Differentially Private Data Generation with Missing Data

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ABSTRACT

We investigate the use of synthetic datasets as a substitute for private data containing sensitive information. While many algorithms generate synthetic data using differential privacy (DP) to protect privacy, we explore the impact of missing values in the private dataset on these approaches. We analyze the quality of synthetic data produced by three broad classes of synthetic data generation algorithms when incomplete data is present. We find that the utility of the synthetic data decreases with missing values, and traditional approaches to deal with missing data are inadequate for generating high-quality synthetic data without compromising privacy. Therefore, we propose three adaptive strategies, one for each algorithm class, to address the missing data issue. Our results demonstrate that these adaptive strategies significantly improve the utility of the synthetic data on four real-world datasets with different types and levels of missing data and privacy requirements. We also investigate the relationship between privacy impact for the complete ground truth data and incomplete data for these DP synthetic data generation algorithms. To obtain tighter upper bounds for ground truth data, we model the missing mechanisms as a sampling process. Overall, this study contributes to a better understanding of the challenges and opportunities for using private synthetic data generation algorithms in the presence of incomplete data.

CCS CONCEPTS

• **Security and privacy → Privacy-preserving protocols**; **Data anonymization and sanitization**; • **Information systems → Data cleaning**.

KEYWORDS

synthetic data generation, missing data, differential privacy, data cleaning

1 INTRODUCTION

Our world as we see it today revolves a lot around private data about our medical, financial, and social information. It is sometimes imperative to query such data for research and advancement of science [8, 45]. Many industries also use statistics from private data to improve their products and user experience. However, reckless data sharing for data-driven applications and research causes great privacy concerns [12, 41] and penalties [1]. As a response, differential privacy (DP) [26] has emerged as a standard data privacy guarantee which has now been adopted by government agencies [4, 40] and companies [30, 38, 43]. Informally, DP guarantees that the output distribution of an algorithm is similar with or without a particular individual in the dataset. A privacy budget is set to limit the total privacy loss and each query (e.g., releasing statistics [4, 17], building prediction models [2, 63], and answering SQL queries [33, 43, 47, 55]) consumes part of the privacy budget, and once that budget is used up, no more queries can be answered directly. An alternative way is to generate a synthetic dataset using the privacy budget. The synthetic dataset, once generated, can be made public, and the analyst can use it for any number of downstream tasks [14, 21, 22, 44, 83].

Despite a number of work [34, 44, 66, 83] that succeed in generating synthetic data with DP guarantees, they only look at a simple scenario where the input data has no missing values. Several prior studies [35, 52] have reported on the prevalence of missing data in various fields. For instance, a study of 9 publicly available healthcare datasets commonly used in machine learning research found that the proportion of missing values ranged from 0.2% to 78.6% [35]. The presence of missing data can be attributed to multiple reasons. One reason is human errors where even the most carefully conducted surveys, aiming to measure treatment or policy effects, may result in responses that suffer from measurement error or that are missing altogether [6]. Yet another reason is privacy regulations such as GDPR [1] which allow people the "right to forget" where one may ask their data to be deleted completely [68]. Simply ignoring such missing data could result in a reduction of statistical power and an increase in estimation bias [59]. In our work, we ask the question: how will missing data affect the quality of the synthetic data generated by mechanisms that offer DP guarantees? Our preliminary study shows that existing differentially private mechanisms have 4%-18.5% decay in the F1-score of downstream ML tasks on the synthetic dataset generated from a dataset with 10% missing values as compared to that when generated from the complete dataset. The decay varies depending on the types of missing mechanisms and the types of data generation processes. Hence, there is a need to understand this practical problem of differentially private synthetic data generation with missing data.

In our work, we formally define the research problem of generating synthetic data for sensitive data with missing values using DP. We consider a missing mechanism that takes complete ground truth data and outputs data with missing values. Under this setup, we can offer DP to either the incomplete data or the ground truth data. For each privacy guarantee, we study how to handle missing data in the synthetic data generation process. We explore several techniques, including the vanilla approach that uses complete rows only, common imputation techniques (e.g., statistical methods, hot and cold deck methods), and machine learning methods. Unfortunately, these methods have their limitations, such as discarding too many rows or incurring high privacy costs, leading to poor-quality synthetic data. To overcome these limitations, we present three ways to amend the existing algorithms for better utility. We call these amended approaches adaptive recourse strategies.

In addition, we show that for these DP synthetic data generation mechanisms, the relationships between the privacy guarantee they offer for the incomplete data and the privacy guarantee they offer for the ground truth data. To do so, we model the missing mechanism as a sampling process and obtain a tighter upper bound for the privacy loss to the ground truth data via sampling amplification techniques [13, 73]. Unlike prior work for sampling amplification that considers a random subset, we make use of the randomness

due to missing values to amplify the privacy for ground truth data. The major contributions of our work are as follows:

- We formalize the problems of DP synthetic data with missing data and differentiate the privacy guarantees for the incomplete data and the ground truth data.
- We benchmark the performance of existing DP synthetic data generation algorithms on different types of missing data. Our results show that these algorithms have a decrease of 5-23% in utility with up to 5% missing values and a decrease of 10-190% with up to 20% missing values.
- We develop three novel adaptive approaches, one for each category of existing DP mechanisms. Our evaluation shows that these approaches improve the utility of the synthetic datasets by up to 15 - 72%. The adaptive approaches sometimes even achieve the same utility as the synthetic data trained on the no-missing ground truth data.
- We are the first to apply amplification due to missing mechanisms and tighten the privacy bound for ground truth data. The amplified ground truth privacy is 0.1-0.65x the privacy achieved for the incomplete data with 10-50% missing values.

2 PRELIMINARIES

We consider a database relation $R = \{A_1, \cdots, A_k\}$ with k attributes, and a database instance D consisting of n rows. We use D_i to refer to the ith row of D, and D_{ij} to refer to the jth attribute of row D_i. We also use $S_{:i}$ to denote all elements from 1 to i in a sequence S.

2.1 Missing Data

For missing data, we define a missingness indicator matrix $M = [\ldots, m_{ij}, \ldots]$ of size $n \times k$, where $i \in [1, n]$, $j \in [1, k]$ and, shorthand m_i to point to i^{th} row of M. Each cell of M has one-to-one relation with D such that, $m_{ij} = 1$ if D_{ij} is missing and $m_{ij} = 0$ otherwise.

Missing data is classified into different types using missing mechanisms. A missing mechanism $M_\phi : \mathcal{D} \rightarrow \mathcal{D}$ takes as input the ground truth dataset $\bar{D}$ and outputs an incomplete dataset D. It is parameterized by Φ, a set of probabilities, which refers to the set of probabilities that control the unknown missing data process. Three missing types can be defined using Φ and the conditional distribution of missing indicator m_i given the dataset D_i [52, 70].

Missing completely at random (MCAR) assumes the probability of missingness is completely independent of the data. Under MCAR, any two rows of the dataset, regardless of their values, for the same attribute have the same probability of having a missing value. Hence, the parameter set Φ consists of $\{\phi_j | j \in [1, k]\}$, where ϕ_j is the probability of any row having a missing value for the jth column. For $j \in [1, k]$ and $i \in [1, n]$, $\Pr[m_{ij}|D] = \Pr[m_{ij}] = \phi_j$. Hence, the probability of a row having no missing values is $\prod_{j=1}^{k}(1 - \phi_j)$.

Missing at random (MAR) captures the scenario when the probability of missingness is independent of the missing values given the observed data. In other words, under MAR how likely a value is to be missing can be estimated based on the non-missing data. Consider examples, 1) Young people have missing IQ (because they haven't taken an IQ test yet), and MAR models the same probability of missing IQ attribute for rows of the same age, regardless of their IQ values; 2) Businessmen are less likely to share their income, and MAR models the same probability for income values for rows that have an occupation as 'Business.' MAR, therefore, is parameterized by a set of conditional probabilities Φ where each $\phi \in |\Phi|$ maps relationships between observed and missing values in the dataset. For $\phi_x \in \Phi$ that models the jth column's missingness, and for $i \in [1, n]$, $\Pr[m_{ij}|D_{i(0)}] = \Pr[m_{ij}|D_{i(0)}, D_{ij}] = \phi_x$, where $D_{i(0)}$ refers to the observed attributes for the ith row.

Missing not at random (MNAR) captures the scenario when given all the observed information, the probability of missingness depends on the unobserved missing values themselves. Consider examples, 1) Students with low scores are unlikely to express their scores, and MNAR models the probability of the missing scores depends on the actual value of the score attribute of the students. 2) People who smoke don't want to mention they smoke. Here MNAR models the probability of missing smokers based on the attribute of smoking. With MNAR missingness, Φ consists of a set of conditional probabilities that map the probability of an attribute to be missing given its own value.

2.2 Differential Privacy

Differential privacy (DP) [28, 29] is used as our measure of privacy.

Definition 2.1 (Approx. Differential Privacy (DP) [29]). A randomized algorithm M achieves (ϵ, δ)-DP if for all $Z \subseteq \text{Range}(\mathcal{M})$ and for any two neighboring databases $D, D' \in \mathcal{D}$ that differ in one row:

$$\Pr[\mathcal{M}(D) \in Z] \leq e^{\epsilon} \Pr[\mathcal{M}(D') \in Z] + \delta.$$

The privacy cost is measured by the parameters (ϵ, δ), often referred to also as the privacy budget. The smaller the privacy parameters, the stronger the offered privacy. When $\delta = 0$, we call the privacy measure pure DP. A common way of achieving DP is by adding noise to the output of the algorithm.

Gaussian mechanism [29] and Laplace mechanism [28] are two such widely used DP algorithms. Given a function $f : \mathcal{D} \rightarrow \mathbb{R}^d$, the Gaussian mechanism adds noise sampled from a Gaussian distribution $\mathcal{N}(0, S_f^2\sigma^2)$ to each component of the query output, where σ is the noise scale and S_f is the L_2 sensitivity of function f, which is defined as $S_f = \max_{D,D' \text{differ in a row}} ||f(D) - f(D')||_2$. For $\epsilon \in (0, 1)$, if $\sigma \geq \sqrt{2\ln(1.25/\delta)}/\epsilon$, the Gaussian mechanism satisfies (ϵ, δ)-DP. Laplace mechanism works similarly but with the noise from the Laplace distribution and the L_1 sensitivity. Both these mechanisms have been applied to answer counting queries [49] and is widely used in estimating low dimensional statistics about the dataset.

Complex DP algorithms can be built from these basic algorithms following two important properties of DP: 1) Post-processing [27] states that for any function g defined over the output of the mechanism $\mathcal{M}$, if $\mathcal{M}$ satisfies (ϵ, δ)-DP, so does $g(\mathcal{M})$; 2) Composability [26] states that if $\mathcal{M}_1, \mathcal{M}_2, \cdots, \mathcal{M}_k$ satisfy (ϵ_1, δ_1)-, (ϵ_1, δ_1)-, $\cdots$, (ϵ_k, δ_k)-DP, then sequentially applying these mechanisms satisfies $(\sum_{i=1}^{k} \epsilon_i, \sum_{i=1}^{k} \delta_i)$-DP.

Stability of transformations Datasets often require certain transformations as a preprocessing step. Examples are SQL operators (e.g., `Where`, `Select`, `Groupby`(, scaling, and imputation. When the dataset is private, these transformations should also be accounted for in the privacy analysis.

Theorem 2.2. *[55] We say a transformation $T(\cdot)$, c-stable, if the distance between $T(D)$ and $T(D')$ is at most c times the distance between D and D'. The composite mechanism $\mathcal{M} \circ T$ then becomes $(c \cdot \epsilon, \delta)$-DP, for any mechanism $\mathcal{M}$ which is (ϵ, δ)-DP.*

2.3 DP Synthetic Data Generation

A common DP study is to generate synthetic data given a fixed privacy budget. The synthetic data, once generated, can be made public and all queries on this dataset come for free due to the post-processing property of DP There are three main approaches for DP synthetic data generation:

Statistical approaches rely on estimating low-dimensional statistics about the dataset such as marginals [69, 78]. These approaches can be made better by finding the correlation between attributes. Techniques for improvement include probabilistic models [46], Bayesian models [50, 67, 83] and undirected graphs [22, 54]. Statistical approaches capture the underlying distribution of the correlated independent attributes very well but fail to imbibe complex relationships between multiple attributes.

Deep learning approaches have shown promise in generating synthetic data [19, 39, 71], especially autoencoders and generative adversarial networks (GAN). Autoencoders map the dataset into a well-behaved low-dimensional feature space, which can be sampled to create synthetic data. GANs contain two neural network structures called the generator and the discriminator. The generator generates fake examples and the discriminator tries to distinguish between a real and fake example. Once they have converged, the generator can then be used to generate synthetic data. A common technique to make these private is by using DPSGD [2, 15, 72, 77]. Multiple deep learning approaches have been thus proposed privately for autoencoders [3, 5, 66] and GANs [32, 44, 76, 79]. They work well on image data but have trouble generating tabular data due to poor encoding schemes for categorical data. Towards this end, conditional GANs [80] and their private counterparts [75] have been proposed in which the generators sample based on the conditional probability of the categorical attribute's density.

Mixed approaches are inspired by both the above approaches and try to preserve both low-dimensional statistics and high-level information. Some techniques include leveraging the dimensionality reduction via random orthonormal (RON) projection, the Gaussian generative model [18], combining denial constraints and attribute-wise embedding models [34] and Gretel.ai statistics [61].

3 PROBLEM STATEMENT

Consider a private dataset that lies behind a privacy firewall with n rows and k attributes. A trusted curator wants to generate a synthetic version fo this dataset of the same size with an end-to-end (ϵ, δ)-DP guarantee while achieving maximum utility (same statistics and correlations) of the synthetic dataset as compared to the original one. In practice, the data collected by the data curator can contain missing values. Prior work does not consider an end-to-end solution for this scenario. Hence, in our work, we consider the case when the private data has missing values. Depending on the consideration of privacy, we formalize two versions of the problem. First, we consider offering a DP guarantee to the incomplete dataset held by the data curator. We formalize this problem as follows.

