

COMPOSITE WESENHEIT BASED GALACTIC BVIJHK LEAVITT LAW CALIBRATION

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Abstract

From the definition of the total-to-selective absorption ratio, composite Wesenheit magnitudes are derived for *BVIJHK* photometry of Galactic Cepheids. Fouque's extinction law (A_λ^V) adjusted to achieve near zero deviation between apparent and true wesenheit magnitudes for all the pair combinations of color indexes when $R_V^{BV} = 3.23$. The adjusted extinction law is used for determining *BVIJHK* extinctions for 94 Galactic Cepheids with available Fernie reddening and Gaia parallax. Metallicity effects are not considered in this study.

Method: Initially, *IJHK* PeriodLuminosity (PL) residuals are correlated with the corresponding PeriodWesenheit (PW) residuals to resolve the degeneracy between band-dependent extinction and distance uncertainties. A star-by-star adjustment of the distance modulus performed to minimize reddening dispersion across bands, yielding a first order distance - reddening correction pair for each individual Cepheid. ($J - K$) based residual correlation provides an accurate estimation of modulus correction. After implementing the first - order corrections in the input data, the procedure is repeated for the *BVIJ* bands to obtain precise reddening corrections using the Wesenheit color ($V - I$). The final corrections are then applied to derive the *BVIJHK* Leavitt Laws and verified with 17 Cluster Cepheids.

Result: Distance - Reddening - Luminosity calibrated V and K bands Galactic Leavitt Law:

$$V_{\text{Gaia}}(\sigma : 0.029) = (-2.268 \pm 0.01) (\log P - 1) + (-3.953 \pm 0.003)$$

$$K_{\text{Gaia}}(\sigma : 0.011) = (-2.971 \pm 0.004) (\log P - 1) + (-5.660 \pm 0.001)$$

The distance moduli to the Magellanic Clouds, estimated from *VIJK* photometry, are found to be 18.365 ± 0.110 mag for the Large Magellanic Cloud (LMC) and 19.068 ± 0.038 mag for the Small Magellanic Cloud (SMC).

Keywords: Interstellar Extinction Law: Wesenheit Magnitude: Galactic Cepheid: Leavitt Law:

Symbols and their descriptions

Symbol	Description	Symbol	Description
m_λ	Apparent Magnitude	α_λ	Period Luminosity Slope
μ	Distance Modulus	γ_λ	Period Luminosity Intercept
M_λ	Absolute Magnitude	ΔW_λ^{12}	Period Wesenheit Residuals
E_{BV}	Interstellar Reddening	α_λ^{12}	Period Wesenheit Slope
A_λ	Interstellar Extinction	γ_λ^{12}	Period Wesenheit Intercept
R_λ^{12}	Total-to-selective Absorption	$\rho_{\kappa\lambda}^{12}$	Residual Correlation Slope
M_λ^0	True Absolute Magnitude	$\Delta_{\kappa\lambda}^{12}$	PL-PW Correlation Residuals
W_λ^{12}	Wesenheit Magnitude	$\delta A_{\kappa\lambda}^{12}$	Extinction Correction
ΔM_λ	Period Luminosity Residuals	$\delta E_{\kappa\lambda}^{12}$	Reddening Correction

1. Definitions

1.1 Distance, Reddening and Luminosity

Luminosity represents the measured photon flux received within a specific wavelength band, here corresponding to the *BVIJHK* bands. The light emitted by a star, characterized by its intrinsic absolute magnitude M_λ^0 , travels over large distances μ and undergoes extinction A_λ as it passes through the interstellar medium (ISM) before reaching the observers detector with the observed intensity m_λ .

In this framework, the relation for the observed apparent magnitude can be expressed as:

$$m_\lambda = \mu + A_\lambda + M_\lambda^0 \quad (1)$$

where m_λ is the apparent magnitude in a given band λ , M_λ^0 is the intrinsic absolute magnitude, μ is the distance modulus, and A_λ is the interstellar extinction in that band.

On the right-hand side, the first term represents the distance modulus, μ . For computational convenience, distances measured in parsecs are converted into dimensionless logarithmic units, expressed in magnitudes. The distances for the 94 Galactic Cepheids in the dataset are determined using two independent methods: (i) the Infrared Surface Brightness (IRSB) method **storm2011**<empty citation>, and (ii) parallax measurements from *Gaia* DR3 **GaiaDR32023**<empty citation>. In this report, calibration is done for *Gaia* distances.

$$\mu = 5 \log D[pc] - 5$$

The interstellar medium (ISM) scatters, absorbs, and emits photons depending on the chemical composition, size, and abundance of its constituent particles, thereby imprinting characteristic signatures on the light spectrum at specific frequencies (or wavelengths). The reduction in light intensity within a given band due to these interactions with the ISM is referred to as *interstellar extinction*.

$$A_\lambda = m_\lambda - m_\lambda^0$$

The wavelength-dependent extinction for each individual Cepheid is determined from the measurement of the color excess, E_{BV} , along the line of sight **fernie1995**<empty citation>. Using the visual Galactic reddening ratio, $R_V = 3.23$ **sandage2004**<empty citation>, within the framework of the Galactic extinction law, A_λ/A_V **fouque2007**<empty citation>, the interstellar extinction for the $BVIJHK$ bands can be estimated as follows:

$$A_\lambda = \frac{A_\lambda}{A_V} \times R_V \times E_{BV}$$

Here, the final factor, the color excess E_{BV} , quantifies the relative extinction difference between two photometric bands. In other words, it can also be expressed as the deviation of the observed color index $(B - V)$ from the intrinsic (or 'true') color index $(B - V)_0$:

$$\begin{aligned} E_{BV} &= (B - V) - (B - V)_0 \\ &= (B - B_0) - (V - V_0) \\ &= A_B - A_V \end{aligned}$$

The intermediate factor, the reddening ratio $R_V = A_V/E_{BV}$, represents the extinction-to-reddening ratio in the visual band. In the most general form, if the color excess between two bands is defined as $E_{12} = A_{m_1} - A_{m_2}$, then the corresponding reddening ratio can be expressed as:

$$R_\lambda^{12} = A_\lambda/E_{12}$$

Color excess from one combination of bands E_{12} can be transformed into any other bands combination as follows:

$$\begin{aligned}
E_{12} &= A_1 - A_2 \\
&= R_1^{BV} * E_{BV} - R_2^{BV} * E_{BV} \\
&= (R_1^{BV} - R_2^{BV}) * E_{BV}
\end{aligned}$$

This leads the transformation law for reddening ratio R :

$$R_\lambda^{12} = \frac{R_\lambda^{BV}}{R_1^{BV} - R_2^{BV}}$$

The values of R_λ^{BV} are calculated using extinction law as mentioned in Table 1.

$$R_\lambda^{BV} = \frac{A_\lambda}{A_V} \times R_V^{BV}$$

1.2 Wesenheit Magnitude

Since the reddening ratio, R_λ^{12} , quantifies the effect of the interstellar medium on the incoming light, it can be used to define a reddening-free magnitude corresponding to the star's true absolute magnitude **madore1982<empty citation>**. This relation can be expressed as:

$$\begin{aligned}
R_\lambda^{12} &= A_\lambda/E_{12} \\
&= \frac{m_\lambda - m_\lambda^0}{(m_1 - m_2) - (m_1 - m_2)_0}
\end{aligned}$$

On rearranging the terms, one can get the Wesenheit function W_λ^{12} corresponding to color $m_1 - m_2$.

$$m_\lambda - R_\lambda^{12}(m_1 - m_2) = m_\lambda^0 - R_\lambda^{12}(m_1 - m_2)_0 \quad (2)$$

$$W_\lambda^{12} = W_0 \quad (3)$$

This implies that, if R is accurately known, the apparent Wesenheit magnitude and the absolute Wesenheit magnitude should be identical. Here, we test the validity of this claim by comparing the deviations between the two versions of the Wesenheit magnitudes, where R is derived from the Fouque extinction law as given in Table 1.

This deviation in wesenheit $W - W_0$ would be zero, when following equality would be

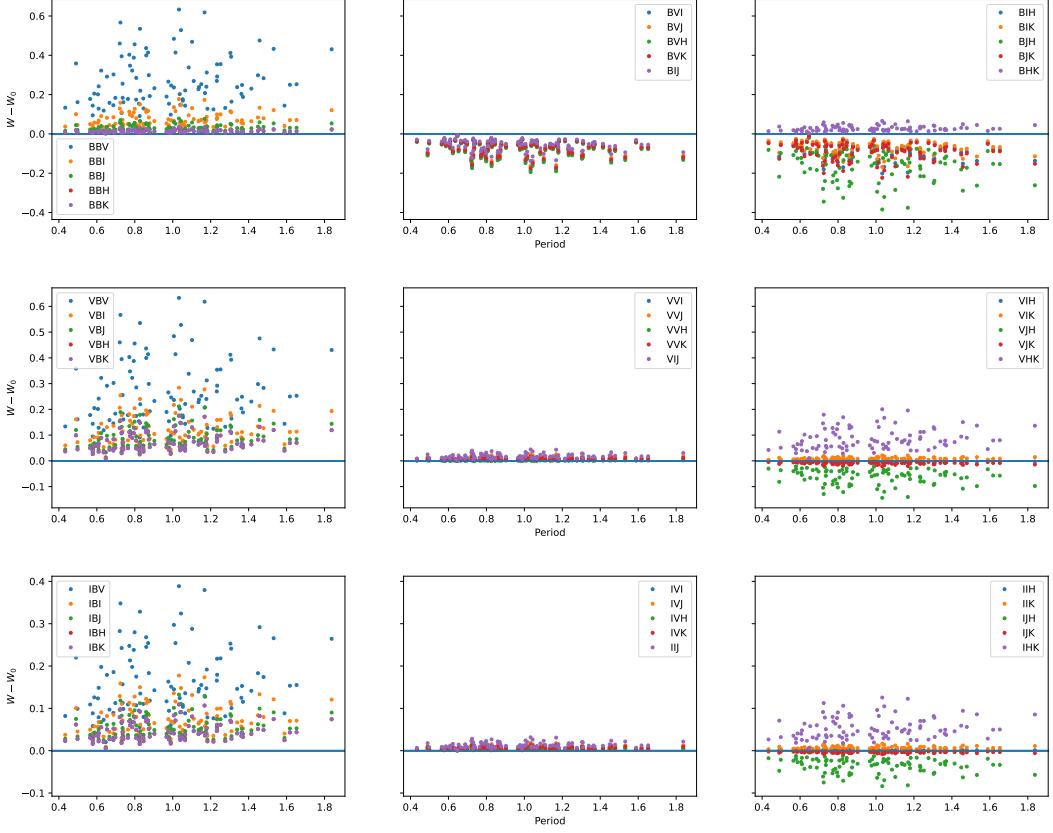


Figure 1: For *BVI* photometry, the deviation between apparent Wesenheit magnitude and absolute Wesenheit magnitude for all the color combination indicates a potential error in the reddening law.

satisfying:

$$\delta A_\lambda - R_\lambda \delta E_{BV} - \delta R E_{BV} = 0 \quad (4)$$

Considering the last term of the above equation, **fouque2007**<empty citation> extinction law have been adjusted to achieve minimum deviation in the wesenheit magnitudes.

