GALACTIC BVIJHK LEAVITT LAW CALIBRATION

Shubham Mamgain[1], Dr. Jesper Storm[2], and Prof. Dr. Maria Rosa Cioni[2]

[1]*Institute of Physics and Astronomy, University of Potsdam*
[2]*Dwarf Galaxy and Galactic Halo, AIP Potsdam*

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Abstract

From the definition of composite wesenheit magnitudes reconstructed the calibration method (Madore) for refining the Galactic Leavitt Laws (LLs) using six-bands photometry. True absolute magnitude of 96 Galactic Cepheids derived using extinction law (Fouque) and IRSB distances (Jesper); also compared results with Gaia parallax driven luminosities. Calibrated K-band relation with IRSB and Gaia distances are:

$K_{IRSB} = -3.016771(logP - 1)(\pm 0.023477) + -5.693494(\pm 0.007047)$
$K_{Gaia} = -2.977082(logP - 1)(\pm 0.074146) + -5.884299(\pm 0.022296)$

Keywords: Reddening: Leavitt Law: Wesenheit Magnitude:

Symbols and their descriptions

Symbol	Description	Symbol	Description
m_λ	Apparent Magnitude	α_λ	Period Luminosity Slope
μ	Distance Modulus	γ_λ	Period Luminosity Intercept
M_λ	Absolute Magnitude	Δ_λ^{12}	Period Wesenheit Residuals
E_{BV}	Interstellar Reddening	α_λ^{12}	Period Wesenheit Slope
A_λ	Interstellar Extinction	γ_λ^{12}	Period Wesenheit Intercept
M_λ^0	True Absolute Magnitude	$\Delta_{\kappa\lambda}^{12}$	PL-PW Correlation Residuals
R_λ^{12}	Selective-to-total Absorption	$\rho_{\kappa\lambda}^{12}$	PL-PW Correlation Slope
W_λ^{12}	Wesenheit Magnitude	$\delta E_{\kappa\lambda}^{12}$	Reddening Correction
Δ_λ	Period Luminosity Residuals	$\delta_{\kappa\lambda\nu..}^{123..}$	Difference Operator

1. Definitions

1.1 Luminosity, Distance and Reddening

Luminosity is the measurement of incoming light flux within a passband, here BVIJHK bands. Light emitted by the stars M^0_λ travel through the large distances μ, pass over the interstellar medium (ISM) A_λ and ultimately reach to the observer's detector m_λ. In this framework, say, star's true absolute magnitude is M^0_λ, then equality for observed apparent magnitude would be:

$$m_\lambda = \mu + A_\lambda + M^0_\lambda$$

On the right hand side, the first term is called distance modulus, μ. In my dataset of 95 Galactic Cepheids, distances to each star calculated by two independent methods: i) InfraRed Surface Brightness (Jesper, 2011) and ii) Parallax (Gaia DR3, 2023). For the ease of calculation, distances measured in the unit of parsec get converted into dimensionless (logarithmic) units called magnitude.

$$\mu = 5 \log D[pc] - 5$$

ISM scatters, absorbs, and emits photons according to the chemical compositions, size and abundance of the particles, imprinting its signature on the spectrum of light at specific energies (or wavelengths). The reduced intensity of light within a band due to interaction with ISM is termed as interstellar extinction.

$$A_\lambda = m_\lambda - m^0_\lambda$$

Extinction over the wavelength, for individual Cepheid, can also be determined from the measurement of color excess E_{BV} along the line-of-sight (Fernie, 1995). Using visual Galactic reddening ratio $R_V = 3.23$ (Sandage, 2004) in the framework of Galactic extinction law, A_λ/A_V (Fouque, 2007), interstellar extinction for BVIJHK bands can be estimated as follow:

$$A_\lambda = \frac{A_\lambda}{A_V} \times R_V \times E_{BV}$$

Here, the last factor - color excess E_{BV} measures the relative extinction difference between any two bands. In other terms it can also be formulated as the deviation of observed color index (B-V) from 'true' color index $(B-V)_0$:

$$\begin{aligned} E_{BV} &= (B-V) - (B-V)_0 \\ &= (B-B_0) - (V-V_0) \\ &= A_B - A_V \end{aligned}$$

The middle factor - reddening ratio, $R_V = A_V/E_{BV}$, is extinction-to-reddening ratio in visual band. In the most generalized form, if the color excess between two bands written as $E_{12} = A_{m_1} - A_{m_2}$, then the reddening ratio would be written as:

$$R_\lambda^{12} = A_\lambda/E_{12}$$

Color excess from one combination of bands E_{12} can be transformed into any other bands combination as follows:

$$\begin{aligned} E_{12} &= A_1 - A_2 \\ &= R_1^{BV} * E_{BV} - R_2^{BV} * E_{BV} \\ &= (R_1^{BV} - R_2^{BV}) * E_{BV} \end{aligned}$$

