Diff TraX

Differential Calculus

→ Lines and curves have slopes

→ You can use those slopes to find solves in real problems

→ There are a set of rules you need to follow to define those slopes

→ There is some underlying theory that helps to underpin how to do this

~~$y = m$~~

Straight line and slopes

$y = mx + c$ ← slope

$$\text{slope} = \frac{\text{Rise}}{\text{Run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Find the derivatives of the function $f(x) = x^2 - 3x + 12$ by first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$f(x+h) = (x+h)^2 - 3x - 3h + 12$
$= x^2 + 2xh + h^2 - 3x - 3h + 12$

$$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h + 12 - x^2 + 3x - 12}{h}$$

$$= \lim_{h \to 0} \frac{(2xh + h^2 - 3h)}{h} = \lim_{h \to 0} 2x + h - 3 = 2x - 3$$

Tangent versus Derivative

Derivative is a way to show rate of change, the amount by which a function is changing at a given point.

→ When we find the derivative of a function we say that we "differentiate" the function and that the function is "differentiable"

$$y = 4$$

$$\frac{dy}{dx} = 4x^0 = 0 \times 4x^{0-1} = 0$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Differentiating polynomial:

$$ax^n$$

$$\frac{d}{dx}(ax^n) = anx^{n-1}$$

Tangent line

tangent line of a functio $f(x)$ at a point $(a, f(a))$ is:

$$\boxed{y = f'(a)(x-a) + f(a)}$$

$f(x) = x + \frac{1}{x}$

$f'(x) = 1 - \frac{1}{x^2}$

$y = f'(a)(x-a) + f(a)$

$f(x) = 4x^6 - 6x^5 + 7x^4 + 8x^3 + 10x^2 - 7x - 1$

$f'(x) = 24x^5 - 30x^4 + 28x^3 + 24x^2 + 20x - 7$

$y = f'(3)(x-3) + f(3)$

Tangent $y = 4427(x-3) + 2129$

Tangents and Normal of a graph:

two lines are ~~norma~~ perpendicular exactly when the product of the slopes $m_1 m_2 = -1$

If ~~a~~ the point of the graph is $(a, f(a))$, and the derivative $f'(a)$ is not equal to zero at this point then the equation ~~the~~ of the line normal to the graph at this point is

$$\boxed{y = -\frac{(x-a)}{f'(a)} + f(a)}$$

Now $f(x) = 12x - 2$

$f'(x) = 12$

equation of line perpendicular to $f(x)$ at $x = -4$?

$$y = -\frac{(x+4)}{f'(-4)} + f(-4)$$

$$= -\frac{x+4}{12} \cancel{-48} - 50$$

~~Now, $f(x) = x$~~
2

Features of function:

Determine if a function is increasing or decreasing

$f(x) = \cancel{8}x^3 - 3x^2 + 4$

$f'(x) = 3x^2 - 6x.$

let $f'(x) = 0$

$3x(x-2) = 0$

$\Rightarrow x = 0$ or $x = 2$

so, $f(x)$ is increasing when $x < 0$, decreasing when $0 < x < 2$ and ~~it~~ is ~~creasi~~ increasing again when $x > 2$

Now

$\cancel{f(x) = -3x^6 - x}$

Stationary points:

$\frac{df}{dx} = 0 \rightarrow$ find $x \rightarrow$ plug it to $f(x)$.

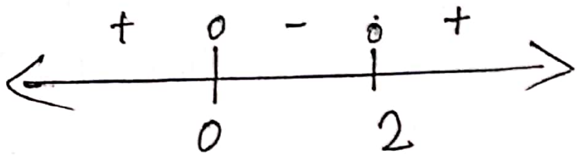
$\cancel{f(x) =}$ $f(0) = 4$

$f(2) = 0$

Categorising stationary points:

- Local maxima
- Local minima

Check using sign diagram:



