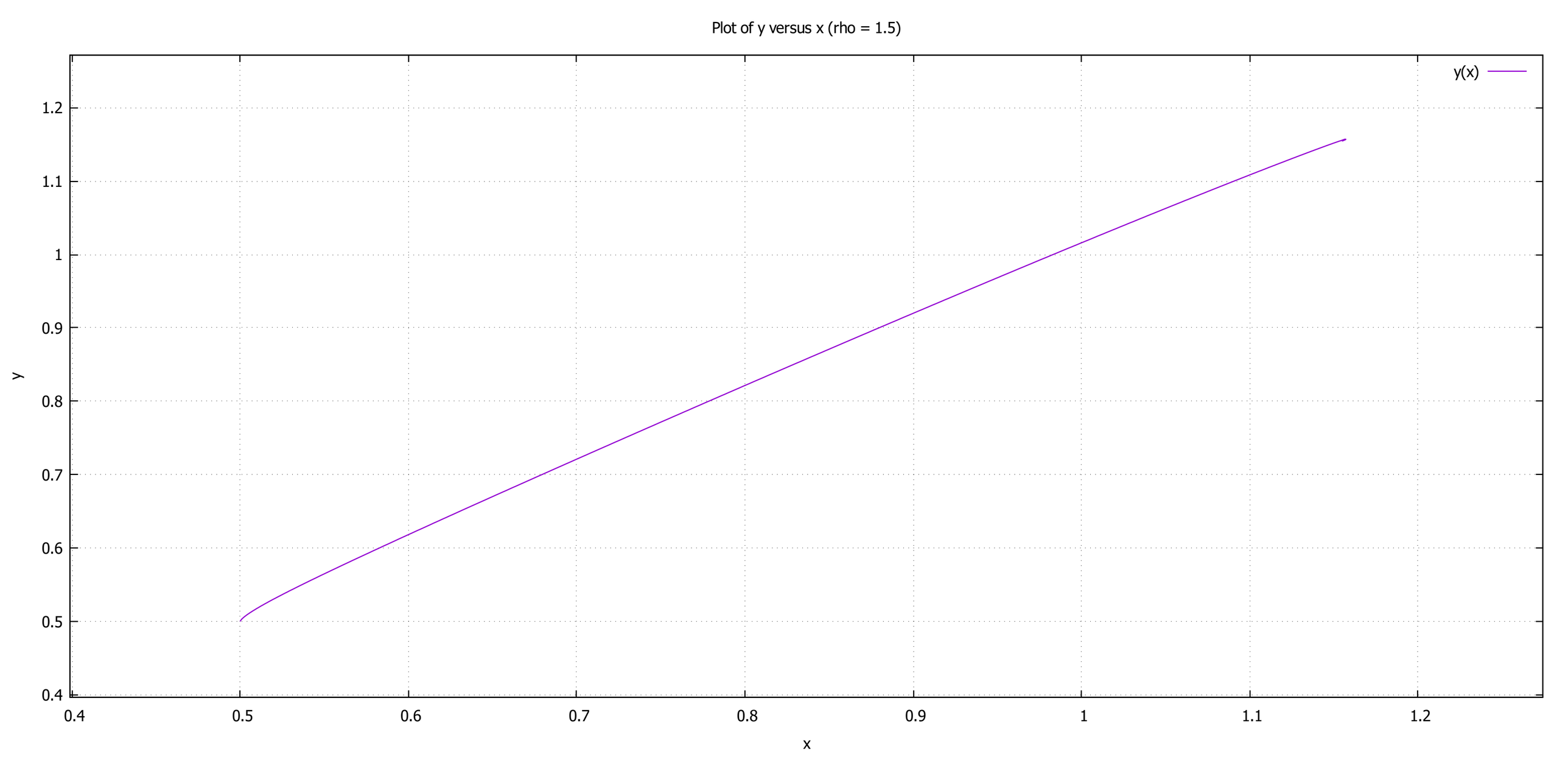

Plot of y versus x (rho = 0.7)
y(x)
0.5
0.4
0.3
0.2
0.1
0
y
0
0.1
0.2
0.3
0.4
0.5
x

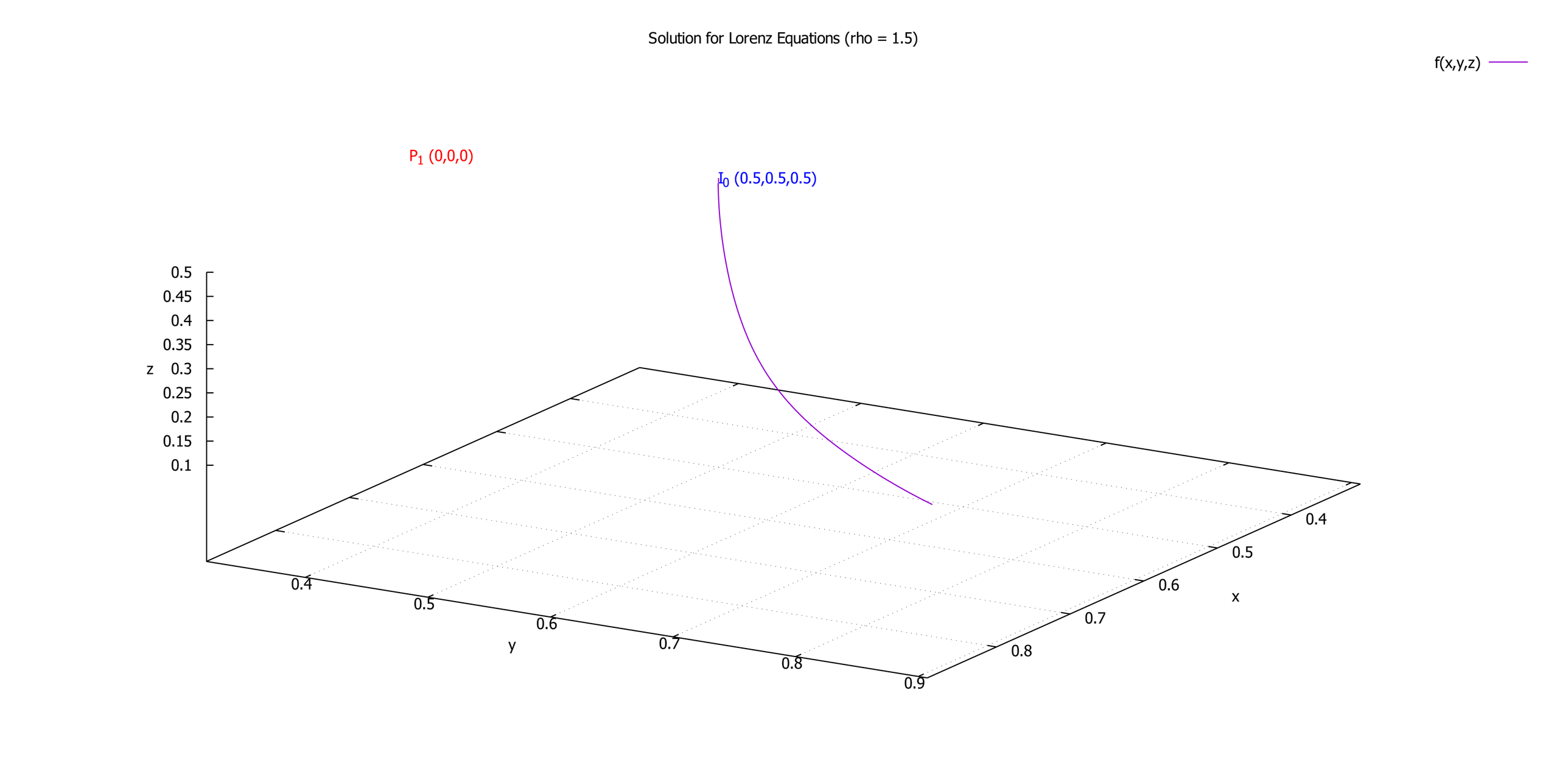

Solution for Lorenz Equations (rho = 0.7)
f(x,y,z)
I_0 (0.5,0.5,0.5)
P_1 (0,0,0)
z
0.5
0.4
0.3
0.2
0.1
0
y
0.5
0.4
0.3
0.2
0.1
0
x
0
0.1
0.2
0.3
0.4
0.5

2 For a point near P_1 ($\rho > 1$)

Following are the plots when initial point is $I_0 \equiv (0.5, 0.5, 0.5)$ and $\sigma = 10, \beta = 2.667, \rho = 1.5$. Here, $P_1 \equiv (0, 0, 0)$

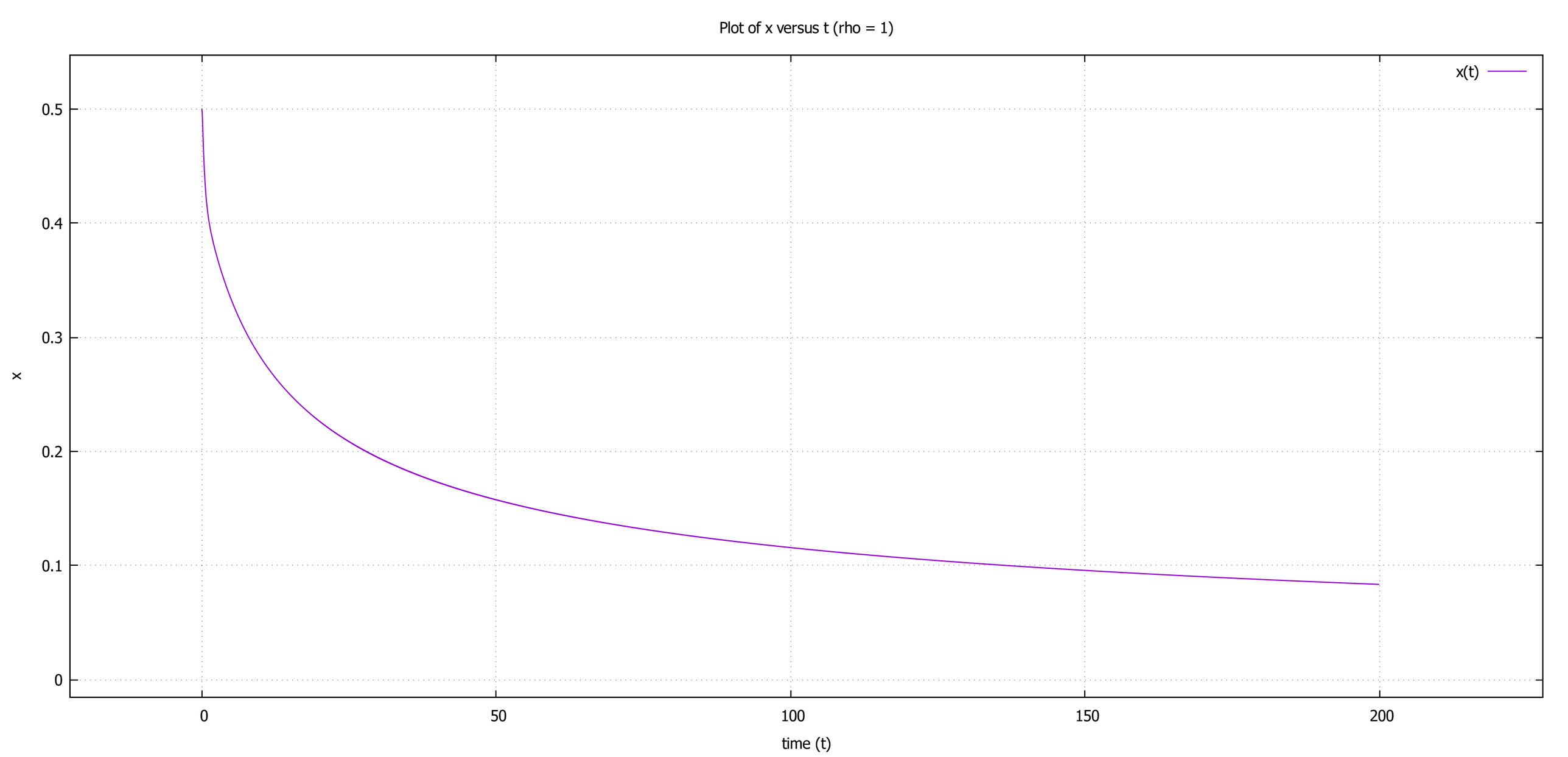

Plot of x versus t (rho = 1.5)
x(t)
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0.4
x
0
50
100
150
200
time (t)

