# Reddenings, Distances and Luminosities

of a new sample of Galactic Cepheids for probing the Milky Way and calibrating the extra-galactic distance scale

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Abstract

**Aim:** Calibration of BVIJHK Galactic Leavitt Law by determining systematic errors in reddening and distance of individual Cepheids.

**Method:** Inspired by Madore's (2017) Leavitt Law calibration algorithm, this research compares the systematic errors yield by four versions of Wesenheit functions based on (B-J), (B-K), (V-J) and (J-K) color indicies. Starting with residual correlation of period-luminosity relations with period-wesenheit relations, bandwise extinction error for given distance moduli trails being calculated for each of the four cases. Distance error trail with the least variance in reddening error across the bands selected as the systematic error pair and adjusted to the luminosities. Calibration with (B-K) and (V-J) based wesenheit yields the tightest Leavitt Law for all the bands. The results demonstrate improved internal consistency in the near-infrared bands and contribute to a more precise calibration of the extragalactic distance scale—thus reinforcing the reliability of the cosmic distance ladder as a tool for precision cosmology.

**Result**: The calibrated BVIJHK Leavitt Laws using 95 Galactic Cepheids are as follows.

**Leavitt Law: BK based**

$$M_B = -1.86(\log P - 1)(\pm 0.011) - 3.22(\pm 0.003)$$
$$M_V = -2.26(\log P - 1)(\pm 0.003) - 3.95(\pm 0.001)$$
$$M_I = -2.57(\log P - 1)(\pm 0.014) - 4.74(\pm 0.004)$$
$$M_J = -2.79(\log P - 1)(\pm 0.011) - 5.22(\pm 0.003)$$
$$M_H = -2.92(\log P - 1)(\pm 0.011) - 5.60(\pm 0.003)$$
$$M_K = -2.97(\log P - 1)(\pm 0.011) - 5.65(\pm 0.003)$$

**Leavitt Law: VJ based**

$$M_B = -1.85(\log P - 1)(\pm 0.009) - 3.22(\pm 0.003)$$
$$M_V = -2.26(\log P - 1)(\pm 0.010) - 3.95(\pm 0.003)$$
$$M_I = -2.56(\log P - 1)(\pm 0.018) - 4.73(\pm 0.005)$$
$$M_J = -2.78(\log P - 1)(\pm 0.010) - 5.22(\pm 0.003)$$
$$M_H = -2.92(\log P - 1)(\pm 0.014) - 5.60(\pm 0.004)$$
$$M_K = -2.97(\log P - 1)(\pm 0.014) - 5.65(\pm 0.004)$$

VIJK Leavitt Law calibrated with (V-J) based wesenheit yields the distances to LMC and SMC as $18.378 \pm 0.114$ and $19.070 \pm 0.032$, respectively.

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# 1 Pulsating Star - Classical Cepheid

## 1.1 Are all stars the same?

Seeing the night sky, one can easily spot that stars come in different colors and luminosities, making it easy to infer that not all stars are the same. But what makes them appear different? With the advent of quantum mechanics and spectroscopic observation, this question was answered by explaining the physical processes occurring inside the star, along with changing chemical potential over the different evolutionary phases of the stars. It has been observed that some stars can be thousands of times larger than others—such as red giants and supergiants — while some undergo radial oscillations periodically, like RR Lyrae, Cepheids, and Mira variables. Some stars may form diffuse gaseous systems, such as protostars and planetary nebulae, while others could be the remnants of dead stars, like white dwarfs or neutron stars. These examples represent different phases in the life cycle of a single star. This chapter delves into these evolutionary stages, examining how astronomers classify stars, discussing their underlying physics, while focusing particularly on Cepheid variables.

### 1.1.1 Discovery of Variable stars

The year 1784 is significant in astronomy, as it was when John Goodricke observed the period variation of a star - Delta Cephei - challenging the centuries-old belief that the sky is static, with all stars being fixed and unchanging. Motivated by such a remarkable discovery, the hunt for more variable stars began. Inheriting the name from the first one, variable stars of a similar class were classified as Cepheid variables. Until the twentieth century, it was unclear to astronomers why Cepheids vary in luminosity. Some suggested a pair of eclipsing binaries [01234567], others proposed interior effects due to stellar magnetic fields [01234567], while some supported the idea of radial pulsation in the star's envelope [01234567]. A notable contribution from Arthur Eddington played a central role in the development of the pulsation theory. Mimicking the physics of a reversible heat engine and conceptualized through a valve mechanism (periodically blocking and releasing heat), Eddington formulated the pulsation theory of Cepheid variables named as $\kappa$-mechanism.