After fine-tuning the Fouqué extinction law for our dataset, R_λ is assumed to be accurately known to the required precision, eliminating the last term in the above equation.

1.3 Error Contribution in Absolute and Wesenheit magnitude

Following are the definitions of 'true' absolute magnitude and wesenheit magnitude for color (B-V):

$$M_\lambda^0 = m_\lambda - R_\lambda^{BV} * E(B - V) - \mu \quad (5)$$

$$W_\lambda^{BV} = m_\lambda - R_\lambda^{BV} * (B - V) - \mu \quad (6)$$

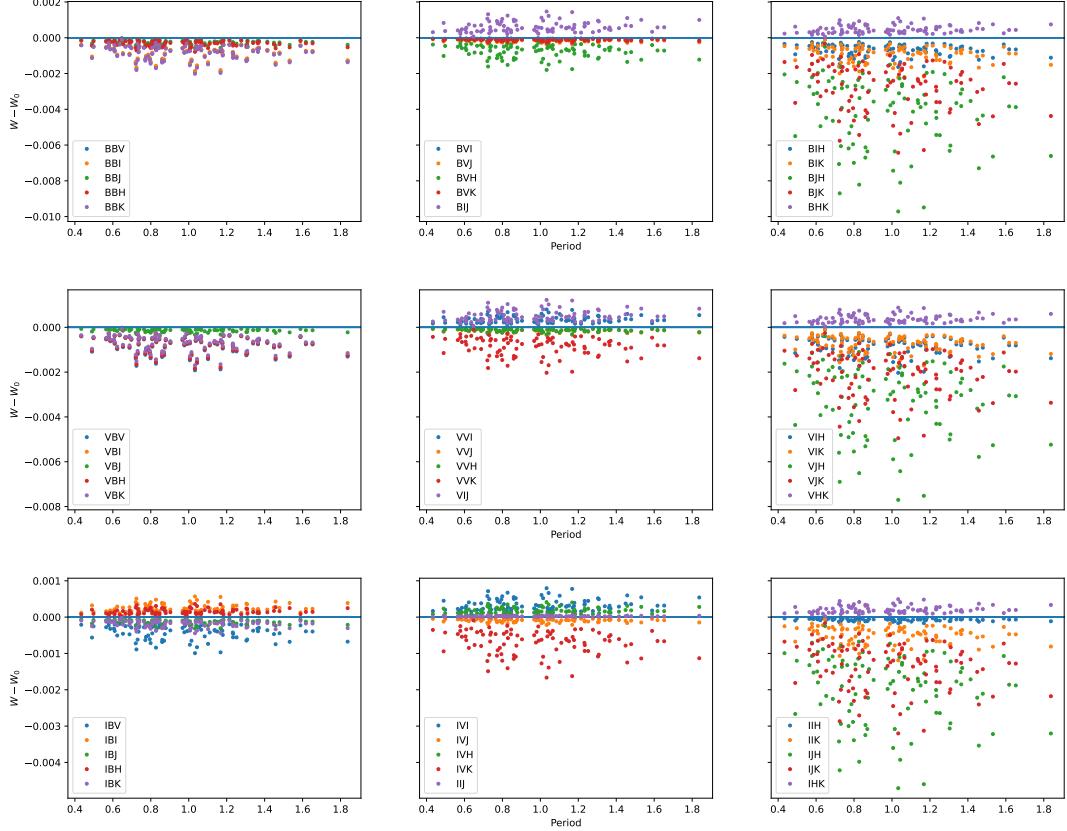


Figure 2: With the adjusted Fouqué extinction law, the deviation between the apparent and absolute Wesenheit magnitudes is reduced by a factor of ten.

Differentiating above equations.

$$\begin{aligned}\delta M_\lambda^0 &= \delta m_\lambda - \delta R_\lambda^{BV} * E(B - V) - R_\lambda^{BV} * \delta E(B - V) - \delta \mu \\ \delta W_\lambda^{BV} &= \delta m_\lambda - \delta R_\lambda^{BV} * (B - V) - R_\lambda^{BV} * \delta(B - V) - \delta \mu\end{aligned}$$

Assume that the apparent luminosities are measured precisely ($\delta m_\lambda \rightarrow 0$) and that the adjusted extinction law is accurate ($\delta R \rightarrow 0$). Even so, the distance modulus ($\delta \mu$) and the color excess (δE_{BV}) may still contain errors for individual Cepheids. Under these conditions, it follows that:

$$\delta M_\lambda = -(R_\lambda^{BV} * \delta E(B - V) + \delta \mu) \quad (7)$$

$$\delta W_\lambda^{BV} = -\delta \mu \quad (8)$$

This pair of equations indicates that both the Wesenheit magnitude and the absolute magnitude are sensitive to errors in distance. However, unlike the absolute magnitude, the Wesenheit magnitude is insensitive to errors in reddening. This key property of the Wesenheit magnitude is particularly useful for decoupling the error budget between distance and reddening when applied to Cepheids.

Table 1: Reddening Ratio metrics for composite photometry.

Extinction Law fouque2007<empty citation>											
A_B^V	1.31	A_V^V	1.0	A_I^V	0.608	A_J^V	0.292	A_H^V	0.181	A_K^V	0.119
Adjusted Extinction Law											
A_B^V	1.2574	A_V^V	1.0	A_I^V	0.609	A_J^V	0.2967	A_H^V	0.1816	A_K^V	0.1231
Reddening Ratio calculated using $R_V^{BV} = 3.23$											
R_B^{12}	R_V^{12}	R_I^{12}	R_J^{12}	R_H^{12}	R_K^{12}						
BBV	4.887	VBV	3.887	IBV	2.367	JBV	1.153	HBV	0.706	KBV	0.478
BBI	1.94	VBI	1.543	IBI	0.939	JBI	0.458	HBI	0.28	KBI	0.19
BBJ	1.309	VBJ	1.041	IBJ	0.634	JBJ	0.309	HBJ	0.189	KBJ	0.128
BBH	1.169	VBH	0.93	IBH	0.566	JBH	0.276	HBH	0.169	KBH	0.114
BBK	1.108	VBK	0.882	IBK	0.537	JBK	0.262	HBK	0.16	KBK	0.108
BVI	3.216	VVI	2.557	IVI	1.557	JVI	0.759	HVI	0.464	KVI	0.315
BVJ	1.788	VVJ	1.422	IVJ	0.866	JVJ	0.422	HVJ	0.258	KVJ	0.175
BVH	1.537	VVH	1.222	IVH	0.744	JVH	0.363	HVH	0.222	KVH	0.15
BVK	1.434	VVK	1.14	IVK	0.694	JVK	0.338	HVK	0.207	KVK	0.14
BIJ	4.025	VIJ	3.201	IIJ	1.95	JIJ	0.95	HIJ	0.581	KIJ	0.394
BIH	2.943	VIH	2.341	IIH	1.425	JIH	0.694	HIH	0.425	KIH	0.288
BIK	2.587	VIK	2.057	IIK	1.253	JIK	0.61	HIK	0.374	KIK	0.253
BJH	10.947	VJH	8.706	IJH	5.302	JJH	2.583	HJH	1.581	KJH	1.071
BJK	7.24	VJK	5.758	IJK	3.506	JJK	1.708	HJK	1.046	KJK	0.708
BHK	21.376	VHK	17.0	IHK	10.353	JHK	5.044	HHK	3.087	KHK	2.091
LMC Extinction Law with $R_V = 3.4$ wang2023<empty citation>											
A_B^V	1.32	A_V^V	1.0	A_I^V	0.65	A_J^V	0.30	A_H^V	0.20	A_K^V	0.15
SMC Extinction Law with $R_V = 2.53$											
A_B^V	1.40	A_V^V	1.0	A_I^V	0.7	A_J^V	0.38	A_H^V	0.28	A_K^V	0.2

2. Galactic Cepheids: Leavitt Law

Classical Cepheids exhibit a linear relationship between their pulsation period ($\log P$) and their true luminosity (M_λ^0), which can be modeled as:

$$\bar{M}_\lambda = \alpha_\lambda(\log P) + \gamma_\lambda \quad (9)$$

This linear regression equation provides the absolute magnitude (\bar{M}_λ) of any Cepheid from its pulsation period. Our dataset includes $BVIJHK$ photometry of Galactic Cepheids, resulting in six Leavitt Laws, each modeled by two parameters: the slope (α_λ) and the intercept (γ_λ).