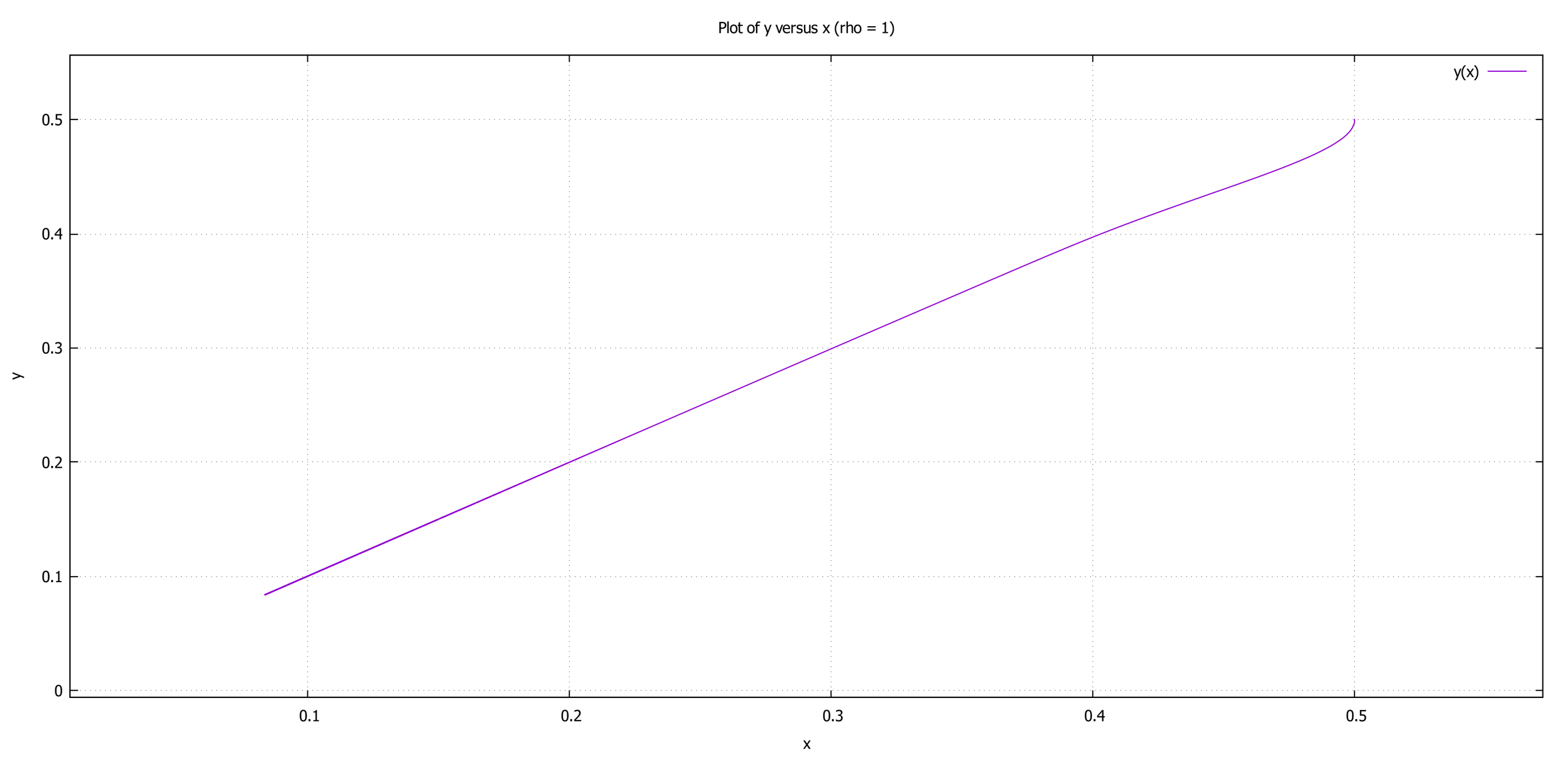

Plot of y versus x (rho = 1.5)
y(x)
y
x
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2

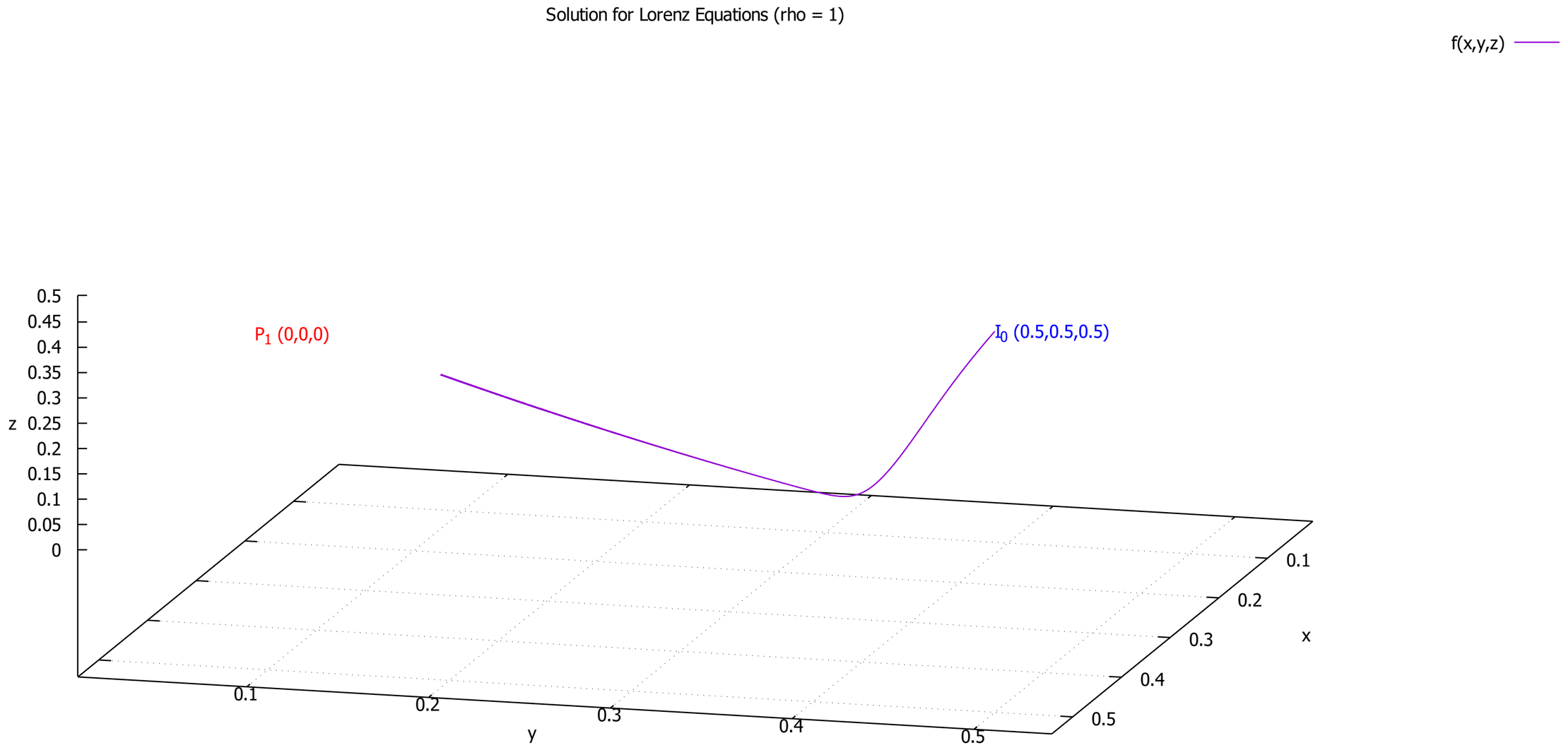

Solution for Lorenz Equations (rho = 1.5)
f(x,y,z)
P1 (0,0,0)
I0 (0.5,0.5,0.5)
z
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
y
0.4
0.5
0.6
0.7
0.8
0.9
x
0.4
0.5
0.6
0.7
0.8

3 For a point near P_1 ($\rho = 1$)

Following are the plots when initial point is $I_0 \equiv (0.5, 0.5, 0.5)$ and $\sigma = 10, \beta = 2.667, \rho = 1$. Here, $P_1 \equiv (0, 0, 0)$

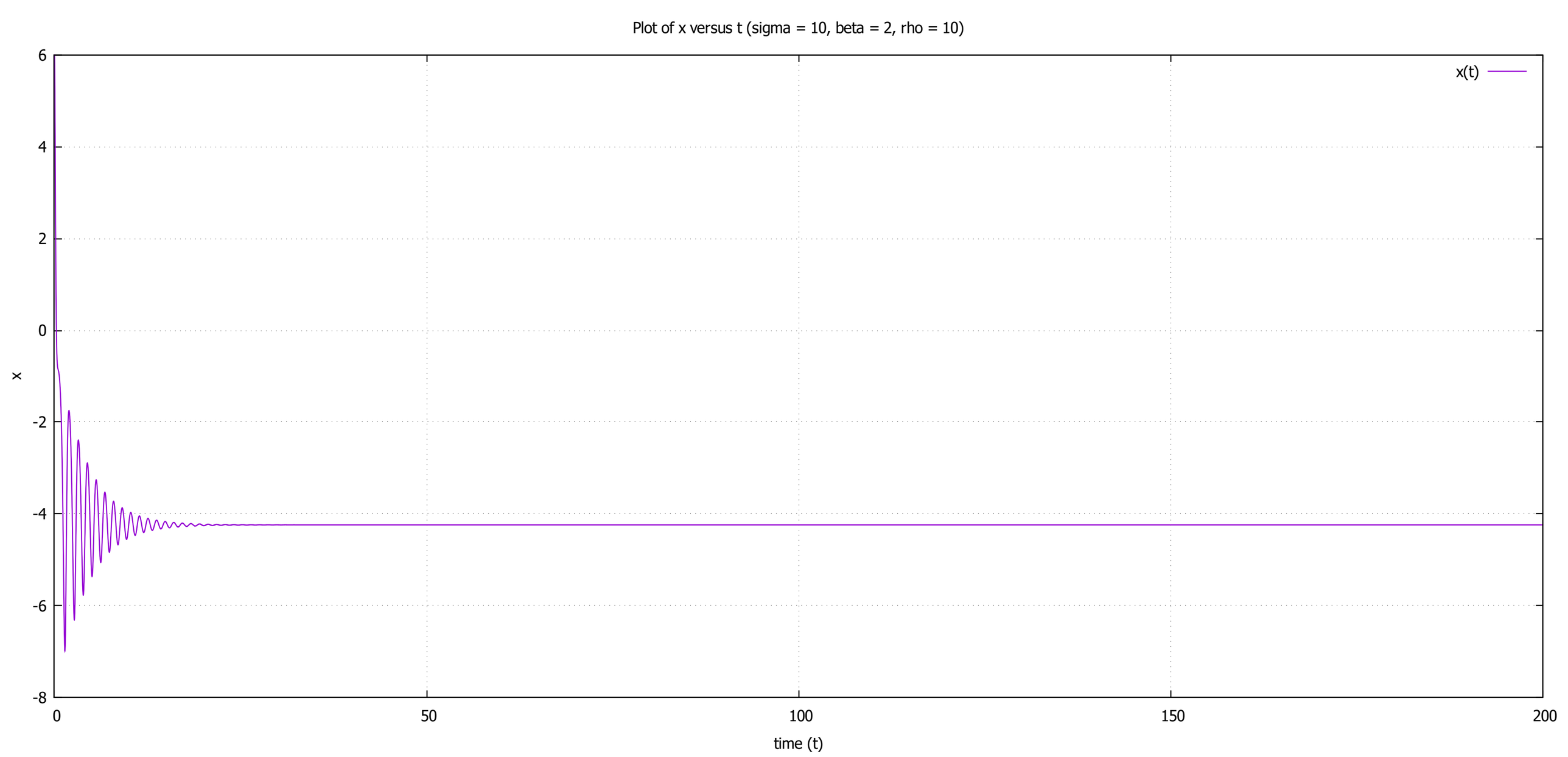

Plot of x versus t (rho = 1)
x(t)
0.5
0.4
0.3
0.2
0.1
0
x
0
50
100
150
200
time (t)