Further developments by Chandrasekhar in the hydrodynamic and magnetohydrodynamic stability provided a foundational approach for modelling stellar pulsation for different masses in various phases [01234567]. Modeling chemical composition and drawing inspiration from Eddington's valve mechanism for stellar pulsations, Zhevakin, in 1953, proposed that the cyclical ionization of helium envelope drives the radial oscillations observed in Cepheid variables (Zhe63). He modeled a spherical shell consisting of an 85%–15% hydrogen–helium mixture (by number of atoms), where helium transitions between the transparent HeI and opaque HeIII states. These ionization transitions modulate the opacity of the stellar envelope, leading to variations in surface brightness and pulsation behavior in Cepheids.

Building upon Chandrasekhar's framework, Cox work on stability of Cepheid variable utilising hydrodynamic theory with thermal condition to yield the equation of state of such radially oscillating system. [01234567] His contibution through non-adiabatic pulsation theory explained $\kappa$- mechanism - how ionized Helium layers changes opacity causing instability region in Hertzsprung-Russell diagram called instability strip.

Over the course of their lifetimes, stars vary in brightness and luminosity depending on their evolutionary phase. Hertzsprung was the first to plot the brightness of a star cluster against its color, revealing an underlying evolutionary trend. Soon after, Russell independently reached similar conclusions. In their honor, this diagram is now called the Hertzsprung-Russell (HR) diagram.

## 1.2 Stellar Evolution

Stars are classified into three broad categories based on their mass: low-mass, intermediate-mass, and massive stars. These categories follow different evolutionary tracks over their lifespan. As the accumulated matter in a star contracts gravitationally, it reaches threshold densities and temperatures, at which point nuclear fusion reactions occur in a natural sequence, starting with hydrogen (1 proton) and progressing to silicon (14 protons + 14 neutrons). The transformation of energy from mass to radiation, heat, neutrino emission, gravitational waves, and other forms is well modeled by quantum physics. In this section, I will briefly summarize the physical processes occurring during the different phases of evolution for each category of star. Following this, we will develop an understanding of the physics behind the pulsations of Cepheid variable stars.

At the point of maximum curvature, the first hydrostatic core forms, with temperatures ranging from 60 to 100 K which halts the continous gravitational collapse due to thermal pressure. According to the virial theorem, as the mass increases, the temperature continues to rise. Once it reaches around 2000 K, molecular hydrogen begins to dissociate which is an endothermic reaction. Continous gravitational collapse temprarity balanced by outward thermal pressure achieving hydrostatic equilibrium. At this stage, a *Hary-Halo object* forms at the core, surrounded by an accretion disk