Problem 1. *[Privacy for Incomplete Data] Consider collecting data from a ground truth data $\bar{D}$ of n rows owned by n individuals, a missing mechanism $M_\Phi : \mathcal{D} \to \mathcal{D}$ is involved that takes in $\bar{D}$ and outputs a dataset D of n rows but with missing values. A trusted data curator uses this dataset D as an input and aims to generate a synthetic data D^* of n rows with a mechanism $M : \mathcal{D} \to \mathcal{D}$ such that D^* share similar statistics and correlations as the ground truth data $\bar{D}$ and M offers (ϵ, δ)-DP to the input data D.*

We consider three options for the data curator to deal with incomplete data. In Section 4, we delineate these options and put forward challenges that come with them, and discuss which option might be the best and when. The first option is a naive adaptation of prior work for DP data generation by simply discarding the rows with missing values [70]. We refer to this approach as *complete row only*. This approach can fail in many cases. For instance, if all rows have some missing values, then there will be no input data for the data generation methods. In Section 4.1, we discuss this approach in detail. A second approach is to impute the missing parts with inferred values from the observed data. We denote this approach as *imputation first approach*. However, as our data is private, the imputation process needs to be privatized as well and the additional incurred privacy cost must be accounted for in the privacy budget (ϵ, δ). In section 4.2, we explore the privacy costs of imputation and show how they can be expensive in practice. As a third approach, rather than having separate processes for imputation and synthetic data generation, we can integrate these two processes into one. This line of thought motivates us to a new approach, which we call *adaptive recourse approach*. In Section 4.3, we improve upon three categories of DP generation approaches and demonstrate their effectiveness in generating synthetic data from incomplete data. These strategies use no extra privacy budget and solely improve by observing available data in the dataset.

The incomplete data D can be modeled as a sample generated from a complete ground truth dataset $\bar{D}$ via a missing mechanism M_Φ. If the privacy goal is to protect the ground truth dataset $\bar{D}$ with DP guarantee, how will the problem and the solution be different? We formalize the second problem as follows.

Problem 2. *[Privacy for Ground Truth Data] Consider the same setup as Problem 1. The trusted data curator uses the incomplete dataset D as input and aims to generate a synthetic data D^* of n rows with a mechanism $M : \mathcal{D} \to \mathcal{D}$ such that D^* share similar statistics and correlations as the ground truth data $\bar{D}$ and $M \circ M_\Phi$ offers $(\bar{\epsilon}, \bar{\delta})$-DP to the ground truth data $\bar{D}$.*

The above problem differs from our first problem only in the last line, where we are trying to achieve DP not for the observed incomplete data but instead for the ground truth data. The missing mechanism allows only partial information to be available to the synthetic data generation process. At first look, these two problems seem the same — guaranteeing privacy for the incomplete data offers privacy to the ground truth data, but they are not equivalent. In Section 5, we look closely into their relationship. We show when privacy for the incomplete data can fail privacy for the ground truth data, and when it cannot. We also present how the missing mechanism can be used as a sampling mechanism to gain in terms of privacy and ultimately reach more useful synthetic data. There

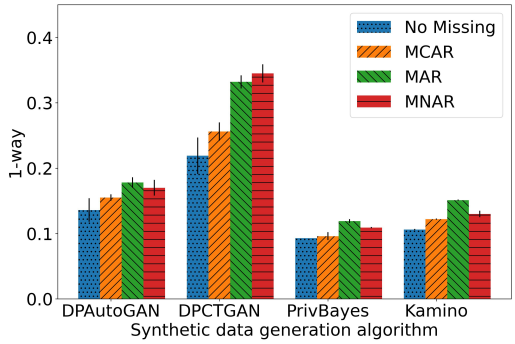


Figure 1: Complete row only approach results in poor results for MAR and MNAR missing mechanism.

are several theorems and lemmas in the paper, the proofs for some of which are available in the longer version of the paper [11].

4 PRIVACY FOR INCOMPLETE DATA

This section examines Problem 1 and explores three methods for generating synthetic data from an incomplete private dataset. The first two methods, which involve discarding rows with missing values and imputations, are found to be ineffective in the DP context due to the loss of valuable information or the high cost of ensuring privacy. Instead, the section recommends adaptive recourse methods, which are novel solutions that address both issues and produce better-quality synthetic data.

4.1 Complete Row Only Approach

Complete row only is a traditional method for handling missing data where incomplete rows are discarded from the dataset. This approach is effective when the missingness is completely at random (MCAR) since the distribution of each attribute remains the same after removing missing rows. However, for other types of missingness, such as missing at random (MAR) and missing not at random (MNAR), the complete row only approach can lead to biased results. Hence, a standard synthetic data generation algorithm that learns directly from the remaining complete rows will result in a biased data distribution that is different from the ground truth data.

Figure 1 illustrates the performance of four different DP synthetic data generation algorithms using the complete row only approach for the Adult dataset with various missing mechanisms. The results indicate that the complete row only approach performs poorly for missing at random (MAR) and missing not at random (MNAR) mechanisms, which introduce bias to the estimated distribution of attributes. This bias can directly affect statistical approaches such as PrivBayes which rely heavily on empirical estimation of marginals. The 1-way distance between the generated synthetic dataset and the original dataset confirms this observation.

Besides the potential bias issue for the complete row only approach, the number of complete rows remaining can be very small. For synthetic data generation methods involving large deep learning models such as GAN, feeding the training process with a small number of complete rows will result in a poor data generation model. This is because the learning process does not converge or/and the noise added for achieving DP overshadows the signals of the training samples. For example, the ground truth Adult dataset which has 32k rows reduces to $\approx$ 5k complete rows 20% MAR and $\approx$ 1k complete rows with 20% MCAR/MNAR missing mechanism respectively. Our results show that the number of complete rows plays a vital role in the performance of the synthetic data generation algorithms. We discuss this in detail in Section 6 where we study several prior work approaches and evaluate them on the different missing data scenarios.

4.2 Imputation First Approach

Imputation is vastly used in practice where the missing data are filled up with values inferred from the observed data. There are multiple ways to impute missing values in the dataset, including:

(1) Statistical methods: Each attribute of the dataset is modeled separately using statistical methods such as mean, median, and mode. The model is then used to fill up the missing values of the attribute [52, 70]. For example, in Figure 2, for the left tables, we use the median of the observed values of column A, to fill up the missing value of the 4th row.
(2) Hot and cold deck methods: This imputation technique replaces every missing value with another value from the same dataset (Hot deck) or from a proxy dataset (Cold deck) [9, 60]. The missing cells of the incomplete row are then filled up from the closest similar row. Similarity metric like cosine distance or ℓ_2 distance can be used.
(3) ML imputation: Machine learning (ML) based approaches are common for missing data imputation [42, 48]. An ML model is trained to predict the missing values of an attribute based on other non-missing attributes in the dataset as training features.

In our setting, as the dataset is private, we need to perform these imputations privately as well. We skip analysis of the cold deck imputation as finding another similar dataset is hard for a private dataset without any prior information. We can perform DP imputation in two ways. The first way is to split the privacy budget and use the first split for imputation and the second split for synthetic data generation. For example, the partial budget can be used to privately train an ML model on the observed features for imputation [2, 15, 72, 77]. However, this approach is staggered due to tuning of the right budget split and choice of imputation algorithm. Additionally, some imputation techniques such as the hot deck imputation that are row specific (replicates the missing value in a row based on some other observed value of a different user) cannot be performed in the DP setting. Randomizing this row to achieve DP introduces too many errors to the dataset.

The second way is to formulate imputation as a transformation of the dataset and calculate the associated privacy cost as an end-to-end algorithm. We use the notion of stability (Theorem 2.2) to calculate the privacy costs of these transformations.

Lemma 4.1. *Consider a transformation $T_A(\cdot)$ for imputing attribute A, which takes in the incomplete dataset D as part of the input and outputs a dataset D' with the complete values for attribute A. Then, the stability of $T_A(\cdot)$ is $c = m_A + 1$ where m_A refers to the number of missing values for the attribute A.*

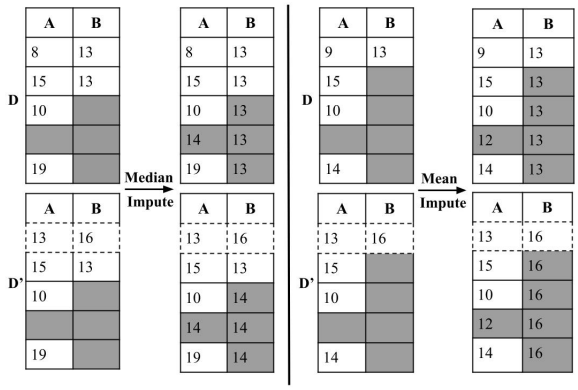


Figure 2: Illustration of worst case statistical imputations.

Proof. As the neighboring databases D and D' differ by a row, $T_A(\cdot)$ uses two different values x and x' to impute the missing values in D and D' respectively. As there are $m_{A(1)}$ rows in both D and D' that have missing values for attribute A, the resulted imputed databases, $T(D)$ and $T(D')$ have $m_{A(1)}+1$ number of rows (include the row that D and D' differ). □Using the above Lemma we can see that applying a sequence of imputation functions over the attributes of a dataset $(\cdots, T_{A_i}, \cdots\})$, the difference in the resulted datasets can be very large when the input dataset differs in a single row. Note that these results hold even if imputation functions for two attributes are not the same.

Theorem 4.2. *The composite mechanism $\mathcal{M} \circ T$ on a dataset D with n rows is $n\epsilon$-DP, where $\mathcal{M}$ is a ϵ-DP mechanism, and T is a sequence of imputation transformation functions performed to each attribute of D.*

Proof. Consider a worst-case scenario: D and D' differ in a single row that does not have any missing values, and the rest of the rows have only one attribute with missing values, i.e., $\sum_i m_{A_i} = n-1$. As T uses the complete row to impute all the missing values, all rows will be affected and the overall cost of $M \circ T$ will be $n\epsilon$-DP. □

Example 4.3. In Figure 2, we illustrate two worst-case toy examples for a dataset with two columns using mean and median imputations. The gray color indicates missing values and dotted lines denote differing rows between the top and the bottom datasets for each example. The missing values in both columns in the left and right examples are filled up with mean and median functions respectively. After applying imputations on the neighboring datasets D and D', the number of rows in column B in both examples starts to differ by 4 rows and 5 respectively including the imputed values and the differing row. In such a scenario, one needs to pay 4ϵ or 5ϵ privacy cost to ensure DP to the incomplete data.

4.3 Adaptive Recourse Approach

We have shown that the aforementioned two approaches suffer from ineffective use of the data or the privacy budget. The complete row only approach ends up discarding several partial rows which could have been used in the learning process while the imputation first approach ends up with a high privacy cost. These issues motivate us to adapt the existing synthetic data generation methods such that they can fully use both the information in the dataset and the privacy budget. We call such an approach *adaptive recourse*. The idea is to utilize the privacy-preserving learning part of the synthetic data generation process for both the imputation and the sampling of the synthetic data. This is beneficial for two reasons. First, the privacy budget is solely spent on the learning of a single model. Second, the imputation process allows more complete training examples to be generated for the iterative learning process, which further improves the utility of the model. For this approach, we present three sub-approaches, depending on the underlying data generation methods, including generative adversarial networks (GAN), partial marginal observation, and column-wise data generation. For each algorithm, we provide an algorithm instance to illustrate their effectiveness in generating synthetic data from an incomplete dataset.

Algorithm 1 DP-MISGAN

Require: Incomplete dataset D, noise scale σ, epochs E, learning rates η_D and η_G, generator interval T_G, batch size B, missing indicator matrix M

Initialize data generator θ_G^D and discriminator θ_D^D
Initialize mask generator θ_G^M and discriminator θ_D^M
for i in $[1, \ldots, E]$ **do**
  Subsample dataset D into $\{S_k\}_{k=1}^{k=(|D|/B)}$ subsets
  for t in $[1, \ldots, |D|/B]$ **do**
    Set real data $x_{data} = S_t$
    Sample real mask x_{mask} from missing indicator $M(S_t)$
    Fill missing values in x_{data} with 0
    Generate fake data and mask
    $y_{data} = \theta_G^D(z)$ where $z \sim \mathcal{N}(0,1)$
    $y_{mask} = \theta_G^M(z)$ where $z \sim \mathcal{N}(0,1)$
    Update $\theta_D^D = \theta_D^D - \eta_D \nabla_{\theta_D^D}[\frac{1}{B}(\theta_D^D; x_{data}) - \frac{1}{B}(\theta_D^D; y_{data})]$
    $\theta_D^M = \theta_D^M - \eta_D \nabla_{\theta_D^M}[\frac{1}{B}(\theta_D^M; x_{mask}) - \frac{1}{B}(\theta_D^M; y_{mask})]$
    if t in interval of T_G **then**
      Generate fake data and mask
      $y_{data} = \theta_G^D(z)$ where $z \sim \mathcal{N}(0,1)$
      $y_{mask} = \theta_G^M(z)$ where $z \sim \mathcal{N}(0,1)$
      Compute data and mask gradient
      $g_d = \nabla_{\theta_G^D}(\theta_D^D; y_{data})$, $g_m = \nabla_{\theta_G^M}(\theta_D^M; y_{mask})$
      Compute noisy gradients $\tilde{g}_{i_d} = \bar{g}_{i_d} + 2\sigma\mathcal{N}(0,1)$
      $\tilde{g}_{i_m} = \bar{g}_{i_m} + 2\sigma\mathcal{N}(0,1)$
      Update generators $\theta_G^D = \theta_G^D - \eta_D \frac{1}{B}\tilde{g}_{i_d}$
      $\theta_G^M = \theta_G^M - \eta_D \frac{1}{B}\tilde{g}_{i_d}$
    end if
  end for
end for
Generate and return synthetic dataset from generator θ_G^D

GAN-based adaptive recourse. In non-private literature, numerous approaches use the GAN framework to deal with missing data [51, 53, 81, 82]. One such leading approach is called MisGAN [51]. The core idea of this approach is to use two generator-discriminator pairs — one for learning the data distribution and

the other for learning the missingness pattern. We develop a DP version of this generation method called *DP-MisGAN*.