This leads the transformation law for reddening ratio R:

$$R_\lambda^{12} = \frac{R_\lambda^{BV}}{R_1^{BV} - R_2^{BV}}$$

Value of R_λ^{BV} are calculated from Fouque (2007) with $R_V^{BV} = 3.23$ (Sandage, 2004):

$$R_\lambda^{BV} = \frac{A_\lambda}{A_V} \times R_V^{BV}$$

Band	R_λ	Band	R_λ
B	4.2313	J	0.94316
V	3.23	H	0.58463
I	1.96384	K	0.38437

1.2 Wesenheit Magnitude

Since reddening ratio, R_λ^{12}, measures the impact of interstellar medium on the incoming light, the ratio itself can be used for defining reddening-free magnitude corresponding to true absolute magnitude. It follows as :

$$\begin{aligned} R_\lambda^{12} &= A_\lambda / E_{12} \\ &= \frac{m_\lambda - m_\lambda^0}{(m_1 - m_2) - (m_1 - m_2)_0} \end{aligned}$$

On rearranging the terms, one can get

$$m_\lambda - R_\lambda^{12}(m_1 - m_2) = m_\lambda^0 - R_\lambda^{12}(m_1 - m_2)_0 \tag{1}$$

$$W_\lambda^{12} = W_0 \tag{2}$$

Comparing the definitions of 'true' absolute magnitude with wesenheit magnitude with respect to color index (B-V):

$$\begin{aligned} M_\lambda^0 &= m_\lambda - R_\lambda^{BV} * E(B - V) - \mu \\ W_\lambda^{BV} &= m_\lambda - R_\lambda^{BV} * (B - V) - \mu \end{aligned}$$

Differentiating above equations. Say, apparent luminosity are precisely measured ($\delta m_\lambda \to 0$), adopted extinction law is correct ($\delta R \to 0$), might distance modulus ($\delta\mu$) and color excess (δE_{BV}) measurements could have error in the case of individual Cepheid. It yields

$$\begin{aligned} \delta M_\lambda &= -(R_\lambda^{BV} * \delta E(B - V) + \delta\mu) \\ \delta W_\lambda^{BV} &= -\delta\mu \end{aligned}$$

This pair of equations suggests that the wesenheit magnitude and absolute magnitude both are sensitive to distance error, though unlike absolute magnitude, wesenheit magnitude is not affected by error in reddening. It is an important feature of wesenheit magnitude which aids in decoupling the error budget of distance and reddenings, when applied to Cepheids.

2. Galactic Cepheids: Leavitt Law

Classical Cepheids follow a linear trend between their pulsation period (logP) and true luminosity (M_λ^0) which can be modelled as :

$$\bar{M}_\lambda = \alpha_\lambda(logP) + \gamma_\lambda$$

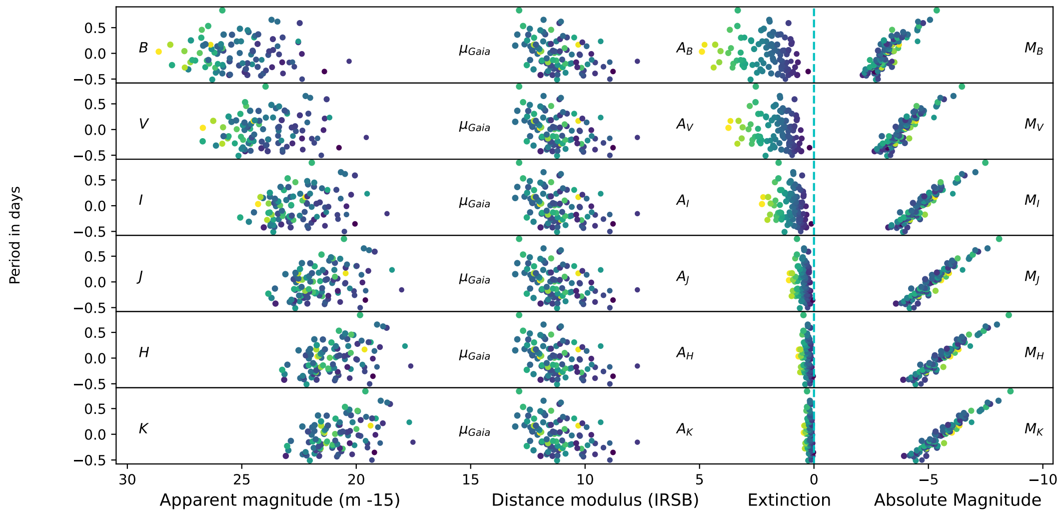


Figure 1: BVIJHK Leavitt Law for 95 Cepheids. Y-axis is period, X-axis starts with apparent magnitude (m) shifted by 15 mag, then distance modulus, calculated extinction in each band and right most is the absolute magnitude.