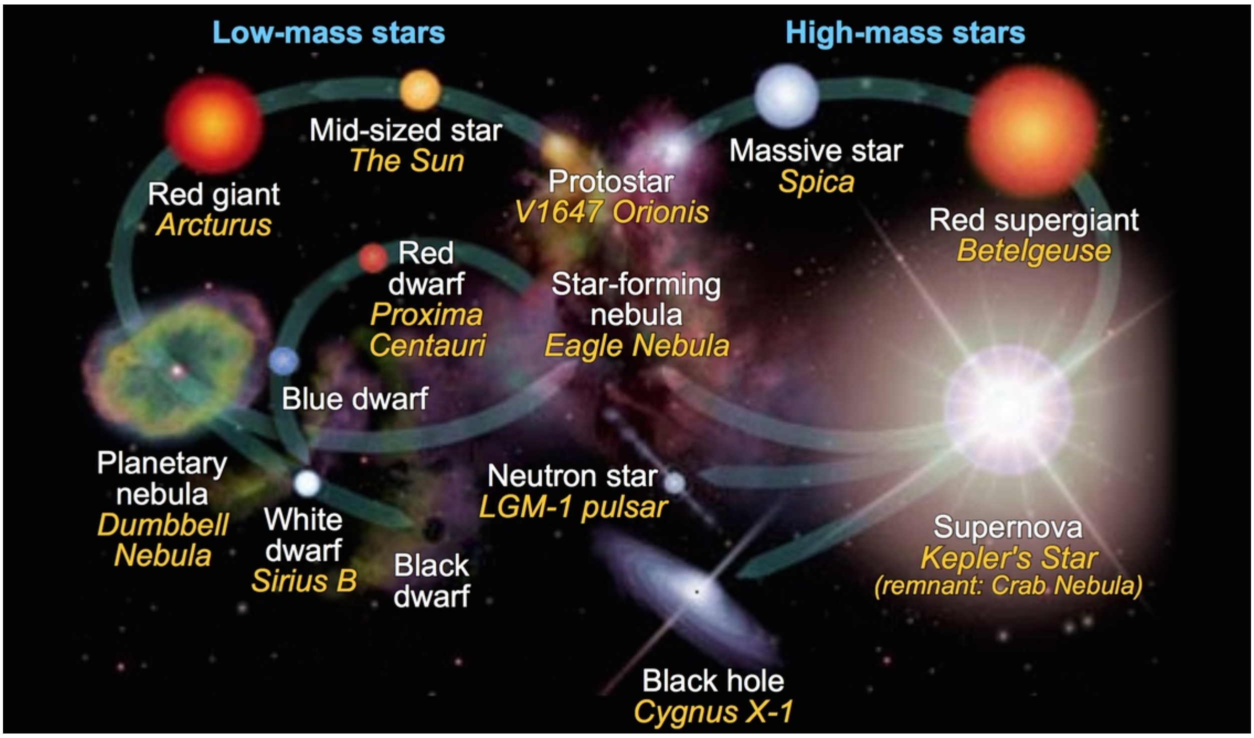


Figure 1.1: Evolutionary cycle for low and high mass star. Low mass stars, like Sun, become a Red Giant when Hydrogen supply to the core stops and fusion takes place in the shell around the core. Afterwards they become a Planetary nebula with a bright core at the center and end their life as a White Dwarf, slowly get faded turning into brown dwarf. Massive stars evolve to Red Supergiant and end their life as Supernova leaving behind either a Neutron star or Black Hole. [cmglee/NASA Goddard Space Flight Center]

of matter that follows the *tendex line* — the trajectory of matter falling into the central core.

Nuclear fusion requires physical contact between atomic nuclei, but they repel each other due to their positive charges. This repulsive force, known as the Coulomb barrier, prevents fusion under normal conditions. To overcome this barrier, nuclei must achieve extremely high kinetic energy, which occurs at the high temperatures and pressures in a star's core. When fusion begins in a protostar—converting four hydrogen atoms into one helium atom—this marks the birth of a star, known as a main sequence star.

The first basic nuclear reaction, the formation of Deuterium (nucleus containing one proton and one neutron), requires a temperature of 10 - 18 million Kelvin. In this senario, two protoniums (hydrogen ions) collide with each other at a very high velocity, surpassing the Coulomb barrier and entering to field of strong nuclear force at femto scale. To stablise the new configuration, one of the proton changes its up quark (+1/3) to down quark (-2/3), releasing positron and an electron antineutrino, to converts into a neutron. This process is called as $\beta^+$ decay process. Both elementary particles, proton and neutron, stick together to form nucleus of Deuturium. Series of such nuclear reactions release an immense amount of energy radiating outwards. Radiation pressure counter balances the continous gravitation collapse of the matter, sustaining the spherical structure of the protostar for thousand to million years. The energy released from fusion reactions in stars calculated by Bethe

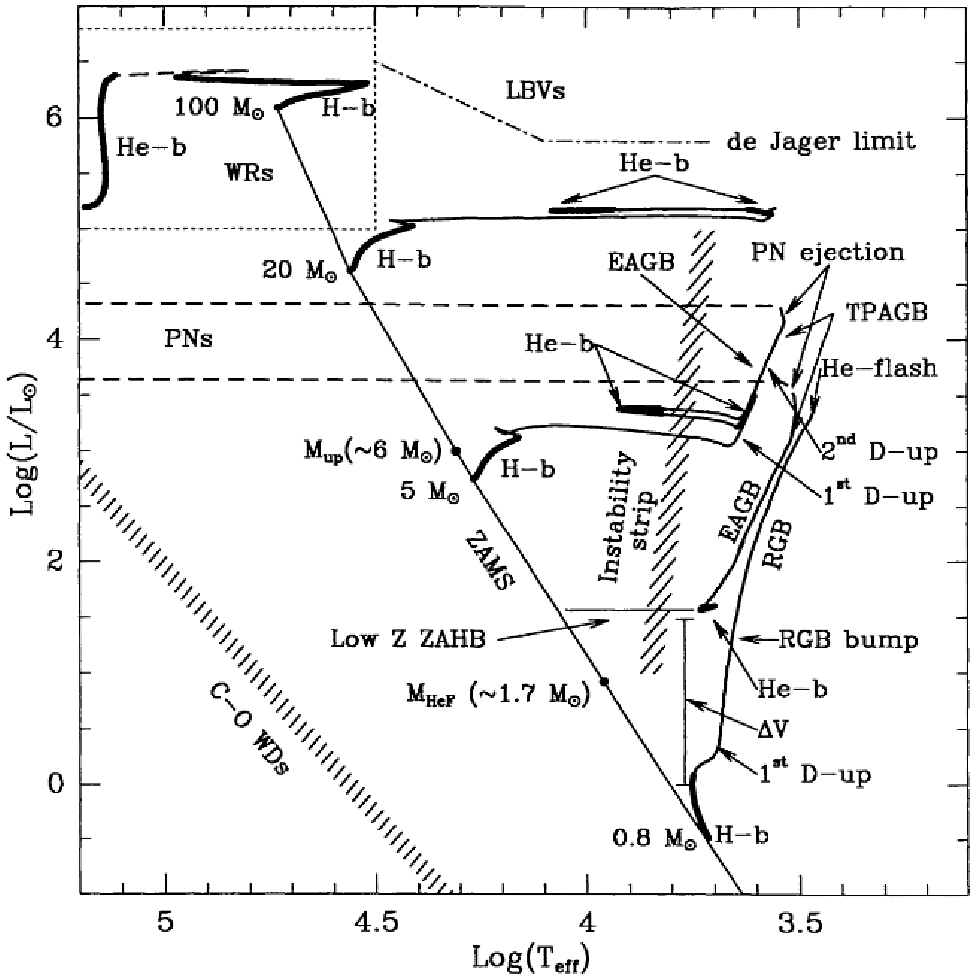


Figure 1.2: (CBB92) HRD

and published under title - 'Energy production in stars'(Bet39). At the heart of these calculation, Einstein's famous equality $E = \Delta mc^2$ underlies which tranforms mass into energy. Here, c is the speed of light and $\Delta m$ is the difference between the mass of reactants and products of the fusion reactions.