A straightforward way to build DPMisGAN is to privatize the non-private optimizer of the discriminator [2, 15, 72, 77]. However, this approach requires adding noise twice, which can harm the algorithm's utility and double the privacy cost. Furthermore, we observe that we only need to publish the generators, not the discriminators. Hence, we resort to the gradient sanitization (GS) approach [20], which perturbs gradients transferred from the discriminators to the generators in order to achieve DP.

We present our DPMisGAN in Algorithm 1. DP-MisGAN, like its non-private counterpart, trains two discriminator-generator pairs. The whole training procedure lasts for E number of epochs and on each epoch, it samples $|D|/B$ subsets without replacement from the given dataset D, where each subset has a size of B assuming the data size is a multiple of B (Line 4). For each subset S_t, DP-MisGAN sets x_{data} as the real data (Line 6) and sets x_{mask} as its corresponding real missing mask (Line 7). The missing mask x_{mask} can be computed from the missing indicator matrix M and corresponds to an array of the same length x_{data} which consists of 1 where the data is missing and 0 otherwise. All missing values in real data x_{data} are filled up with 0s (Line 8). DP-MisGAN then creates two fake examples y_{data} and y_{mask} by sampling two vectors of random initial noise from a Gaussian distribution $\mathcal{N}(0, 1)$ and passing it through the data and mask generators respectively (Line 9). The previously computed variables x_{data} and x_{mask} are then used as input for both these GANs and updated using gradient descent. Each iteration t involves two phases. In the first phase, the discriminators learn the true mask and data distributions (Line 10). The second phase trains the two generators and is executed only in the user-inputted generator intervals T_G (Line 11). In each generator interval, two fake samples y_{data} and y_{mask} are again generated similarly from the two generators (Line 12). These fake samples are used to compute gradients from the discriminators (Line 13) and carefully noised to ensure privacy (Line 14). Finally after the training is completed for E epochs, the discriminators are thrown away and the privately learnt data generator is used to sample synthetic data. In our work, we calculate the final ϵ value using a privacy accountant [2, 7, 37] for the specified number of noise additions done during the generator update interval.

DPMisGAN benefits in two ways by applying gradient sanitization (GS) to achieve DP. First, noise is added only to the gradients of the generators in the second generator training phase (Line 14). Second, the training of MisGAN assumes an underlying Wasserstein-Gan (WGAN) [10] framework that enforces a gradient penalty term in its loss function making the ℓ_2-norm of the discriminator gradients naturally close to 1. This allows to us get rid of the gradient clipping procedure that is used to bound the sensitivity of $\|g\|_2 < C$ by replacing the vector g by $g/max(1, \|g\|_2/C)$ in the standard DPSGD procedure. We expect DP-MisGAN to perform better than the naive GAN approaches because it learns from both the complete rows as well as the incomplete rows of the dataset. Furthermore, as DP-MisGAN learns the missing data pattern of the incomplete dataset, we anticipate that it will capture more information in complex MAR and MNAR missing mechanisms.

Partial marginal observation-based adaptive recourse. This approach can be applied to any algorithm that uses low dimensional marginal queries [54, 83]. In this approach, instead of discarding all the partially missing rows, only the rows with missing cells in the queried attributes are removed. Such a strategy is most helpful when only a subset of attributes have missing data. For example, with MAR missing mechanism, partial marginal observation can learn from all the non-missing columns.

We extend PrivBayes [83] using this strategy and call it PrivBayes enhanced or PrivBayesE in short (Algorithm 2). Similar to PrivBayes, a Bayesian network is learned to know the correlated columns (Lines 1-8), and each time a marginal query is asked, PrivBayesE learns from all non-missing information of the attribute/(s) (Lines 9-15). The improvement as compared to the complete row only approach is because the missing rows are discarded on a smaller set of attributes rather than from the whole dataset. This is especially beneficial for the missing completely at random (MCAR) where looking at more data helps better estimate the true distribution of the marginals and the missing at random (MAR) scenario where some marginals are completely available and the distribution over such attributes can be estimated as learning on complete data. The privacy analysis of PrivBayesE is also similar to PrivBayes as the same marginals are computed in both algorithms and no additional queries are asked to the dataset.

Algorithm 2 PrivBayes Enhanced (PrivBayesE)

Require: Incomplete dataset D, Attributes $\mathcal{A}$, Privacy budget ϵ_1, ϵ_2

1: Initialize Bayesian network B of degree k and $V = \phi$
2: Sample X_1 from $\mathcal{A}$ and add (X_1, ϕ) to B; add X_1 to V
3: **for** $i = 2 \ldots |\mathcal{A}|$ **do**
4: Initialize $\Omega = 0$
5: For each $X \in \mathcal{A} \setminus V$ and each $\Psi \in \binom{V}{k}$; add (X, Ψ) to Ω
6: Select a pair (X_i, Ψ_i) from Ω with maximal mutual information in complete rows for attributes X_i in D using exponential mechanism of budget $\epsilon_1/|\mathcal{A}|$
7: Add (X_i, Ψ_i) to B; add X_i to V
8: **end for**
9: Initialize synthetic dataset D^*
10: **for** $i = 1 \ldots |\mathcal{A}|$ **do**
11: Compute distribution from non-missing values $\Pr[X_i, \Psi_i]$ from the complete rows of X_i in D
12: Learn $\Pr^*[X_i, \Psi_i]$ with Laplace mechanism at budget ϵ_2
13: Set negative values to 0 and normalize
14: Sample from $\Pr^*[X_i, \Psi_i]$ and add to D^*
15: **end for**
16: Return D^*

Column-wise data generation-based adaptive recourse. This approach can be applied to any algorithm that uses column-wise intermediate models to learn the data distribution. In such algorithms, a sequence of attributes is decided and starting with the second attribute in sequence, a model is learnt to predict the current attribute using previously learnt ones. We show that such an algorithmic architecture can be used to impute values as the learning proceeds. Each time a model is trained to predict the next attribute,

the same model can be used to impute missing values. There are no additional privacy costs involved in such an algorithm because only the existing models are used for imputation. This strategy is most appropriate for learning missing data when the missing data has correlation with other attributes in the dataset (e.g, MAR).

Algorithm 3 Kamino Impute (KaminoI)

Require: Incomplete dataset D, Attributes $\mathcal{A}$, Constraints Ψ, Privacy budget ϵ_1, ϵ_2

1: Build sequence S of attributes $\mathcal{A}$ using constraints Ψ
2: Initialize synthetic dataset D^*
3: Compute distribution of first attribute $H = \Pr[S_1]$ using all non-missing values
4: Generate DP H^* by adding Gaussian noise of budget ϵ_1
5: Sample from H^* to populate $D^*[S_1]$
6: **for** $i = 2 \dots |\mathcal{A}|$ **do**
7: Load training features $X = S_{:j}$, and target label $Y = S_j$
8: Train model $M = \theta(X, Y)$ privately with budget $\frac{\epsilon_2}{|\mathcal{A}|-1}$
9: Impute missing values in dataset $D[S_j]$ using M
10: Predict synthetic values $\Omega = M(D^*[S_{:j}])$ and fill $D^*[S_j] = \Omega$
11: **end for**
12: Return D^*

We extend Kamino [34] using this strategy and call in Kamino impute or KaminoI in short (Algorithm 3). A sequence of attributes within the additional constraints Ψ is decided to start the algorithm (Line 1). The distribution of the first attribute in Kamino's computed sequence is learnt using all the non-missing cells (Line 3). This computed distribution is noised (Line 4) and values are sampled to populate the synthetic dataset (Line 5). The other attributes are learnt in sequence using a private intermediate model (Lines 7-8) and missing values are simultaneously imputed (Line 9). The same intermediate model is also used to fill up the synthetic dataset (Line 10). The improvement as compared to the complete row only approach comes from the fact that none of the missing values are discarded and instead used to learn the intermediate models.

5 PRIVACY FOR GROUNDTRUTH DATA

In this section, we shift our focus to exploring the privacy implications for the ground truth data, which we approach as a distinct problem that closely relates to Problem 1. We find that privacy for ground truth can be attained by ensuring that the probability of missing values in a row is independent of the other rows in the dataset. Furthermore, we demonstrate that certain missing mechanisms, such as MCAR, allow a tighter privacy analysis.

5.1 Relationship to Problem 1

We have proposed multiple synthetic data generation algorithms M which train on the incomplete dataset D and achieve (ϵ, δ)-DP as solutions to problem 1. However, this incomplete dataset D is the result of a missing mechanism M_ϕ on the ground truth dataset $\bar{D}$. In problem 2, we study the same mechanisms M which train on the D but discuss their privacy impact on $\bar{D}$. We do so by combining the missing mechanism M_ϕ and the synthetic data generation process M as a composite mechanism $M \circ M_\phi$.

It's important to note that just because M is a DP mechanism for incomplete data, it doesn't necessarily mean that $M \circ M_\phi$ is DP for the ground truth data. For example in Figure 3, consider a census dataset D with missing income values due to a missing not at random (MNAR) mechanism. If a neighboring dataset D' differs in the last row with an income of 80k, the MNAR mechanism results in three rows having missing values. Such missing mechanisms may potentially generate n different rows for the neighboring ground truth, meaning that an ϵ-DP mechanism for incomplete data cannot guarantee the same level of privacy for the ground truth data.

The example above does not provide a strong privacy guarantee for the ground truth data because the probability of a row having missing values depends on the values of other rows. However, we can show that if M_ϕ enforces independent probabilities for each row to have missing values, a strong privacy guarantee applies to the ground truth data.

Theorem 5.1. *Let the missing mechanism M_Φ has independent randomness to hide the values of each row and $D = M_\phi(\bar{D})$. If M achieves (ϵ, δ)-DP for D, then $M \circ M_\Phi$ satisfies $(\bar{\epsilon}, \bar{\delta})$-DP for $\bar{D}$, where $\bar{\epsilon} \le \epsilon, \bar{\delta} \le \delta$.*

Proof. (sketch) As M_Φ has independent randomness to hide values of each row, given the ground truth data $\bar{D}$ and a possible incomplete dataset D, we have $\Pr[D|\bar{D}] = \prod_l \Pr[D_l|\bar{D}]$, where D_l refers to the value taken by the lth row. Consider neighboring groundtruth datasets $\bar{D}$ and $\bar{D}'$ differ in the ith row and any possible output O of $M \circ M_\Phi$. Let $\mathcal{D}$ be all possible incomplete datasets that can be outputted by M_Φ from $\bar{D}$ or $\bar{D}'$. We partition $\mathcal{D}$ into $\{\cdots, \mathcal{D}_j, \cdots\}$ such that all datasets with the same row values except the ith row are in the same group $\mathcal{D}_j$. Hence, for all $D \in \mathcal{D}_j$, they have the same probability for $\prod_{l \neq i} \Pr_{M_\Phi}[D_l|\bar{D}]$. Now we have

$$
\begin{aligned}
& \Pr[O|\bar{D}] \\
= & \sum_{\mathcal{D}_j} \sum_{D \in \mathcal{D}_j} \Pr_M[O|D] \Pr_{M_\Phi}[D|\bar{D}] \\
= & \sum_{\mathcal{D}_j} \sum_{D \in \mathcal{D}_j} (\Pr_M[O|D] \Pr_{M_\Phi}[D_i|\bar{D}] \cdot \prod_{l \neq i} \Pr_{M_\Phi}[D_l|\bar{D}]) \\
= & \sum_{\mathcal{D}_j} (\prod_{l \neq i} \Pr_{M_\Phi}[D_l|\bar{D}] \cdot \sum_{D \in \mathcal{D}_j} \Pr_M[O|D] \Pr_{M_\Phi}[D_i|\bar{D}]) \\
\le & \sum_{\mathcal{D}_j} (\prod_{l \neq i} \Pr_{M_\Phi}[D_l|\bar{D}'] \cdot \sum_{D' \in \mathcal{D}_j} (e^\epsilon \Pr_M[O|D'] + \delta) \Pr_{M_\Phi}[D'_i|\bar{D}']) \\
= & e^\epsilon \sum_{\mathcal{D}_j} \sum_{D' \in \mathcal{D}_j} \Pr_M[O|D'] \Pr_{M_\Phi}[D'|\bar{D}] + \delta \sum_{\mathcal{D}_j} \sum_{D' \in \mathcal{D}_j} \Pr_{M_\Phi}[D'|\bar{D}'] \\
= & e^\epsilon \sum_{\mathcal{D}_j} \sum_{D' \in \mathcal{D}_j} \Pr_M[O|D'] \Pr_{M_\Phi}[D'|\bar{D}] + \delta
\end{aligned}
$$

The inequality above is based on for any neighbors D and D', we have $\Pr_M[O|D] \le e^\epsilon \Pr_M[O|D'] + \delta$ and $\sum_{D \in \mathcal{D}_j} \Pr_{M_\Phi}[D_i|\bar{D}] = \sum_{D' \in \mathcal{D}_j} \Pr_{M_\Phi}[D'_i|\bar{D}'] = 1$. □

Theorem 5.1 says that the privacy bound for the ground truth dataset is lesser than equal to the bound of the incomplete dataset for a synthetic data generation algorithm if each row in the ground truth dataset has an independent probability of having missing

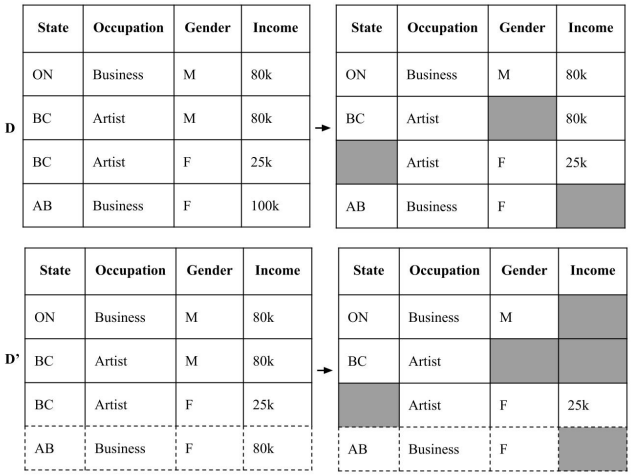


Figure 3: Example of private incomplete datasets generated from two neighbouring ground truth datasets. Gray denotes missing cells and dotted lines represent the differing row.

values. Next we illustrate how to obtain a tighter privacy bound for the missing completely at random (MCAR) mechanism.