$f'(-1) = +ve$

$f'(1) = -ve$

$f'(3) = +ve$

Now

$f(x) = 5x - 7 + 8x^{-1}$

$f'(x) = 5 - \frac{8}{x^2}$

Now, if $f'(x) = 0$

$\frac{5x^2 - 8}{x^2} = 0$

$x = 0, \; x = \sqrt{\frac{8}{5}}$

$f(0) = -7$

$f\left(\sqrt{\frac{8}{5}}\right) = -7 + 4\sqrt{10}$

For this point we get maxima

Concavity

Concave up → look like a cup

Concave down ⇒ look like a cap

$f(x) = x^3 - 3x^2 + 4$

$f'(x) = 3x^2 - 6x$

$f''(x) = 6x - 6$

if $f''(x) > 0$ on a region then the graph of $f(x)$ is concave up on that region

if $f''(x) < 0$ on a region then the graph of $f(x)$ is concave down on that region

However, if the value of $f''(x)$ is zero at a number c and the sign $f''(x)$ changes from positive to negative at c then the function $f(x)$ has a point of inflection at $(c, f(c))$

Point of inflection: A point where the graph is concave up on one side and concave down on the other.

	increasing	decreasing
Concave up		
Concave down		

Now

$\cancel{f(x) = 4x^4 - 4x^3 + 2x^2 - 9x + 9}$

$f(x) = 4x^4 - 4x^3 + 2x^2 - 9x + 16$

$f'(x) = 16x^3 - 12x^2 + 4x - 9$

$f''(x) = 48x^2 - 24x + 4$

To sum up everything

- First derivative test for increasing/ decreasing
  - $f'(x) > 0$ on interval $\rightarrow$ $f(x)$ increasing on interval
  - $f'(x) < 0$ on interval $\rightarrow$ $f(x)$ decreasing on interval
- Second derivative test for concavity
  - $f''(x) > 0$ on interval $\rightarrow$ $f(x)$ concave up on interval
  - $f''(x) < 0$ on interval $\rightarrow$ $f(x)$ concave down on interval

- First derivative test for stationary points

$f'(x) = 0$ at $x = a$ → $f(x)$ has stationary point at $x=a$

- Sign diagram for first derivative at stationary points

$+ \quad a \quad -$ $f'(x)$	$f(x)$ has local maximum at $x=a$
$- \quad a \quad +$ $f'(x)$	$f(x)$ has local minimum at $x=a$

- Second derivative test for local maximum/minimum

$f''(a) > 0$, $x=a$ stationary point → $f(x)$ has local min at $x=a$

$f''(a) < 0$, $x=a$ stationary point → $f(x)$ has local max at $x=a$

- Sign diagram for second derivative at an inflection point

$- \quad a \quad +$ $f''(x)$	$f(x)$ has a point at $x=a$
$+ \quad a \quad -$ $f''(x)$	$f(x)$ has a point at $x=a$

Graphing whole function:

$f(x) = x^4 - 24x^2$

$f'(x) = 4x^3 - 48x$

$f''(x) = 12x^2 - 48$

x. intercepts:

if $f(x) = 0$

$x^4 - 24x^2 = 0$

$\Rightarrow x^2(x^2 - 24) = 0$

$x = 0$; $x = \pm 2\sqrt{6}$

turning points

if $f'(x) = 0$

$4x^3 - 48x$

$\Rightarrow 4x(x^2 - 12) = 0$

$\Rightarrow x = 0 \quad x = \pm 2\sqrt{3}$

inflection points:

if $f''(x) = 0$

$12x^2 = 48$

$\Rightarrow x = \pm 2$

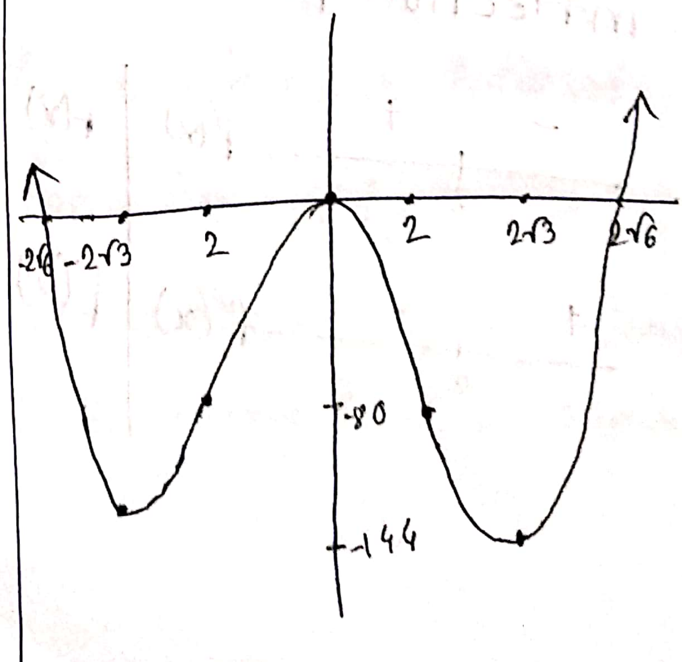
Sign digram for $f'(x)$

x		$-2\sqrt{3}$		0		$2\sqrt{3}$	
$f'(x)$	$-$		$+$		$-$		$+$

Sign digram for $f''(x)$

x		-2		2	
$f''(x)$	$+$		$-$		$+$

sketch



Cubic polynomials

A cubic has a point of inflection at $b/3a$

$f(x) = ax^3 + bx^2 + cx + d$

$f'(x) = 3ax^2 + 2bx + c$

$f''(x) = 0 \Leftrightarrow 6ax + 2b = 0$

$\Rightarrow x = -\dfrac{b}{3a}$

Derivative of sums of functions

$h(x) = x^2 + x^{1/3}$

$f(x) = x^2$

$g(x) = x^{1/3}$

- $h(x) = f(x) + g(x)$
- $h'(x) = f'(x) + g'(x)$

Chainrule Products Quotient

$h(x) = f(x)\, g(x)$

$h'(x) = f'(x)\, g(x) + g'(x)\, f(x)$ [Product Rule]

$h(x) = k\, f(x)$

$h'(x) = k\, f'(x)$

Now,

$g(x) = 3x-9$ $\qquad g'(x) = 3$

$h(x) = 8x^{-7}-2$ $\qquad h'(x) = -56x^{-8}$

$f(x) = g(x)\,h(x)$

$f'(x) = h'(x)\,g(x) + g'(x)\,h(x)$

$= \left(-56x^{-8}\right)\left(3x-9\right) + 3\left(8x^{-7}-2\right)$

$$g(x) = \frac{f'(x) - g'(x)\,h(x)}{h'(x)}$$

$h(x) = \dfrac{f(x)}{g(x)}$ [Quotient Rule]

$$h'(x) = \frac{f'(x)\,g(x) - f(x)\,g'(x)}{g(x)^2}$$

Rules for Reciprocal

$$\frac{d}{dx}\left(\frac{1}{g(x)}\right) = -\frac{g'(x)}{g(x)^2}$$

Now,

$g(x) = 5x^5+2 \qquad g'(x) = 25x^4$

$h(x) = -8x^{-97/24} - 5 \qquad h'(x) = \frac{97}{3} x^{-\frac{121}{24}}$.

$$f(x) = \frac{g(x)}{h(x)}$$

$$f'(x) = \frac{g'(x)\,h(x) - h'(x)\,g(x)}{h(x)^2}$$

$$= \frac{(25x^4) \times \left(-8x^{-\frac{97}{24}} - 5\right) - \left(\frac{97}{3} x^{-\frac{121}{24}}\right) \times \left(5x^5+2\right)}{\cancel{\left(\frac{97}{3} x^{-\frac{121}{24}}\right)^2} \left(-8x^{-97/24} - 5\right)^2}$$

$$\Rightarrow \cancel{\frac{f'(x)\,h(x)^2}{g(x)}} = \frac{\cancel{f'(x)\,h(x)^2 - g'(x)\,h(x)}}{}$$

$$\Rightarrow g(x) = \frac{g'(x)\,h(x) - f'(x)\,h(x)^2}{h'(x)}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