Note 1.2.1: Nuclear Fusion Reactions in Main Sequence Star

Accumulation of Deuterium opens possibility for fusing with one more proton to form Helion (2 protons and 1 neutron). Rising temperature fuses two Helion atoms to form Helium (2 protons and 2 neutrons) and emits two Hydrogen atoms. This series of reactions from Hydrogen to Helium is called p-p chain (pp I branch) reaction. Other rarer reactions can also occur, for instance, pp II branch when Helion fuses with Helium to form Beryllium (4 protons and 3 neutrons). Then either Beryllium decays to Lithium (3 protons and 4 neutron), Lithium fuses with proton and breakdown into two Helium atoms. Otherwise, in pp III branch, Beryllium fuses with proton to form Boron (4 proton and 4 neuton) which is unstable and breaks down into two Helium. The most rare case is pp IV branch, when Helion captures a proton and form Helium.

Proton-Proton Reactions: Efficient for 1 $M_0$ star's core.

Branch I: 83.30% Helium production, at 10-18 MK

$$^{1}\mathrm{H} \xrightarrow[\mathrm{e}^{+},\nu_e]{\mathrm{p}} {}^{2}\mathrm{D} \xrightarrow[\gamma]{^{1}\mathrm{H}} {}^{3}\mathrm{He} \xrightarrow[2\,^{1}\mathrm{H}]{^{3}\mathrm{He}} {}^{4}\mathrm{He}$$

0.42 5.49 12.85 (Energy release in MeV)

Branch II: 16.70 % Helium production, at 18-25 MK

$$^{4}\mathrm{He} \xrightarrow[\gamma]{\mathrm{He}} {}^{7}\mathrm{Be} \xrightarrow[\nu_e]{\mathrm{e}^{-}} {}^{7}\mathrm{Li} \xrightarrow{\mathrm{H}} 2\,^{4}\mathrm{He}$$

1.59 0.861 17.35 (Energy release in MeV)

Branch III: 0.12% Helium production, above 25 MK

$$^{4}\mathrm{He} \xrightarrow[\gamma]{\mathrm{He}} {}^{7}\mathrm{Be} \xrightarrow[\gamma]{\mathrm{H}} {}^{8}\mathrm{B} \xrightarrow[\mathrm{e}^{+},\nu_e]{} {}^{8}\mathrm{Be} \longrightarrow 2\,^{4}\mathrm{He}$$

1.59 5.59 23.29 (Energy release in MeV)

Branch IV: 0.00002% Helium generation in Sun

$$^{3}\mathrm{He} \xrightarrow[\mathrm{e}^{+},\nu_e]{\mathrm{H}} {}^{4}\mathrm{He}$$

19.795 (Energy release in MeV)

All these four branches are called pp chain reaction.

The fusion of elements releases vast amounts of heat, radiation and neutrino flux. A star's evolution depends primarily on its initial mass. Massive stars experience stronger gravitational pressure, higher core temperatures, and faster fusion rates, causing them to burn through their fuel more quickly and live shorter lives compared to lower-mass stars. Stars progress through several phases during their evolution, driven by their mass, as shown in Figure 1.1.

### 1.2.1 Main sequence star and Turn-off point

The color of the star gives a rough idea about the surface temperature as well as evolutionary phase of the star. Newly born stars are rich in Hydrogen and fuse it rigorously resulting very high surface temperature making it appear bluer. After millions of years, helium as byproduct accumulates at the core which blocks the supply of Hydrogen and reduces the rate of fusion due to which star cools down and turn yellowish. It is worth to say that, high mass stars exert high pressure at core and burn its fuel quickly as compare to low mass stars. Where lifespan of a high mass star (8-10 $M_\odot$ ) lies around 10-15 million years, for low mass star it can be 15-20 billion years. In its life journey, any star spends most of the time in Main Sequence phase.

For the case, when the initial mass of star remained under half of solar mass, after a few billion years, enough Helium get depleted at the core which blocks the hydrogen supply, ultimately reducing the rate of fusion reaction. Star gets cooler with time, finally become redder then a brown dwarf and its life as a black dwarf after trillions of year. Such star never leaves the main sequence but only changes its spectral type as it evolves. For star of one solar mass, when Helium core forms which reduces the rate of fusion reaction, the gravitation pull begins to contract the core and core's temperature increases again. With higher temperature, Hydrogen begins to fuse around the Helium core in a shell, making the star move away from the main sequence and marks the turn-off point in the HR diagram.