5.2 Privacy Amplification Due To MCAR

Missing completely at random (MCAR) enforces independent probability of having missing rows for each attribute in the dataset. We use these probabilities to tighten the privacy bounds for ground truth data when the missing mechanism is MCAR. The technique developed by us is inspired by the seminal work of privacy amplification due to sampling [13]. The premise of privacy amplification by subsampling is that we run a DP algorithm on some random subset of the data (e.g, sampled Gaussian mechanism, DP-SGD). The subset introduces additional uncertainty, which benefits privacy. In particular, there is some probability that your data is not included in the analysis, which can only enhance your privacy. Privacy amplification due to subsampling has been shown to work for many sampling methods (e.g, Poisson sampling, sampling with/without replacement) and for neighbouring datasets which may differ with replacement or substitution. Privacy amplification by subsampling theorem 5.2 makes this intuition precise.

Theorem 5.2 (Sampling Amplification Theorem [13, 73]). *Consider an algorithm $M : \mathcal{D} \rightarrow \mathcal{D}$ that satisfies (ϵ, δ)-DP and a sampling mechanism $S(D)$ that samples a random subset U from dataset D of n samples. If $p = max_{i \in [n]} \Pr_U[i \in U]$, then the composite mechanism $M(S(D))$ offers (ϵ', δ')-DP where $\epsilon' = \log(1 + p(e^{\epsilon} - 1))$, $\delta' = p\delta$. For small values of ϵ, we have $\epsilon' = \log(1 + p(e^{\epsilon} - 1)) \approx p\epsilon$.*

In our missing data context, we note that for synthetic data generation algorithms that train on incomplete data, many rows are naturally discarded due to the presence of missing cells. We exploit this natural throwing out of rows as a sampling mechanism and show that it can be used to amplify privacy. Recall from Section 2.1 that MCAR enforces independent probability of having missing cells in the dataset for each attribute $\phi_1, \cdots, \phi_k$. We use these probabilities to propose our amplification results in Proposition 5.3.

Proposition 5.3. *Consider an MCAR mechanism $M_\Phi : \mathcal{D} \rightarrow \mathcal{D}$ with missing probabilities $\{\phi_1, \ldots, \phi_k\}$ over attributes $\{A_1, \ldots A_k\}$ of the input ground truth data $\bar{D}$ and outputs an incomplete dataset D. If an algorithm $M : \mathcal{D} \rightarrow \mathcal{D}$ takes in rows in D which have no missing values on attributes $\mathcal{A}_M \subseteq \{A_1, \ldots, A_k\}$, then $M \circ M_\Phi$ offers $(p\epsilon, p\delta)$-DP to the ground truth data $\bar{D}$ where $p_M = \prod_{A_i \in \mathcal{A}_M}(1 - \phi_i)$. We call $\mathcal{A}_M$ an amplification attribute set for M and p_M the amplification factor of $\mathcal{A}_M$.*

Proof. A row in MCAR has $\prod_{i=1}^{i=l}(1 - \phi_i)$ probability of having no missing values and plugging in to Theorem 5.2. □

We note three important facts. First, if an algorithm M takes in rows with no missing values over an attribute set $\mathcal{A}_M$, then M also takes in rows with no missing values over an attribute set $\mathcal{A}'_M \subset \mathcal{A}_M$. In other words, if $\mathcal{A}_M$ is an amplification attribute set for M, then any subset of $\mathcal{A}_M$ is an amplification attribute set for M with amplification factor greater than that of $\mathcal{A}_M$. Second, when $\mathcal{A}_M = \emptyset$, $p_{\mathcal{A}_M} = 1$. Third and more importantly, as the dataset is read only once, each attribute can only be used once as an amplification factor. We can now use Proposition 5.3 and Theorem 5.2 in conjunction to show the privacy amplifications for the different algorithms we have discussed so far in our paper.

Use case 1: Privacy amplification for complete row only approach. Here we show how to apply Proposition 5.3 to all complete row only approaches (PrivBayes, Kamino and GAN based approaches). As these approaches take as input all attributes, the probability of seeing a row without missing values is $\prod_{i=1}^{i=k}(1 - \phi_i)$. The following example illustrates how this probability can be used to obtain a tighter privacy bound for the ground truth data.

Example 5.4 (MCAR amplfication for complete row only approach). Consider the incomplete dataset from Figure 3. Lets assume that the missing data comes from a MCAR mechanism where the missing probabilities are $\phi_{State} = \frac{1}{4}, \phi_{Occupation} = 0, \phi_{Gender} = \frac{1}{4}, \phi_{Income} = \frac{1}{4}$. Given 4 DP sub-algorithms M_1, M_2, M_3, M_4 that each offer DP guarantee to the incomplete dataset D at budget $\frac{\epsilon}{4}$. M_1 computes the marginals of the complete rows over attribute <State>, M_2 over <Occupation>, M_3 over <Gender> and M_4 over <Gender, Income>. As all sub-algorithms take as input only the complete rows, using Proposition 5.3, the amplification is $\prod_i(1 - \phi_i) = 0.421$ and using Theorem 5.5 the final privacy is $\bar{\epsilon} = 4 * 0.421\frac{\epsilon}{4} = 0.421\epsilon$.

Use case 2: Privacy amplification for partial marginal observation approach. For partial marginal observation methods (e.g. PrivBayesE), calculating the amplification privacy cost is more complex. These methods involve multiple low-dimensional marginals with overlapping attributes. To determine the overall amplification for such algorithms, it is necessary to calculate the amplification for each marginal and carefully compose them. The complexity of this calculation arises from the optimal selection of the amplification attribute set for each marginal, which maximizes amplification while ensuring that each attribute is used only once. First, we consider a simple case that the amplification factors of all marginals are disjoint. In this scenario, we can compose the total privacy cost using Theorem 5.5 and demonstrate using Example 5.6.

Theorem 5.5. *Consider an MCAR mechanism M_Φ, and a sequence of j mechanisms $M_1, \ldots, M_j$ with DP guarantees of $\epsilon_1, \ldots, \epsilon_j$ to D and amplification attribute set $\mathcal{A}_{M_1}, \ldots, \mathcal{A}_{M_j}$ respectively. If their amplification attribute sets do not overlap, then these mechanisms offers DP to the ground truth data $\bar{D}$ at a cost of $\bar{\epsilon} = \sum_{i=1} p_{\mathcal{A}_{M_i}} \epsilon_i$.*

Proof. As all mechanisms M_i work on disjoint sets of attributes, their amplification attribute sets $\mathcal{A}_i$ are also disjoint. Furthermore as the missing probabilities are always ≤ 1, we always use all attributes in $\mathcal{A}_i$ amplify marginal M_i. We can then use Theorem 5.2 to calculate the final amplified privacy cost $\bar{\epsilon} = \sum_{i=1} p_{\mathcal{A}_{M_i}} \epsilon_i$. □

Example 5.6. Continuing from Example 5.4, lets assume we have the same dataset but use a partial observation algorithm. We consider only the sub-algorithms M_1, M_2 and M_4 for this example. The marginals for these sub-algorithms do not overlap and allow us to consider all engaging attributes as their amplification attribute set. Hence, by Theorem 5.5, M_1 is amplified using $p_{M_1} = 1 - \phi_{state} = \frac{3}{4}$, M_2 is amplified using $p_{M_2} = 1 - \phi_{occupation} = 1$ and M_4 using $p_{M_4} = (1 - \phi_{gender})(1 - \phi_{income}) = \frac{9}{16}$. The amplified privacy cost would thus be $\frac{3}{4}\frac{\epsilon}{3} + \frac{\epsilon}{3} + \frac{9}{16}\frac{\epsilon}{3} = 0.77\epsilon$.

The problem however gets more nuanced when two marginals have overlapping attributes. We show this in Example 5.7 by first showing a naïve composition and then an optimized one.

Example 5.7. Let's continue Example 5.4 by considering all 4 sub-algorithms and a partial observation algorithm. The marginals for sub-algorithms M_3 and M_4 overlap in the 'Gender' attribute with amplification factors $p_{M_3} = (1 - \phi_{Gender}) = \frac{3}{4}$ and $p_{M_4} = (1 - \phi_{Gender})(1 - \phi_{Income}) = \frac{9}{16}$ respectively. We cannot apply Theorem 5.5 on M_3 and M_4's entire attribute set as the corresponding amplification attribute sets would then overlap on the 'Gender' attribute. Instead, we resort to other ideas. A naïve solution to this problem would be to amplify the DP-mechanism which has the most amplification and skip the others. In our example, we would therefore amplify only M_4 with amplification of $\frac{9}{16}$ and skip amplification for M_3. The total amplified privacy cost would thus be $\bar{\epsilon} = \frac{3}{4}\frac{\epsilon}{4} + \frac{\epsilon}{4} + \frac{\epsilon}{4} + \frac{9}{16}\frac{\epsilon}{4} = 0.83\epsilon$. However, a better bound can be calculated if overlapping mechanisms were grouped together and amplified using the intersecting attribute. For instance, both M_3 and M_4 can be amplified by an amplification factor of $\frac{3}{4}$ using amplification attribute set 'Gender', resulting a total privacy loss of $\frac{3}{4}\frac{\epsilon}{4} + \frac{\epsilon}{4} + \frac{3}{4}(\frac{\epsilon}{4} + \frac{\epsilon}{4}) = 0.81\epsilon$ i.e. tighter than 0.83ϵ.

In a more general setting, solving this problem requires us to make group overlapping mechanisms and make sure that each group has distinct amplification factors.

Problem 3. *Consider an MCAR mechanism M_Φ and a sequence of j mechanisms $M_1, \ldots, M_j$ with DP guarantees of $\epsilon_1, \ldots, \epsilon_j$ to D, where M_i computes a marginal over attributes $\mathcal{A}_i$. We would like to find amplification attribute sets $\{\mathcal{A}_{M_1} \subseteq \mathcal{A}_1, \ldots, \mathcal{A}_{M_j} \subseteq \mathcal{A}_k\}$ and their corresponding amplification factor $p_1, \ldots, p_j$ for $M_1, \ldots M_j$, that gives the smallest DP cost to the ground truth data $\bar{D}$.*

One way to solve the above problem is by creating valid partitions of marginals and assigning each group in the partition an amplification attribute set such that all groups have disjoint attribute sets and all marginals from the same group are amplified using their own amplification attribute set.

Definition 5.8 (Valid partition). Given DP mechanisms $M_1, \ldots, M_j$ for computing marginals over attribute sets $\mathcal{A}_1, \ldots, \mathcal{A}_j$, a partition of these mechanisms $P = \{G_1, \ldots G_i\}$ is considered valid if it satisfies these conditions: (1) All mechanisms in the same partition are amplified with the same set of amplification attribute set and with the same amplification factors; and (2) The amplification attribute set of all partitions are disjoint.

The privacy cost for this valid partition is $\bar{\epsilon} = \sum_{G_l \in P} p_{\mathcal{A}_{G_l}} \sum_{M_j \in G_l} \epsilon_j$, where $\mathcal{A}_{G_l}$ is the amplification attribute set take by the mechanisms grouped into G_l and $p_{\mathcal{A}_{G_l}}$ is the corresponding amplification factor.

A valid partition ensures that the amplification attribute set of each group is disjoint and hence, all attributes in the dataset are only considered once in the amplification analysis. To solve Problem 3, we simply need to choose the partition which is both valid and also has the least privacy cost. Therefore, all DP sub-mechanisms are amplified using the best disjoint amplification attribute set. However, we note that enumerating all possible valid partition is intractable as there are exponentially many partitions that we can possibly make from our DP sub-mechanisms[1]. If the cardinality of $\mathcal{A}$ and the number of DP mechanisms j are small then one can enumerate all possible solutions and choose the best one. However, for datasets with a large number of attributes, we show an initial pruning method to trim away bad solutions using Lemma 5.9.