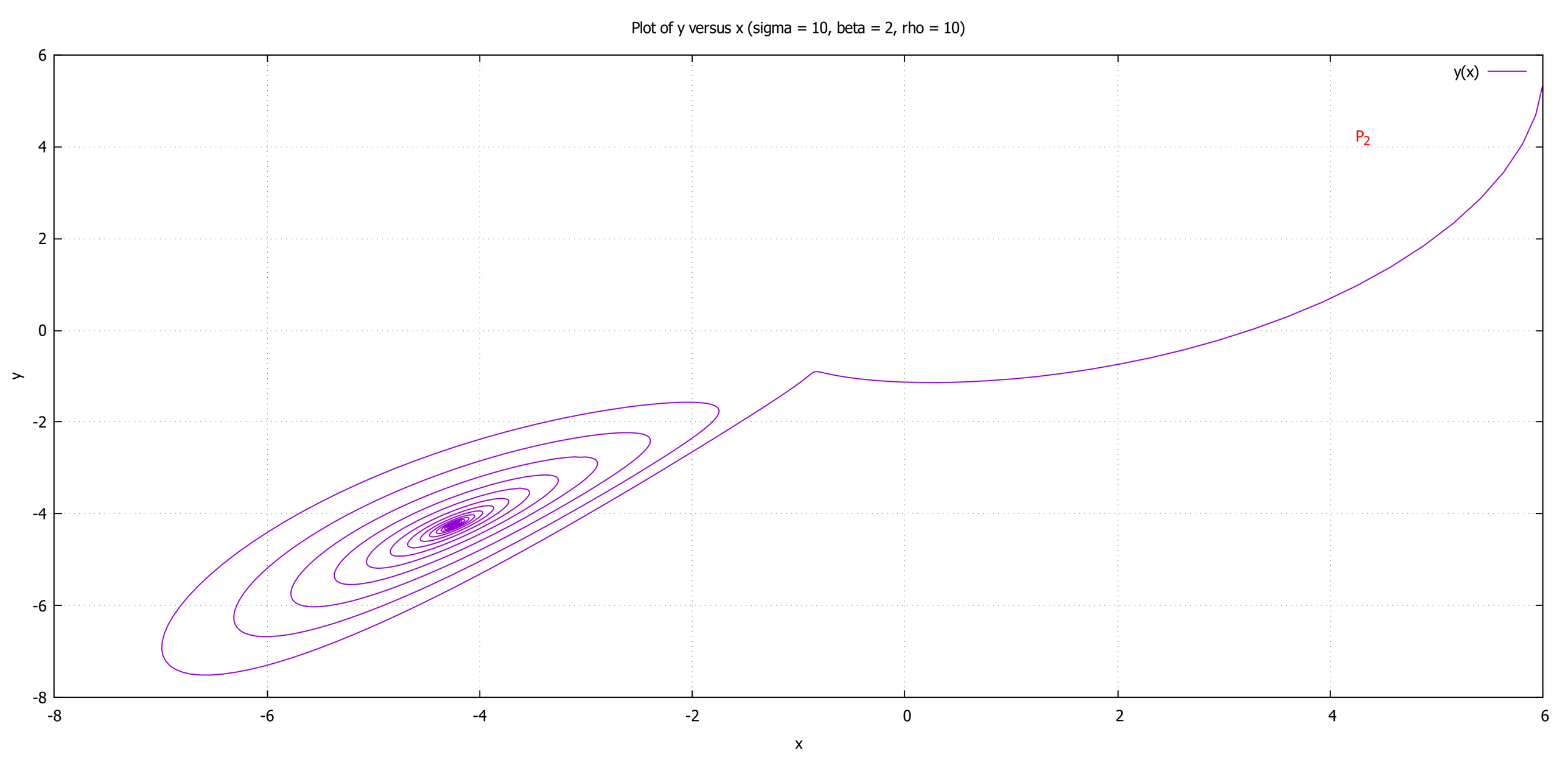

Plot of y versus x (rho = 1)
y(x)
0.5
0.4
0.3
0.2
0.1
0
y
0.1
0.2
0.3
0.4
0.5
x

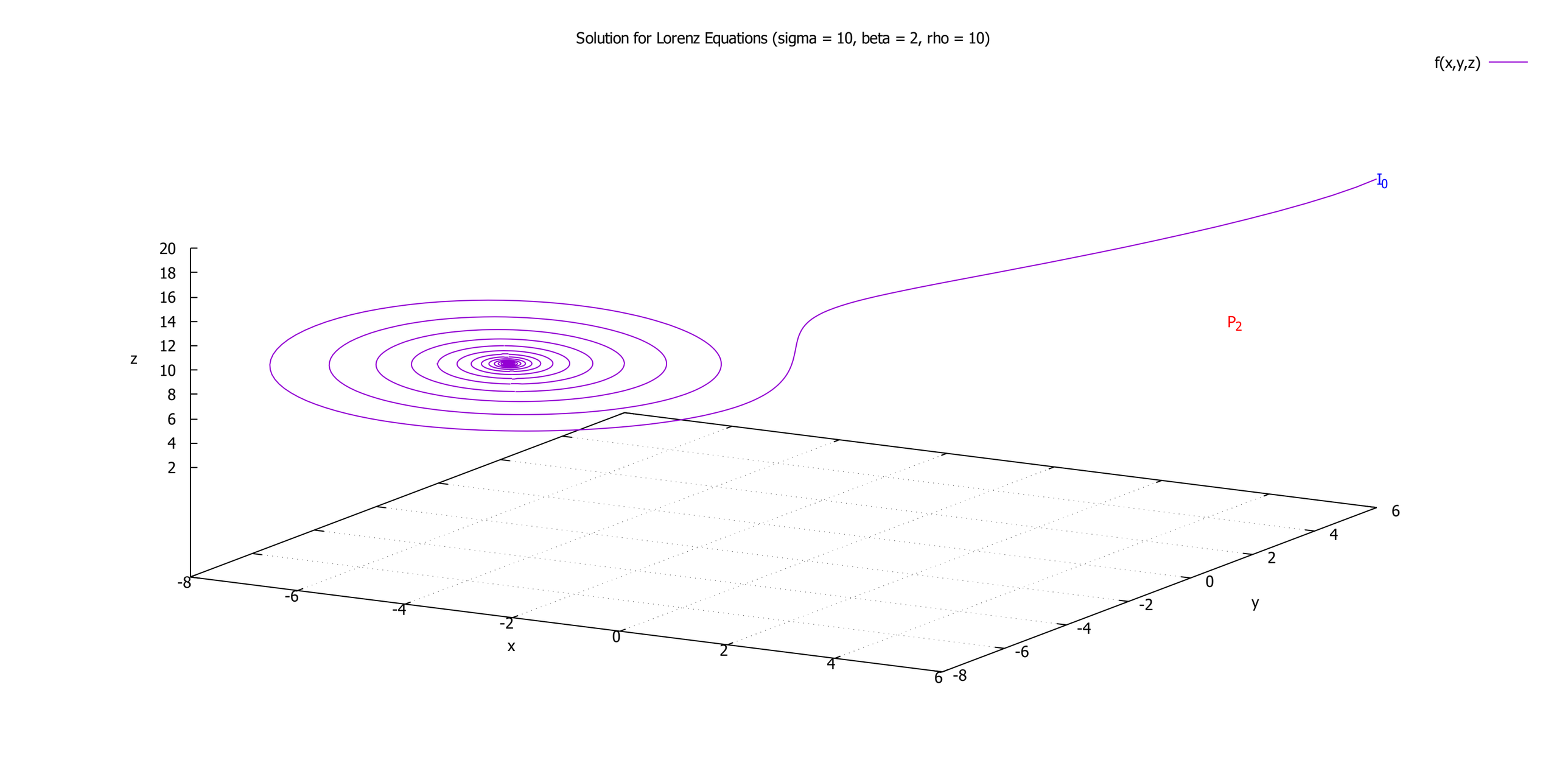

Solution for Lorenz Equations (rho = 1)
f(x,y,z)
P1 (0,0,0)
I0 (0.5,0.5,0.5)
z
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
y
0.1
0.2
0.3
0.4
0.5
x
0.1
0.2
0.3
0.4
0.5

4 For a point near P_2 ($\rho < \rho_0$)

Following are the plots when initial point is $I_0 \equiv (6, 6, 20)$ and $\sigma = 10, \beta = 2, \rho = 10$. Here $P_2 \equiv (4.24, 4.24, 9)$ and $\rho_0 = 21.42$

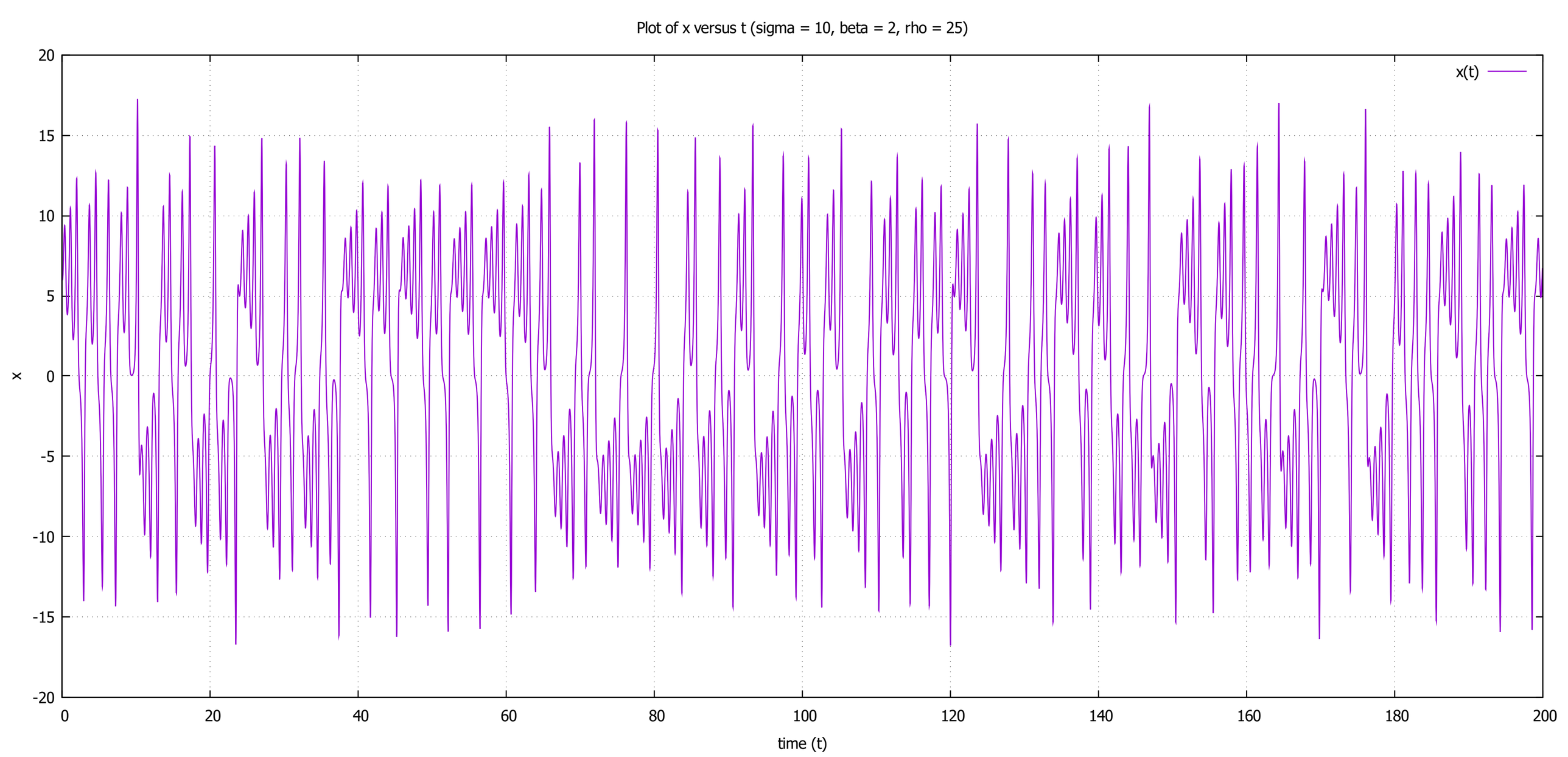

Plot of x versus t (sigma = 10, beta = 2, rho = 10)
x(t)
x
time (t)
6
4
2
0
-2
-4
-6
-8
0
50
100
150
200

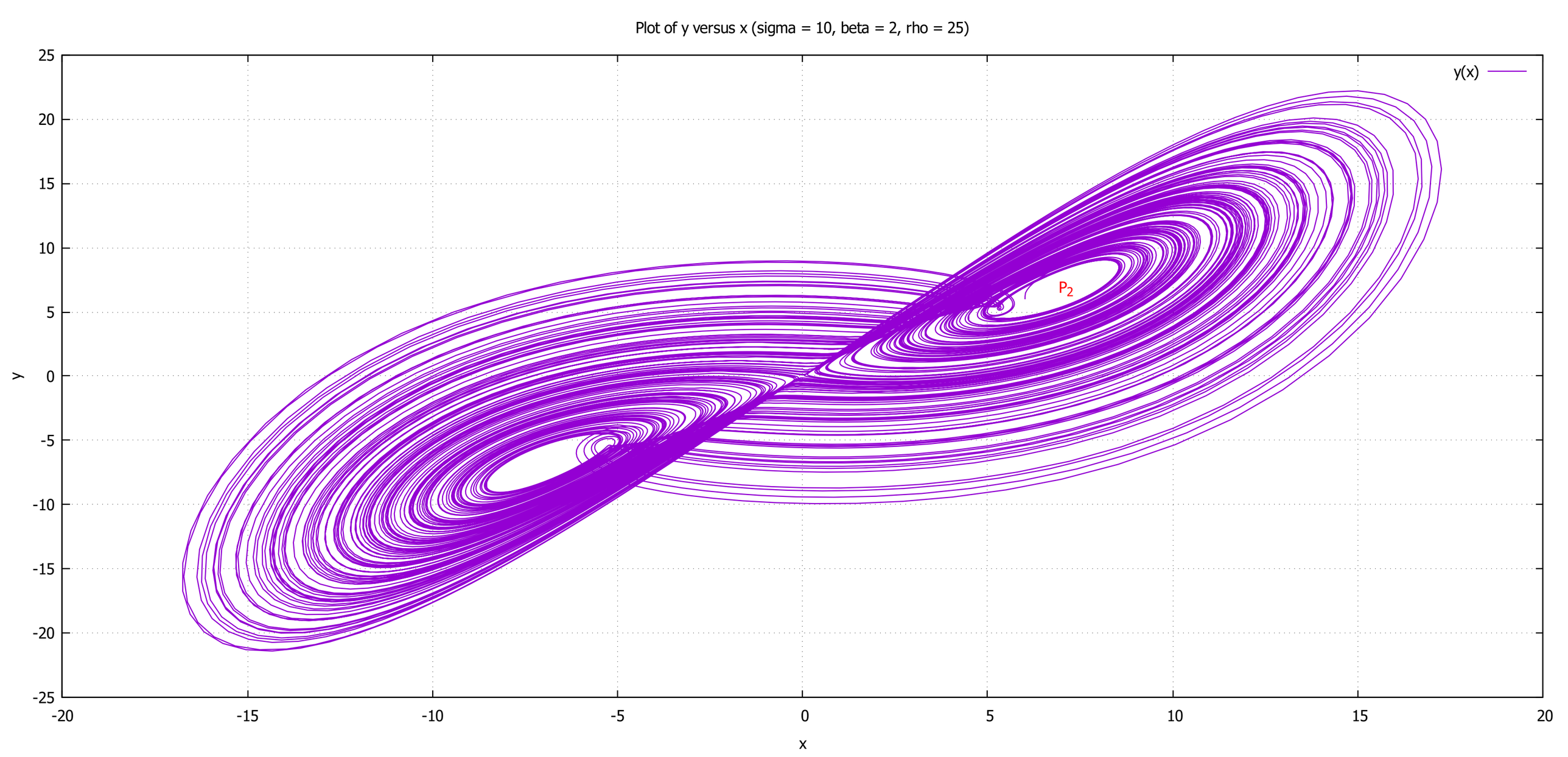

Plot of y versus x (sigma = 10, beta = 2, rho = 10)
y(x)
P2
y
x
6
4
2
0
-2
-4
-6
-8
-8
-6
-4
-2
0
2
4
6