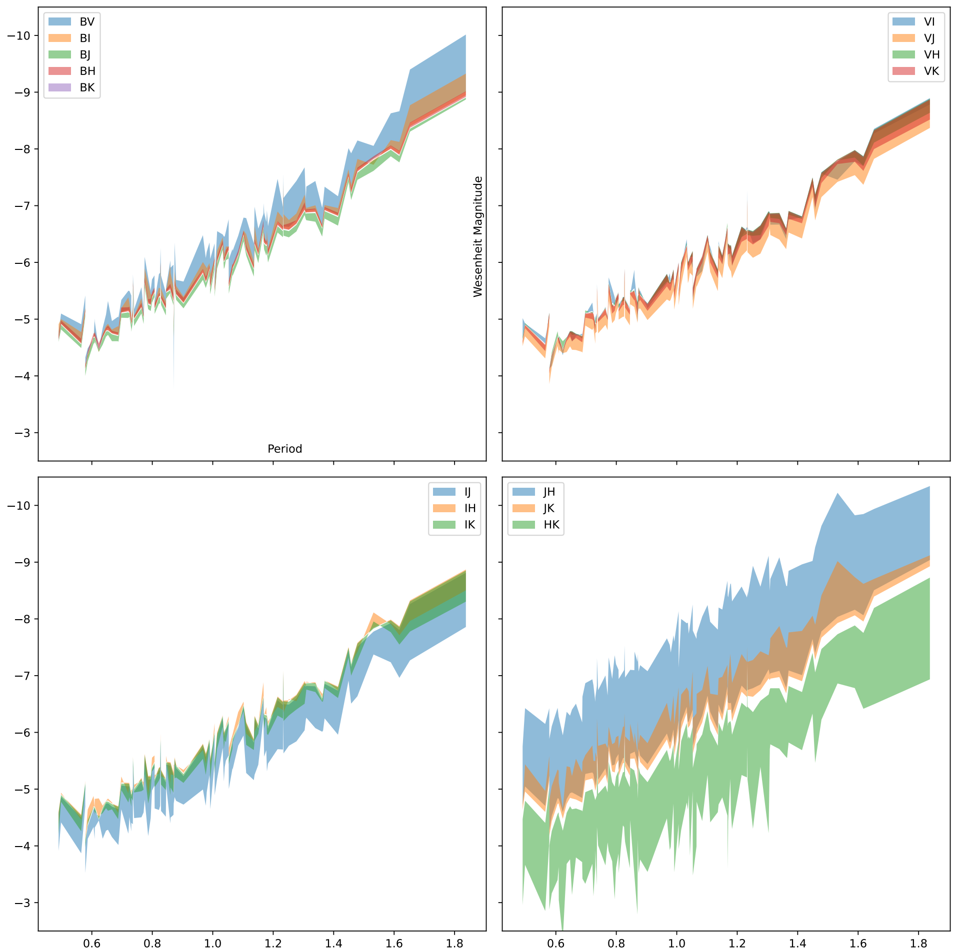


Figure 2: BVIJHK Period Wesenheit relations with respect to different color indexes. Scattered colored region covers area between PW_B and PW_K relations.

The model equation yields the absolute magnitude ($\bar{M}_\lambda$) of any distant Cepheid from its pulsation period. My sample data contains BVIJHK bands photometry of a sample of Galactic Cepheids. Resulting 6 Leavitt Laws are modelled by two parameters: slope (α_λ) and intercept (γ_λ). Wesenheit magnitude based Leavitt Law could be derived by choosing a reference color index ($m_1 - m_2$). For each band, reddening free instances of Leavitt Laws would be modelled as:

$$\bar{W}_\lambda^{12} = \alpha_\lambda^{12}(logP) + \gamma_\lambda^{12}$$

In this way, there can be 15 pair combinations of color indexes for BVIJHK photometry. For six observed bands with each color index gives 90 composite wesenheit magnitudes (W_λ^{12}).

The deviation of observed luminosity (M_λ^0) from the modelled value ($\bar{M}_\lambda$) is denoted by ΔM_λ

$$\Delta_\lambda = M_\lambda^0 - \bar{M}_\lambda$$

Similarly, deviation of wesenheit luminosity from the PW relation would be:

$$\Delta_\lambda^{12} = W_\lambda^{12} - \bar{W}_\lambda^{12}$$

To get an overview, two kinds of magnitudes calculated for each band. The resulting magnitudes (6+90) are correlated with the pulsation period of Cepheids which yields a linear equation of two variables with slope, intercept and residual of each Cepheid from the model line.

$$\text{PL relation: } \alpha_\lambda \mid \gamma_\lambda \mid \Delta_\lambda \mid \implies 6$$
$$\text{PW relation: } \alpha_\lambda^{12} \mid \gamma_\lambda^{12} \mid \Delta_\lambda^{12} \implies 90$$

2.1 PL-PW Residual

Crucial difference between PW and PL relation is PW driven parameters are free from reddening errors, but not the PL ones. Correlation of PW vs PL residues ($\Delta - \Delta$) gives quantitative insights about the reddening and modulus error budget. To develop a mathematical framework, two cases need to be considered:

i) When there is error only in distance, ($\delta\mu \neq 0$, $\delta E_{12} = 0$):

$$\text{Deviation in PL} : \Delta_\lambda + \delta\mu$$
$$\text{Deviation in PW} : \Delta_\lambda^{12} + \delta\mu$$

Error in distance deviates the star along the both axes equally from its original position, it means data with least / no reddening error would perfectly aligned to the line with slope 1. This can be observed with infrared bands as reddening errors must be the least for longer wavelength.

ii) When there is error only in reddening, ($\delta E_{12} \neq 0$, $\delta\mu = 0$):

$$\text{Deviation in PL} : \Delta_\lambda + \delta E_{12}$$
$$\text{Deviation in PW} : \Delta_\lambda^{12}$$

In the reddening error case, only PL relation got affected but not the PW relation, as the wesenheit magnitude is reddening independent by definition. It means, star would move along the PL residue axis, but not to the other one. This implies *vertical shift of the star in $\Delta - \Delta$ plots comes from the reddening error.*

3. Δ_λ - Δ_W Correlation

In the generalised form, residuals of PL relations (ΔM_λ) plotted against the PW residuals (ΔW_κ^{12}), yielding slopes for the regression lines ($\rho_{\kappa\lambda}^{12}$) with zero intercept. Residuals with respect to regression line is denoted by:

$$\Delta_{\kappa\lambda}^{12} = \Delta_\lambda - \rho_{\kappa\lambda}^{12} \times \Delta_\kappa^{12}$$