The deviation of the observed luminosity (M_λ^0) from the modeled value (\bar{M}_λ) is denoted as ΔM_λ .

$$\Delta M_\lambda = M_\lambda^0 - \bar{M}_\lambda$$

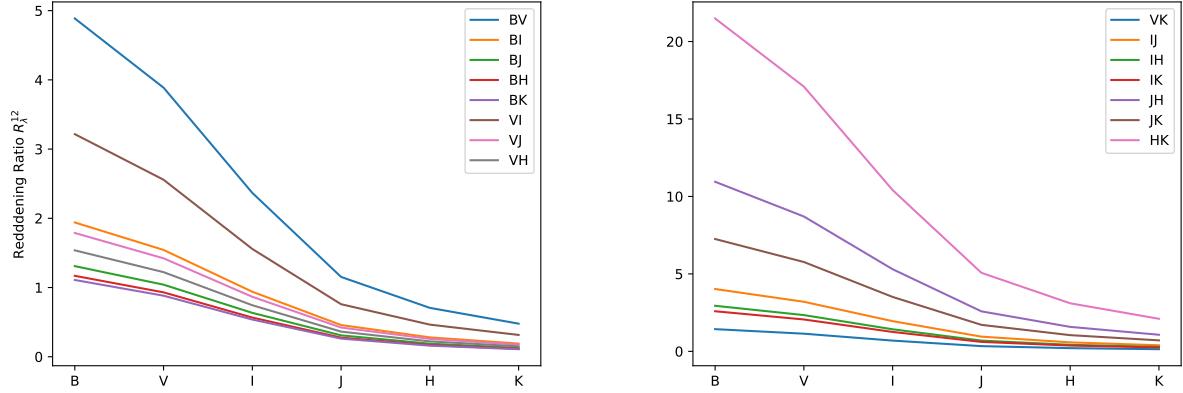


Figure 3: Variation in reddening ratio for different color index

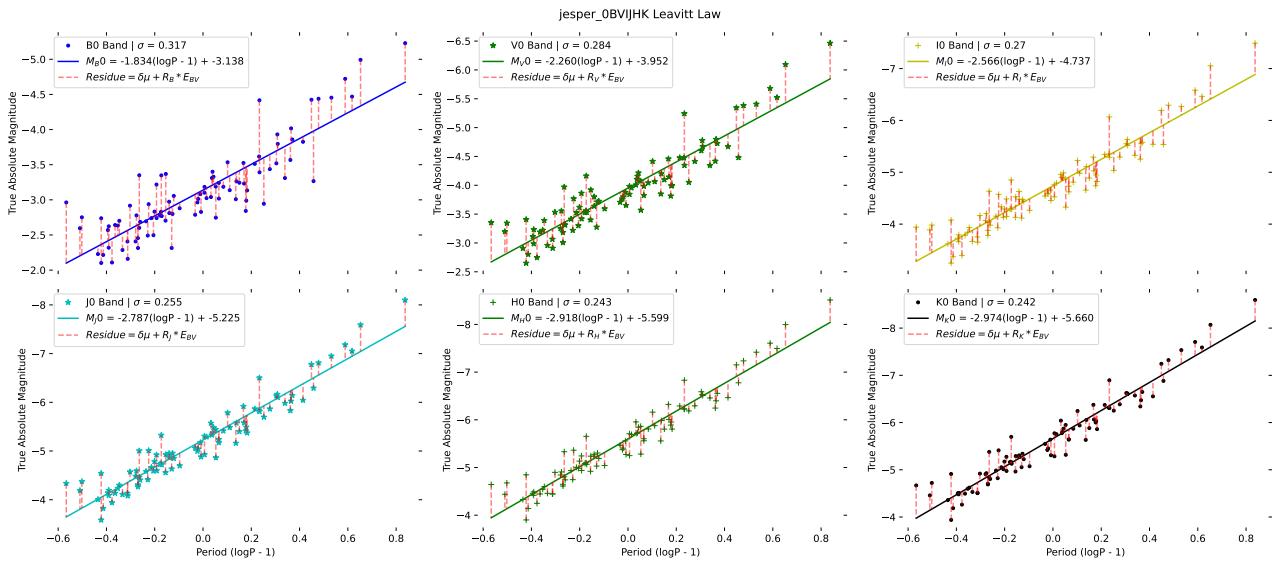


Figure 4: BVIJHK Leavitt Law prior to calibration

The Wesenheit magnitude - based Leavitt Law can be derived using a reference color index ($m_1 - m_2$). For each combination of band and color index, the reddening-free form of the Leavitt Law can be modeled as:

$$\bar{W}_\lambda^{12} = \alpha_\lambda^{12}(\log P) + \gamma_\lambda^{12} \quad (10)$$

For $BVIJHK$ photometry, there are 15 possible combinations of color indices. Considering six observed bands with each color index results in 90 composite Wesenheit magnitudes (W_λ^{12}). The deviation of the Wesenheit magnitude from the corresponding Period-Wesenheit (PW) relation is given by:

$$\Delta W_\lambda^{12} = W_\lambda^{12} - \bar{W}_\lambda^{12}$$

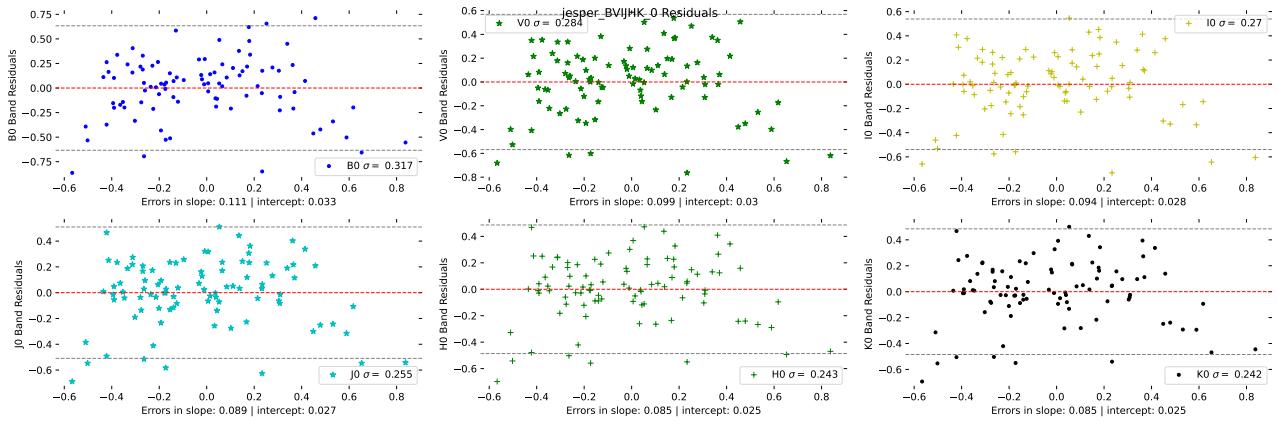


Figure 5: Residuals of the $BVIJHK$ Leavitt Laws show a decreasing scatter with increasing wavelength, indicating a dependence on interstellar extinction.

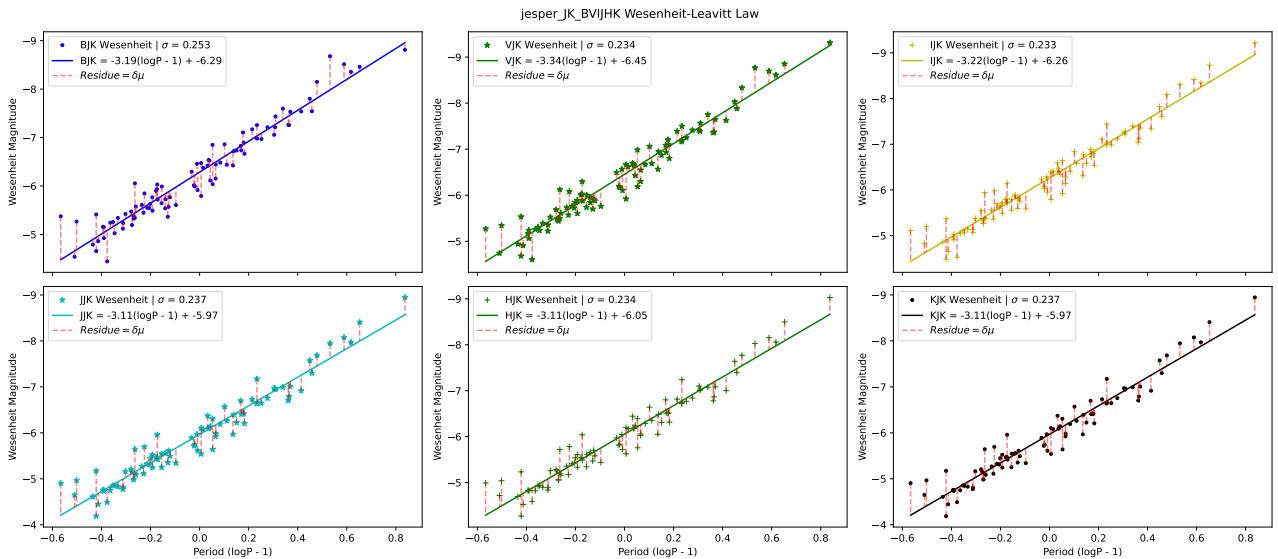


Figure 6: BVIJHK Period-Wesenheit Relation for (J-K) color.

Correlation between pulsation period and luminosity (true absolute and wesenheit) of Cepheids produces 6 + 90 linear equations, each characterized by a slope and intercept, along with the residuals for each Cepheid. A summary is provided below in Table ??

$$\begin{aligned} \text{PL relation: } & \alpha_\lambda \mid \gamma_\lambda \mid \Delta M_\lambda \mid \implies 6 \text{ equations} \\ \text{PW relation: } & \alpha_\lambda^{12} \mid \gamma_\lambda^{12} \mid \Delta W_\lambda^{12} \implies 90 \text{ equations} \end{aligned}$$

2.1 PL-PW Residuals

A crucial difference between the two types of relations is that the residuals of the Period-Wesenheit (PW) relation are free from reddening errors, whereas those of the Period-Luminosity (PL) relation are not. The correlation between the PL and PW residuals ($\Delta_{\text{PL}} - \Delta_{\text{PW}}$ correlation) provides quantitative insights into the error budget associated with reddening and distance modulus. To develop a mathematical framework, two cases must be considered:

- i) When there is error only in distance, ($\delta\mu \neq 0, \delta E_{12} = 0$):

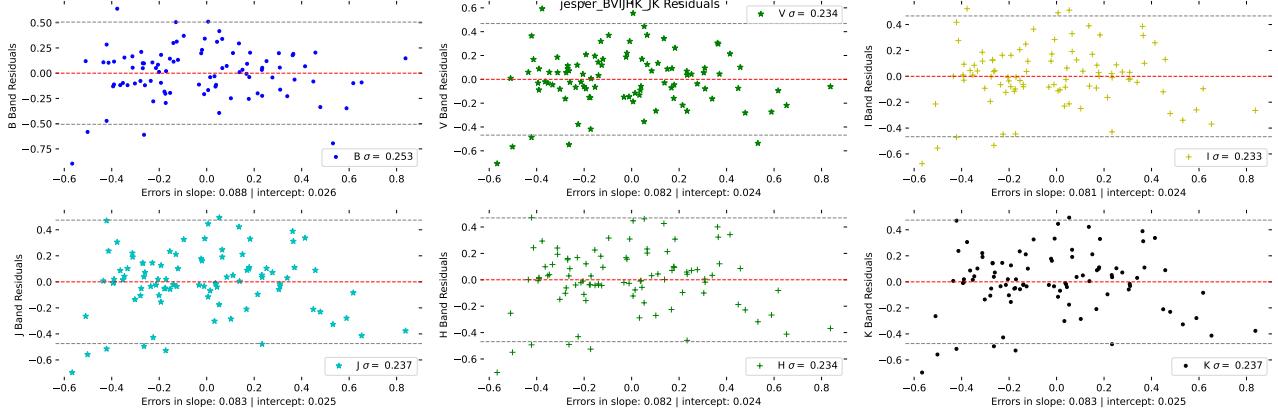


Figure 7: Residuals of the $BVIJHK$ Wesenheit-Leavitt Law for the $(J - K)$ color show an approximately constant scatter across all bands, indicating that the Wesenheit magnitude is insensitive to interstellar extinction. Therefore, scatter is attributed primarily to errors in the distance modulus.