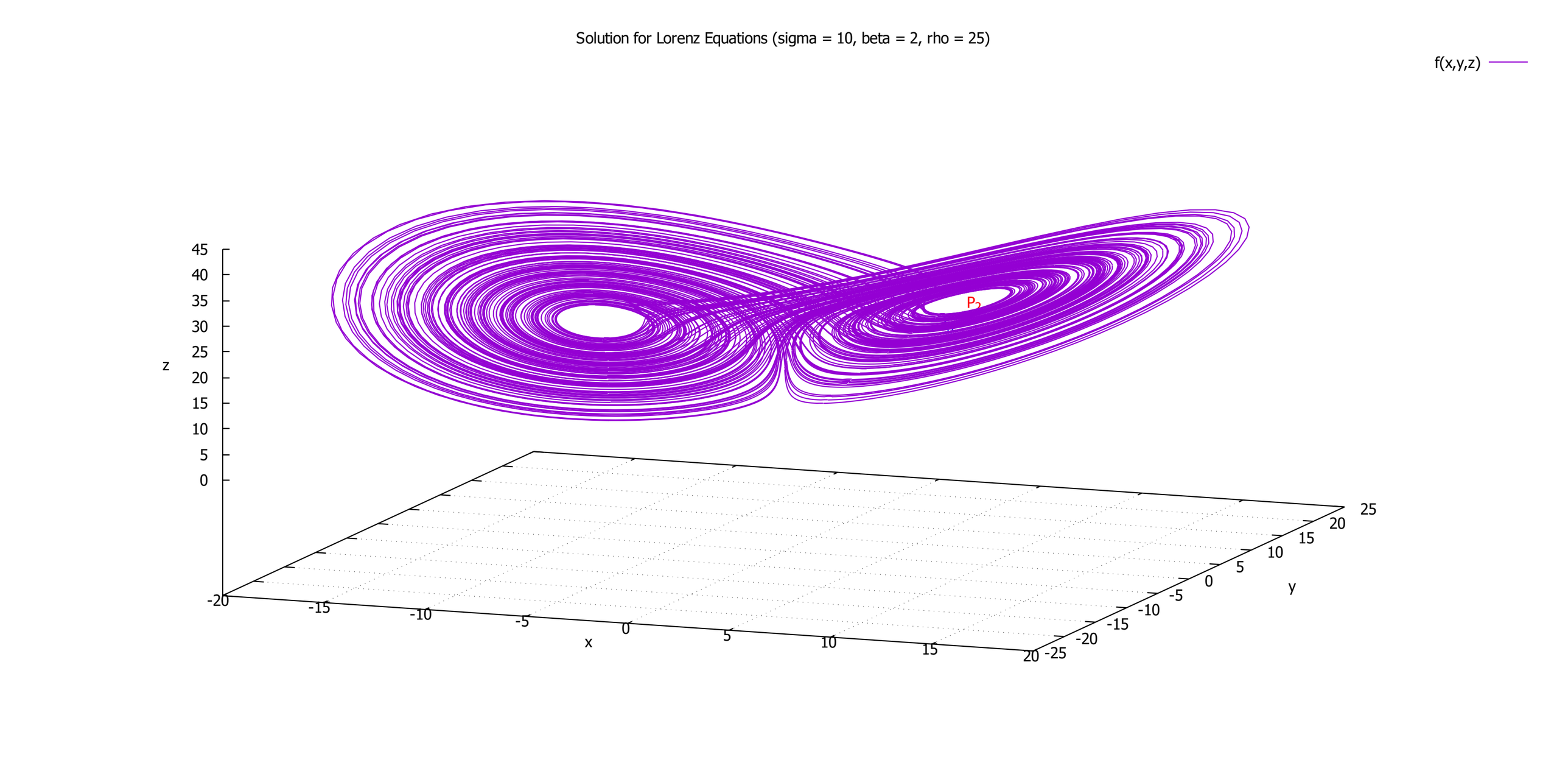

Solution for Lorenz Equations (sigma = 10, beta = 2, rho = 10)
f(x,y,z)
I0
P2
z
20
18
16
14
12
10
8
6
4
2
x
-8
-6
-4
-2
0
2
4
6
y
-8
-6
-4
-2
0
2
4
6

5 For a point near P_2 ($\rho > \rho_0$)

Following are the plots when initial point is $I_0 \equiv (6, 6, 20)$ and $\sigma = 10, \beta = 2, \rho = 25$. Here $P_2 \equiv (6.92, 6.92, 24)$ and $\rho_0 = 21.42$

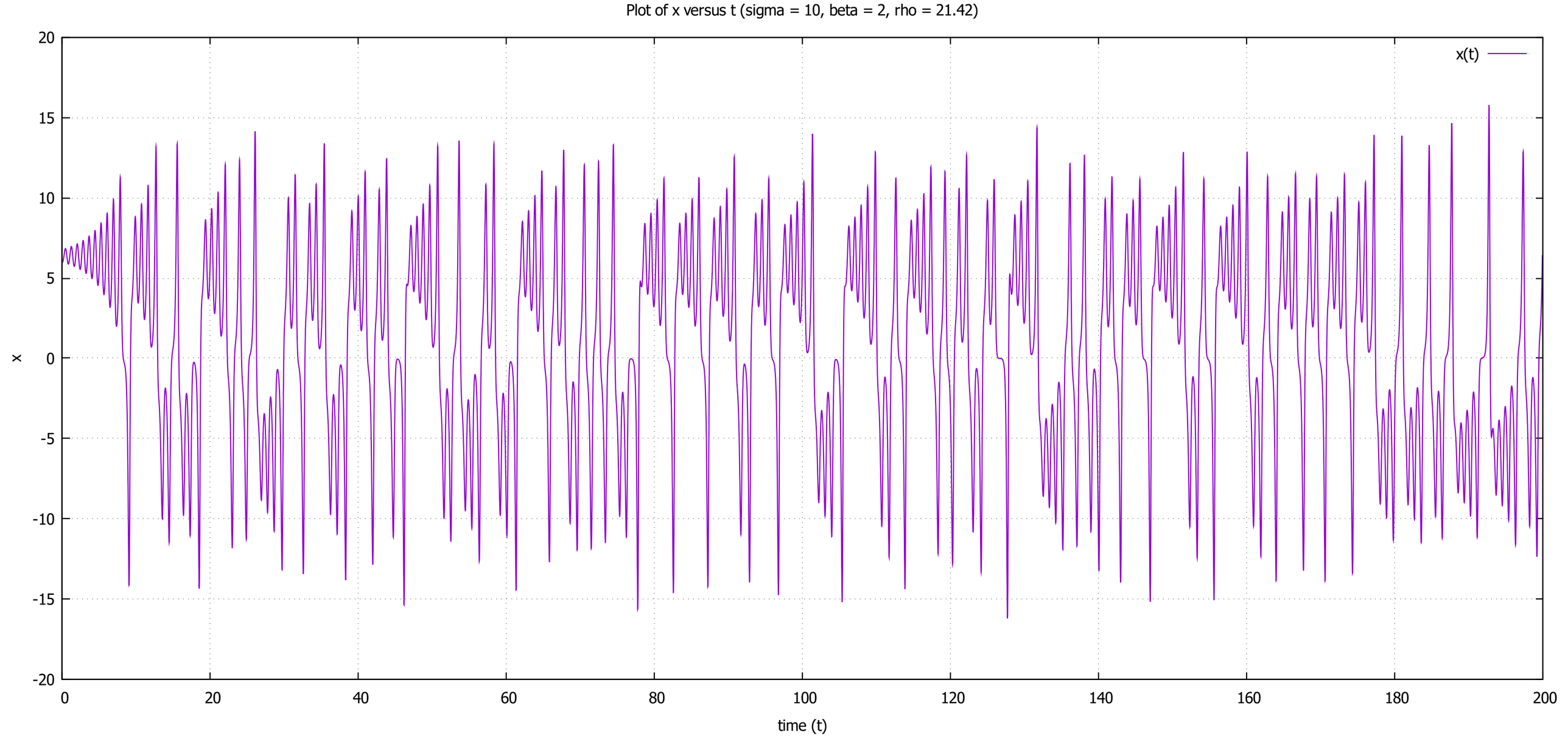

Plot of x versus t (sigma = 10, beta = 2, rho = 25)
x(t)
x
time (t)
20
15
10
5
0
-5
-10
-15
-20
0
20
40
60
80
100
120
140
160
180
200

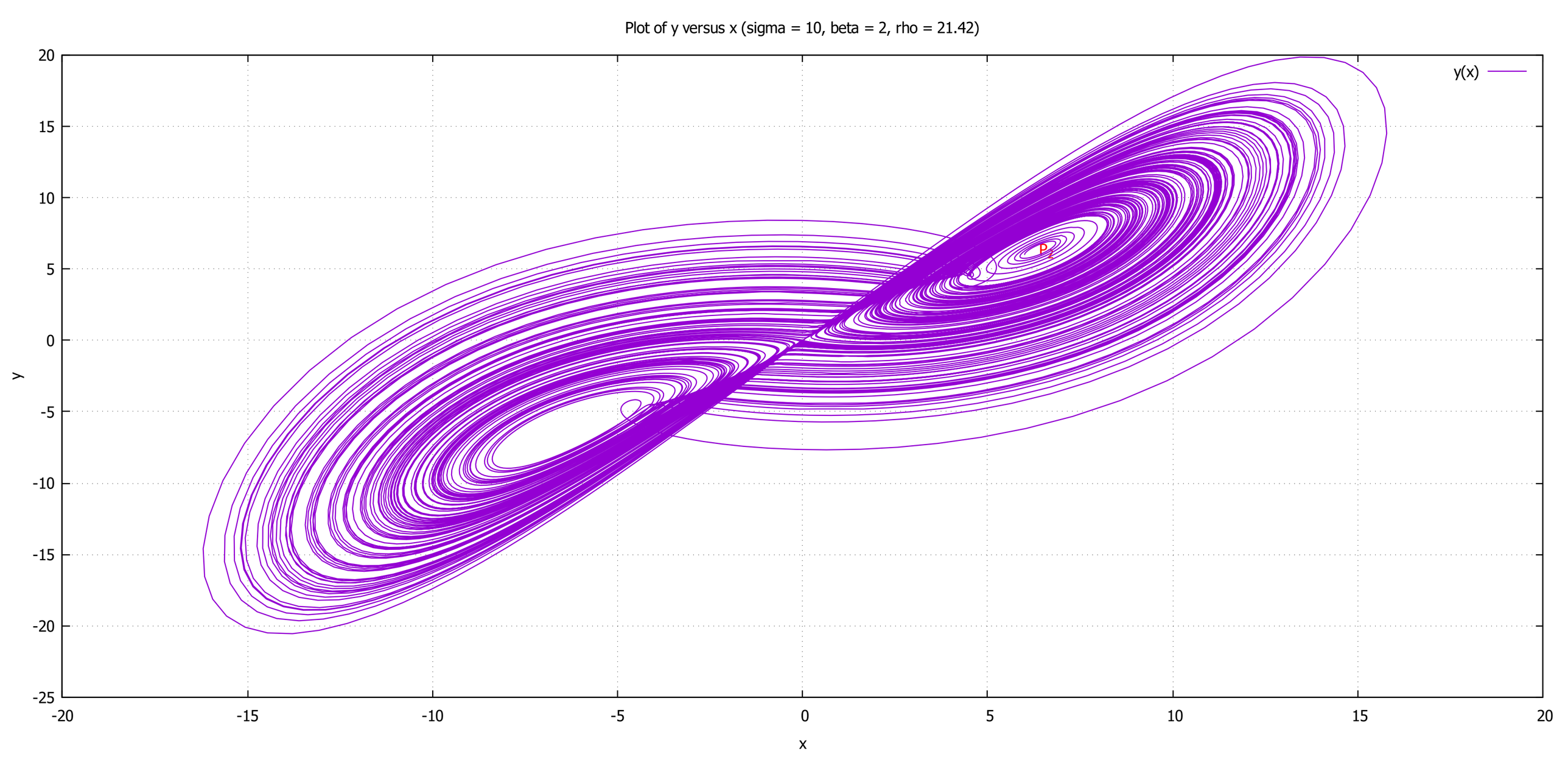

Plot of y versus x (sigma = 10, beta = 2, rho = 25)
y(x)
P2
y
x
25
20
15
10
5
0
-5
-10
-15
-20
-25
-20
-15
-10
-5
0
5
10
15
20

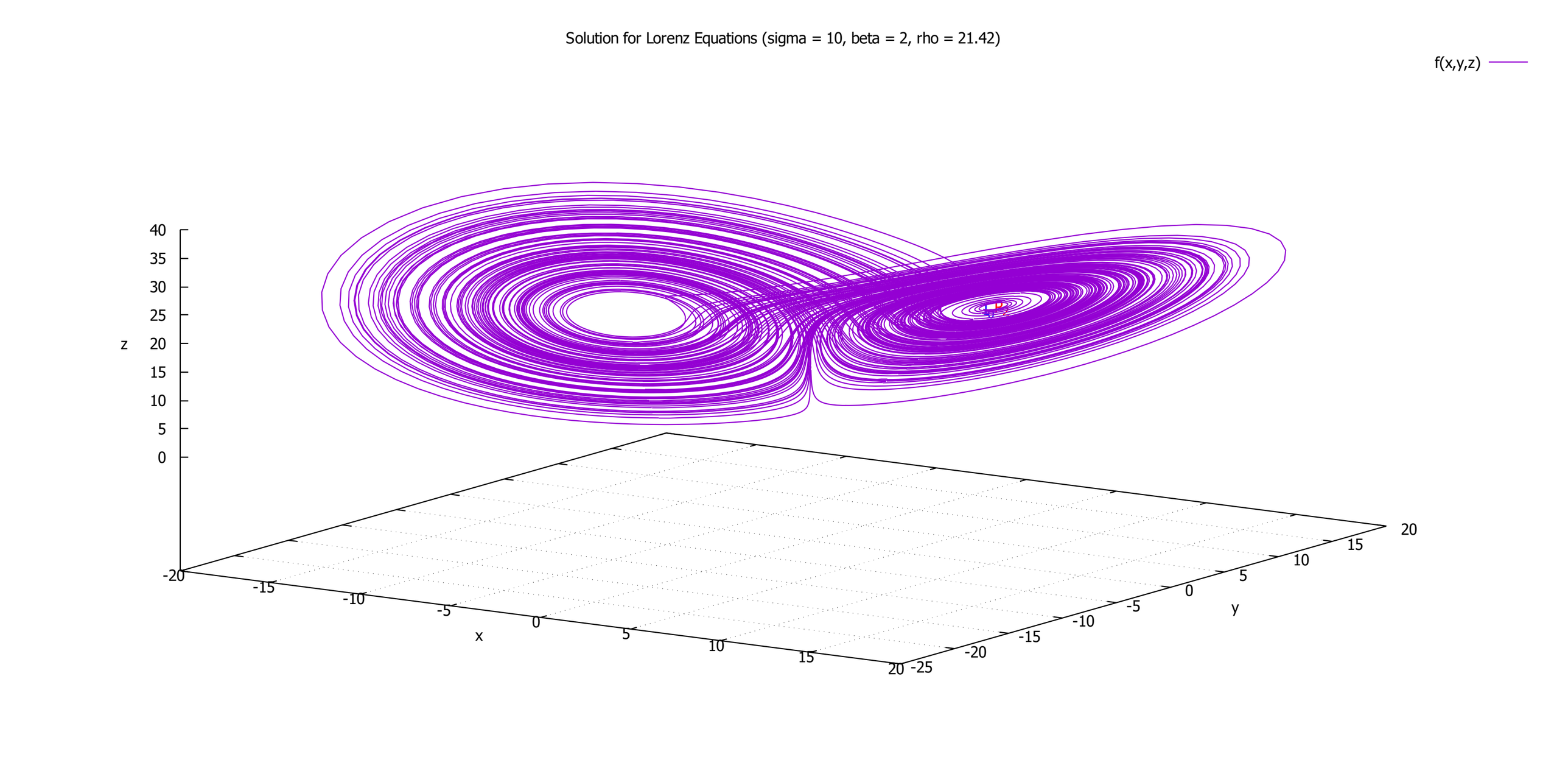

Solution for Lorenz Equations (sigma = 10, beta = 2, rho = 25)
f(x,y,z)
P2
z
45
40
35
30
25
20
15
10
5
0
-20
-15
-10
-5
0
5
10
15
20
x
-25
-20
-15
-10
-5
0
5
10
15
20
25
y