* Find the equation of the line tangent to $y = \sin(x)$

$x = \pi/4$

$$\left.\frac{dy}{dx}\right|_{\pi/4} = \cos(\pi/4) = \frac{1}{\sqrt{2}}$$

$$y = \frac{x}{\sqrt{2}} + C$$

Now,

$x = \pi/4$

$y = \sin(\pi/4) = \frac{1}{\sqrt{2}}$

So, $C = y - \frac{x}{\sqrt{2}} = \frac{1}{\sqrt{2}} - \frac{\pi}{4\sqrt{2}}$

$$= \frac{4-\pi}{4\sqrt{2}}$$

So, $y = \frac{x}{\sqrt{2}} + \frac{4-\pi}{4\sqrt{2}}$

$$\Rightarrow y = \frac{4x+4-\pi}{4\sqrt{2}}$$

Motivation for the derivative of sine and cosine:

$\cos(x) = \sin(x+\pi/2)$

$\cos(x+\pi/2) = -\sin(x)$

Find the stationary and inflection points of the function

$g(x) = x + \cos(x)$

Stationary Points

$g'(x) = 1 - \sin(x)$

$g''(x) = -\cos(x)$

Stationary points

$1 - \sin(x) = 0$

$\Rightarrow \sin(x) = 1$

$\Rightarrow x = \pi/2 + 2\pi k$ for $k \in \mathbb{Z}$

$y = \pi/2 + \cos(\pi/2)$

$y = \pi/2$ $\quad (\pi/2, \pi/2)$

Inflection points

$-\cos(x) = 0$

$\Rightarrow x = \pi/2 + \pi m$ for $m \in \mathbb{Z}$

- why $g(x) = 2x - \cos(x)\sin(x)$ is alway increasing

$g'(x) = 2 - (\cos^2 - \sin^2 x)$

$= 2 - \cos 2x$

$-1 \leq \cos 2x \leq 1$;

$\therefore g'(x) \geq 1 > 0$

$g'(x)$ is always positive

- $f(x) = -0.7 \cos(x) - 7.3$

$f'(x) = 0.7 \sin(x)$

$f'(-8) = 0.7 \sin(-8)$

$m = -\frac{1}{f'(-8)}$

$= \cancel{-0.974}\ 1.443$

$y = \cancel{-0.974x + c}$ $= \cancel{-0.974x + -15.785}$

$1.443x + c$ $= 1.443x + 4.345$

$x = -8$

$y = -\cancel{7.993}\ -7.198$

$C = \cancel{-15.785}\ 4.345$

$g(x) = 12\sin x$ $\qquad g'(x) = 12\cos(x)$

$h(x) = x^9$ $\qquad h'(x) = 9x^8$

$f(x) = g(x)\,h(x)$

$= g'(x)\,h(x) + g(x)\,h'(x)$

$= 12\cos(x)\,x^9 + 12\sin(x).\,9x^8$

$= 12x^8\left(x\cos(x) + 9\sin(x)\right)$

Derivative of tangent

$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\frac{d}{dx}(\tan(x)) = \sec^2(x) = \frac{1}{\cos^2(x)}$$ [Quotient Rule]