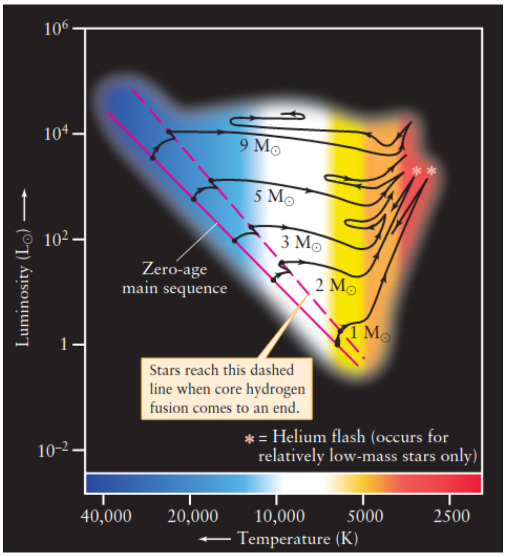


Figure 1.3: Isochrones of stars with different masses depicting their evolutionary traids. Source: Universe (VIII edition) by Roger A. Freedman & William J. Kaufmann III

### 1.2.2 Red Giant Star and Helium Flash

Shell burning Hydrogen generates even more energy than core burning phases. Released energy pushes the envelope of matter outwards which expands the surface of the star enormously and it becomes a giant. With increased surface area and shell H-burning, luminosity of the star increases by the order 3 to 4, however, due to expansion, surface of the star cools down making it red in color. This physical characteristics gives it name - Red Giant. Further accumulation of Helium at the core due to H-shell burning continuous. Star having mass more than 3 solar mass, gradually star helium burning at the core, however stars with mass between 0.3 to 3 solar mass attain Helium fusion spontaneously, called as Helium Flash. For the latter case, central region of the core becomes incompressible but the rest of core still collapses causing further rise in temperature at center, but not sufficiently enough that helium could fuse, so the central core takes the form of degenerate matter to balance the gravitation collapse.

Around 300 million K, the degenerate matter spontaneously fuses into heavier elements, Carbon and Oxygen, by triple alpha process leaving behind a flash of high energy which rises the temperature exponentially high. Degenerate matter fuses rapidly rising the temperature even higher which accelerate the reaction rate. This thermal runaway reaction increases the energy production of the star to 100 billion times for a very short time, which is comparable to entire galaxy's energy output. Eventually thermal pressure becomes dominant and the core expands. This phase in HR diagram can be observed as the tip of the red gaint branch on the right-upper side for intermidiate mass (0.4 to 3 solar mass). Just after this state, star rearranges its structure rapidly and attain an equilibrium state with a lower luminosity but higher temperature than before. This spontaneous process shifts the star from red gaint branch to horizontal branch while leaving a discontnuity in between the two regions in HR diagram. [1]

### 1.2.3 Horizontal branch and Variable stars

While evolving through horizontal branch, stars fuse Helium at the core and Hydrogen in a shell. Moving towards right side, star enters to the blue edge of instability strip of HR diagram where it experience a dynamic equilibrium between gravitational contraction of outer envelope and its thermal expansion due to energy generation from internal processes. This translated as a radial oscillatory motion of the surface, giving birth to a new category of stars called variable stars. In 1784, John Goodricke had discovered a variable star, $\delta$ Cephei, in Cepheus constellation and named the star as $\delta$ Cephei. After the discovery of a few more variable stars, the family of $\delta$ Cephei type stars called as Cepheid stars.

Initially, it was assumed that the periodic variation of luminosity arising because the star is a member of eclipsing binary system. In 1917, Arthur Eddington rejected the prior theory and gave a clever explanation behind the radial pulsation of star by purposing a valve mechanism. His purposed mechanism known as Kappa mechanism as kappa stand for coefficient of absorption of stellar material. After excessive burn of hydrogen, helium becomes the next abundant element on the surface. Due to high temperature at the compressed state of Cepheid star, Helium releases its both electrons and get ionized $He^{2+}$. Opaque $He^{2+}$ blocks the outcoming radiation making the star appear dimmer. The radiant energy develops a pressure and expands the star's surface. The expansion increases the surface area illuminating the star. Expansion also cools down the surface so the ionized Helium captures the free electrons and back to its ground state He. Neutral Helium is a transparent gas consequently, all the trapped radiation releases making the star even more luminous. As the radiation pressure decreases, the stellar surface begin to collapse again due to gravitational pull and shrinks down to original state. This brings the process to its initial state.