6 For a point near P_2 ($\rho = \rho_0$)

Following are the plots when initial point is $I_0 \equiv (6, 6, 20)$ and $\sigma = 10, \beta = 2, \rho = 21.42$. Here $P_2 \equiv (6.39, 6.39, 20.42)$ and $\rho_0 = 21.42$

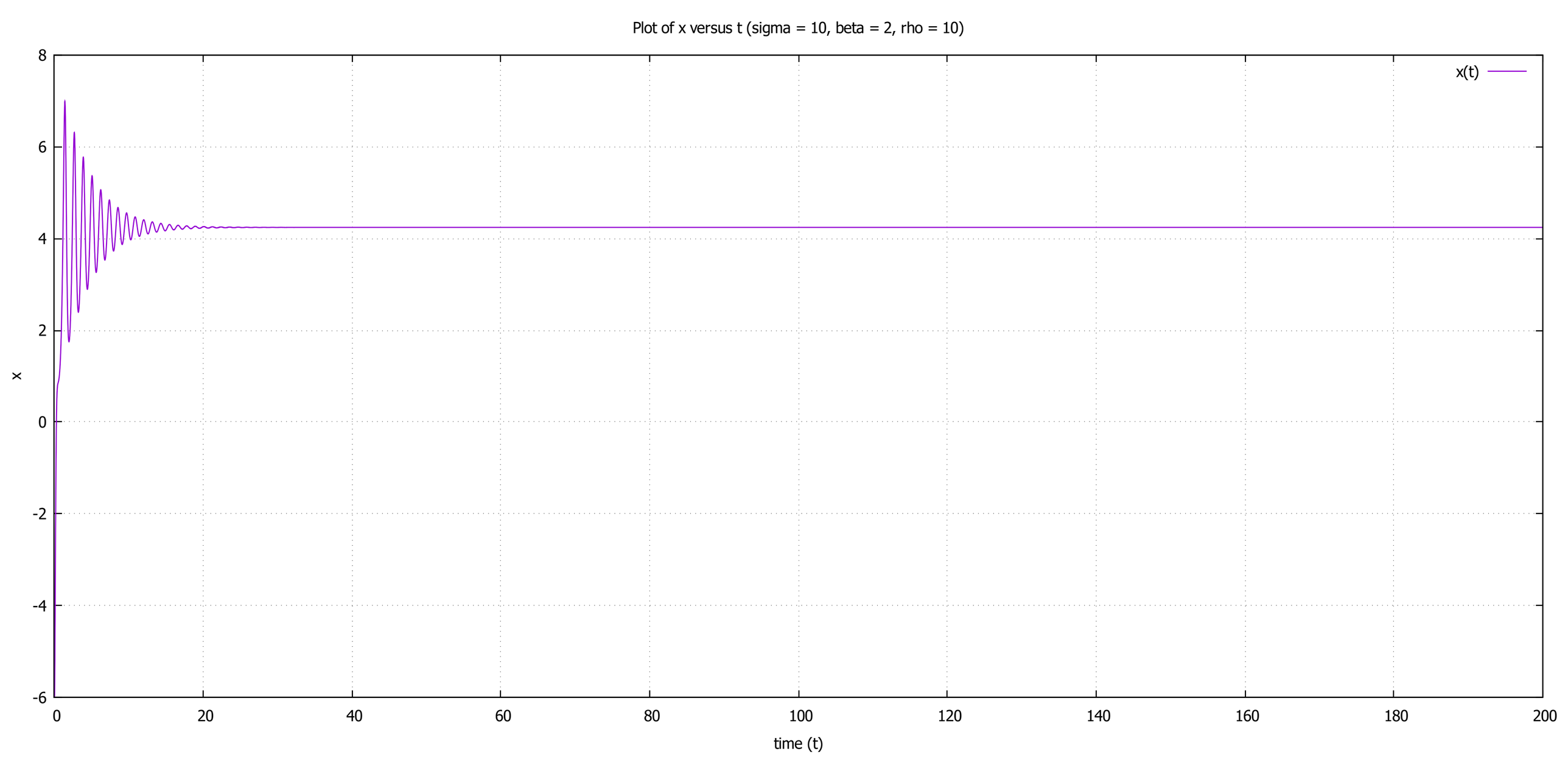

Plot of x versus t (sigma = 10, beta = 2, rho = 21.42)
x(t)
x
time (t)
20
15
10
5
0
-5
-10
-15
-20
0
20
40
60
80
100
120
140
160
180
200

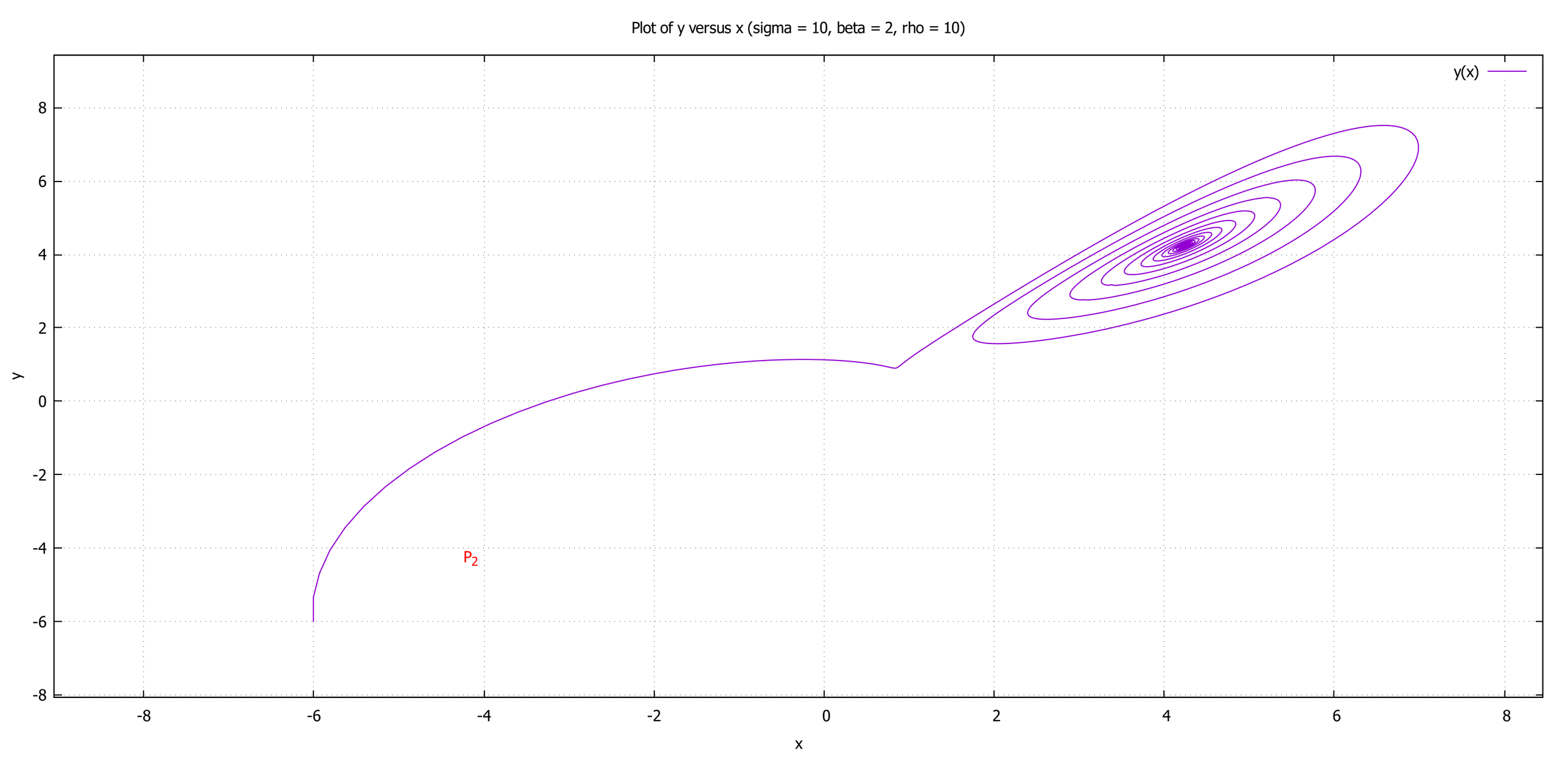

Plot of y versus x (sigma = 10, beta = 2, rho = 21.42)
y(x)
P2
20
15
10
5
0
-5
-10
-15
-20
-25
y
-20
-15
-10
-5
0
5
10
15
20
x

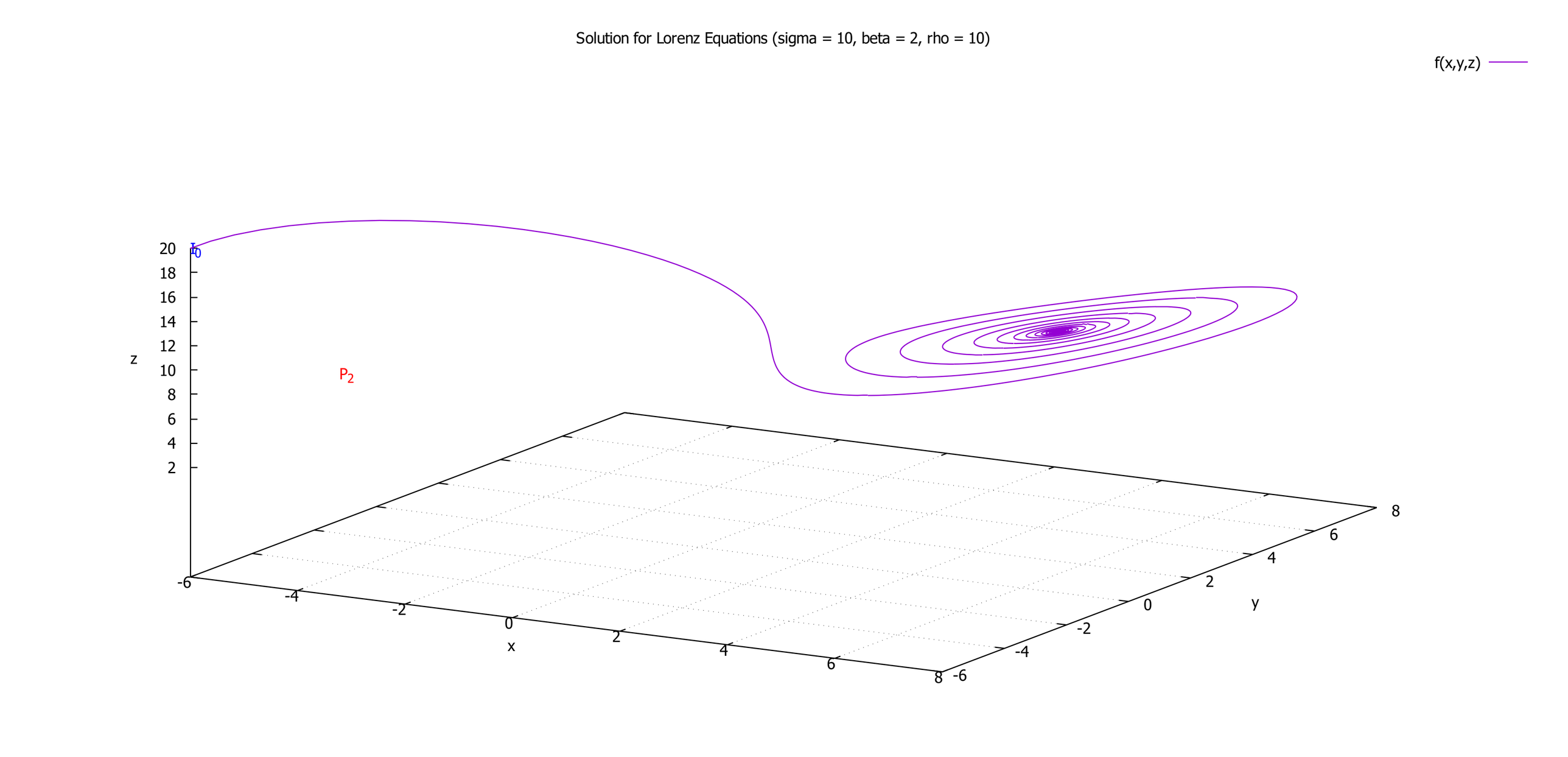

Solution for Lorenz Equations (sigma = 10, beta = 2, rho = 21.42)
f(x,y,z)
z
40
35
30
25
20
15
10
5
0
-20
-15
-10
-5
0
5
10
15
20
x
-25
-20
-15
-10
-5
0
5
10
15
20
y

