Supporting Information for Microburst Size Distribution Derived with AeroCube-6

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Introduction

Text S1: Analytic Derivation of $\bar{F}(s)$ Here we derive the integral form of $\bar{F}(s)$ under the following assumptions:

1. microbursts are circular with radius r

2. microbursts are randomly and uniformly distributed around AC6.

First recall the area $A(r,s)$, given in Eq. 4 in the main text and copied here for convenience

$$A(r,s) = 2r^2 \cos^{-1}\left(\frac{s}{2r}\right) - \frac{s}{2}\sqrt{4r^2 - s^2}. \quad (1)$$

A circular microburst who's center lies in $A(r, s)$ will be observed by both AC6 units and is counted in $\bar{F}(s)$. Now we derive the integral form of $\bar{F}(s)$ that accounts for the different spacecraft separations and microburst sizes that are distributed by a hypothesized PDF $p(r, \theta)$.

First we will account for the effects of various spacecraft separation, assuming all microbursts are one size. For reference choose of radius, r_0 and spacecraft separation, s_0 such that $A(r_0, s_0) > 0$ which implies that some number of microbursts, n_0 will be simultaneously observed. Now, if for example the spacecraft separation (or microburst radius) is changed such that the area doubles, the second assumption implies that the number of microbursts observed during the same time interval must double as well. This can be expressed as

$$\frac{n_0}{A(r_0, s_0)} = \frac{n}{A(r, s)} \tag{2}$$

and interpreted as the conservation of the microburst area density. By rewriting Eq. 2 as

$$n(r, s) = \Big(\frac{n_0}{A(r_0, s_0)}\Big) A(r, s) \tag{3}$$

it is more clear that the number of microbursts of size r observed at separation s is just $A(r, s)$ scaled by the reference microburst area density. The cumulative number of microbursts observed above s is then

$$N(r, s) = \int_s^\infty n(r, s') ds' = \Big(\frac{n_0}{A(r_0, s_0)}\Big) \int_s^\infty A(r, s') ds'. \tag{4}$$

Lastly, $\bar{F}(s)$ for a single r is then

$$\bar{F}(s) = \frac{N(s)}{N(0)} = \frac{\int_s^\infty A(r,s')ds'}{\int_0^\infty A(r,s')ds'} \tag{5}$$

To incorporate a continuous microburst PDF such as $p(r) = p_1\delta(r-r_1)+p_2\delta(r-r_2)+...$ we sum up the weighted number of microbursts that each size contributes to $N(s)$ i.e.

$$N(s) = \Big(\frac{n_0}{A(r_0,s_0)}\Big)\Big(\int_s^\infty p_1 A(r_1,s')ds' + \int_s^\infty p_2 A(r_2,s')ds' + ...\Big) \tag{6}$$

The last step is to convert the sum of Dirac Delta functions into a continuous PDF $p(r)$ after which

$$N(s) = \Big(\frac{n_0}{A(r_0,s_0)}\Big)\int_s^\infty \int_0^\infty A(r,s')p(r)drds'. \tag{7}$$

With these considerations, $\bar{F}(s)$ is then given by

$$\bar{F}(s,\theta) = \frac{\int_s^\infty \int_0^\infty A(r,s')p(r,\theta)drds'}{\int_0^\infty \int_0^\infty A(r,s')p(r,\theta)drds'} \tag{8}$$

Text S2: Comparison of microburst to whistler mode chorus $\bar{F}(s)$

TBD