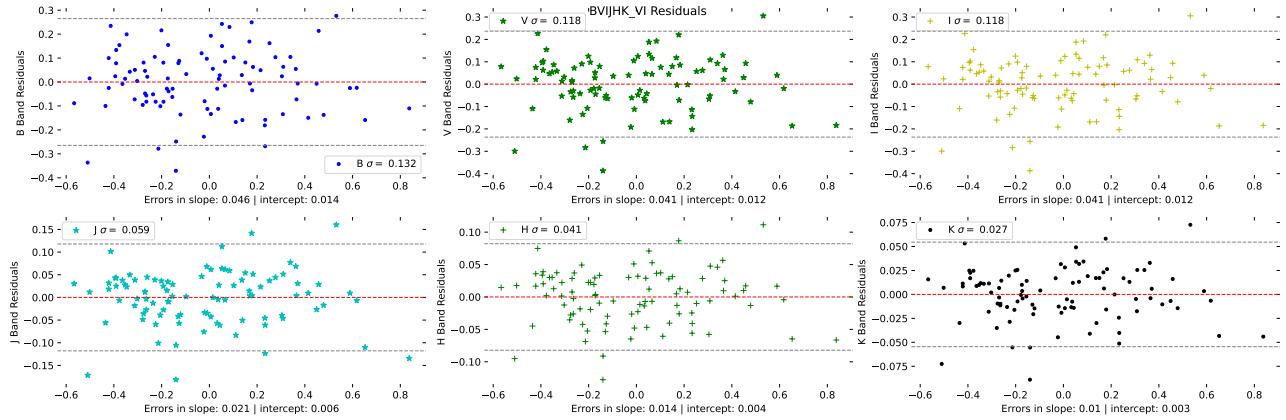


Figure 8: Residuals of BVIJHK Wesenheit-Leavitt Law for $(V-I)$ color.

$$\text{Deviation in PL : } \Delta M_\lambda = \delta \mu$$

$$\text{Deviation in PW : } \Delta W_\lambda^{12} = \delta \mu$$

In the residual correlation plots, *an error in distance shifts the star equally along both axes from its original position*, meaning that Cepheid with minimal or no reddening error would lie perfectly along a line with slope 1. This behavior is clearly evident in the infrared bands, where reddening errors are expected to be minimal at longer wavelengths.

ii) When there is error in reddening alone, ($\delta E_{12} \neq 0$, $\delta \mu = 0$):

$$\text{Deviation in PL : } \Delta M_\lambda = R_\lambda^{12} \delta E_{12}$$

$$\text{Deviation in PW : } \Delta W_\kappa^{12} = 0$$

In this case, only the PL residuals are affected, whereas the PW residuals remain unchanged, since the Wesenheit magnitude is reddening - independent by definition. This means that the star shifts along the PL residual axis but not along the PW residual axis. Consequently, *a vertical displacement of the star in $\Delta\Delta$ plots originates from the reddening error*.

Table 2: Slope and Intercept of BVIJHK Leavitt Law using 94 Galactic Cepheids

	B (err)	V (err)	I (err)	J (err)	H (err)	K (err)
Slope and Intercept of Period-Luminosity Relation						
Slope	-1.83 (0.159)	-2.26 (0.131)	-2.57 (0.105)	-2.79 (0.091)	-2.92 (0.086)	-2.97 (0.086)
Intercept	-4.45 (0.048)	-5.00 (0.039)	-5.37 (0.031)	-5.53 (0.027)	-5.79 (0.026)	-5.79 (0.026)
Slope of Period-Wesenheit Relations						
BV	-3.92 (0.132)	-3.92 (0.132)	-3.57 (0.113)	-3.28 (0.095)	-3.22 (0.089)	-3.18 (0.087)
BI	-3.25 (0.101)	-3.39 (0.105)	-3.25 (0.100)	-3.12 (0.091)	-3.12 (0.087)	-3.11 (0.086)
BJ	-3.08 (0.089)	-3.25 (0.094)	-3.17 (0.094)	-3.08 (0.089)	-3.10 (0.086)	-3.10 (0.085)
BH	-3.10 (0.085)	-3.27 (0.091)	-3.18 (0.092)	-3.09 (0.088)	-3.10 (0.085)	-3.10 (0.085)
BK	-3.10 (0.085)	-3.27 (0.090)	-3.18 (0.091)	-3.09 (0.088)	-3.10 (0.085)	-3.10 (0.085)
VI	-2.82 (0.097)	-3.04 (0.097)	-3.04 (0.097)	-3.02 (0.090)	-3.06 (0.086)	-3.07 (0.085)
VJ	-2.78 (0.088)	-3.01 (0.088)	-3.02 (0.090)	-3.01 (0.088)	-3.05 (0.085)	-3.07 (0.085)
VH	-2.84 (0.083)	-3.06 (0.084)	-3.05 (0.088)	-3.03 (0.087)	-3.06 (0.084)	-3.07 (0.084)
VK	-2.86 (0.084)	-3.07 (0.084)	-3.06 (0.088)	-3.03 (0.087)	-3.07 (0.084)	-3.07 (0.084)
IJ	-2.72 (0.102)	-2.97 (0.094)	-3.00 (0.088)	-3.00 (0.088)	-3.05 (0.085)	-3.06 (0.085)
IH	-2.87 (0.089)	-3.09 (0.085)	-3.07 (0.083)	-3.03 (0.086)	-3.07 (0.083)	-3.08 (0.084)
IK	-2.89 (0.090)	-3.10 (0.085)	-3.08 (0.084)	-3.04 (0.086)	-3.07 (0.083)	-3.08 (0.084)
JH	-3.27 (0.081)	-3.40 (0.079)	-3.26 (0.079)	-3.13 (0.081)	-3.13 (0.081)	-3.11 (0.082)
JK	-3.19 (0.088)	-3.34 (0.082)	-3.22 (0.081)	-3.11 (0.083)	-3.11 (0.082)	-3.11 (0.083)
HK	-3.04 (0.159)	-3.22 (0.131)	-3.15 (0.105)	-3.07 (0.091)	-3.09 (0.086)	-3.09 (0.086)
Intercept of Period-Wesenheit Relations						
BV	-7.11 (0.039)	-7.11 (0.039)	-6.66 (0.034)	-6.16 (0.029)	-6.17 (0.027)	-6.05 (0.026)
BI	-6.24 (0.030)	-6.42 (0.031)	-6.24 (0.030)	-5.96 (0.027)	-6.05 (0.026)	-5.96 (0.026)
BJ	-5.87 (0.027)	-6.12 (0.028)	-6.06 (0.028)	-5.87 (0.027)	-5.99 (0.026)	-5.93 (0.025)
BH	-6.02 (0.025)	-6.24 (0.027)	-6.13 (0.027)	-5.90 (0.026)	-6.02 (0.025)	-5.94 (0.025)
BK	-5.94 (0.025)	-6.18 (0.027)	-6.09 (0.027)	-5.89 (0.026)	-6.00 (0.025)	-5.94 (0.025)
VI	-5.66 (0.029)	-5.96 (0.029)	-5.96 (0.029)	-5.82 (0.027)	-5.96 (0.026)	-5.91 (0.025)
VJ	-5.42 (0.026)	-5.76 (0.026)	-5.84 (0.027)	-5.76 (0.026)	-5.93 (0.025)	-5.88 (0.025)
VH	-5.67 (0.025)	-5.96 (0.025)	-5.96 (0.026)	-5.82 (0.026)	-5.96 (0.025)	-5.91 (0.025)
VK	-5.59 (0.025)	-5.90 (0.025)	-5.93 (0.026)	-5.80 (0.026)	-5.95 (0.025)	-5.90 (0.025)
IJ	-5.10 (0.031)	-5.51 (0.028)	-5.69 (0.026)	-5.69 (0.026)	-5.88 (0.025)	-5.85 (0.025)
IH	-5.68 (0.027)	-5.97 (0.025)	-5.97 (0.025)	-5.82 (0.026)	-5.97 (0.025)	-5.91 (0.025)
IK	-5.53 (0.027)	-5.85 (0.025)	-5.89 (0.025)	-5.79 (0.026)	-5.94 (0.025)	-5.89 (0.025)
JH	-7.23 (0.024)	-7.21 (0.024)	-6.72 (0.024)	-6.19 (0.024)	-6.19 (0.024)	-6.06 (0.024)
JK	-6.29 (0.026)	-6.46 (0.024)	-6.27 (0.024)	-5.97 (0.025)	-6.05 (0.024)	-5.97 (0.025)
HK	-4.45 (0.048)	-5.00 (0.039)	-5.37 (0.031)	-5.53 (0.027)	-5.79 (0.026)	-5.79 (0.026)

3. Distance - Reddening Calibration Methodology

To decouple the reddening and distance errors for individual Cepheids, the residuals of the PL relations (ΔM_λ) are plotted against the corresponding PW residuals (ΔW_κ^{12}). Fitting a regression line with slope $\rho_{\kappa\lambda}^{12}$ and zero intercept yields the residuals for each band and choice of Wesenheit color. The resulting $\Delta\Delta$ residuals are denoted as:

$$\Delta_{\kappa\lambda}^{12} = \Delta M_\lambda - \rho_{\kappa\lambda}^{12} \times \Delta W_\kappa^{12}$$

This vertical deviation from the regression line indicates the extinction correction for the

corresponding band, independent of any correction to the distance modulus.

$$\delta A_{\kappa\lambda}^{12}(0) = \Delta_{\kappa\lambda}^{12}$$

By normalizing with R_λ^{12} , the extinction corrections ($\delta A_{\kappa\lambda}^{12}$) are converted into reddening corrections ($\delta E_{\kappa\lambda}^{12}$) as follows:

$$\delta E_{\kappa\lambda}^{12}(0) = \frac{\delta A_{\kappa\lambda}^{12}}{R_\lambda^{12}}$$

If the distances of individual targets are precisely known, then reddening is the sole source of error. This implies that the corrections indicated by each band should be consistent with one another.

$$\delta E_{\kappa B}^{12} = \delta E_{\kappa V}^{12} = \dots = \delta E_{\kappa K}^{12} = \delta E_{12}^*$$

If distances are accurately known, the reddening corrections derived from different bands would show no dispersion, as discussed above. In contrast, an incorrect distance for a given Cepheid would result in discrepancies among the reddening corrections estimated from the various bands.