7 For a point near P_3 ($\rho < \rho_0$)

Following are the plots when initial point is $I_0 \equiv (-6, -6, 20)$ and $\sigma = 10, \beta = 2, \rho = 10$. Here $P_3 \equiv (-4.24, -4.24, 9)$ and $\rho_0 = 21.42$

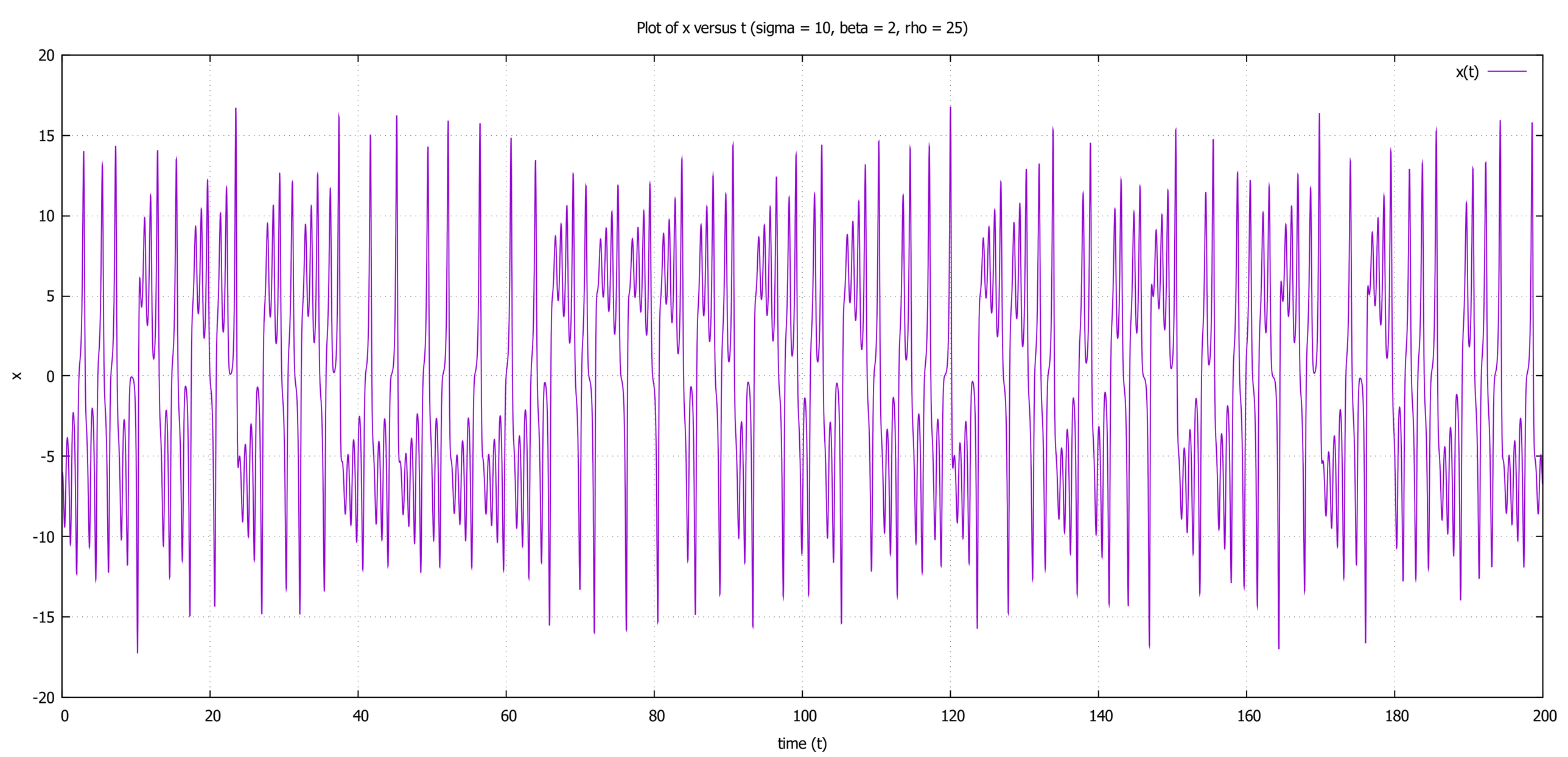

Plot of x versus t (sigma = 10, beta = 2, rho = 10)
x(t)
x
time (t)
8
6
4
2
0
-2
-4
-6
0
20
40
60
80
100
120
140
160
180
200

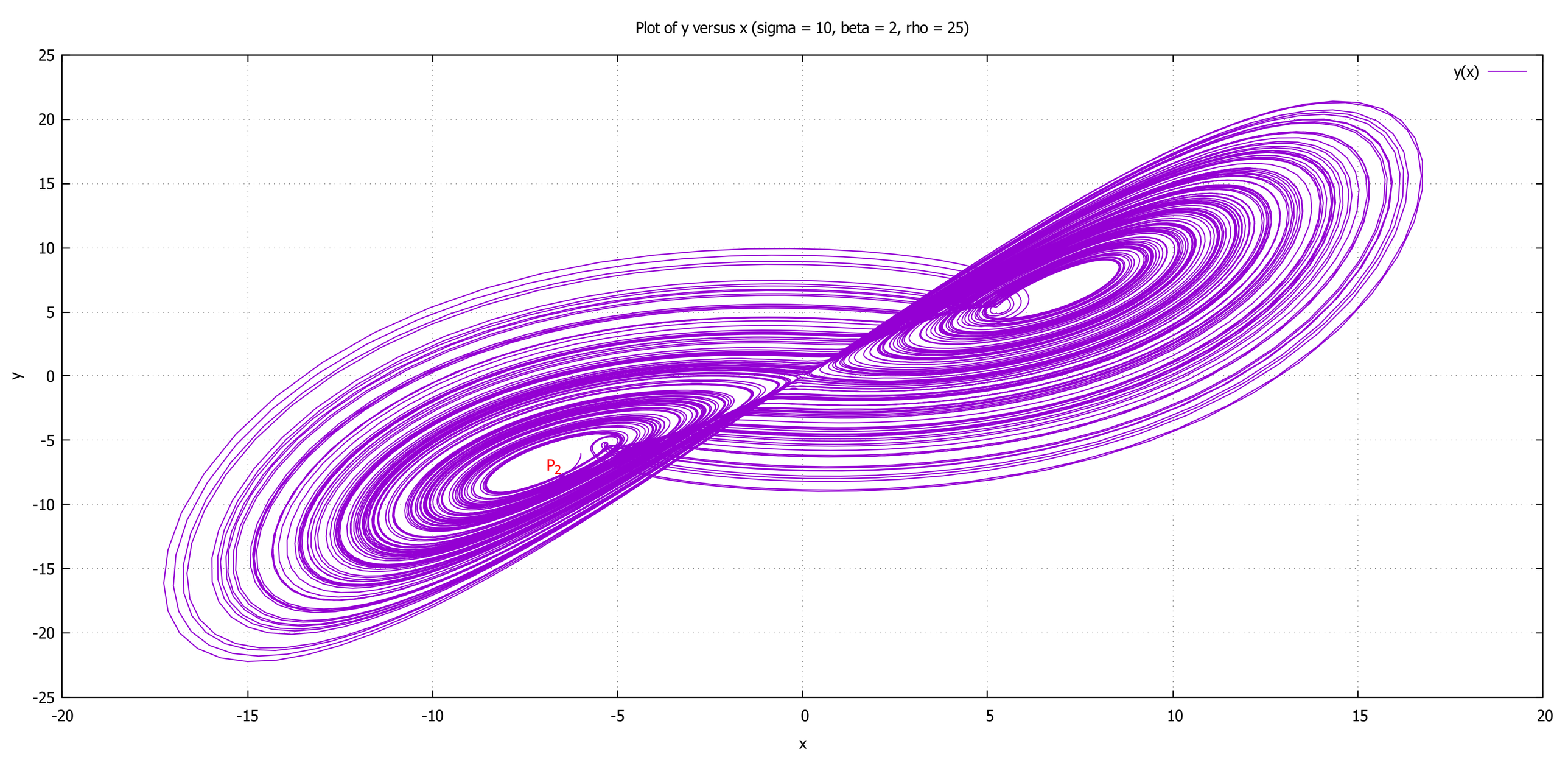

Plot of y versus x (sigma = 10, beta = 2, rho = 10)
y(x)
P2
x
y
-8
-6
-4
-2
0
2
4
6
8

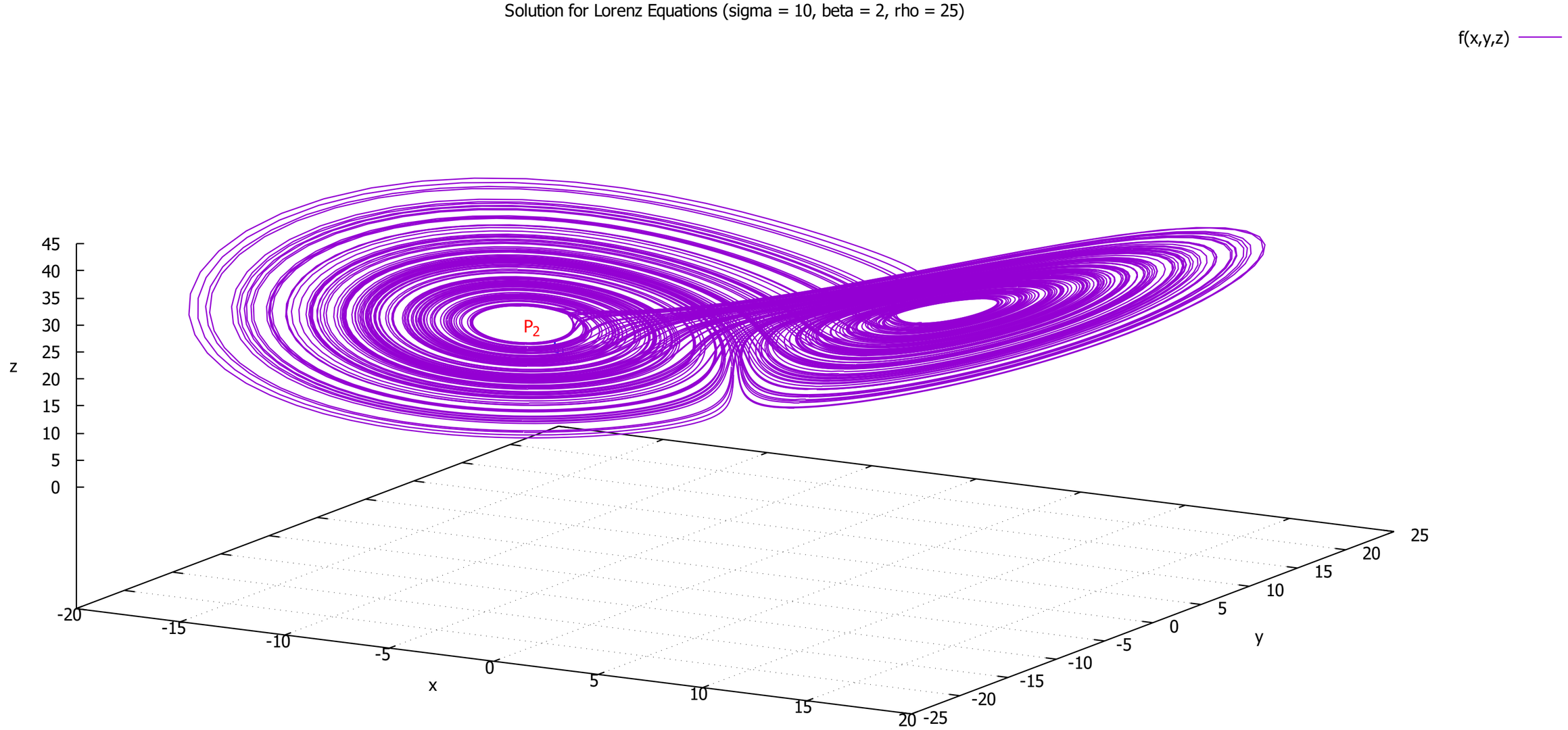

Solution for Lorenz Equations (sigma = 10, beta = 2, rho = 10)
f(x,y,z)
z
x
y
P0
P2
20
18
16
14
12
10
8
6
4
2
-6
-4
-2
0
2
4
6
8

8 For a point near P_3 ($\rho > \rho_0$)

Following are the plots when initial point is $I_0 \equiv (-6, -6, 20)$ and $\sigma = 10, \beta = 2, \rho = 25$. Here $P_3 \equiv (-6.92, -6.92, 24)$ and $\rho_0 = 21.42$