This vertical deviation from the regression line estimates the extinction correction required in the respective band, without considering any correction in the distance modulus.

$$\delta A_{\kappa\lambda}^{12}(0) = \Delta_{\kappa\lambda}^{12}$$

On normalising with R_λ^{12}, extinction corrections ($\delta A_{\kappa\lambda}^{12}$) transformed into reddening corrections $\delta E_{\kappa\lambda}^{12}$ as follow:

$$\delta E_{\kappa\lambda}^{12}(0) = \frac{\delta A_{\kappa\lambda}^{12}}{R_\lambda^{12}}$$

Assuming adopted Galactic extinction law is true and distances of individual targets are precisely known. For such case, reddening is the only source of error when estimating absolute magnitude and the corrections suggested by each band must be agreeing with each other.

$$\delta E_{\kappa B}^{12} = \delta E_{\kappa V}^{12} = ... = \delta E_{\kappa K}^{12} = \delta E_{12}^*$$

For the correct distances, there will be no dispersion in reddening corrections deduced from different bands. However, existing deviation in estimated reddening corrections from different bands suggest a correction in distance modulus.

3.1 Distance - Reddening Correction

To estimate the error in distances, a range of correction ($\delta\mu$) possibilities are assumed. For each possibility, reddening corrections for each band get estimated. That particular modulus correction for which estimated reddening corrections being consistent in all the bands (or with least variation); physically, that will be the most optimal correction solution pair.

Variation in extinction ($\delta A^{12}_{\kappa\lambda}$) due to distance modulus correction would be calculated as following:

$$\begin{aligned}\delta A^{12}_{\kappa\lambda}(\delta\mu) &= (\Delta_\lambda + \delta\mu) - \rho^{12}_{\kappa\lambda} \times (\Delta^{12}_\kappa + \delta\mu)\\ &= \delta A^{12}_{\kappa\lambda}(0) + \delta\mu(1 - \rho^{12}_{\kappa\lambda})\end{aligned}$$

Corresponding reddening corrections would be

$$\delta E^{12}_{\kappa\lambda}(\delta\mu) = \delta E^{12}_{\kappa\lambda}(0) + \frac{\delta\mu(1 - \rho^{12}_{\kappa\lambda})}{R^{12}_\lambda}$$

A suitable distance-reddening correction pair determined by selecting that value of $\delta\mu$ for which dispersion in estimated reddening corrections from BVIJHK bands would be the least.

$$RMS(\delta E(\delta\mu)) = \frac{1}{6}\sum_\lambda (\delta E^{12}_{\kappa\lambda}(\delta\mu) - < \delta E^{12}_{\kappa\lambda}(\delta\mu) >_\lambda)^2$$

$$min(RMS(\delta E(\delta\mu))) \implies (\delta\mu^*, \delta E^*_{BV})$$

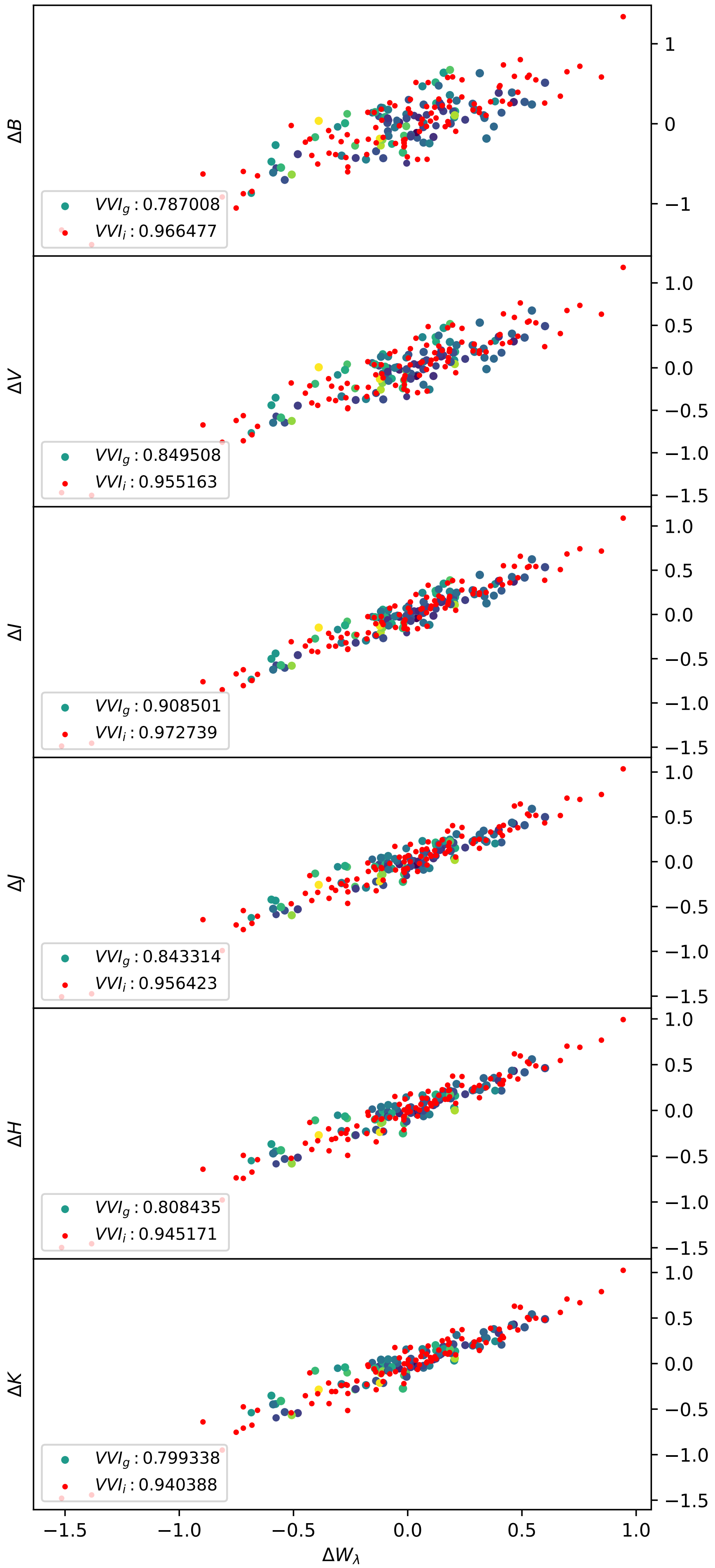
These corrections will be adjusted with the original data as follow.

