## I. EQUATION FOR THE SPECTRAL FUNCTIONS

We use the following parametrization for equilibrium GF[1]

$$\hat{g} = \cos\theta\hat{\tau}_3 + \sin\theta\hat{\tau}_1 \tag{1}$$

Then we get the following equations for the parameter $\theta$

$$(\omega + ih)\sin\theta + \Delta\cos\theta + \partial_x\left(\frac{D}{2}\partial_x\theta\right) = 0 \tag{2}$$

$$\partial_x\theta(x = 0) = 0 \tag{3}$$

with the boundary conditions for the case of isolator

$$\partial_x\theta(x = 0) = 0 \tag{4}$$

or for the case of strong ferromagnet

$$\theta(x = 0) = 0 \tag{5}$$

We solve Eq.5 by iterations, linearising it at each step $\theta_{k+1} = \theta_k + \tilde{\theta}$

$$2\left[(\omega \pm ih)\cos\theta_k - \Delta\sin\theta_k\right]\tilde{\theta} + \partial_x(D\partial_x\tilde{\theta}) = 0 \tag{6}$$

This equation is solved by the sweeping method.

## II. SELF-CONSISTENCY CONDITIONS

After solving equation for spectral function we solve self-consistency equations for the order parameter and effective field

$$\Delta = 2\pi T\rho\sum_n[\mathrm{Re}\sin\theta(\omega_n) - \frac{\Delta}{\omega_n}] + [1 - \rho\ln(T/T_c)]\Delta \tag{7}$$

$$h = h_0 + G_0(h + m) \tag{8}$$

$$m = 2\pi T\sum_n \mathrm{Im}\cos\theta(\omega_n) \tag{9}$$

where $\omega_n = \pi(2n + 1)T$ and we have used the relation $2\pi T\rho\sum_{n=0}^{N_D} 1/\omega_n = 1 - \rho\ln(T/T_c)$ and $\rho$ is the coupling constant, $G_0$ is the Landau parameter, $m$ is the quasi-calssical part of magnetization determined by the vicinity of the Fermi level. The total magnetization is $M = h+m$, where $h$ is the total exchange field. The external exchange field is $h_0$. Note that Eq.11 is different in the sign of $m$ from Eq.81 in Ref.2


[1] W. Belzig, F. K. Wilhelm, C. Bruder, G. Schön, and A. D. Zaikin, Superlattices and microstructures **25**, 1251 (1999).

[2] J. Alexander, T. Orlando, D. Rainer, and P. Tedrow, Physical Review B **31**, 5811 (1985).