Lemma 5.9. *If a valid partition P is an optimal solution to Problem 3, then each group $G \in P$ should have a non-empty amplification attribute set.*

Proof. Suppose there are a sequence of DP sub-mechanisms $M_1, \ldots, M_j$ with privacy cost of $\epsilon_1, \ldots, \epsilon_j$. A valid partition P over $M_1, \ldots, M_j$ consists of a group G that has an empty amplification attribute set. Then the amplification factor of group G will be $p_G = 1$ and the overall privacy cost of this partition can be calculated using Definition 5.8.

$$\begin{aligned}\bar{\epsilon}_P &= \sum_{G_l \in P} p_{\mathcal{A}_{G_l}} \sum_{M_j \in G_l} \epsilon_j \\ &= \sum_{G_l \in \{P-G\}} p_{\mathcal{A}_{G_l}} \sum_{\mathcal{A}_j \in G_l} \epsilon_j + \sum_{M_j \in G} \epsilon_j \\ &= L + X\end{aligned}$$

, where $L = \sum_{G_l \in \{P-G\}} p_{\mathcal{A}_{G_l}} \sum_{\mathcal{A}_j \in G_l} \epsilon_j$ and $X = \sum_{M_j \in G} \epsilon_j$. We can always define another valid partition P' that has the groups as P but group G has amplification attribute set A. Therefore, the privacy cost of partition P' can be calculated as:

$$\begin{aligned}\bar{\epsilon}_{P'} &= \sum_{G_l \in P'} p_{\mathcal{A}_{G_l}} \sum_{M_j \in G_l} \epsilon_j \\ &= \sum_{G_l \in \{P'-G\}} p_{\mathcal{A}_{G_l}} \sum_{\mathcal{A}_j \in G_l} \epsilon_j + p_A \sum_{M_j \in G} \epsilon_j \\ &= L' + p_A X\end{aligned}$$

, where $L' = \sum_{G_l \in \{P'-G\}} p_{\mathcal{A}_{G_l}} \sum_{\mathcal{A}_j \in G_l} \epsilon_j$. Note that as amplification attribute set A is used for G in P', the privacy cost of the other groups can at most be increased by p_A. Assuming the worst case,

[1]The total number of partitions of a set is given by the Bell number.

$L' = p_A L$ and that the privacy cost of P is always higher than that of P'.

$$L + X - L' - p_A X \geq 0$$
$$L + X - p_A L - p_A X \geq 0$$
$$L(1 - p_A) + X(1 - p_A) \geq 0$$
$$(L + X)(1 - p_A) \geq 0$$

The above inequality if always true as the first term is always positive (L and X are privacy costs) and the $p_A \leq 1$ as all missing percentages are ≤ 1. □

For large datasets where we are left with multiple partitions even after pruning, we use a brute force search as described in Algorithm 4. In Line 1, we enumerate all possible disjoint amplification attribute sets that we can make from $\mathcal{A}$ and store in a variable $\mathcal{P}_{\mathcal{A}}$. Then, we loop through each possible disjoint attribute set $P_{\mathcal{A}} \in \mathcal{P}_{\mathcal{A}}$, and calculate the cost of each amplification attribute set $\mathcal{A}_l \in P_{\mathcal{A}}$ in Line 2-3 using Proposition 5.3. We initialize the cost of the partition $c_{P_{\mathcal{A}}} = 0$ in Line 4. Then we loop through each pair of marginal S_i and its corresponding DP mechanism (M_i, ϵ_i) and find the candidate amplification attribute sets that are contained in the marginal S_i in Line 6. If no such candidate set is for S_i, then we can prune the entire partition $P_{\mathcal{A}}$ using Lemma 5.9 and loop back to Line 2. Otherwise, in Line 7-8, we find the candidate attribute set which has the best amplification cost for M_i and add its corresponding cost the final cost $c_{P_{\mathcal{A}}}$. Finally, the partition with the minimum sum cost is returned in Line 10. In Example 5.10 we show how Algorithm 4 can be used to find the valid partition for our running example which has the least privacy cost.

Example 5.10. Consider the same setup in Example 5.7. There are total of 4 attributes and Algorithm 4 starts by enumerating all 15 possible disjoint amplification attribute sets — {State | Occupation | Gender | Income}, {State Occupation | Gender | Income}, ... , {State Occupation Gender | Income}, ... , {State Occupation Gender Income}. We then iterate through each of these disjoint sets. Lets consider the disjoint sets $\mathcal{A}_1 : \{State\}, \mathcal{A}_2 : \{Occupation\}, \mathcal{A}_3 : \{Gender\}, \mathcal{A}_4 : \{Income\}$. For each attribute set we calculate its corresponding amplification factor, $c_{\mathcal{A}_1} : (1-\phi_{State}) = 3/4, c_{\mathcal{A}_2} : (1-\phi_{Occupation}) = 0, c_{\mathcal{A}_3} : (1 - \phi_{Gender}) = 3/4, c_{\mathcal{A}_4} : (1 - \phi_{Income}) = 3/4$. We then iterate through all marginals and choose the best amplification attribute for each marginal. Therefore, M_1 is amplified using $\mathcal{A}_1$, M_2 using $\mathcal{A}_2$, M_3 and M_4 both using $\mathcal{A}_3$. The final privacy cost therefore is $\bar{\epsilon} = \frac{3}{4}\frac{\epsilon}{4} + \frac{\epsilon}{4} + \frac{3}{4}\frac{\epsilon}{4} + \frac{3}{4}\frac{\epsilon}{4} = 0.81\epsilon$. This partition also happens to be the best partition amongst the 15 partitions.

Use case 3: Privacy amplification for column-wise imputation algorithms. Column wise imputation algorithms (e.g. KaminoI) learn one attribute at a time in predefined sequence S over the attributes $\{A_1, \ldots, A_k\}$. The first attribute in sequence S_1 is learnt using its observed distribution and the rest $S_2, \ldots, S_k$ are learnt using intermediate models $M_2, \ldots, M_k$ with privacy costs $\epsilon_2, \ldots, \epsilon_k$. At each i^{th} iteration, the S_i attribute is learnt using the intermediate model M_i which takes the previously learnt attributes $S_{:i}$ as feature input and learns the target attribute S_i. After the training is complete, M_i is used to sample the synthetic dataset and also used to impute the incomplete values of the target attribute $D[S_i]$. Thus for the next model M_{i+1}, the previously learnt attributes in sequence are either complete or imputed. In other words, the total number of complete rows fed to an intermediate model M_i solely depends on the number of complete values in $D[S_i]$. Therefore, the amplified privacy for ground truth data for M_i is $\bar{\epsilon_i} = p_{S_i}\epsilon_i$ where $p_{S_i} = 1 - \phi_i$ is the amplification factor of attribute S_i. However we note that for training model M_i, every attribute $S_{:i-1}$ is considered for amplification using the same probability as S_i. And as each attribute can be used once, we can only amplify a single model and calculate the privacy for ground truth accordingly.

Algorithm 4 Optimal amplified privacy cost

Require: Marginals $\mathcal{S} = \{S_1, \ldots, S_j\}$ over attributes $\mathcal{A}$, DP mechanisms $\mathcal{M} = \{(M_1, \epsilon_1), \ldots, (M_j, \epsilon_j)\}$ for $\mathcal{S}$, Missing probabilities $\Phi = \{\Phi_A | A \in \mathcal{A}\}$ for MCAR

1: Find all possible partitions of $\mathcal{A}$ and store in $\mathcal{P}_{\mathcal{A}}$
2: **for** each attribute partition $P_{\mathcal{A}}$ in $\mathcal{P}_{\mathcal{A}}$ **do**
3: Calculate amplification factor $c_{\mathcal{A}_l} = \prod_{A \in \mathcal{A}_l}(1 - \phi_A)$ for $\mathcal{A}_l \in P_{\mathcal{A}}$
4: Initialize disjoint set cost $c_{P_{\mathcal{A}}} = 0$
5: **for** each $S_i \in \mathcal{S}$ with its corresponding $((M_i, \epsilon_i))$ **do**
6: Skip $P_{\mathcal{A}}$ if $\{\mathcal{A}_l \subseteq S_i | \mathcal{A}_l \in P_{\mathcal{A}}\} = \emptyset$
7: Find the best amplification attribute set for M_i: $\mathcal{A}_{l*} \leftarrow argmin_{\mathcal{A}_l \in P_{\mathcal{A}} \wedge \mathcal{A}_l \subseteq S_i} c_{\mathcal{A}_l}$
8: Add the best amplified cost $c_{P_{\mathcal{A}}} = c_{P_{\mathcal{A}}} + \epsilon_i \cdot c_{\mathcal{A}_{l*}}$
9: **end for**
10: **end for**
11: Return the attribute partition $P_{\mathcal{A}}$ with minimum cost $c_{P_{\mathcal{A}}}$

Theorem 5.11. *Consider an MCAR mechanism M_Φ and a sequence of attributes S and k mechanisms $M_1, \ldots, M_k$ with privacy cost of $\epsilon_1, \ldots, \epsilon_k$ to D, where M_i is an intermediate model that trains A_i as target and $A_{:i}$ as features. If model M_i is used for imputation of attribute A_i, then the overall process offers DP to the ground truth data $\bar{D}$ at a cost of $\bar{\epsilon} = p_{S_j}\epsilon_j + \sum_{i=1, i\neq j}^{k} \epsilon_i$, where $j = \max_{i=1}^{k}(1 - \phi_i)$.*

Proof. As all cells of an attribute are imputed at previous iterations, the number of complete rows fed to a model at the ith iteration depends on the missing cells of the S_i attribute. The amplification for the ith model can therefore be calculated at $\epsilon_i p_{S_i}$, where $p_{S_i} = 1 - \phi_i$ is the amplification factor of attribute S_i. To calculate the overall privacy of the algorithm, we would like to compose the amplified privacy of all sub models. However, the model M_i trains on every attribute $S_{:i-1}$ and each attribute uses the same probability as S_i. Furthermore as each attribute can only be used once, we can only amplify a single model. Hence, we choose to amplify the model with the maximum amplification M_j where $j = \max_{i=1}^{k}(1 - \phi_i)$ and the other models are left un-amplified. □

Example 5.12. Consider the incomplete dataset from Figure 3. Lets assume we have a column based imputation algorithm learns this dataset. Each attribute is learnt using the equal privacy budget $\frac{\epsilon}{4}$. The attributes 'State', 'Gender' and 'Income' has an amplification factor $\frac{3}{4}$ whereas the attribute 'State' has a factor of 1. Therefore, the amplified privacy budget is $\bar{\epsilon} = \frac{3}{4}\frac{\epsilon}{4} + 3\frac{\epsilon}{4} = 0.9375\epsilon$.

Table 1: Dataset Characteristics

Dataset	Cardinality	#Numerical Attr	#Categorical Attr
Adult	32561	5	10
Bank	45211	3	14
BR2000	38000	3	11
National	15012	6	14

6 EVALUATION

We thoroughly experiment with DP synthetic data generation algorithms on missing data. First, we benchmark existing DP methods to demonstrate how missing data affects the utility of the generated synthetic data. Next, we evaluate the effectiveness of our proposed adaptive recourse methods and analyze the impact of varying missing data percentages, missing mechanisms, and privacy budgets for each method. Finally, we show how missingness amplifies privacy for ground truth data.

6.1 Experimental Setup

Datasets. We run our experiments on four tabular datasets: (i) Adult dataset [25], which contains information about 32561 individuals from the 1994 US Census (ii) Bank dataset [57], which has 45211 rows about direct marketing campaigns of a Portuguese banking institution (iii) BR2000 [83], which consists of 38,000 census records collected from Brazil in the year 2000, (iv) National dataset [74] from NIST Diverse Community Excerpts which contains information about 15012 individuals US census data. Each dataset has a combination of numerical and categorical columns which are preprocessed according to the synthetic data generation algorithm as discussed in their respective research paper – numerical attributes are discretized into 10 uniform bins or scaled between 0 to 1 and the categorical attributes are encoded using one-hot or ordinal encoding.

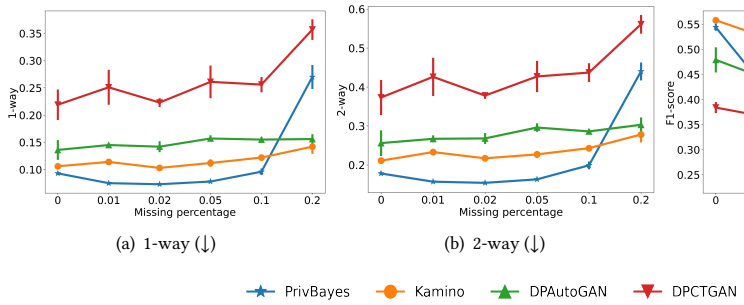
Baselines. We study four baselines which follow different approaches to generate synthetic data and have been published in well-known conferences with their code readily available. PrivBayes from statistical approaches, DPCTGAN, DPAutoGAN from deep learning techniques and Kamino which is a mixed approach. These algorithms rely upon multiple tunable hyperparameters, which are crucial to their outcome. We try our best to tune these parameters by running grid searches. We don't account for the privacy cost for tuning but in practice, these parameters need to be tuned privately as well [56, 64]. We also notice that deep learning methods need a larger privacy budget than other methods to get sensible results. Therefore by default, we assign PrivBayes and Kamino $\epsilon = 1$, and the GAN approaches $\epsilon = 3$. The adaptive methods for these baselines have the same budget as their non-adaptive counterparts. Furthermore, PrivBayes works on the notion of pure DP ($\delta = 0$), and the others use approx. DP. The δ value is roughly set to one magnitude lower than that of $1/|D|$ to the nearest exponent of 10.

Missing data. We implement a pipeline that can generate different categories of missing data at different missing percentages, using the approach from Muzellec et al. [58]. MCAR missing data is generated by masking the original dataset using a realization of a Bernoulli variable with a fixed parameter such that there is exactly the required number of missing values. To generate MAR data, we first use 50% of the attributes in the dataset as features for a logistic regression model. The other attributes then have missing values according to random weights in the logistic model. A bias term is fitted using line search to attain the desired proportion of missing values. The MNAR approach works similarly to MAR, with the difference that the 50% non-missing attributes are masked by an MCAR mechanism. This imposes the logistic model to depend potentially on missing values, hence enforcing MNAR missingness. As 50% of the columns in the MAR mechanism has complete values, it comparatively has more complete rows in comparison to MCAR and MNAR. We run experiments on all kinds of missing types for every dataset and go up to 30% missing values except the national dataset, where we stop at 20% due to the lower number of rows in the original dataset.