To estimate the distance error, a range of possible distance corrections ($\delta\mu^i$) is considered. For each assumed correction, the reddening corrections for all bands are computed. The particular distance correction for which the reddening corrections are most consistent across all bands (i.e., exhibiting minimal variation) is adopted as the optimal distance reddening correction pair for the individual Cepheid.

The variation in extinction ($\delta A_{\kappa\lambda}^{12}$) resulting from a distance modulus correction is calculated as follows:

$$\begin{aligned}\delta A_{\kappa\lambda}^{12}(\delta\mu) &= (\Delta M_\lambda + \delta\mu) - \rho_{\kappa\lambda}^{12} \times (\Delta W_\kappa^{12} + \delta\mu) \\ &= \delta A_{\kappa\lambda}^{12}(0) + \delta\mu(1 - \rho_{\kappa\lambda}^{12})\end{aligned}$$

The corresponding reddening corrections are then given by:

$$\delta E_{\kappa\lambda}^{12}(\delta\mu) = \delta E_{\kappa\lambda}^{12}(0) + \frac{\delta\mu(1 - \rho_{\kappa\lambda}^{12})}{R_\lambda^{12}} \quad (11)$$

For the optimal distance - reddening correction pair, the dispersion in the estimated reddening corrections from the BVIJHK bands is minimized.

$$RMS(\delta E(\delta\mu))_\lambda = \frac{1}{6} \sum_{\lambda} (\delta E_{\kappa\lambda}^{12}(\delta\mu) - <\delta E_{\kappa\lambda}^{12}(\delta\mu)>_\lambda)^2$$

$$\min(RMS(\delta E(\delta\mu))_\lambda)_\mu \implies (\delta\mu^*, \delta E_{12}^*)$$

These corrections are then applied to the original data as follows:

$$M_\lambda^* = M_\lambda^0 + \delta A_\lambda^* + \delta\mu^* \quad (12)$$

3.1 Madore Approach

In his analysis, **madore2017** used W_V^{VI} as a reference for the reddening - free magnitude. He correlated the PW residuals (ΔW_V^{VI}) with the BVRIJHK PL residuals (ΔM_λ), yielding the corresponding slopes ($\rho_{V\lambda}^{VI}$) and residuals ($\Delta_{V\lambda}^{VI}$).

Let $\delta\mu^*$ denote the correction required in the distance modulus. Then, the reddening corrections $\delta E_{V\lambda}^{VI}$, as derived by Madore (2017), are given by:

$$\delta E_{V\lambda}^{VI}(\delta\mu^*) = \frac{\delta A_{V\lambda}^{VI}(\delta\mu^*)}{R_\lambda^{VI}}$$

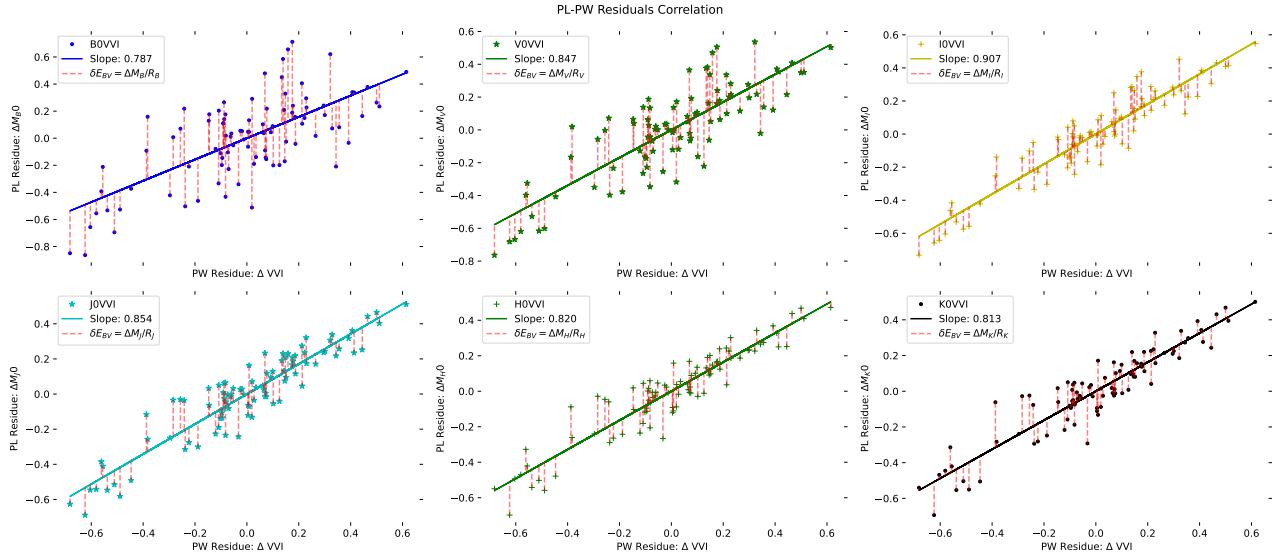
Converting E_{VI} into E_{BV}

$$\begin{aligned} \delta E_{V\lambda}^{BV}(\delta\mu^*) &= \frac{\delta A_{V\lambda}^{VI}(\delta\mu^*)}{(R_V^{BV} - R_I^{BV})R_\lambda^{VI}} \\ &= \frac{\delta A_{V\lambda}^{VI}(\delta\mu^*)}{R_\lambda^{BV}} \end{aligned}$$

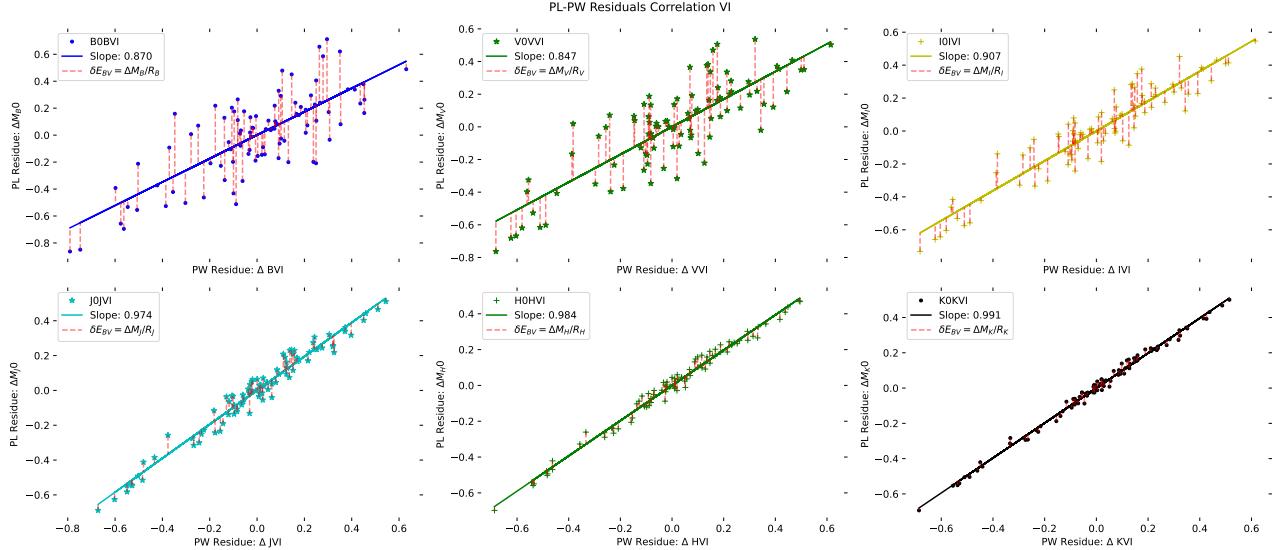
By implementing these reddening and distance error corrections in his dataset of 59 Cepheids, he derived the Leavitt Laws as given in Table ??:

3.2 My Approach

Instead of keeping ΔW_V^{VI} fixed, I have used ΔW_λ^{VI} the reddening free magnitude for the respective band m_λ . This choice follows directly from the definition of the Wesenheit function: the reddening - free magnitude for a given band must be derived using R_λ^{12} , not R_V^{12} . Specifically, R_V^{VI} gives the wesenheit magnitude corresponding to the V band only for the $(V - I)$ color index. For the B band, $R_B^{VI}(V - I)$ must be subtracted from B to obtain the corresponding Wesenheit magnitude. Consequently, the PW residuals along x-axis in my approach will be



(a) Residual correlation according to Madore (2017), where the y -axis represents the residuals of the PL relation (ΔM_λ) and the x -axis, fixed for all six bands, represents the residuals of the PW relation (ΔW_V^{VI}).



(b) Residual correlation where both the axis varies with bands.

Figure 9: Comparison of the slopes between the two cases shows that the slope of the regression line in the bottom panel approaches 1 more rapidly with increasing wavelength.