[1] Kristen. B. W. McQuinn et al 2019 ApJ 880 63

### 1.2.4 AGB Stars and Planetary Nebulae

In a time scale of 100 million years, star evolve through the instability strip and exits from red edge of the instability strip towards right in HR diagram. At this stage, Helium fusion process continues in a shell around the Carbon/Oxygen core making star luminous then Red Giant phase and even bigger in size, comparable to the orbit of the Earth. In HR diagram, it moves alongside the Red Giant branch which gives it name Asymptotic Giant Branch. After Helium shell runs out of fuel, outer layer of Hydrogen burning becomes the dominant source of energy and produces Helium. After 100 thousand years, Helium accumulates and ignites again spontaneously which expands the outer surface of the star even more. Expansion cools down the temperature which halts the Hydrogen shell burning. In the next hundreds of years, Helium burning stops and Hydrogen burning take place to produce more Helium. This cycle of energy production in shells along with Helium shell flash processes expel 50% to 70% of star's mass in the interstellar space, bringing AGB stars to its end as a planetary nebula. The radius of planetary nebula can reach upto 30 light years (nearest star to the Sun, Proxima Centauri is about 4.2 lightyears away).

### 1.2.5 Super Novae and Compact Objects

After fusing Helium and Hydrogen in shell, Carbon and Oxygen rich core forms at the center. By passing through AGB and then planetary nebula phase, star release its most of mass as stellar wind and the core remains as the remnant of dead star, countering the gravitational collapse by electron degneracy pressure as there is not active nuclear fusion going on. Such an object is called White Dwarf. S. Chandrasekhar, in 1930s, developed a theory about such compact object and purposed that such object can not exceed its mass more than 1.44 times of solar mass. This number is known as Chandrasekhar limit.

As per the initial mass of the star, it could reach to three end points of its life - a) White dwarf, b) Neutron star or c) Black Hole. White dwarf is the remnant of a dead star whose core is mainly composed of degenerate matter. At this stage, there is no active nuclear fusion process going inside white dwarfs as there is not enough mass remaining to rise the core temperature for further fusion process. To sustain the system, the electron degeneracy pressure balances the gravitational collapse in White Dwarfs.

If a white dwarf exceeds its mass than 1.44 solar mass, then gravitational pull dominates the electron degeneracy pressure and spontaneous thermal runaway reaction initiates the fusion process of carbon to heavier elements like neon, oxygen, magnesium, silicon, sulfur, argon, calcium, titanium, chromium, iron to nickel. This process emits an immense amount of energy and the event is called as Type Ia Super Nova, where Ia is the luminosity class. There are other types of super novae (or novae) also occur from different range of masses of stars, however, Type Ia SNe are set as standard