$f(x) = 2.7\tan(x) - 18.4$

$$f'(x) = \frac{2.7}{\cos^2(x)}$$

$$m = -\frac{1}{f'(-9)}$$

$= -0.3075$

$x = -9,\ y = -17.1787$

$y = -0.3075x + c$

$c = -19.946$

$y = -0.3075x - 19.946$

Derivative of 2^x

$f(x) = a^x$

$f'(x) = a^x \ln(a)$

Now,

$f(x) = x\, 5^x$

$f'(x) = x5^x \ln(5) + 5^x$

$\cancel{= 5^x(\ln(5)+1)}$

$f''(x) = 25^x \ln(5) + (\ln 5)^2\, x 5^x$

$f(x) = 4.2 \times 12^x - 6.6$

$m = f'(x) = 4.2 \times \ln(12) \cdot \times 12^x = 10.437 \times 12^x$

$y = mx + c$ $\qquad$ $x = -18$ $\quad$ $y = -6.552$

$c = y - mx$

$= -6.552$

$\cancel{6.552}$

$y = 10.437 \times 12^x x \cancel{+} - 6.552$

* $g(x) = 9\sin(x)$ $\quad$ $g'(x) = 9\cos(x)$

$h(x) = -10 \times 0.8^x$ $\quad$ $h'(x) = -10 \times \ln(0.8) \times (0.8)^x$

$$f(x) = \frac{g(x)}{h(x)}$$

$$= \frac{g'(x)h(x) - g(x)h'(x)}{h(x)^2}$$

$$= \frac{-90\cos(x)\,0.8^x + 90\,(0.8)^x \sin(x)\ln(0.8)}{100 \times 0.8^{2x}}$$

$$= \frac{9}{10} \times \frac{\ln(0.8)\sin(x) - \cos(x)}{0.8^x}$$

Derivative of e^x

$$\frac{d}{dx}(e^x) = e^x$$

$f(x) = e^x + mx$

a) show that $f(x)$ is always concave up for any values of m

b) For which values of m does the function have a turning point

c) If $f(x)$ has a turning point, then how many does it have and are they maxima or minima?

(a) $f'(x) = e^x + m$

$f''(x) = e^x$ [always concave up] [always positive]

(b) At turning point $f'(x) = 0$

$e^x + m = 0$

$\Rightarrow e^x = -m$ Now $e^x > 0$, $-m > 0$, $m < 0$

Then $x = \ln(-m)$

this is actually positive

Turning point at $x = \ln(-m)$ for $m < 0$

(c) Concave up, this turning point is a local min

$f(x) = 4.2e^{x} - 12.7$ $\quad m = f'(x) = 4.2e^{x}$

$x = -1.8$ $\quad y = -\frac{1.8}{4.2e^{-1.8}} + c$

$y = -12.006$ $\quad c = -9.413$

$y = m(x+1.8) + c$

$y = \cancel{0.694x}$

$-1.44404x - 9.413$

(e)

$g(x) = -10e^{x}$ $\quad g'(x) = -10e^{x}$

$h(x) = 12e^{x}$ $\quad h'(x) = 12e^{x}$

$f(x) = \frac{g(x)}{h(x)}$

$f'(x) = \frac{g'(x)h(x) - h'(x)g(x)}{h(x)^2}$

$= \frac{(-10e^{x})(12e^{x}) - (12e^{x})(-10e^{x})}{144e^{2x}}$

Derivative of $\ln(x)$ and $\log_a(x)$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$\boxed{y = m(x - x_0) + c}$$

Now,

$$f(x) = -8.4 \log_5(x) - 12.7$$

$x = 12$

$y = -25.6693$

$$y = -0.435x - 20.451$$

$$f'(x) = \frac{-8.4}{x \ln(5)}$$

$m = f'(x) = -0.4349$

$c = y - mx = -20.4505$

Chain Rule

needed to differentiate composite functions

$$h(x) = f(g(x))$$
$$h'(x) = f'(g(x)).g'(x)$$

Now,

$$f(x) = \cos(x^2+x+2)$$
$$f'(x) = -\sin(x^2+x+2)\frac{d}{dx}(x^2+x+2)$$
$$= -\sin(x^2+x+2)(2x+1)$$
$$= -(2x+1)\sin(x^2+x+2)$$

