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I. EQUATION FOR THE SPECTRAL FUNCTIONS

We find the spectral functions from the stationary Usadel equation

$$[(i\hat{\sigma}_z h + \omega)\hat{\tau}_3, \hat{g}] + \partial_x(D\hat{g}\partial_x\hat{g}) = \Delta[\hat{\tau}_1, \hat{g}] - [\hat{\Sigma}_{so}, \hat{g}] \quad (1)$$

with the boundary conditions for the case of isolator

$$D\hat{g}\partial_x\hat{g}(x=0) = 0 \quad (2)$$

or for the case of strong ferromagnet we assume the superconductivity is completely suppressed at the interface

$$\hat{g}(x=0) = \hat{\tau}_3 \quad (3)$$

We assume the case of no spin-orbital relaxation and consider equations for spin-up(down) components when $\sigma_z\hat{g} = \pm\hat{g}$. It is enough to consider only e.g. spin-up component.

Using the normalization condition $\hat{g}^2 = 1$ we use the following parametrization for equilibrium GF¹

$$\hat{g} = \cos\theta\hat{\tau}_3 + \sin\theta\hat{\tau}_1 \quad (4)$$

Then we get the following equations for the parameter θ

$$(\omega + ih)\sin\theta + \Delta\cos\theta + \partial_x\left(\frac{D}{2}\partial_x\theta\right) = 0 \quad (5)$$

$$\partial_x\theta(x=0) = 0 \quad (6)$$

with the boundary conditions for the case of isolator

$$\partial_x\theta(x=0) = 0 \quad (7)$$

or for the case of strong ferromagnet

$$\theta(x=0) = 0 \quad (8)$$

We solve Usadel equation by iterations, linearising it at each step $\theta_{k+1} = \theta_k + \tilde{\theta}$

$$2[(\omega \pm ih)\cos\theta_k - \Delta\sin\theta_k]\tilde{\theta} + \partial_x(D\partial_x\tilde{\theta}) = 0 \quad (9)$$

This equation is solved by the sweeping method.

II. SELF-CONSISTENCY CONDITIONS

After solving equation for spectral function we solve self-consistency equations for the order parameter and effective field

$$\Delta = 2\pi T\rho \sum_n [\text{Re}\sin\theta(\omega_n) - \frac{\Delta}{\omega_n}] + [1 - \rho\ln(T/T_c)]\Delta \quad (10)$$

$$h = h_0 + G_0(h + m) \quad (11)$$

$$m = 2\pi T \sum_n \text{Im}\cos\theta(\omega_n) \quad (12)$$

where $\omega_n = \pi(2n+1)T$ and we have used the relation $2\pi T\rho \sum_{n=0}^{N_D} 1/\omega_n = 1 - \rho\ln(T/T_c)$ and ρ is the coupling constant, G_0 is the Landau parameter, m is the quasicalssical part of magnetization determined by the vicinity of the Fermi level. The total magnetization is $M = h+m$, where h is the total exchange field. The external exchange field is h_0 . Note that Eq.11 is different in the sign of m from Eq.81 in Ref.2

¹ W. Belzig, F. K. Wilhelm, C. Bruder, G. Schön, and A. D. Zaikin, Superlattices and microstructures **25**, 1251 (1999).

² J. Alexander, T. Orlando, D. Rainer, and P. Tedrow, Phys-

ical Review B **31**, 5811 (1985).