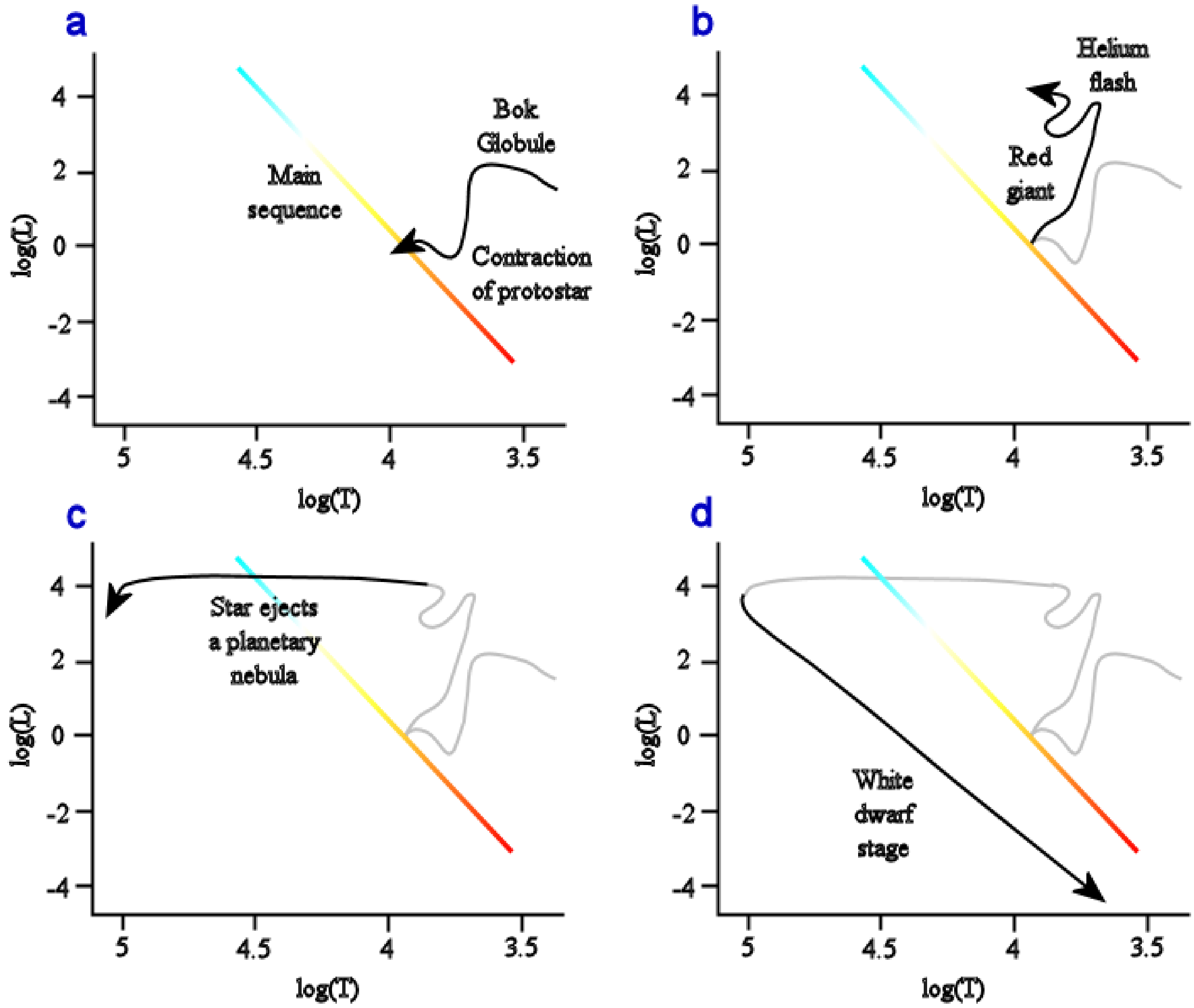


Figure 1.4: **Stellar Evolution of Sun-type star: a)** Gravitational collapse of a gas cloud forms a protostar which begin to fuse Hydrogen and arrived to the main sequence. **b)** After billions of years, star leaves the main sequence and become a Red Giant. At the tip of the Red Giant branch, star begin to fuse Helium after the episode of Helium flash and slowly moves to AGB phase **c)** Evolved star expanded upto the size of Jupiter orbit and loses its envelope to form a planetary nebula at the age of 12 billion years. **d)** Ejecting way the outer envelope, the remnant of dead star shines due to the degeneracy pressure of its electron and called as White Dwarf. Source: Stars and Nebula by William J. Kaufmann, III

candle with absolute luminosity of -19 mag approx. Formation of Neutron star and Black hole results from more massive stars compare to the progenitor of white dwarfs.

## 1.3 Pulsation of Cepheid

More classical studies done long ago by Cox, King, and Tabor (1973) show that the Cepheids will not pulsate unless the helium content is at least Y=0.25 (CKT73)

The absolute magnitude is a definite function of the period. This was first shown by Miss Leavitt from a study of the variables in the Lesser Magellanic Cloud. A full confirmation was obtained by H. Shapley from the variables in globular clusters. In the same cluster the absolute magnitude differs from the apparent magnitude by a *constant* (depending on the unknown distance of the cluster) so that the period-luminosity relation is given directly without the intervention of parallax. It is found that the period determines the absolute magnitude to within a probable error of $\pm 0^{m} \cdot 25$. Having thus found the period-magnitude curve applicable to all Cepheids except for the unknown constant, we proceed to anchor the curve by combining our knowledge of the mean luminosity of the nearer Cepheids (derived from their parallactic and cross motions) into a single mean determination of the constant.

There is a progression of spectral type in the direction from *A* towards *M* as the period (and luminosity) increases.

The Cepheids are more luminous than ordinary giant stars of the same spectral class, although some giant stars of high luminosity, called pseudo-Cepheids, are found which seem to resemble them very closely without showing any light-variation. Cepheids and pseudo-Cepheids are sometimes described as "super-giants."

Figure 1.5: (Edd26) Chapter Eight of The internal constitution of the stars.

**II. Establishment of the Nature of the Variations and the Search for the Excitation Mechanism**

The concept of radial pulsations of stellar masses as the explanation of the observed cepheid phenomena was first given a firm mathematical foundation by Eddington (1926). He assumed that the oscillations were very small and essentially adiabatic, and derived and studied solutions of the adiabatic wave equation for such oscillations. Perhaps the most important and far-reaching result of these studies was the theoretical derivation of the well-known "period-mean density" relation which is apparently obeyed by actual pulsating stars:

$$\Pi(\bar{\rho}/\bar{\rho}_{\odot})^{\frac{1}{2}} = Q \quad , \tag{1}$$

where $\Pi$ is the period; $\bar{\rho}$ and $\bar{\rho}_{\odot}$ are the mean densities of, respectively, the star and the sun; and $Q$ is the "pulsation constant," a slowly varying function of the properties and internal structure (particularly the central mass concentration) of the stellar model.

Figure 1.7: (? ) Understanding pulsation mechanism in fourty years

*If the leakage of heat decreases during compression and increases during expansion, then driving of pulsation is possible.* (Noe98)

* The chief arguments against the binary theory and in favour of the pulsation theory were put forward by H. Shapley (*Astrophys. Journ.* **40**, p. 448 (1914)). The pulsation theory was previously advocated by H. C. Plummer, chiefly because deviations from elliptic motion were detected of a kind impossible to ascribe to gravitational perturbations by a third body (*Monthly Notices*, **73**, p. 665; **74**, p. 662).