Again

$$f(x) = 2^{x^2}$$
$$\Rightarrow f'(x) = 2^{x^2}\ln(2).2x$$

Again $f(x) = (x^2+1)^7$
$$f'(x) = 7(x^2+1)^6\, 2x$$
$$= 14x(x^2+1)^6$$

Again $f(x) = (\sin(x)+e^x)^5$
$$= 5(\sin(x)+e^x)^4 (\cos(x)+e^x)$$

Derivative

$\sin(g(x)) \longrightarrow \cos(g(x)) \cdot g'(x)$

$\cos(g(x)) \longrightarrow -\sin(g(x)) \cdot g'(x)$

$e^{g(x)} \longrightarrow e^{g(x)} \cdot g'(x)$

$\ln(g(x)) \longrightarrow \frac{g'(x)}{g(x)}$

Now

$f(x) = 2x\sin 2x$

$f'(x) = \cancel{2\left(x \frac{d}{dx}\right.}$

$2.x.\cos 2x.2 + 2.\sin 2x$ [chain rule and ~~multiplicative~~ product rule]

$= 4x\cos 2x + 2\sin 2x$

why $\frac{d}{dx}(a^x) = a^x \ln a$ is true using chain rule

$a = e^{\ln(a)}$

$a^x = \left(e^{\ln(a)}\right)^x$

$a^x = e^{x\ln a}$

$\frac{d}{dx}(a^x) = e^{x\ln a} \cdot \ln a$

$\frac{d}{dx}(a^x) = a^x \ln a$ [QED]

Tangent of $f(x)$ at $a \Rightarrow y = f'(a)(x-a) + f(a)$

Normal of $f(x)$ at $a \Rightarrow y = -(x-a)/f'(a) + f(a)$

Now,

$$f(x) = 2^{2(\cos(x))^2 - 3\cos(x) - 3}$$

$$= (\ln 2)\, 2^{2\cos(x)^2 - 3\cos(x) - 3} \cdot (-4\cos(x)\sin(x) + 3\cos(x) + 3)$$

Now, $f(x) = e^{-0.2x^2 + 1.7x - 0.6}$

$$f'(x) = \left(e^{-0.2x^2 + 1.7x - 0.6}\right)(-0.4x + 1.7)$$

Displacement, ~~Vecto~~ Velocity and Acceleration

$s(t)$ = displacement

$$v(t) = \frac{s}{t}$$

$$a(t) = \frac{v}{t}$$

Simple harmonic motion

$$x(t) = A\cos\left(\sqrt{\frac{k}{m}}\,t\right) + B\sin\left(\sqrt{\frac{k}{m}}\,t\right)$$

A spring, with a spring constant $k = 50 kg/s^2$ and mass $m = 2 kg$ has a natural length of 50 cm. The spring is stretched to a length of 70cm and then ~~released~~ and then released with init velocity 0 cm/s

$$x(t) = A\cos\left(\sqrt{\frac{k}{m}}\,t\right) + B\sin\left(\sqrt{\frac{k}{m}}\,t\right)$$

to find the values of the constants A and B

$$\sqrt{\frac{k}{m}} = \sqrt{\frac{50}{2}} = 5$$

$$x(t) = A\cos(5t) + B\sin(5t)$$

$x(t)$ = length spring stretched or compressed beyond its natural length at time t

$x(0) = 20\,cm$: $20 = A\cos(0) + B\sin(0) \Rightarrow A = 20$

$x'(0) = 0\,cm/s$ $\therefore x(t) = 20\cos(5t) + B\sin(5t)$

$$x'(t) = -100\sin(5t) + 5B\cos(5t)$$

$$0 = -100\sin(0) + 5B\cos(0)$$

$$\Rightarrow 0 = 5B \Rightarrow B = 0$$

$$\therefore x(t) = 20\cos(5t)$$

A spring with spring constant $k = 1539 kg/s^2$ is attached to a weight mass $m = 19kg$ and is stretched beyond it natural length by 8cm. The spring is then released with an initial velocity of $-9 cm/s$. The position of the end of the spring at time t relative to its natural length is given by

$$x(t) = A\cos\left(\sqrt{\frac{k}{m}}\,t\right) + B\sin\left(\sqrt{\frac{k}{m}}\,t\right)$$