Table 3: Madore (2017): Slope and Intercept of Galactic BVIJHK Leavitt Law

Band	Slope	σ (mag)	Zero	σ (mag)
B	2.277	0.121	3.214	0.036
V	2.670	0.095	3.944	0.028
R	2.874	0.075	4.396	0.022
I	2.983	0.064	4.706	0.019
J	3.198	0.054	5.258	0.016
H	3.333	0.055	5.558	0.017
K	3.377	0.055	5.659	0.016
W, VI	3.476	0.037	5.889	0.011
W, BI	3.600	0.033	5.997	0.010

varying with the respective PL residuals ΔM_λ . The difference between Madore's and my PW residuals is represented by the operator δ_V^λ and is calculated as follows:

$$\begin{aligned}
\delta_V^\lambda \Delta W_\lambda^{VI} &= \Delta W_\lambda^{VI} - \Delta W_V^{VI} \\
&= (W_\lambda^{VI} - \bar{W}_\lambda^{VI}) - (W_V^{VI} - \bar{W}_V^{VI}) \\
&= (W_\lambda^{VI} - W_V^{VI}) - (\bar{W}_\lambda^{VI} - \bar{W}_V^{VI}) \\
&= ((M_\lambda - M_V) - (R_\lambda^{VI} - R_V^{VI})(V - I)) - ((\alpha_\lambda^{VI} - \alpha_V^{VI}) \log P + (\gamma_\lambda^{VI} - \gamma_V^{VI}))
\end{aligned}$$

This deviation in the PW residuals affects the slope of the regression line, ρ , which implies that the correlation residuals, $\Delta_{\kappa\lambda}^{12}$, will also be affected. In Madore's approach, the $\Delta - \Delta$ correlation residuals are expressed as $\Delta_{V\lambda}^{VI}$, since he correlated ΔM_λ with ΔW_V^{VI} . In contrast, I have correlated ΔM_λ with ΔW_λ^{VI} , so that the resulting residuals take the form $\Delta_{\lambda\lambda}^{VI}$.

Difference between both kinds of correlation residuals is:

$$\begin{aligned}
\delta_{V\lambda}^{\lambda\lambda} \Delta_{\lambda\lambda}^{VI} &= \Delta_{\lambda\lambda}^{VI} - \Delta_{V\lambda}^{VI} \\
&= \delta A_\lambda^S(0) - A_\lambda^M(0) \\
&= (\Delta M_\lambda - \rho_{\lambda\lambda}^{VI} \times \Delta W_\lambda^{VI}) - (\Delta M_\lambda - \rho_{V\lambda}^{VI} \times \Delta W_V^{VI}) \\
&= \rho_{V\lambda}^{VI} \times \Delta W_V^{VI} - \rho_{\lambda\lambda}^{VI} \times \Delta W_\lambda^{VI}
\end{aligned}$$

3.3 Physical Significance of Residual-Slope

The slope of the PL - PW residual correlations quantifies the contributions error sources. In this study, the absence of extinction errors, the distance remains the only source of error, which affects both axes equally, resulting in a slope of 1.

Table 4: Variation of the $\Delta - \Delta$ correlation slope, $\rho_{\kappa\lambda}^{12}$, for fifteen Wesenheit colors, comparing Madore's approach (white) with my approach (yellow). The $(V - I)$ case is highlighted in red. The continuous decrease in slope-error with increasing wavelength, aligning along a slope of 1, is a strong indication of larger distance error with respect .

ρ	B (err)	V (err)	I (err)	J (err)	H (err)	K (err)
BV	0.243 (0.084)	0.398 (0.067)	0.471 (0.056)	0.479 (0.050)	0.478 (0.045)	0.471 (0.046)
	0.243 (0.084)	0.398 (0.067)	0.653 (0.054)	0.870 (0.035)	0.929 (0.024)	0.962 (0.017)
BI	0.652 (0.092)	0.758 (0.066)	0.831 (0.045)	0.798 (0.040)	0.773 (0.035)	0.765 (0.036)
	0.652 (0.092)	0.683 (0.068)	0.832 (0.045)	0.944 (0.025)	0.968 (0.016)	0.982 (0.011)
BJ	0.881 (0.090)	0.945 (0.060)	0.975 (0.041)	0.972 (0.021)	0.938 (0.014)	0.933 (0.016)
	0.881 (0.090)	0.835 (0.066)	0.920 (0.041)	0.972 (0.021)	0.981 (0.014)	0.990 (0.009)
BH	0.917 (0.096)	0.983 (0.065)	1.015 (0.045)	1.017 (0.025)	0.988 (0.014)	0.984 (0.014)
	0.917 (0.096)	0.859 (0.070)	0.941 (0.043)	0.983 (0.022)	0.988 (0.014)	0.995 (0.009)
BK	0.945 (0.094)	0.996 (0.064)	1.022 (0.045)	1.023 (0.024)	0.992 (0.014)	0.994 (0.009)
	0.945 (0.094)	0.880 (0.069)	0.951 (0.042)	0.986 (0.021)	0.990 (0.013)	0.994 (0.009)
VI	0.787 (0.086)	0.847 (0.060)	0.907 (0.037)	0.854 (0.035)	0.820 (0.032)	0.813 (0.034)
	0.870 (0.077)	0.847 (0.060)	0.907 (0.037)	0.974 (0.020)	0.984 (0.013)	0.991 (0.009)
VJ	0.938 (0.087)	0.980 (0.057)	1.000 (0.039)	0.994 (0.017)	0.956 (0.010)	0.953 (0.011)
	1.053 (0.072)	0.980 (0.057)	0.990 (0.034)	0.994 (0.017)	0.994 (0.011)	0.996 (0.007)
VH	0.951 (0.094)	1.004 (0.064)	1.031 (0.045)	1.031 (0.023)	1.001 (0.012)	0.998 (0.012)
	1.087 (0.082)	1.004 (0.064)	1.011 (0.037)	1.006 (0.018)	1.001 (0.012)	1.002 (0.008)
VK	0.965 (0.093)	1.007 (0.063)	1.030 (0.045)	1.030 (0.023)	0.998 (0.013)	1.001 (0.008)
	1.080 (0.080)	1.007 (0.063)	1.013 (0.037)	1.008 (0.018)	1.001 (0.011)	1.001 (0.008)
IJ	0.939 (0.087)	0.969 (0.060)	0.969 (0.047)	0.985 (0.023)	0.948 (0.017)	0.946 (0.017)
	0.764 (0.079)	0.814 (0.069)	0.969 (0.047)	0.985 (0.023)	0.992 (0.014)	0.996 (0.010)
IH	0.956 (0.096)	1.005 (0.066)	1.024 (0.049)	1.036 (0.026)	1.007 (0.015)	1.005 (0.014)
	0.856 (0.095)	0.891 (0.079)	1.024 (0.049)	1.013 (0.023)	1.007 (0.015)	1.007 (0.010)
IK	0.969 (0.093)	1.008 (0.064)	1.025 (0.048)	1.032 (0.025)	1.001 (0.015)	1.005 (0.010)
	0.868 (0.090)	0.906 (0.076)	1.025 (0.048)	1.013 (0.023)	1.007 (0.014)	1.005 (0.010)
JH	0.941 (0.104)	1.008 (0.073)	1.044 (0.054)	1.053 (0.034)	1.032 (0.021)	1.030 (0.020)
	0.477 (0.133)	0.654 (0.112)	0.977 (0.072)	1.053 (0.034)	1.032 (0.021)	1.027 (0.014)
JK	0.969 (0.096)	1.011 (0.067)	1.035 (0.049)	1.038 (0.029)	1.010 (0.018)	1.016 (0.012)
	0.678 (0.110)	0.788 (0.096)	1.000 (0.060)	1.038 (0.029)	1.024 (0.018)	1.016 (0.012)
HK	0.962 (0.090)	0.984 (0.063)	0.998 (0.048)	0.997 (0.030)	0.964 (0.024)	0.976 (0.016)
	0.375 (0.061)	0.445 (0.064)	0.703 (0.058)	0.911 (0.035)	0.964 (0.024)	0.976 (0.016)

$$\lim_{\lambda \rightarrow \infty} \rho_{\lambda\lambda}^{12} = 1 \quad (13)$$

This behavior is particularly evident at longer wavelengths, such as the K band shown in Table ???. The yellow-highlighted cases represent the slopes, ρ , obtained using my approach. This is not observed in Madore's approach, as it uses a fixed PW residual for all bands along the y -axis, resulting in deviation from slope 1. To illustrate this more clearly, consider the case of an incorrect reddening law, R_λ^{12} , where ΔW_λ^{12} takes the form:

$$\Delta W_\lambda^{12} = -\delta\mu - \delta R_\lambda^{12}(m_1 - m_2)$$

which affects ρ as follows:

Madore (for $(V - I)$):

$$\rho_{V\lambda}^{VI} = \frac{-\delta\mu - \delta A_\lambda}{-\delta\mu - \delta R_V^{VI}(V - I)} \quad (14)$$

As the wavelength increases, δA_λ decreases, while δR_V^{VI} remains fixed, causing the resulting ratio to be smaller than 1 even at longer wavelengths.

Shubham (for $(V - I)$):

$$\rho_{\lambda\lambda}^{VI} = \frac{-\delta\mu - \delta A_\lambda}{-\delta\mu - \delta R_\lambda^{VI}(V - I)} \quad (15)$$

Here, both δA_λ and δR_λ^{VI} decrease with increasing wavelength, causing the numerator and denominator to converge, and the slope approaches 1 at longer wavelengths.

Since the deviation between $\rho_{V\lambda}^{VI}$ and $\rho_{\lambda\lambda}^{VI}$ is non-zero, the derived reddening corrections will also differ from those of Madore. The difference between the two reddening errors, prior to distance correction, can be expressed as:

$$\begin{aligned} \delta E_{\lambda\lambda}^{VI}(0) - \delta E_{V\lambda}^{VI}(0) &= \frac{\delta A_{\lambda\lambda}^{VI}(0) - \delta A_{V\lambda}^{VI}(0)}{R_\lambda^{VI}} \\ &= \frac{\Delta_{\lambda\lambda}^{VI} - \Delta_{V\lambda}^{VI}}{R_\lambda^{VI}} \\ &= \frac{\Delta W_\lambda^{VI} \times \rho_{\lambda\lambda}^{VI} - \Delta W_V^{VI} \times \rho_{V\lambda}^{VI}}{R_\lambda^{VI}} \end{aligned}$$

Determination of distance correction ($\delta\mu^*$) requires scaling factor $(1 - \rho_{\kappa\lambda}^{12})$, implies distance correction would also be different.