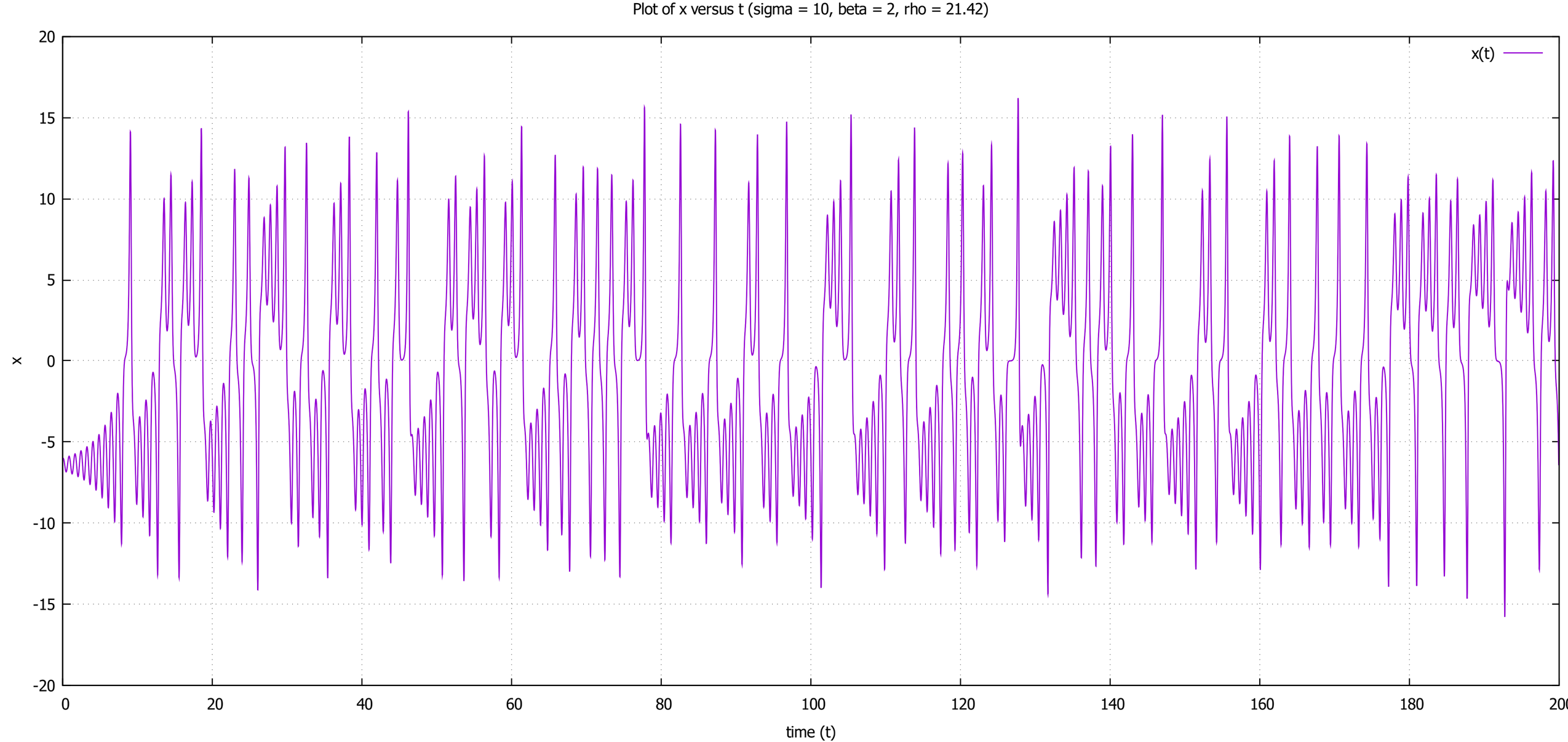

Plot of x versus t (sigma = 10, beta = 2, rho = 25)
x(t)
x
time (t)
20
15
10
5
0
-5
-10
-15
-20
0
20
40
60
80
100
120
140
160
180
200

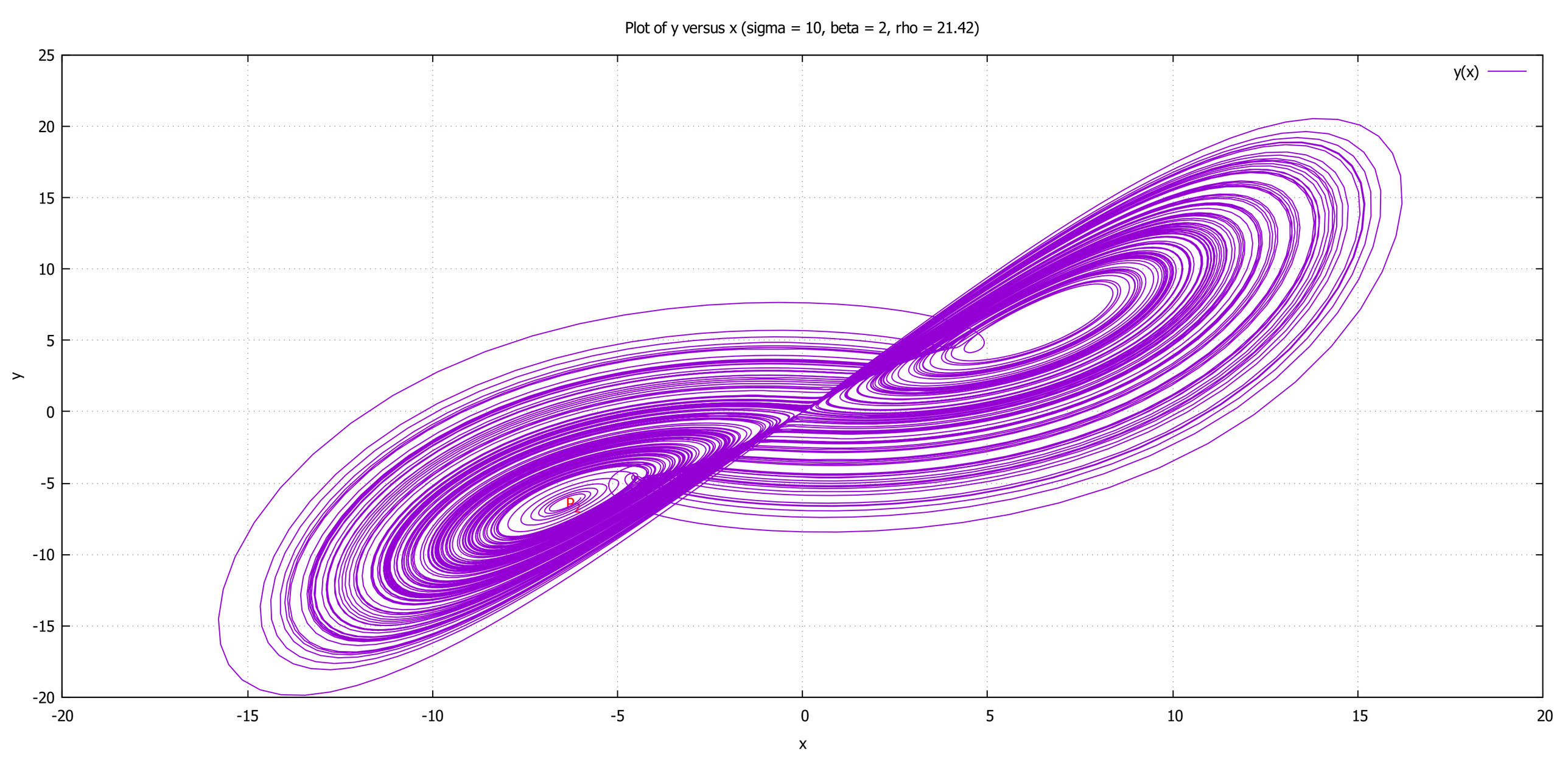

Plot of y versus x (sigma = 10, beta = 2, rho = 25)
y(x)
P2
25
20
15
10
5
0
-5
-10
-15
-20
-25
-20
-15
-10
-5
0
5
10
15
20
y
x

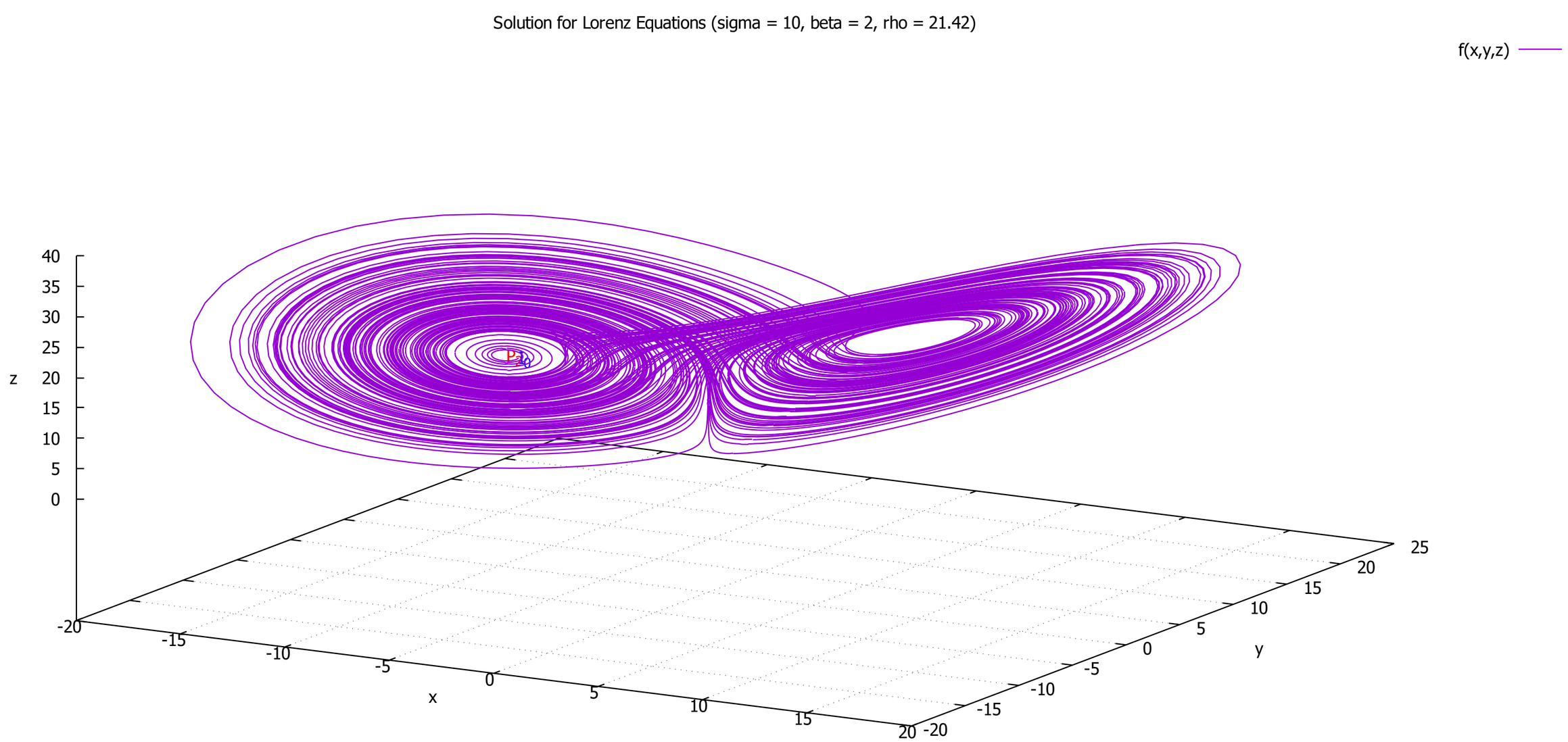

Solution for Lorenz Equations (sigma = 10, beta = 2, rho = 25)
f(x,y,z)
P2
z
45
40
35
30
25
20
15
10
5
0
-20
-15
-10
-5
0
5
10
15
20
x
-25
-20
-15
-10
-5
0
5
10
15
20
25
y

9 For a point near P_3 ($\rho = \rho_0$)

Following are the plots when initial point is $I_0 \equiv (-6, -6, 20)$ and $\sigma = 10, \beta = 2, \rho = 21.42$. Here $P_3 \equiv (-6.39, -6.39, 20.42)$ and $\rho_0 = 21.42$

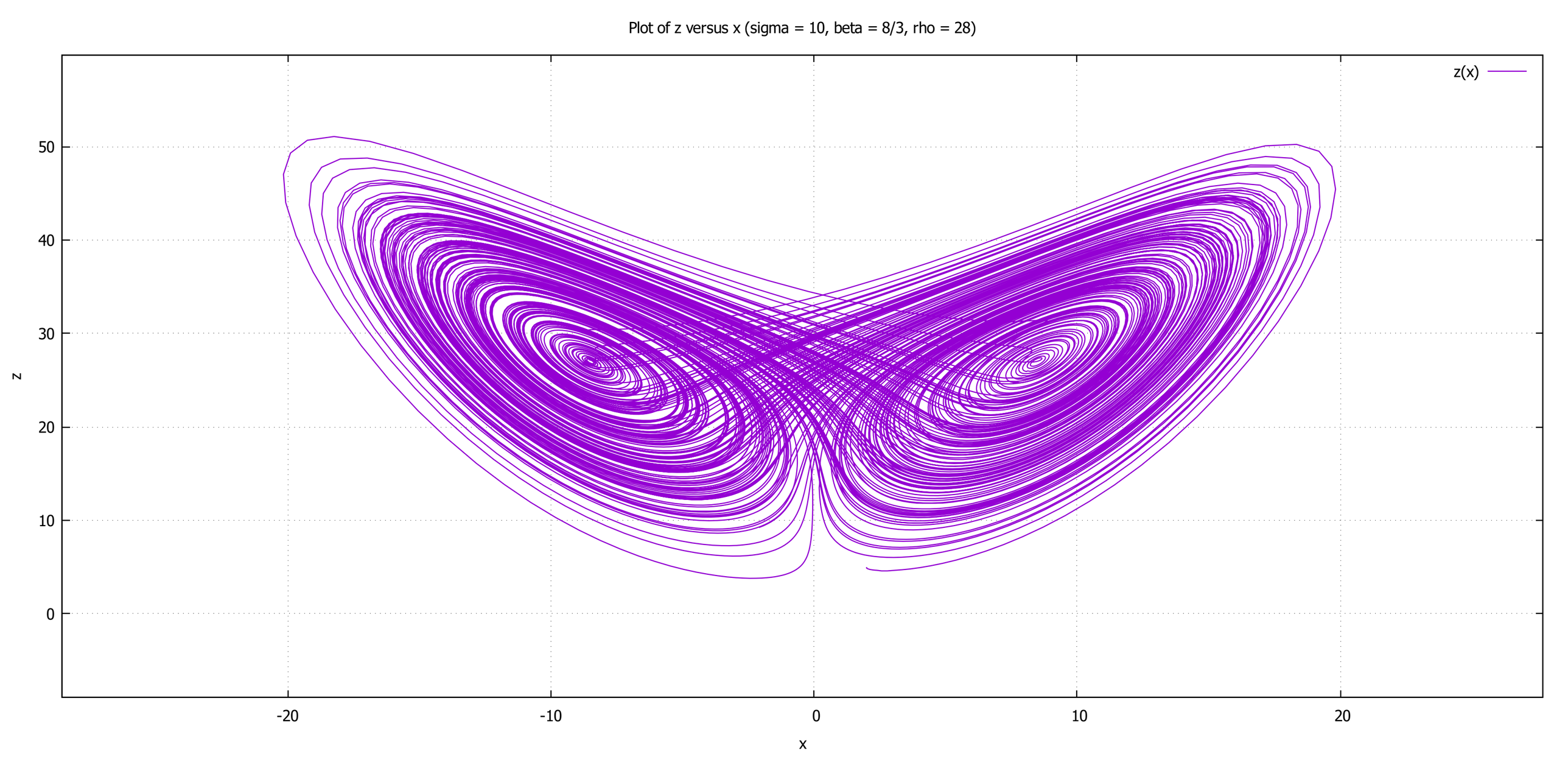

Plot of x versus t (sigma = 10, beta = 2, rho = 21.42)
x(t)
x
time (t)
20
15
10
5
0
-5
-10
-15
-20
0
20
40
60
80
100
120
140
160
180
200