$$\sqrt{\frac{k}{m}} = \sqrt{\frac{1539}{19}} = 9$$

$x(t) = A\cos 9t + B\sin 9t \Rightarrow x(0) = A \Rightarrow A = 8cm$

$x'(t) = -9A\sin 9t + 9B\cos 9t$ $\Rightarrow -9 = -72\sin 9t + 9B\cos 9t$

$\Rightarrow \cancel{B = \frac{72\sin 9t - 9}{}}$

$x(0) = 8cm$ $\Rightarrow B\cancel{\cos 9t} = \frac{72\sin(0) - 9}{9\cos 0}$

$x'(0) = -9cm/s$

$\Rightarrow B = -1$

$x(2.8) = 8\cos(9\times 2.8) - \sin(9\times 2.8)$

Now,

$f(t) = A_0 e^{kt}$ $\qquad$ $f(0) = 0.3B$

$f'(t) = A_0 k e^{kt}$ $\qquad$ $f(2020) = 7.7B$

$f(0) = A_0 e^{k(0)}$

$A_0 = 0.3$

$f(t) = 0.3e^{kt}$

$f(2020) = 0.3e^{2020k}$

$7.7 = 0.3e^{2020k}$

$\Rightarrow e^{2020k} = \frac{7.7}{0.3}$

$\Rightarrow 2000k = \ln\left(\frac{7.7}{0.3}\right)$

$\Rightarrow k = \frac{1}{2020}\ln\left(\frac{7.7}{0.3}\right)$

$f(t) = 0.3e^{\frac{t}{2020}\ln(7.7/0.3)}$

$f'(t) = 0.3 \times \frac{1}{2020} \times \ln\left(\frac{7.7}{0.3}\right) \cdot e^{\frac{t}{2020}\ln\left(\frac{7.7}{0.3}\right)}$

Modelling exponential decay with caffeine intake

$f(t) = A_0 e^{kt}$

Healthy Half life 5 hours

$f(0) = 1$ (intiallywe got 100%)

$f(0) = A_0 e^{k(0)} = A_0 = 1$

$f(t) = e^{kt}$

$f(5) = e^{5k} = \frac{1}{2}$

$\Rightarrow 5k = \ln\left(\frac{1}{2}\right) = \ln(1) - \ln(2) = -\ln(2)$

$\Rightarrow k = -\frac{\ln(2)}{5}$

$f_h(t) = e^{-\frac{\ln(2)}{5}t}$

Pregnatin, $f(0) = 1$

$f(18) = 1/18$

$\Rightarrow k = -\frac{\ln(2)}{18}$

$f_p(t) = e^{-\frac{\ln(2)}{8}t}$

$\frac{d_H f}{dt} = e^{-\frac{\ln(2)}{5}t} \times \left(-\frac{\ln(2)}{5}\right)$

$\frac{d_p f}{dt} \quad e^{-\frac{\ln(2)}{18}t}\left(-\frac{\ln(2)}{18}\right)$

Compound interest

$$A(t) = I\left(1 + \frac{0.01 \times r}{n}\right)^{nt}$$

$$\cancel{A'(t) = I}$$

$\cancel{IA(0)} = 650$

$r = 4.57$

$n = 52$

$$A(t) = 650\left(1 + \frac{0.0457}{52}\right)^{52t}$$

$$A'(t) = 33800 \ln\left(1 + \frac{0.0457}{52}\right) \times \left(1 + \frac{0.0457}{52}\right)^{52t}$$

$h(0) = 1.6$

$$h(t) = 8.1 \ln(1.76t + 1) + 1.6$$

$$\cancel{h'(t) = 8.1 \times \frac{1}{1.76}}$$

$$43.44 = 8.1 \ln(1.76t + 1) + 1.6$$

$$\ln(1.76t + 1) = \frac{4.34 - 1.6}{8.1}$$

$$h'(t) = \frac{8.1 \times 1.76}{1.76t + 1}$$

0.2