Figure 1.6: (Edd26) abandoning eclipsing binary theory.

### 1.3.1 Light Curve

progression of curve shapes with period is called Hertzsprung sequence (SL81)

### 1.3.2 Kappa Mechanism

The kappa mechanism is a key process that drives the pulsations in Cepheid stars.

It operates within the outer layers of the star, specifically in the partially ionized regions where the opacity, represented by the Greek letter kappa ($\kappa$), changes significantly with temperature.

The pulsations in Cepheids are characterized by radial expansion and contraction of the outer layers, causing the star to periodically brighten and dim.

The kappa mechanism is responsible for triggering these pulsations by influencing the balance between gravity and radiation pressure in the star's interior.

Opacity and Temperature Sensitivity: Opacity refers to the ability of a medium to absorb and scatter radiation. In Cepheid stars, the opacity is affected by the degree of ionization within the outer layers. At certain temperatures, the opacity experiences a sharp increase, leading to a localized region called the opacity bump.

Expansion and Contraction: During the pulsation cycle, the outer layers of the Cepheid star expand during maximum brightness and contract during minimum brightness. As the star expands, it cools down due to the decrease in temperature. Conversely, during contraction, the temperature increases.

Partial Ionization: As the star expands, the temperature decreases in the outer layers, causing some previously ionized atoms to recombine with electrons, resulting in partial ionization. This ionization state is crucial for the kappa mechanism to operate effectively.

Opacity Bump Effect: Within the partially ionized regions of the Cepheid star, the temperature reaches a point where the opacity is relatively high. This increased opacity affects the balance between gravity and radiation pressure. When the opacity is high, the radiation pressure becomes less effective at counteracting gravity, causing the outer layers to contract.

Energy Transfer: The contraction of the outer layers increases the temperature, reducing the degree of ionization. As a result, the opacity decreases, allowing the radiation pressure to become more efficient. This increased radiation pressure then pushes against gravity, causing the outer layers to expand.

Feedback Loop: The expansion of the outer layers leads to a decrease in temperature, triggering recombination and an increase in opacity. This increased opacity restricts radiation pressure, causing the outer layers to contract again. This cycle repeats, creating the pulsations observed in Cepheid stars.

## 1.4 Hydrogen Ionization Front:

At base of HIF, opacity increases sharply limiting the depth of Cepheids photosphere. Hydrogen ionizes at 6000K (Kan95)

Motion HIF over envolopes (KM71)

## 1.5 Leavitt Law

To obtain the total luminosity LL of a star (total power emitted across all wavelengths and directions), you integrate over the star's entire surface area and over all wavelengths:

$$L = \int_0^\infty (4\pi R^2 F_\lambda(T))d\lambda \tag{1.1}$$

$$=4\pi^2 R^2 \int_0^\infty B_\lambda(T)d\lambda \tag{1.2}$$

$$=4\pi R^2 \sigma T^4 \tag{1.3}$$

On plotting a scatter plot in between period and luminosity,a linear relation in between both the parameters with a certain scatter

The period-luminosity relation (also known as the Leavitt Law or the Cepheid period-luminosity relation) is an empirical relationship that exists between the pulsation period and the intrinsic luminosity of Cepheid variable stars. This relationship allows astronomers to determine the distance to Cepheids and calibrate the cosmic distance ladder.

The period-luminosity relation was first discovered by American astronomer Henrietta Leavitt in the early 20th century while studying the brightness variations of Cepheid stars in the Small Magellanic Cloud. Leavitt found that there was a consistent relationship between the periods and the average luminosities of these stars.

The period-luminosity relation states that the longer the period of a Cepheid star's pulsation, the more luminous it is. In other words, there is a direct correlation between the pulsation period and the intrinsic brightness of the star. This relationship allows astronomers to use the observed period of a Cepheid star to determine its intrinsic luminosity.

Once the intrinsic luminosity is known, the apparent brightness (or magnitude) of the star can be measured. By comparing the intrinsic luminosity with the apparent brightness, astronomers can calculate the distance to the Cepheid star using the inverse square law of light.

The period-luminosity relation is valuable because it provides a reliable and relatively straightforward method for determining distances to Cepheids and, subsequently, to other celestial objects. By measuring the period of a Cepheid star's pulsation, astronomers can estimate its intrinsic luminosity, and from that, they can determine its distance by comparing it to the observed brightness.

The period-luminosity relation has been refined over the years through extensive observations and analysis of Cepheid stars in various galaxies. It has become an essential tool for measuring cosmic distances and has played a vital role in our understanding of the scale of the universe, the expansion rate of the universe (Hubble's Law), and the calibration of other distance indicators, such as Type Ia supernovae.

# Part II

# Calibration Method and Dataset

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