4. Tweaking Distance - Reddening Calibration Method

Given multiple photometric bands, shorter wavelengths are more sensitive to reddening uncertainties, while distance errors are largely wavelength-independent. To mitigate this, distance and reddening are estimated in two stages. First, the longer-wavelength $IJHK$ bands provide an initial distance - reddening correction via an infrared Wesenheit color, offering robust distance estimates with minimal reddening adjustment. These corrected values are then refined using the shorter-wavelength $BVIJ$ bands and the $(V - I)$ Wesenheit index, which improves reddening accuracy with little effect on distance. The final corrections produce a tightly constrained Galactic Leavitt Law across the $BVIJHK$ bands, with dispersions of 0.087 mag in B and 0.011 mag in K . The next section details the calibration procedure.

4.1 Distance correction using IJHK photometry

To derive the distance correction, I used the longer-wavelength $IJHK$ bands, which are weakly affected by extinction, excluding the more sensitive B and V bands to avoid bias from optical reddening. The correction was estimated using Wesenheit indices based on $(I - J)$, $(I - H)$, $(I - K)$, and $(J - K)$ colors, yielding a robust distance estimate with minimal reddening contamination.

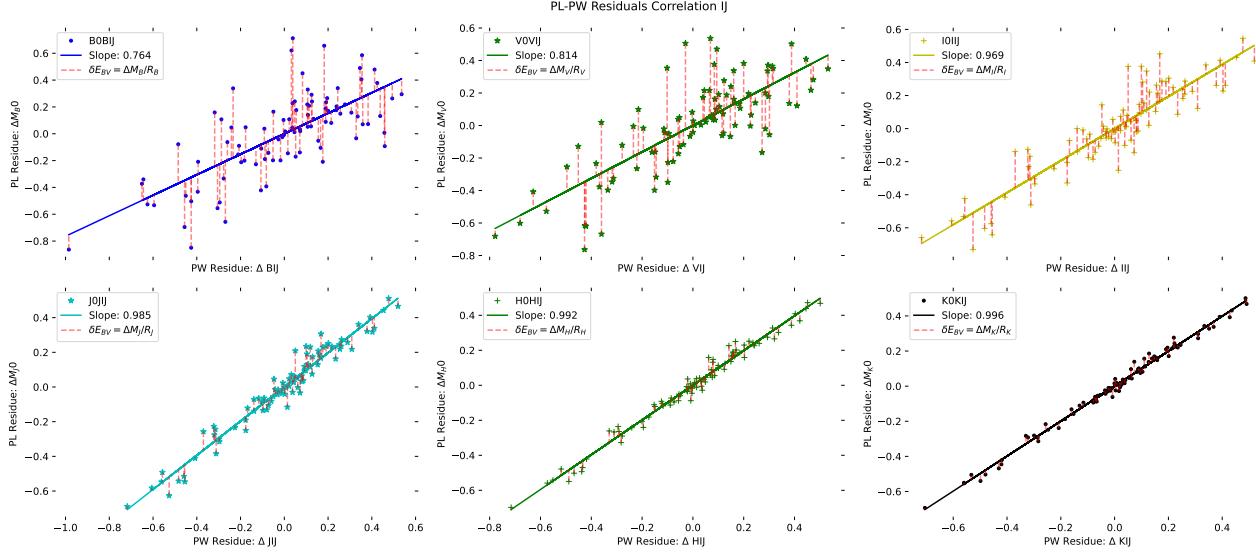


Figure 10: Optical-Infrared Wesenheit based $\Delta M_\lambda - \Delta W_\lambda^{IJ}$ correlation. Notice the relative scatter in between BV and $IJHK$ bands.

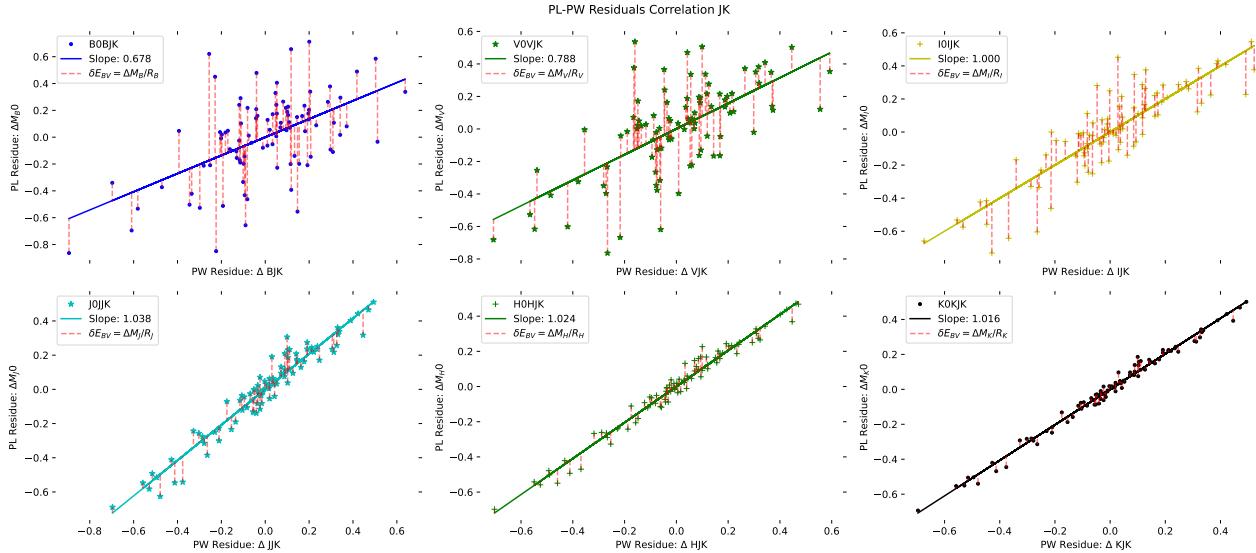


Figure 11: Intrared Wesenheit based $\Delta M_\lambda - \Delta W_\lambda^{JK}$ correlation.

To illustrate the decoupling of distance and reddening errors, residual-correlation plots for the $(I - J)$ and $(J - K)$ colors are shown. For the colors $(I - J)$, $(I - H)$, $(I - K)$, and $(J - K)$, the Wesenheit magnitudes remain effectively reddening - free even in the presence of

intrinsic uncertainties in the extinction law, as described by Equation ???. This is supported by the highly consistent values of $\delta\mu$ across all four colors (see Fig. ??).

Among these, $(J - K)$ combines two closely spaced near-infrared bands that are only weakly affected by interstellar extinction. The extinction difference between them is small ($A_J - A_K \approx 0.2$), whereas optical infrared combinations such as $(I - K)$ or $(I - J)$ are more sensitive to extinction variations. Consequently, $(J - K)$ serves as an almost reddening - independent indicator, enabling a more precise determination of intrinsic luminosity and distance. The $(J - K)$ -based distance - calibrated Leavitt Law will subsequently be refined to incorporate the final reddening corrections.

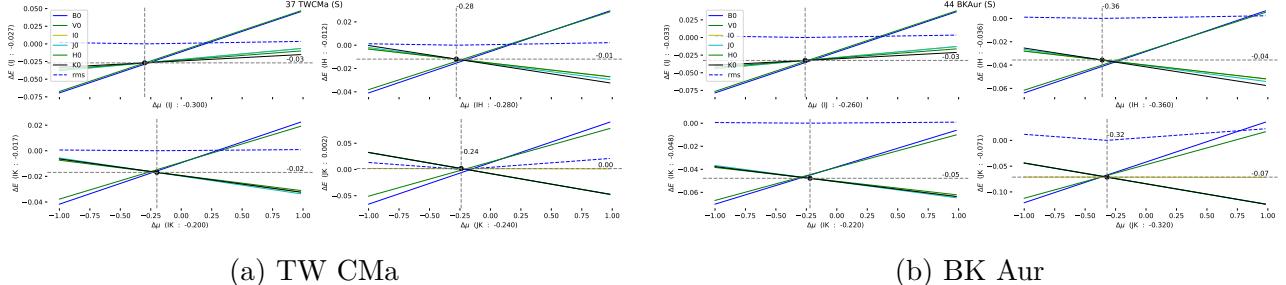


Figure 12: $(I - J)$, $(I - H)$, $(I - K)$, and $(J - K)$ based modulus correction. The least dispersion among I , J , H , and K bands gives the correction pair $(\delta\mu, \delta E)$. The B and V bands are excluded.

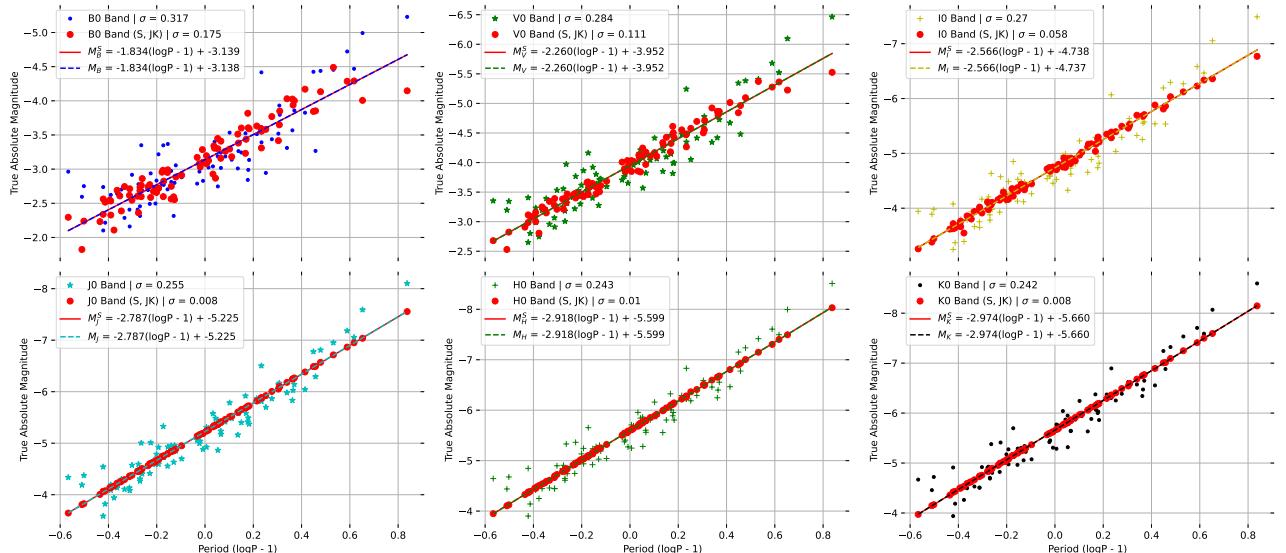


Figure 13: Distance calibration using the $(J - K)$ -based Wesenheit index, with the B and V bands excluded, yields minimal reddening effects and high accuracy in distance estimates. The resulting distance correction is then applied uniformly across all bands. | Colored: Original Data | Red : Corrected Data

4.2 Reddening correction using BVIJ photometry

To obtain an accurate reddening correction, the optical and near - infrared bands ($BVIJ$) are used, while the H and K bands are excluded from the analysis. Using the first - order

distance reddening corrected data, the PL PLW residual correlation is re - evaluated. Since the remaining distance correction is expected to be minimal, the residual slope in this second - order correction should not approach unity as rapidly as observed in the first-order case.