Metrics. We use two utility metrics for evaluation. The first metric is k-way marginal distance. We calculate all k-sized combinations of attributes in the dataset [34, 83], and then report the average variational distance between the true vs the synthetic marginals. For each k-sized combination set of attributes in $A \in \mathcal{A}$, we calculate the marginal, $h : \mathcal{D} \rightarrow \mathbb{R}^{|\mathcal{D}(A)|}$ and report the average variational distance between as $\max_{a \in \mathcal{D}(A)} |h(D')[a] - h(\bar{D})[a]|$ where D' and $\bar{D}$ are the synthetic data and ground true data respectively [34, 83]. In our evaluation, we set $k = 1, 2$. Smaller values of this metric show more closeness between the true and synthetic data. Our second metric is model training. We consider 9 classification models (LogisticRegression, AdaBoost, GradientBoost, XGBoost, RandomForest, BernoulliNB, DecisionTree, Bagging, and MLP) to classify each attribute in the dataset (e.g., income is more than 50k or not, the loan should be given or not to user) using all other attributes as features. Each target attribute is processed to be a binary attribute, and we try to balance the two classes as much as possible. The quality of the learning task is represented by the average of all models across all attributes. The F1 score is reported for learning quality. Each model is trained using 70% of the synthetic data, and the F1 score is evaluated using 30% of the true data [34]. Larger values of this metric show better performance. For all our experiments, we run 3 times and report the mean with standard deviation.

6.2 Experimental Findings

6.2.1 Benchmark Existing DP Methods on Missing Data. We demonstrate the impact of missing values on DP synthetic data generation algorithms. Figure 4 shows the performance of the baselines on the Adult dataset with varying levels of missing completely at random (MCAR) data (x-axis). Our results indicate that missing data up to 5% minimally affects the quality of the synthetic data with 1-way and 2-way metrics experiencing 5-19% impact, and F1-score experiencing 1-13% impact. However, as missing data increases, the quality of synthetic data generated by all algorithms degrades drastically. With 20% missing data, the 1-way metric is affected by 14-190%, the 2-way by 18-147%, and the F1-score is impacted by 6-28%. With high amounts of missing data ($\geq$ 10%), PrivBayes and DPCTGAN exhibit the most decrease in utility. However, Kamino and DPAutoGAN show a consistent level of performance even with more missing data indicating a high degree of stability in the output. The stability of these methods can be attributed to the fact that



(a) 1-way (↓) (b) 2-way (↓) (c) F1-score (↑)

Figure 4: The effect of missing data on DP synthetic data generation algorithms.

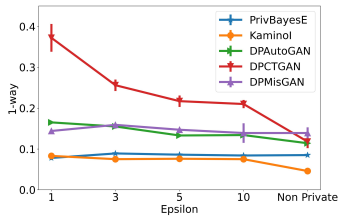
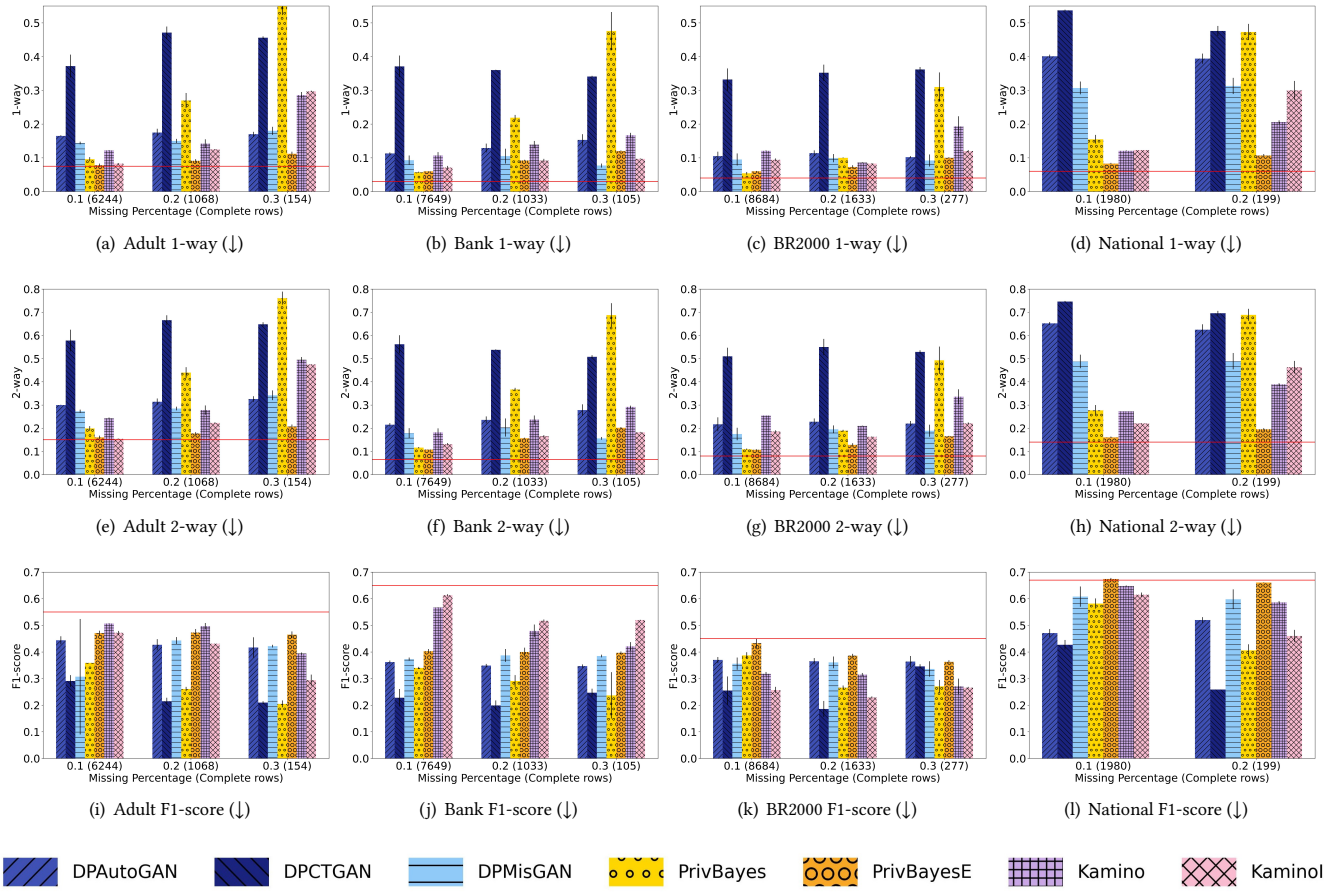


Figure 5: Adaptive methods vs baselines with MCAR missingness at varying privacy budget.



(a) Adult 1-way (↓) (b) Bank 1-way (↓) (c) BR2000 1-way (↓) (d) National 1-way (↓)

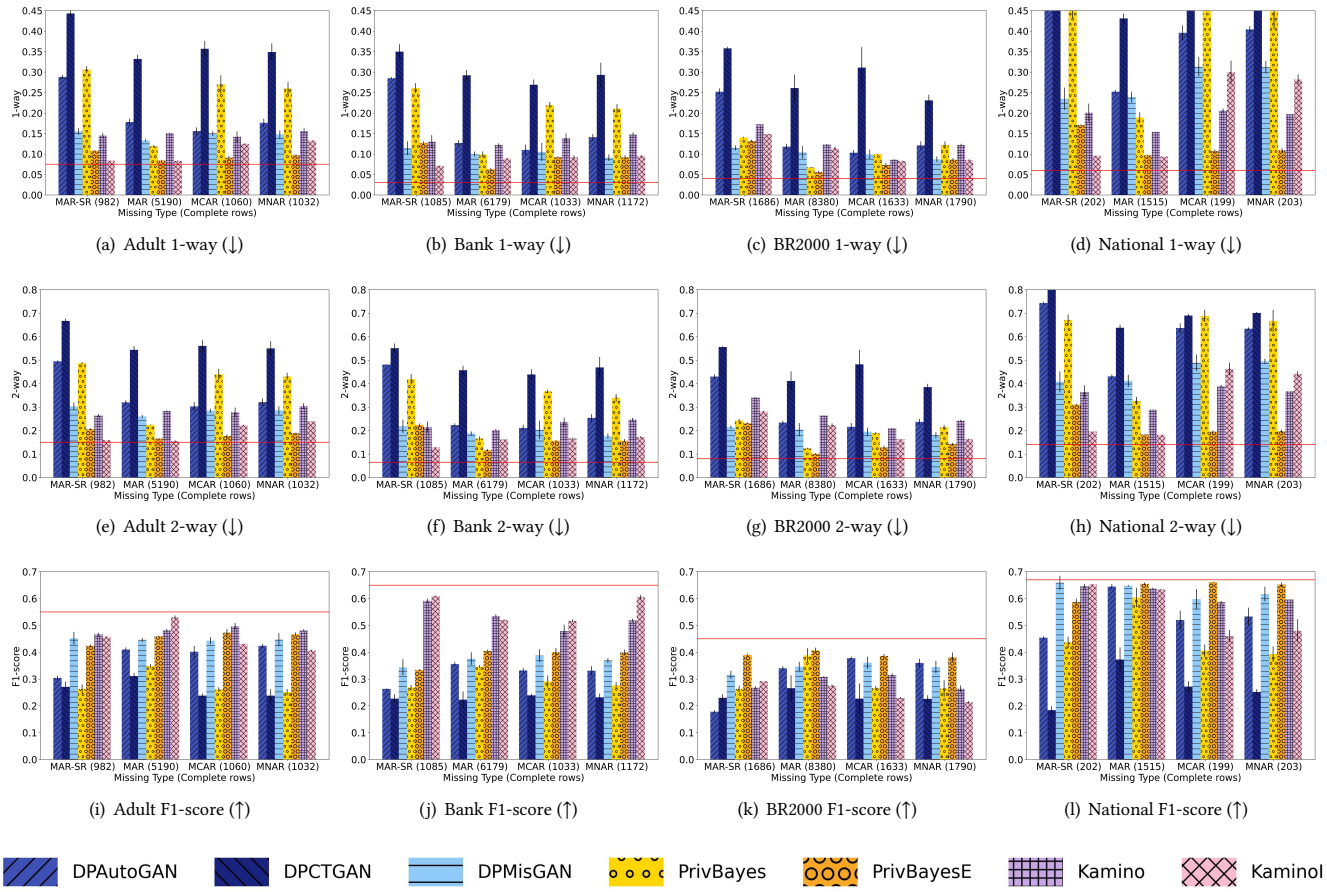
(e) Adult 2-way (↓) (f) Bank 2-way (↓) (g) BR2000 2-way (↓) (h) National 2-way (↓)

(i) Adult F1-score (↓) (j) Bank F1-score (↓) (k) BR2000 F1-score (↓) (l) National F1-score (↓)

Figure 6: Adaptive methods vs baselines with MCAR missingness at $\epsilon = 1$. Algorithms of the same category are colored with the same shade. The adaptive methods result in better quality synthetic data. The red line denotes the best no missing baseline.

these methods tend to extract more information from the available data. Kamino learns the functional dependencies of the dataset and tries to preserve them while generating the synthetic data and DPAutoGAN trains an autoencoder as a preprocessing step to learn the low-dimensional statistics of the data.

6.2.2 Evaluate Adaptive Recourse Approach. We evaluate adaptive recourse approaches at various experimental configuarations. **Varying missing percentage.** In this experiment, we compare the different synthetic data generation algorithms to their improved adaptive methods from Section 4.3. We repeat our experiment on



(a) Adult 1-way (↓) (b) Bank 1-way (↓) (c) BR2000 1-way (↓) (d) National 1-way (↓)

(e) Adult 2-way (↓) (f) Bank 2-way (↓) (g) BR2000 2-way (↓) (h) National 2-way (↓)

(i) Adult F1-score (↑) (j) Bank F1-score (↑) (k) BR2000 F1-score (↑) (l) National F1-score (↑)

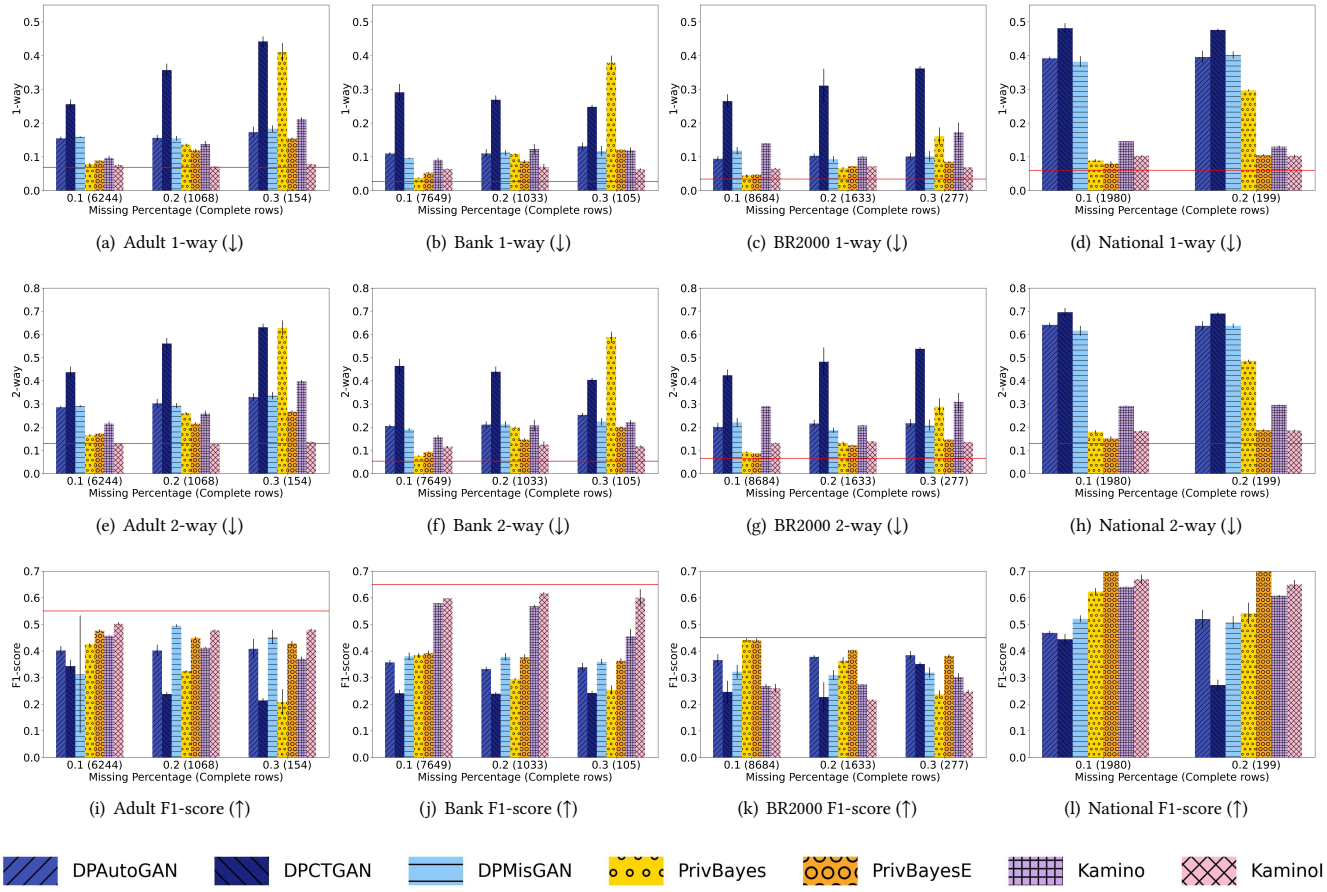
Figure 7: Adaptive methods vs non-adaptive methods on different missing mechanisms. The red line denotes the no missing baseline.

four different datasets with varying amount of MCAR data at two privacy levels ($\epsilon = 1, 3$).