$$M^*_\lambda = M^0_\lambda + \delta A^*_\lambda + \delta\mu^*$$

3.2 Madore Approach

In his analysis, Madore (2017) used W^{VI}_V as a reference for reddening free magnitude. He correlated the PW residuals (Δ^{VI}_V) with BVRIJHK PL residuals (Δ_λ) which corresponds to slopes ($\rho^{VI}_{V\lambda}$) and residuals ($\Delta^{VI}_{V\lambda}$).

Say, $\delta\mu^*$ is the correction required in distance modulus, then extinction corrections derived by Madore (2017) are:


ΔB
ΔV
ΔI
ΔJ
ΔH
ΔK
ΔW_λ
$VVI_g: 0.787008$
$VVI_i: 0.966477$
$VVI_g: 0.849508$
$VVI_i: 0.955163$
$VVI_g: 0.908501$
$VVI_i: 0.972739$
$VVI_g: 0.843314$
$VVI_i: 0.956423$
$VVI_g: 0.808435$
$VVI_i: 0.945171$
$VVI_g: 0.799338$
$VVI_i: 0.940388$

$$\delta A_{V\lambda}^{VI}(\delta\mu^*) = \delta A_{V\lambda}^{VI}(0) + \delta\mu^*(1 - \rho_{V\lambda}^{VI})$$

By dividing with R_λ^{VI}, residuals transformed into reddening corrections $\delta E_{V\lambda}^{VI}$

$$\delta E_{\lambda\lambda}^{VI}(\delta\mu^*) = \frac{\delta A_{\lambda\lambda}^{VI}(\delta\mu^*)}{R_\lambda^{VI}}$$

Converting E_{VI} into E_{BV}

$$\begin{aligned}\delta E_{\lambda\lambda}^{BV}(\delta\mu^*) &= \frac{\delta A_{\lambda\lambda}^{VI}(\delta\mu^*)}{(R_V^{BV} - R_I^{BV})R_\lambda^{VI}} \\ &= \frac{\delta A_{\lambda\lambda}^{VI}(\delta\mu^*)}{R_V^{BV}}\end{aligned}$$

Implimenting these error corrections of reddenings and distances in his dataset of 59 Cepheids, he derived the Leavitt Laws as follow:

3.3 My Approach

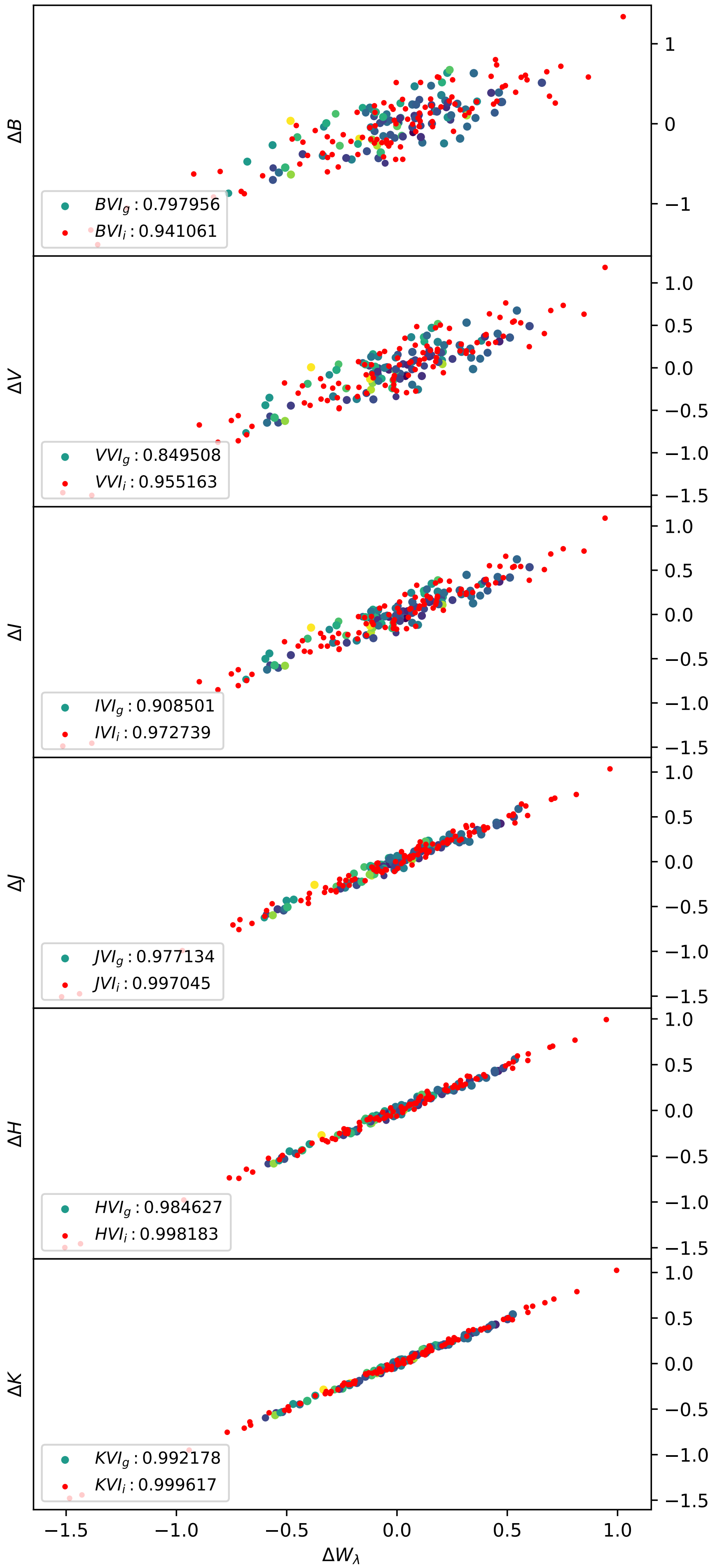
Instead of W_V^{VI}, I have used W_λ^{VI} - reddening free magnitude for respective band m_λ. The reason is the definition of wesenheit function. Reddening free magnitude for a given band must be derived from R_λ^{12}, not from R_V^{12}. R_V^{VI} gives reddening free magnitude corresponds to V band only for $(V - I)$ color index. For B band weseheit magnitude, $R_B^{VI}(V - I)$ would be subtracted from B.

This imples PW residuals in my approach for each band Δ_λ^{VI} will be slighly different than Δ_V^{VI} such that:

$$\Delta_\lambda^{VI} = \Delta_V^{VI} + \delta_V^\lambda \Delta_\lambda^{VI}$$

Difference between the Madore and my PW residues is represented by the difference operator δ_V^λ and calculated as follow:

$$\begin{aligned}\delta_V^\lambda \Delta_\lambda^{VI} &= \Delta_\lambda^{VI} - \Delta_V^{VI} \\ &= (W_\lambda^{VI} - \bar{W}_\lambda^{VI}) - (W_V^{VI} - \bar{W}_V^{VI}) \\ &= (W_\lambda^{VI} - W_V^{VI}) - (\bar{W}_\lambda^{VI} - \bar{W}_V^{VI}) \\ &= ((M_\lambda - M_V) - (R_\lambda^{VI} - R_V^{VI})(V - I)) - ((\alpha_\lambda^{VI} - \alpha_V^{VI}) \log P + (\gamma_\lambda^{VI} - \gamma_V^{VI}))\end{aligned}$$



ΔB
ΔV
ΔI
ΔJ
ΔH
ΔK
$BVI_g: 0.797956$
$BVI_i: 0.941061$
$VVI_g: 0.849508$
$VVI_i: 0.955163$
$IVI_g: 0.908501$
$IVI_i: 0.972739$
$JVI_g: 0.977134$
$JVI_i: 0.997045$
$HVI_g: 0.984627$
$HVI_i: 0.998183$
$KVI_g: 0.992178$
$KVI_i: 0.999617$
−1.5 −1.0 −0.5 0.0 0.5 1.0
ΔW_λ

This deviation in PW residues affect the slope of delta-delta plot ρ, which means correlation residuals $\Delta^{12}_{\kappa\lambda}$ will also be affected. Madore's correlation residuals are denoted by $\Delta^{VI}_{V\lambda}$ as he was correlating ΔM_λ with ΔW^{VI}_V. Instead, I have used $\Delta^{VI}_{\lambda\lambda}$ to derive the corrections as I have correlated ΔM_λ with ΔW^{VI}_λ. Both terms can be related as:

$$\Delta^{VI}_{\lambda\lambda} = \Delta^{VI}_{V\lambda} + \delta^{\lambda\lambda}_{V\lambda}\Delta^{VI}_{\lambda\lambda}$$

Difference between both kinds of correlation residues is:

$$\begin{aligned}
\delta^{\lambda\lambda}_{V\lambda}\Delta^{VI}_{\lambda\lambda} &= \Delta^{VI}_{\lambda\lambda} - \Delta^{VI}_{V\lambda} \\
&= (\Delta_\lambda - \rho^{VI}_{\lambda\lambda} \times \Delta^{VI}_\lambda) - (\Delta_\lambda - \rho^{VI}_{V\lambda} \times \Delta^{VI}_V) \\
&= \rho^{VI}_{V\lambda} \times \Delta^{VI}_V - \rho^{VI}_{\lambda\lambda} \times \Delta^{VI}_\lambda
\end{aligned}$$