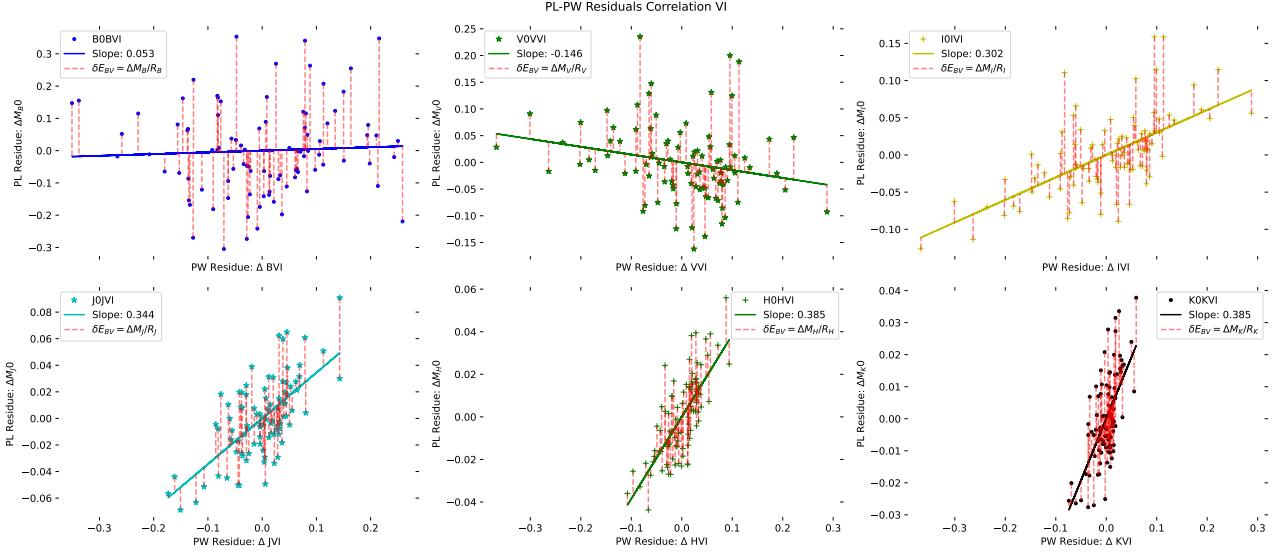


Figure 14: Residual correlation for $V - I$. High variation in slope indicating absence of systematic effect of distance error.

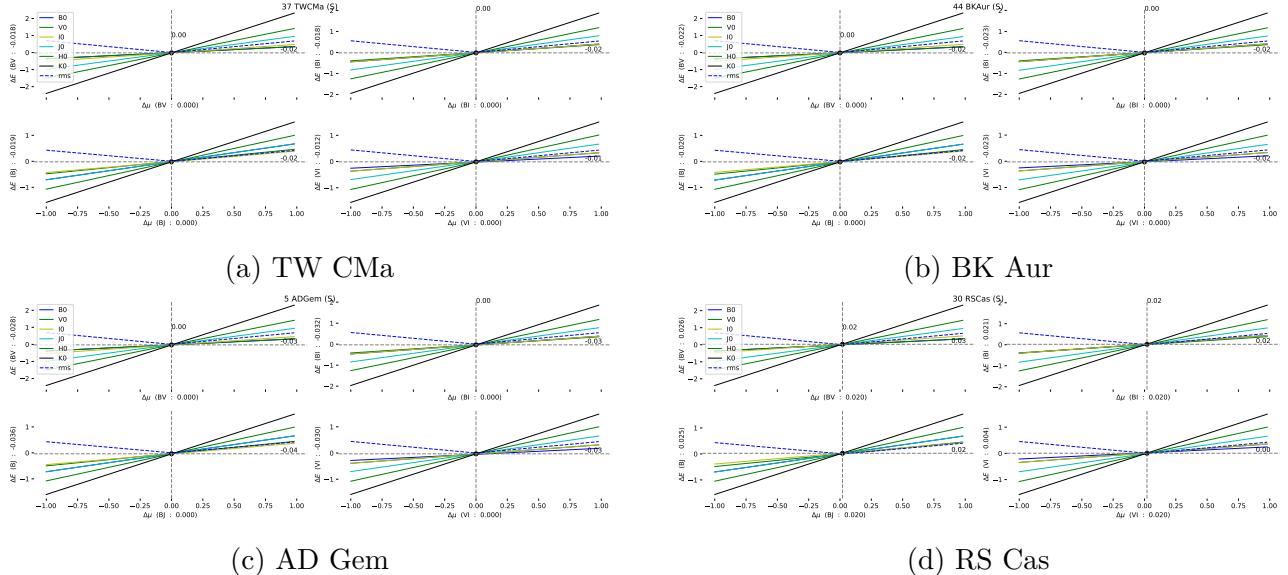


Figure 15: $(B - V)$, $(B - I)$, $(B - J)$, and $(V - I)$ based reddening correction. The H and K bands are excluded. Notice the consistency in reddening and modulus correction accross the colors.

The high dispersion in the residual slopes for the second-order correction clearly demonstrates the effectiveness of the first-order distance correction, as the systematic trend with a slope of unity is no longer followed with increasing wavelength. It is also noteworthy that the range of PL residuals (y - axis) decreases rapidly across the bands, indicating significant contamination from extinction errors still present. The reddening uncertainty can thus be quantified by identifying the distance solution that minimizes dispersion across the bands.

The robustness of this approach is illustrated for a sample of four stars, as shown in Figure ???. For the given sample of Cepheids, the reddening errors derived from different colors show strong mutual agreement, requiring little to no additional distance correction across all bands. For individual Cepheids, applying these optical-based corrections yields the final calibrated results.

5. Calibrated Leavitt Law

Each Cepheid in the dataset is corrected for distance and reddening, yielding calibrated true absolute magnitudes in the *BVIJHK* bands. This produces a calibrated Leavitt Law, for which the ($J - K$)based modulus and ($V - I$)based reddening exhibit the least scatter.

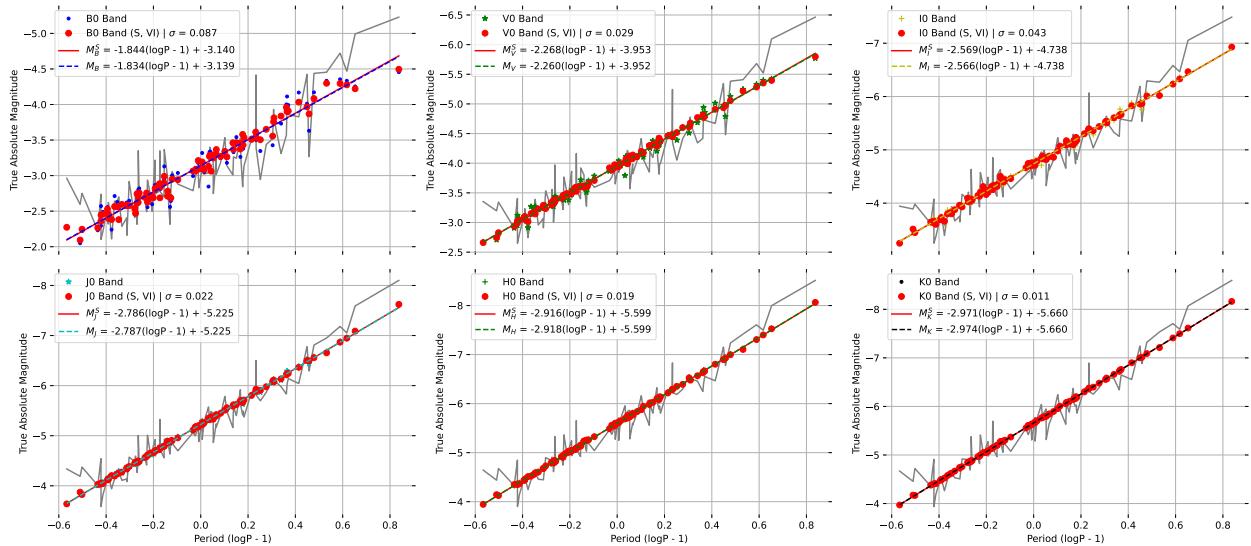


Figure 16: *BVIJ* based reddening correction (second - order) estimated with ($V - I$)based Wesenheit index, applied across all bands to yield the final *BVIJHK* Leavitt Laws. Background continuous line represents original data.

Calibrated Leavitt Law are:

$$B = (-1.844 \pm 0.030)(\log P - 1) - 3.140 \pm 0.009$$

$$V = (-2.268 \pm 0.010)(\log P - 1) - 3.953 \pm 0.003$$

$$I = (-2.569 \pm 0.015)(\log P - 1) - 4.738 \pm 0.004$$

$$J = (-2.786 \pm 0.008)(\log P - 1) - 5.225 \pm 0.002$$

$$H = (-2.916 \pm 0.007)(\log P - 1) - 5.599 \pm 0.002$$

$$K = (-2.971 \pm 0.004)(\log P - 1) - 5.660 \pm 0.001$$