We plot the algorithms in different shades of color depending on their type. GAN-based approaches are colored in shades of blue, statistical methods (PrivBayes and PrivBayesE) in yellow and mixed approaches (Kamino and KaminoI) in pink. Algorithms of the same shade of color are compared (e.g, PrivBayesE (Orange) is compared against PrivBayes (Yellow)). The red line denotes the best corresponding utility metric achieved by any of the baselines on the no missing ground truth data. We observe in general that adaptive methods (DPMisGAN, PrivBayesE and KaminoI) result in better quality synthetic data then their non-adaptive counterparts (DPAutoGAN, DPCTGAN, PrivBayes, Kamino). Across all datasets, the 1-way scores are improved by up to 68%, 2-way by up to 66% and F1-scores of up to 24%. Furthermore, we observe that the adaptive methods achieve the same utility as the no missing baseline often with 10% missing data (e.g. 1-way and 2-way for KaminoI and PrivBayesE on the Adult dataset at 10% missing data).

We also observe that there is no algorithm which performs the best overall. In the $\epsilon = 1$ case, we note that PrivBayesE beats the other mechanisms in most cases. However, in the $\epsilon = 3$ case, KaminoI significantly beats PrivBayesE for the Adult and Bank datasets. We attribute this phenomenon to the fact that higher privacy noise affects the learning of the intermediate models in Kamino. Each intermediate model in Kamino receives ϵ/k (where k is the number of attributes) budget to learn due to which a lot of noise is added in the low privacy regime.

Varying missing mechanisms. In Figure **??**, we compare adaptive vs non-adaptive methods with different missing mechanisms. We repeat our experiment for all 4 datasets and at 10% missing data for all missing mechanisms. The adaptive methods (DPMisGAN, PrivBayesE, and KaminoI) beat their non-adaptive counterparts across all missing mechanisms. The experiment conducted here is similar to the one (shown in Figure 1) in Section 4.1, but there are slight differences in the results. In this experiment, we kept the missing percentage constant, while in Section 4.1, the number of rows was constant. As missing values are added only to half of the attributes, missing at random (MAR) has more complete rows (the number in brackets on the x-axis labels) as compared to the other mechanisms. It is interesting to note that if we increase the missing



(a) Adult 1-way (↓) (b) Bank 1-way (↓) (c) BR2000 1-way (↓) (d) National 1-way (↓)

(e) Adult 2-way (↓) (f) Bank 2-way (↓) (g) BR2000 2-way (↓) (h) National 2-way (↓)

(i) Adult F1-score (↑) (j) Bank F1-score (↑) (k) BR2000 F1-score (↑) (l) National F1-score (↑)

Figure 8: Baselines vs Adaptive methods at $\epsilon = 3$. The adaptive methods perform better than their respective baselines.

percentage for MAR and plot it with same rows (MAR-SR), the algorithms start performing poorly. Hence, we make the conclusion that the number of complete of rows makes a more vital impact compared to the missing mechanism itself.

Varying privacy budget. We show the impact of the privacy budget in Figure 9 for the adaptive methods on the Adult dataset with 10% MCAR missing data. We vary the privacy budget $\epsilon \in [1, 3, 5, 10, \infty]$ where $\epsilon = \infty$ refers to the non-private run. First, we note that increasing the privacy budget improves the utility of the synthetic dataset across all algorithms. Second, we observe that PrivBayesE performs better at smaller epsilon values ($\epsilon = 1$) but KaminoI has better performance with higher epsilon ($\epsilon \geq 3$). We attribute to the fact that KaminoI trains multiple intermediate models and these models require more higher privacy budget.

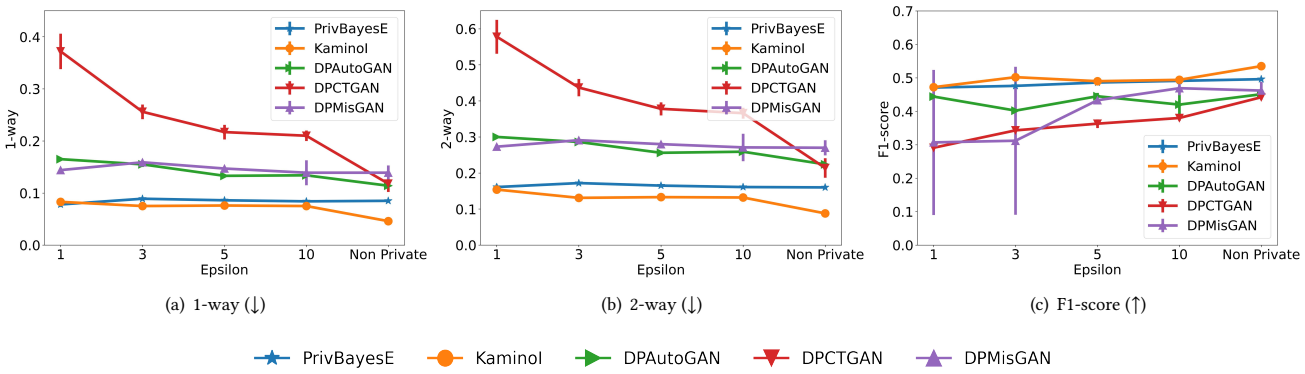
6.2.3 Amplification Due To Missingness. In this experiment, we show the amplified privacy budget for ground truth data. We experiment with PrivBayesE that runs on the incomplete dataset with 10 - 50% MCAR missing data. For each run, we allocate a privacy budget of $\epsilon = 1$ and observe the marginals calculated by PrivBayesE. We assume a uniform privacy budget for each marginal calculated by PrivbayesE and run Algorithm 4 to calculate the best valid partition of these marginals. In Table 2, we plot the amplified privacy cost $\bar{\epsilon}$ based on the best partition found by the algorithm. We repeat the experiment for two datasets – Adult and BR2000. Our results show that the amplified privacy cost decreases almost linearly from 0.88x to 0.44x for Adult and 0.83x to 0.31x for the BR2000 dataset.

Table 2: Amplified privacy for ground truth data.

Dataset	MCAR missing %				
	0.1	0.2	0.3	0.4	0.5
Adult	0.88	0.77	0.65	0.47	0.44
BR2000	0.83	0.68	0.55	0.41	0.31

7 RELATED WORK

Differentially private synthetic data generation has been studied vastly in prior literature [16, 31, 62, 84]. The prior works generate synthetic data via statistical approaches which estimate low-dimensional marginal distributions [69, 78], deep learning approaches



(a) 1-way (↓) (b) 2-way (↓) (c) F1-score (↑)

Figure 9: Varying privacy budget and comparing utilities of our approaches

such as generative adversarial networks (GAN) [36], or the combination of the two [34]. However, all of these algorithms focus on the no missing data setting. Some prior work look into the differentially private missing data setting. Patki et al. [65] propose the synthetic data vault framework which identifies and repairs inconsistencies in the generated synthetic data from incomplete data. Other works focus on privately imputing missing values. Clifton et. al. [23] introduce imputation for missing data using differentially private k-nearest neighbours algorithm for use cases involving SQL queries. Das et. al. [24] analyze the privacy bounds when adding privacy to the imputation stage and analysis stage implemented from a non-private imputation algorithm using OLS regression.

8 CONCLUSION

In conclusion, our research paper presents a comprehensive study on differentially private synthetic data generation algorithms for private datasets with missing values. Our proposed adaptive recourse methods outperform classical approaches and strike a balance between privacy and utility. We also provide techniques for calculating privacy bounds and demonstrate the effectiveness of our methods through extensive experiments on real-world datasets. Our findings have important implications for privacy-preserving data sharing and analysis, and can facilitate the development of more effective methods for generating synthetic data in various applications.

REFERENCES

[1] 2016-04-27. Regulation (EU) 2016/679 of the European Parliament and of the Council of 27 April 2016 on the protection of natural persons with regard to the processing of personal data and on the free movement of such data, and repealing Directive 95/46/EC (General Data Protection Regulation). *OJ* (2016-04-27).

[2] Martín Abadi, Andy Chu, Ian J. Goodfellow, H. Brendan McMahan, Ilya Mironov, Kunal Talwar, and Li Zhang. 2016. Deep Learning with Differential Privacy. In *CCS*. ACM, 308–318.

[3] Nazmiye Ceren Abay, Yan Zhou, Murat Kantarcioglu, Bhavani Thuraisingham, and Latanya Sweeney. 2018. Privacy preserving synthetic data release using deep learning. In *Joint European Conference on Machine Learning and Knowledge Discovery in Databases*. Springer, 510–526.

[4] John M. Abowd. 2018. The U.S. Census Bureau Adopts Differential Privacy. In *KDD*. 2867.

[5] Gergely Acs, Luca Melis, Claude Castelluccia, and Emiliano De Cristofaro. 2018. Differentially private mixture of generative neural networks. *IEEE Transactions on Knowledge and Data Engineering* 31, 6 (2018), 1109–1121.

[6] Anish Agarwal and Rahul Singh. 2021. Causal inference with corrupted data: Measurement error, missing values, discretization, and differential privacy. *arXiv preprint arXiv:2107.02780* (2021).

[7] Wael Alghamdi, Shahab Asoodeh, Flavio P Calmon, Juan Felipe Gomez, Oliver Kosut, Lalitha Sankar, and Fei Wei. 2022. The Saddle-Point Accountant for Differential Privacy. *arXiv preprint arXiv:2208.09595* (2022).

[8] Elena Andreou, Eric Ghysels, and Andros Kourtellos. 2013. Should macroeconomic forecasters use daily financial data and how? *Journal of Business & Economic Statistics* 31, 2 (2013), 240–251.

[9] Rebecca R Andridge and Roderick JA Little. 2010. A review of hot deck imputation for survey non-response. *International statistical review* 78, 1 (2010), 40–64.

[10] Martin Arjovsky, Soumith Chintala, and Léon Bottou. 2017. Wasserstein generative adversarial networks. In *International conference on machine learning*. PMLR, 214–223.

[11] Anonymous authors. 2023. Differentially Private Data Generation with Missing Data. http://link.com.

[12] Brooke Auxier, Lee Rainie, Monica Anderson, Andrew Perrin, Madhu Kumar, and Erica Turner. 2019. Americans and Privacy - Concerned Confused and Feeling Lack of Control Over Their Personal Information. *Pew Research Center* (2019).

[13] Borja Balle, Gilles Barthe, and Marco Gaboardi. 2018. Privacy amplification by subsampling: Tight analyses via couplings and divergences. *Advances in Neural Information Processing Systems* 31 (2018).

[14] Boaz Barak, Kamalika Chaudhuri, Cynthia Dwork, Satyen Kale, Frank McSherry, and Kunal Talwar. 2007. Privacy, accuracy, and consistency too: a holistic solution to contingency table release. In *PODS*. 273–282.

[15] Raef Bassily, Adam D. Smith, and Abhradeep Thakurta. 2014. Private Empirical Risk Minimization: Efficient Algorithms and Tight Error Bounds. In *FOCS*. 464–473.

[16] Claire McKay Bowen and Fang Liu. 2020. Comparative Study of Differentially Private Data Synthesis Methods. *Statist. Sci.* 35, 2 (May 2020), 280–307. https://doi.org/10.1214/19-sts742

[17] U.S. Census Bureau. Accessed on 2020-11-30. LEHD Origin-Destination Employment Statistics (2002-2017). https://onthemap.ces.census.gov/

[18] Thee Chanyaswad, Changchang Liu, and Prateek Mittal. 2019. Ron-gauss: Enhancing utility in non-interactive private data release. *Proceedings on Privacy Enhancing Technologies* 2019, 1 (2019), 26–46.

[19] R. Chawla. 2019. Deepfakes : How a pervert shook the world. *International Journal for Advance Research and Development* 4 (2019), 4–8.

[20] Dingfan Chen, Tribhuvanesh Orekondy, and Mario Fritz. 2020. Gs-wgan: A gradient-sanitized approach for learning differentially private generators. *Advances in Neural Information Processing Systems* 33 (2020), 12673–12684.

[21] Qingrong Chen, Chong Xiang, Minhui Xue, Bo Li, Nikita Borisov, Dali Kaafar, and Haojin Zhu. 2018. Differentially Private Data Generative Models. *CoRR* abs/1812.02274 (2018).