Since deviation is non-zero, derived extinction corrections would be different and could be formulated as:

$$\begin{aligned}
\delta A^{VI}_{\lambda\lambda}(\delta\mu^*) &= \Delta^{VI}_{\lambda\lambda} + \delta\mu^*(1 - \rho^{VI}_{\lambda\lambda}) \\
&= \Delta^{VI}_{V\lambda} + \delta\mu^*(1 - \rho^{VI}_{\lambda\lambda}) + \delta^{\lambda\lambda}_{V\lambda}\Delta^{VI}_{\lambda\lambda} \\
&= \delta A^{VI}_{V\lambda}(\delta\mu^*) + \delta^{\lambda\lambda}_{V\lambda}(\Delta^{VI}_{\lambda\lambda} - \delta\mu^* \times \rho^{VI}_{\lambda\lambda})
\end{aligned}$$

The conversion of extinction error into reddening error required scaling with R^{-1}. Hence reddening correction would be:

$$\delta E^{BV}_{\lambda\lambda}(\delta\mu^*) = \frac{\delta A^{VI}_{\lambda\lambda}(\delta\mu^*)}{R^{BV}_V}$$

To compare, Madore's result were:

$$\delta E^{BV}_{V\lambda}(\delta\mu^*) = \frac{\delta A^{VI}_{V\lambda}(\delta\mu^*)}{R^{BV}_\lambda}$$

The difference between both reddening errors is:

$$
\begin{aligned}
\delta E^{VI}_{\lambda\lambda}(\delta\mu^*) - \delta E^{VI}_{V\lambda}(\delta\mu^*) &= \frac{\delta A^{VI}_{\lambda\lambda}(\delta\mu^*) - \delta A^{VI}_{V\lambda}(\delta\mu^*)}{R^{VI}_{\lambda}} \\
&= \frac{(\Delta^{VI}_{\lambda\lambda} + \delta\mu^*(1-\rho^{VI}_{\lambda\lambda})) - (\Delta^{VI}_{V\lambda} + \delta\mu^*(1-\rho^{VI}_{V\lambda})}{R^{VI}_{\lambda}} \\
&= \frac{(\Delta^{VI}_{\lambda} \times \rho^{VI}_{\lambda\lambda} - \Delta^{VI}_{V} \times \rho^{VI}_{V\lambda}) - \delta\mu^* \times (\rho^{VI}_{\lambda\lambda} - \rho^{VI}_{V\lambda})}{R^{VI}_{\lambda}} \\
&= \frac{(\Delta^{VI}_{\lambda} - \delta\mu^*) \times \rho^{VI}_{\lambda\lambda} - (\Delta^{VI}_{V} - \delta\mu^*) \times \rho^{VI}_{V\lambda}}{R^{VI}_{\lambda}}
\end{aligned}
$$

It can be noticed that the deviation in reddening corrections coming from the choice of wesenheit function.

3.4 Physical Significance of ρ

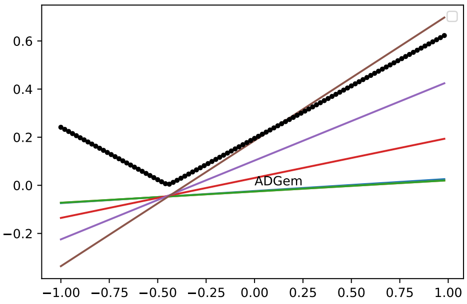
Slope of PL-PW residual correlation plots provides crucial information about the error contributions. In the absence of extinction error, distance remains the only source of error, it implies both PL and PW residual affected equally making then align along the slope 1. This can be observed with longer wavelength data, like in K band.

$$\lim_{\lambda \to K} \rho(\lambda) = 1$$

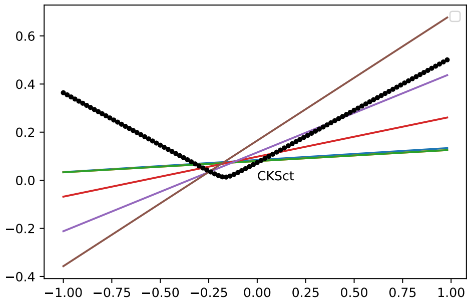
This can be noticed for $\rho^{VI}_{\lambda\lambda}$ in below table but not for $\rho^{VI}_{V\lambda}$

$\Delta_\lambda - \Delta^{VI}_{\lambda}$	$\rho^{VI}_{\lambda\lambda}$ (Gaia)	Error	$\rho^{VI}_{\lambda\lambda}$ (IRSB)	Error
B,BVI	0.797956	0.077717	0.941061	0.050271
V,VVI	0.849508	0.060454	0.955163	0.039847
I,IVI	0.908501	0.036756	0.972739	0.024227
J,JVI	0.977134	0.019786	0.997045	0.011922
H,HVI	0.984627	0.012834	0.998183	0.007469
K,KVI	0.992178	0.008549	0.999617	0.004928
$\Delta_\lambda - \Delta^{VI}_{V}$	$\rho^{VI}_{V\lambda}$ (Gaia)	Error	$\rho^{VI}_{V\lambda}$ (IRSB)	Error
B,VVI	0.787008	0.081522	0.966477	0.054158
V,VVI	0.849508	0.060454	0.955163	0.039847
I,VVI	0.908501	0.036756	0.972739	0.024227
J,VVI	0.843314	0.035688	0.956423	0.024787
H,VVI	0.808435	0.032532	0.945171	0.023823
K,VVI	0.799338	0.033288	0.940388	0.024455

