Supporting Information for Microburst Size Distribution Derived with AeroCube-6

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Introduction

Text S1: Analytic Derivation of $\bar{F}(s)$ Here we derive the integral form of the $\bar{F}(s)$ under the following assumptions:

1. microbursts are circular with radius r
2. microbursts are randomly and uniformly distributed around AC6.

First recall the area $A(r, s)$ given in Eq. 4 in the main text and copied here for convenience

$$A(r, s) = 2r^2 \cos^{-1}\left(\frac{s}{2r}\right) - \frac{s}{2}\sqrt{4r^2 - s^2}. \tag{1}$$

A circular microburst who's center lies in $A(r,s)$ will be observed by both AC6 units and is counted in $\bar{F}(s)$. For a given microburst radius r and reference spacecraft separation, s_0 chosen such that $A(r,s_0) > 0$, there will be n_0 microbursts simultaneously observed. Now for example if the spacecraft separation is changed such that the area doubles, the second assumption implies that the number of microbursts observed during the same time interval is doubled as well. This can be written as

$$\frac{n_0}{A(r,s_0)} = \frac{n}{A(r,s)} \tag{2}$$

and can be interprted as the microburst density. Equation 2 which can then be rewritten as

$$n(r,s) = \Big(\frac{n_0}{A(r,s_0)}\Big)A(r,s) \tag{3}$$

so if s is changed such that e.g. $A(r,s)$ doubles, then n must also double.

To calculate the cumulative number of microbursts observed above s, we define

$$N(r,s) = \int_s^\infty n(r,s')ds' = \Big(\frac{n_0}{A(r,s_0)}\Big)\int_s^\infty A(r,s')ds' \tag{4}$$

which works for a one-size microburst distribution of radius r. $\bar{F}(s)$ for a single r is then

$$\bar{F}(s) = \frac{\int_s^\infty A(r,s')ds'}{\int_0^\infty A(r,s')ds'} \tag{5}$$

Now how

Quarantine this section Now we develop the analytic $\bar{F}(s)$ given $p(r)$. For now we assume a one-sized $p(r)$ i.e., a delta function in r. Then the ratio of the circle intersection areas at two distinct AC6 separations s_1 and s_2 is the ratio of the number of microbursts observed at those two separations

$$\frac{n_1}{n_2} = \frac{A(r, s_1)}{A(r, s_2)}. \tag{6}$$

Similar to the MC model Move derivation to SI

Then the analytic $\bar{F}(s)$ represents how quickly the fraction of microbursts observed by AC6 decreases as a function of s.

Furthermore the hypothesized $p(r)$ is the relative occurrence of microbursts at various r and is effectively a weighting factor on $A(r, s)$ which must integrated out (marginalized) since AC6 is observing the cumulative effect of microbursts of all r. Lastly, a cumulative integral over a dummy variable s' is applied to the normalized areas to calculate $\bar{F}(s)$. With these considerations the analytic $\bar{F}(s)$ is given by

$$\bar{F}(s, \theta) = \frac{\int_{s}^{\infty} \int_{0}^{\infty} A(r, s') p(r, \theta) dr ds'}{\int_{0}^{\infty} \int_{0}^{\infty} A(r, s') p(r, \theta) dr ds'} \tag{7}$$

where as in the Monte Carlo model the denominator normalizes $\bar{F}(s)$ to unity. End of quarantine

Mention rain bucket analogy

Text S2: Comparison of microburst to whistler mode chorus $\bar{F}(s)$