[22] Rui Chen, Qian Xiao, Yu Zhang, and Jianliang Xu. 2015. Differentially Private High-Dimensional Data Publication via Sampling-Based Inference. In *SIGKDD*. 129–138.

[23] Chris Clifton, Eric J. Hanson, Keith Merrill, and Shawn Merrill. 2022. Differentially Private *k*-Nearest Neighbor Missing Data Imputation. *ACM Trans. Priv. Secur.* 25, 3 (2022), 16:1–16:23. https://doi.org/10.1145/3507952

[24] Soumojit Das, Jorg Dreschler, Keith Merrill, and Shawn Merrill. 2022. Imputation under Differential Privacy. *CoRR* abs/2206.15063 (2022). https://doi.org/10.48550/arXiv.2206.15063

[25] Dheeru Dua and Casey Graff. 2017. UCI Machine Learning Repository. http://archive.ics.uci.edu/ml

[26] Cynthia Dwork. 2006. Differential Privacy. In *ICALP*, Vol. 4052. Springer, 1–12.

[27] Cynthia Dwork, Krishnaram Kenthapadi, Frank McSherry, Ilya Mironov, and Moni Naor. 2006. Our Data, Ourselves: Privacy Via Distributed Noise Generation. In *EUROCRYPT*, Vol. 4004. Springer, 486–503.

[28] Cynthia Dwork, Frank McSherry, Kobbi Nissim, and Adam Smith. 2006. Calibrating Noise to Sensitivity in Private Data Analysis. In *Proceedings of the 3rd Conference on Theory of Cryptography (TCC '06)*. 265–284.

[29] Cynthia Dwork and Aaron Roth. 2014. The Algorithmic Foundations of Differential Privacy. *Foundations and Trends in Theoretical Computer Science* 9, 3-4 (2014), 211–407.

[30] Úlfar Erlingsson, Vasyl Pihur, and Aleksandra Korolova. 2014. RAPPOR: Randomized Aggregatable Privacy-Preserving Ordinal Response. In *CCS*. ACM, 1054–1067.

[31] Liyue Fan. 2020. A Survey of Differentially Private Generative Adversarial Networks. In *The AAAI Workshop on Privacy-Preserving Artificial Intelligence*.

[32] Lorenzo Frigerio, Anderson Santana de Oliveira, Laurent Gomez, and Patrick Duverger. 2019. Differentially Private Generative Adversarial Networks for Time Series, Continuous, and Discrete Open Data. In *SEC*. 151–164.

[33] Chang Ge, Xi He, Ihab F. Ilyas, and Ashwin Machanavajjhala. 2019. APEx: Accuracy-Aware Differentially Private Data Exploration. In *SIGMOD*. 177–194.

[34] Chang Ge, Shubhankar Mohapatra, Xi He, and Ihab F Ilyas. 2021. Kamino: Constraint-aware differentially private data synthesis. *Proceedings of the VLDB Endowment* 14, 10 (2021), 1886–1899.

[35] Kylie Getz, Rebecca A Hubbard, and Kristin A Linn. 2023. Performance of Multiple Imputation Using Modern Machine Learning Methods in Electronic Health Records Data. *Epidemiology* (2023), 10–1097.

[36] Ian J. Goodfellow, Jean Pouget-Abadie, Mehdi Mirza, Bing Xu, David Warde-Farley, Sherjil Ozair, Aaron C. Courville, and Yoshua Bengio. 2014. Generative Adversarial Networks. *CoRR* abs/1406.2661 (2014).

[37] Sivakanth Gopi, Yin Tat Lee, and Lukas Wutschitz. 2021. Numerical composition of differential privacy. *Advances in Neural Information Processing Systems* 34 (2021), 11631–11642.

[38] Andy Greenberg. 2016. Apple's 'Differential Privacy' Is About Collecting Your Data—But Not *Your* Data. *Wired* (2016).

[39] Rahul Gupta. 2019. Data Augmentation for Low Resource Sentiment Analysis Using Generative Adversarial Networks. In *ICASSP*. IEEE, 7380–7384.

[40] Michael B. Hawes. 2020. Implementing Differential Privacy: Seven Lessons From the 2020 United States Census. *Harvard Data Science Review* (30 4 2020). https://doi.org/10.1162/99608f92.353c6f99 https://hdsr.mitpress.mit.edu/pub/dgg03vo6.

[41] IBM. 2020. Cost of a Data Breach Report. (2020).

[42] José M Jerez, Ignacio Molina, Pedro J García-Laencina, Emilio Alba, Nuria Ribelles, Miguel Martín, and Leonardo Franco. 2010. Missing data imputation using statistical and machine learning methods in a real breast cancer problem. *Artificial intelligence in medicine* 50, 2 (2010), 105–115.

[43] Noah M. Johnson, Joseph P. Near, and Dawn Song. 2018. Towards Practical Differential Privacy for SQL Queries. *PVLDB* 11, 5 (2018), 526–539.

[44] James Jordon, Jinsung Yoon, and Mihaela van der Schaar. 2019. PATE-GAN: Generating Synthetic Data with Differential Privacy Guarantees. In *ICLR*.

[45] Nancy E Kass, Marvin R Natowicz, Sara Chandros Hull, Ruth R Faden, Laura Plantinga, Lawrence O Gostin, and Julia Slutsman. 2003. The use of medical records in research: what do patients want? *The Journal of Law, Medicine & Ethics* 31, 3 (2003), 429–433.

[46] Daphne Koller and Nir Friedman. 2009. *Probabilistic Graphical Models - Principles and Techniques*. MIT Press.

[47] Ios Kotsogiannis, Yuchao Tao, Xi He, Maryam Fanaeepour, Ashwin Machanavajjhala, Michael Hay, and Gerome Miklau. 2019. PrivateSQL: A Differentially Private SQL Query Engine. *PVLDB* 12, 11 (2019), 1371–1384.

[48] Kamakshi Lakshminarayan, Steven A Harp, Robert P Goldman, Tariq Samad, et al. 1996. Imputation of Missing Data Using Machine Learning Techniques.. In *KDD*, Vol. 96.

[49] Chao Li, Gerome Miklau, Michael Hay, Andrew McGregor, and Vibhor Rastogi. 2015. The matrix mechanism: optimizing linear counting queries under differential privacy. *VLDB J.* 24, 6 (2015), 757–781.

[50] Haoran Li, Li Xiong, Lifan Zhang, and Xiaoqian Jiang. 2014. DPSynthesizer: Differentially Private Data Synthesizer for Privacy Preserving Data Sharing. *Proc. VLDB Endow.* 7, 13 (2014), 1677–1680.

[51] Steven Cheng-Xian Li, Bo Jiang, and Benjamin Marlin. 2019. Misgan: Learning from incomplete data with generative adversarial networks. *arXiv preprint arXiv:1902.09599* (2019).

[52] Roderick JA Little and Donald B Rubin. 2019. *Statistical analysis with missing data*. Vol. 793. John Wiley & Sons.

[53] Yonghong Luo, Ying Zhang, Xiangrui Cai, and Xiaojie Yuan. 2019. E2gan: End-to-end generative adversarial network for multivariate time series imputation. In *Proceedings of the 28th international joint conference on artificial intelligence*. AAAI Press, 3094–3100.

[54] Ryan McKenna, Daniel Sheldon, and Gerome Miklau. 2019. Graphical-model based estimation and inference for differential privacy. In *ICML*, Vol. 97. 4435–4444.

[55] Frank McSherry. 2009. Privacy integrated queries: an extensible platform for privacy-preserving data analysis. In *SIGMOD*, Ugur Çetintemel, Stanley B. Zdonik, Donald Kossmann, and Nesime Tatbul (Eds.). ACM, 19–30.

[56] Shubhankar Mohapatra, Sajin Sasy, Xi He, Gautam Kamath, and Om Thakkar. 2021. The Role of Adaptive Optimizers for Honest Private Hyperparameter Selection. *arXiv preprint arXiv:2111.04906* (2021).

[57] Sérgio Moro, Paulo Cortez, and Paulo Rita. 2014. A data-driven approach to predict the success of bank telemarketing. *Decision Support Systems* 62 (2014), 22–31.

[58] Boris Muzellec, Julie Josse, Claire Boyer, and Marco Cuturi. 2020. Missing data imputation using optimal transport. In *International Conference on Machine Learning*. PMLR, 7130–7140.

[59] Shinichi Nakagawa and Robert P Freckleton. 2008. Missing inaction: the dangers of ignoring missing data. *Trends in ecology & evolution* 23, 11 (2008), 592–596.

[60] Eric Schulte Nordholt. 1998. Imputation: methods, simulation experiments and practical examples. *International Statistical Review* 66, 2 (1998), 157–180.

[61] AH Noruzman, NA Ghani, and NSA Zulkifli. [n. d.]. Gretel. ai: Open-Source Artificial Intelligence Tool To Generate New Synthetic Data. ([n. d.]).

[62] National Institute of Standards and Technology. 2018. Differential Privacy Synthetic Data Challenge. https://www.nist.gov/ctl/pscr/open-innovation-prize-challenges/past-prize-challenges/2018-differential-privacy-synthetic

[63] Nicolas Papernot, Shuang Song, Ilya Mironov, Ananth Raghunathan, Kunal Talwar, and Úlfar Erlingsson. 2018. Scalable Private Learning with PATE. In *ICLR*.

[64] Nicolas Papernot and Thomas Steinke. [n. d.]. Hyperparameter Tuning with Renyi Differential Privacy. In *International Conference on Learning Representations*.

[65] Neha Patki, Roy Wedge, and Kalyan Veeramachaneni. 2016. The Synthetic Data Vault. In *2016 IEEE International Conference on Data Science and Advanced Analytics (DSAA)*. 399–410. https://doi.org/10.1109/DSAA.2016.49

[66] NhatHai Phan, Yue Wang, Xintao Wu, and Dejing Dou. 2016. Differential privacy preservation for deep auto-encoders: an application of human behavior prediction. In *Thirtieth AAAI Conference on Artificial Intelligence*.

[67] Haoyue Ping, Julia Stoyanovich, and Bill Howe. 2017. DataSynthesizer: Privacy-Preserving Synthetic Datasets. In *SSDBM*. ACM, 42:1–42:5.

[68] Eugenia Politou, Efthimios Alepis, and Constantinos Patsakis. 2018. Forgetting personal data and revoking consent under the GDPR: Challenges and proposed solutions. *Journal of cybersecurity* 4, 1 (2018), tyy001.

[69] Wahbeh H. Qardaji, Weining Yang, and Ninghui Li. 2014. PriView: practical differentially private release of marginal contingency tables. In *SIGMOD*. 1435–1446.

[70] Donald B Rubin. 1976. Inference and missing data. *Biometrika* 63, 3 (1976), 581–592.

[71] Ashish Shrivastava, Tomas Pfister, Oncel Tuzel, Joshua Susskind, Wenda Wang, and Russell Webb. 2017. Learning from Simulated and Unsupervised Images through Adversarial Training. In *CVPR*. IEEE Computer Society, 2242–2251.

[72] Shuang Song, Kamalika Chaudhuri, and Anand D. Sarwate. 2013. Stochastic gradient descent with differentially private updates. In *GlobalSIP*. 245–248.

[73] Thomas Steinke. 2022. Composition of Differential Privacy & Privacy Amplification by Subsampling. *arXiv preprint arXiv:2210.00597* (2022).

[74] Streat D. Simpson A. Task C., Bhagat K. and Howarth G.S. 2023. NIST Diverse Communities Data Excerpts, National Institute of Standards and Technology. https://doi.org/10.18434/mds2-2895

[75] Reihaneh Torkzadehmahani, Peter Kairouz, and Benedict Paten. 2019. Dp-cgan: Differentially private synthetic data and label generation. In *Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition Workshops*. 0–0.

[76] Reihaneh Torkzadehmahani, Peter Kairouz, and Benedict Paten. 2020. DP-CGAN: Differentially Private Synthetic Data and Label Generation. *CoRR* abs/2001.09700 (2020).

[77] Oliver Williams and Frank McSherry. 2010. Probabilistic Inference and Differential Privacy. In *NIPS*. 2451–2459.

[78] Xiaokui Xiao, Guozhang Wang, and Johannes Gehrke. 2011. Differential Privacy via Wavelet Transforms. *IEEE Trans. Knowl. Data Eng.* 23, 8 (2011), 1200–1214.

[79] Liyang Xie, Kaixiang Lin, Shu Wang, Fei Wang, and Jiayu Zhou. 2018. Differentially Private Generative Adversarial Network. *CoRR* abs/1802.06739 (2018).

[80] Lei Xu, Maria Skoularidou, Alfredo Cuesta-Infante, and Kalyan Veeramachaneni. 2019. Modeling tabular data using conditional gan. *Advances in Neural Information Processing Systems* 32 (2019).

[81] Yungang Xu, Zhigang Zhang, Lei You, Jiajia Liu, Zhiwei Fan, and Xiaobo Zhou. 2020. scIGANs: single-cell RNA-seq imputation using generative adversarial networks. *Nucleic acids research* 48, 15 (2020), e85–e85.

[82] Jinsung Yoon, James Jordon, and Mihaela Schaar. 2018. Gain: Missing data imputation using generative adversarial nets. In *International conference on machine learning*. PMLR, 5689–5698.

[83] Jun Zhang, Graham Cormode, Cecilia M. Procopiuc, Divesh Srivastava, and Xiaokui Xiao. 2014. PrivBayes: private data release via bayesian networks. In *SIGMOD*. 1423–1434.

[84] Tianqing Zhu, Gang Li, Wanlei Zhou, and Philip S. Yu. 2017. Differentially Private Data Publishing and Analysis: A Survey. *IEEE Trans. Knowl. Data Eng.* 29, 8 (2017), 1619–1638.

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