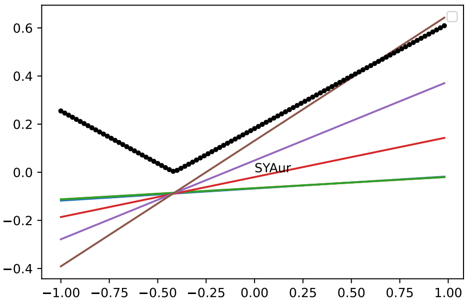
4. Correction Analysis



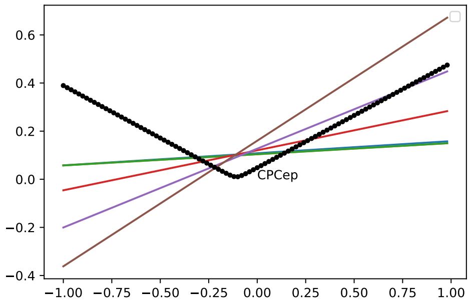
(a) AD Gem



(b) CK Sct



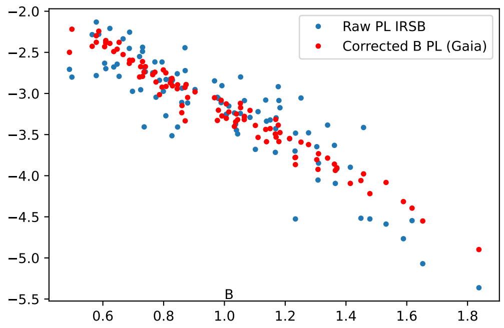
(c) SY Aur



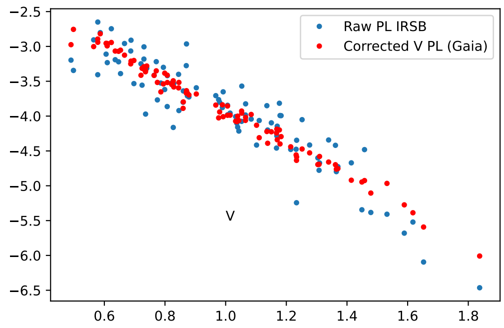
(d) CP Cep

5. Corrected PL relation

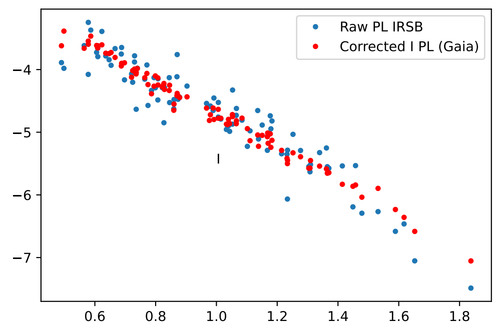
$$B_g = -1.915405(logP - 1)(0.037904) + -3.209270(0.011101)$$
$$V_g = -2.315307(logP - 1)(0.030725) + -3.936832(0.008998)$$
$$I_g = -2.616377(logP - 1)(0.032859) + -4.720667(0.009623)$$
$$J_g = -2.839497(logP - 1)(0.012398) + -5.203807(0.003631)$$
$$H_g = -2.972906(logP - 1)(0.007202) + -5.584565(0.002109)$$
$$K_g = -3.025780(logP - 1)(0.003460) + -5.639665(0.001013)$$



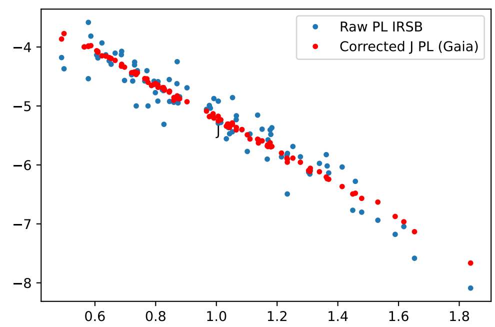
(a) PLB



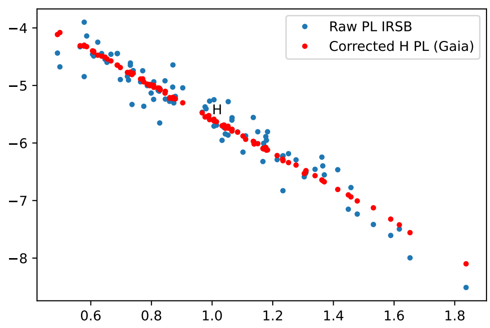
(b) PLV



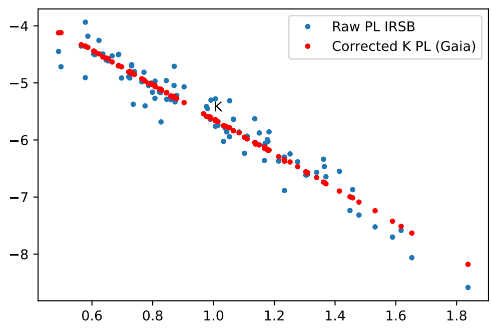
(c) PLI



(d) PLJ



(e) PLH



(f) PLK