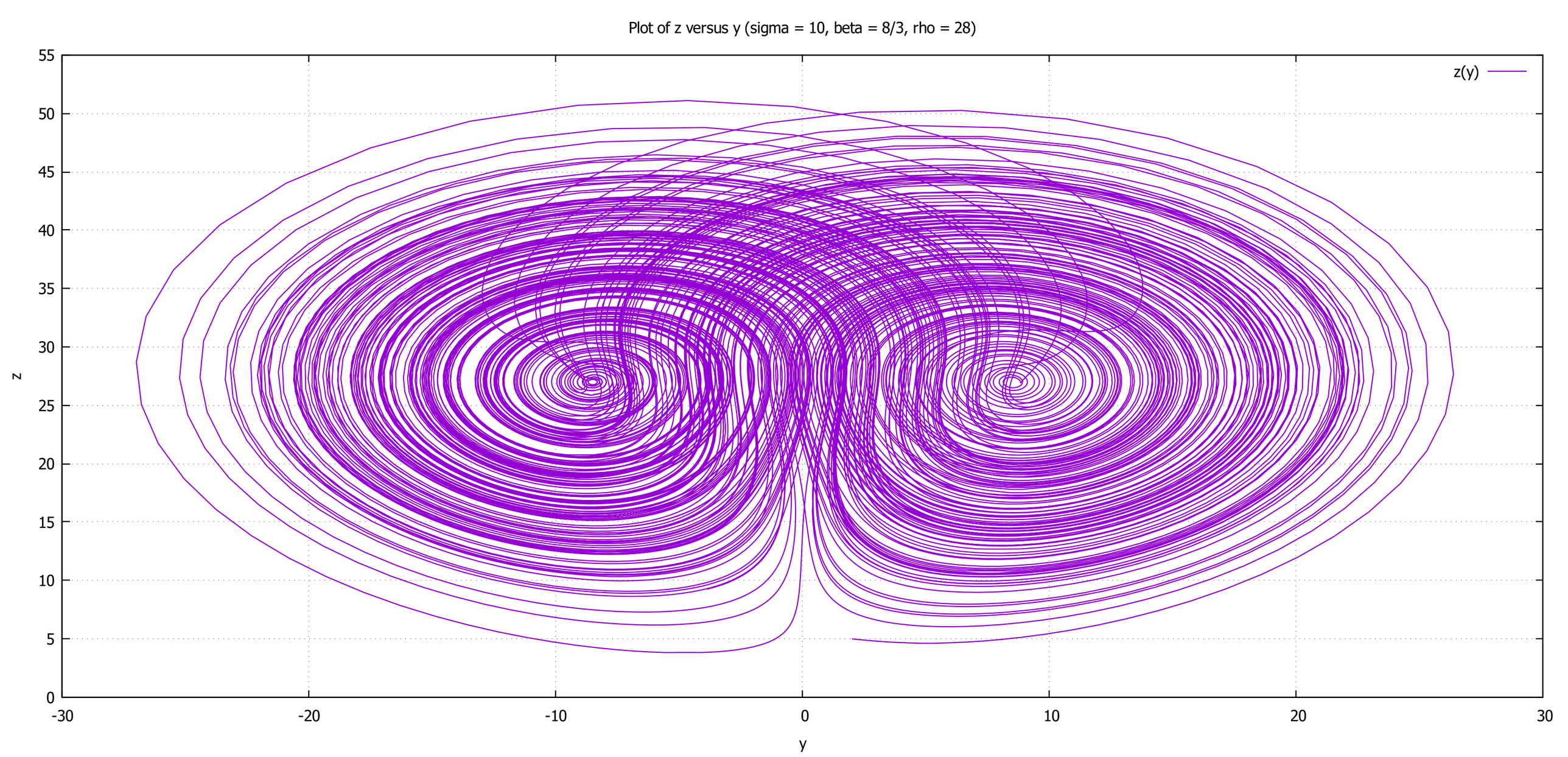

Plot of y versus x (sigma = 10, beta = 2, rho = 21.42)
y(x)
x
y
25
20
15
10
5
0
-5
-10
-15
-20
-20
-15
-10
-5
0
5
10
15
20

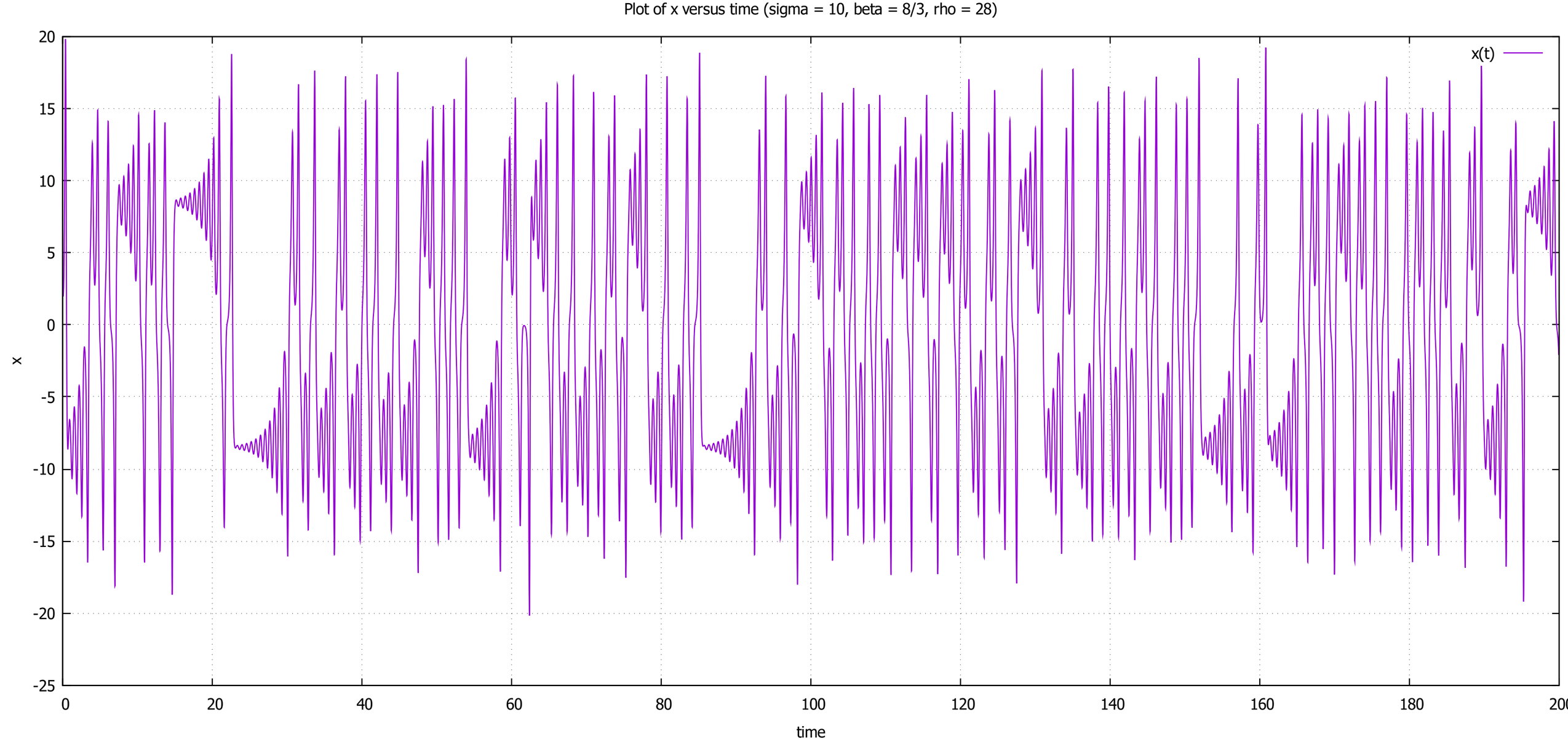

Solution for Lorenz Equations (sigma = 10, beta = 2, rho = 21.42)
f(x,y,z)
z
40
35
30
25
20
15
10
5
0
x
-20
-15
-10
-5
0
5
10
15
20
y
-20
-15
-10
-5
0
5
10
15
20
25

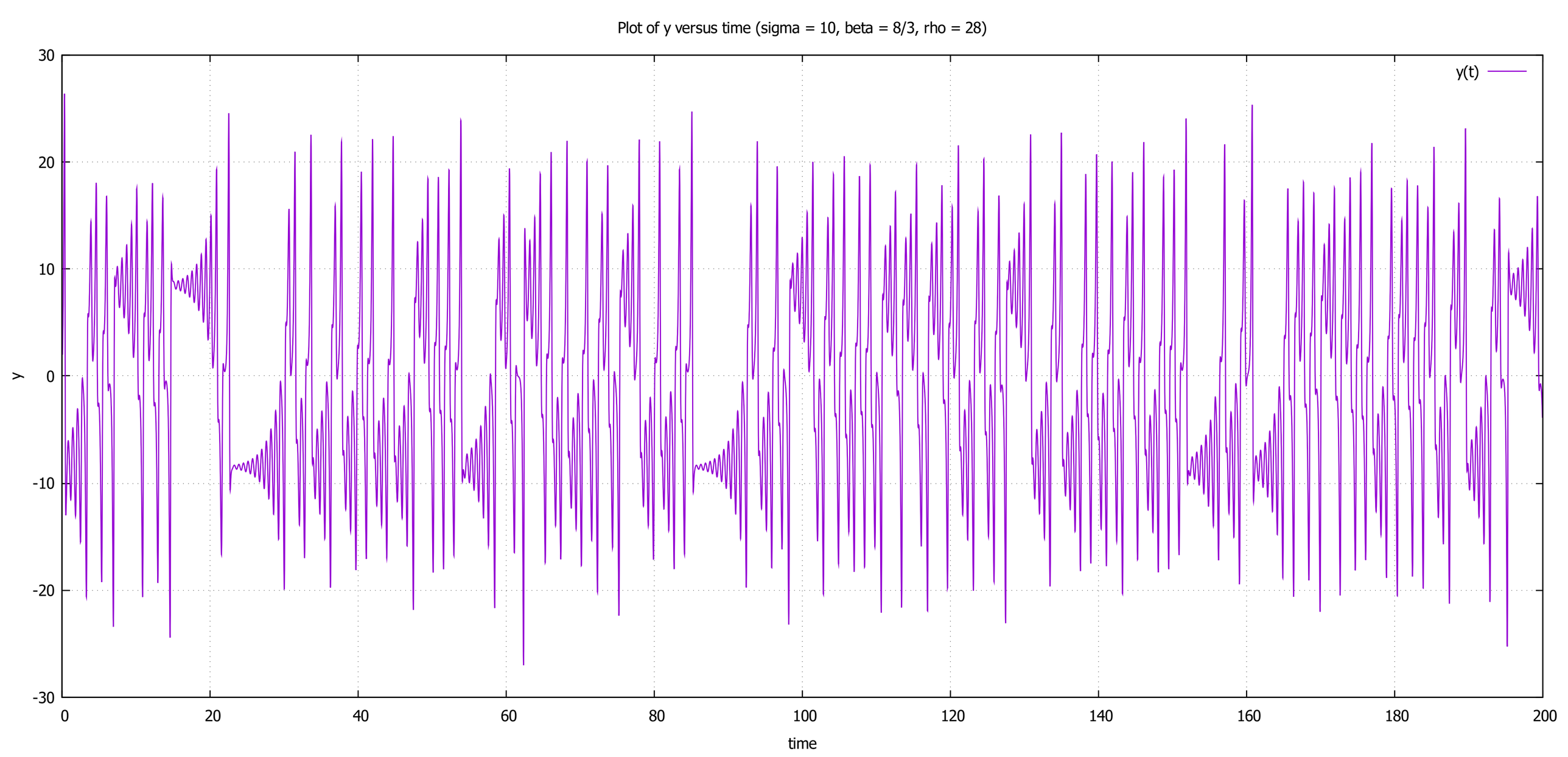
10 For $\sigma = 10, \beta = 8/3, \rho = 28$

Following are the plots when initial point is $I_0 \equiv (2, 2, 5)$ and $\sigma = 10, \beta = 8/3, \rho = 28$.


Plot of z versus x (sigma = 10, beta = 8/3, rho = 28)
z(x)
50
40
30
20
10
0
z
-20
-10
0
10
20
x


Plot of z versus y (sigma = 10, beta = 8/3, rho = 28)
z(y)
z
y
55
50
45
40
35
30
25
20
15
10
5
0
-30
-20
-10
0
10
20
30


Plot of x versus time (sigma = 10, beta = 8/3, rho = 28)
x(t)
x
time
20
15
10
5
0
-5
-10
-15
-20
-25
0
20
40
60
80
100
120
140
160
180
200


Plot of y versus time (sigma = 10, beta = 8/3, rho = 28)
y(t)
y
time
30
20
10
0
-10
-20
-30
0
20
40
60
80
100
120
140
160
180
200