Assignment 4

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1A

Using $\theta_1 = 1.5$ and $\theta_2 = 2$ we draw a sample of size 1000 using the independence Metropolis Hastings algorithm with gamma distribution as the proposal density.

```r
theta_1 = 1.5 # true value theta1
theta_2 = 2 # true value theta2
mean_z1 = sqrt(theta_2/theta_1)
mean_z2 = sqrt(theta_1/theta_2) + 1/(2*theta_2)

# hyperparams
b = 2.5
a = mean_z1*b
#M-H Algorithm
MH_alg1 = function(N){
  MH_samples = rep(NA, N)
  count = 0
  current_z = 1.0
  for(i in 1:N){
    curr_p = pdf_z(current_z)
    z_new = rgamma(1, a, b)
    p_new = pdf_z(z_new)

    accept = exp(p_new + dgamma(current_z,a,b,log = T) - p_new - dgamma(z_new,a,b,log = T))
    if(runif(1) < accept){
      current_z = z_new
      count = count + 1
    }
    MH_samples[i] = current_z
  }
  return(list(MH_samples=MH_samples,count=count))
}
```

After trying several hyperparameters for different Gamma distributions, the best sample obtains a mean, $E(Z)$, of

```
## [1] 1.763996
```

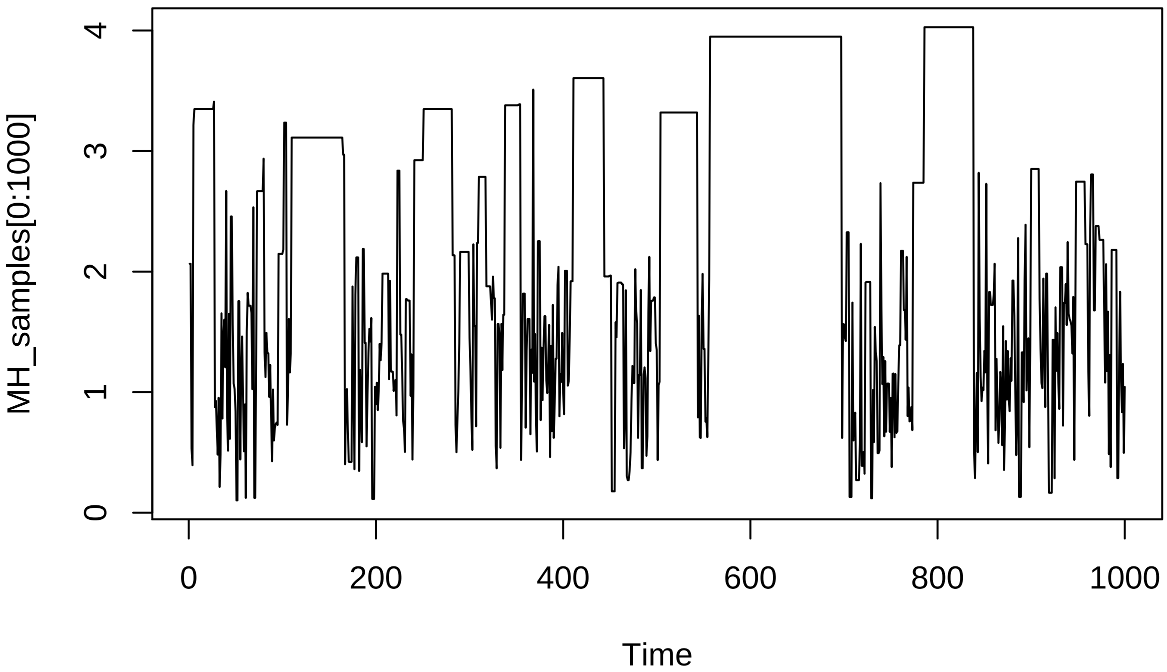
$E(1/Z)$

```
## [1] 1.537128
```

and an accuracy of

```
## [1] 0.513
```

The traceplot for the samples for Metropolis-Hastings is shown below:



1B

The density of $W = log(Z)$ is given by

$$f_W(w) \propto \exp\left\{-\frac{3}{2}w - \theta_1 exp\{w\} - \frac{\theta_2}{\exp(w)}\right\}\exp(w)$$

We draw a sample of size 1000 using the random-walk Metropolis algorithm with this density.

```
v = 0.01
MH_RW = function(N){
  N = N
  MH_RW = rep(NA, N)
  a_count = 0
  z_curr = 1.0
  for (i in 1:N) {
    p_curr = pdf_z2(z_curr)
    z_new = exp(log(z_curr) + rnorm(1,0,sqrt(v)))
    p_new = pdf_z2(z_new)
    acceptance = exp(p_new - p_curr)
    if(runif(1) < acceptance){
      z_curr = z_new
      a_count = a_count+1
    }
    MH_RW[i] = z_curr
  }
  return(list(MH_RW=MH_RW, a_count=a_count))
}
```

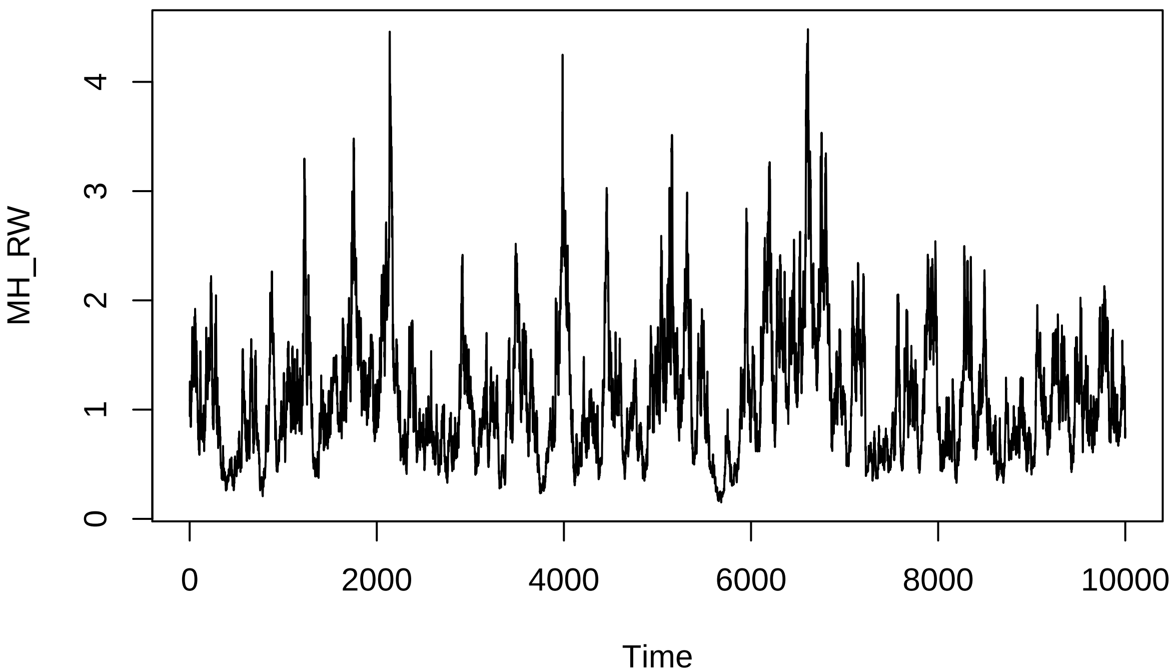
The mean for the samples is

```
## [1] 1.114488
```

And the accuracy is

```
## [1] 0.9363
```

If we use 10000 metropolis hastings random ralk samples, the traceplot is shown below



2A

$$x_i|\nu,\theta \sim Gamma(\nu,\theta)$$
$$\nu \sim Gamma(a,b)$$
$$\theta \sim Gamma(\alpha,\beta)$$

The joint posterior for θ and ν

$$\pi(\theta,\nu,\boldsymbol{x}) \propto \frac{\left(\prod_{i=1}^n x_i\right)^{\nu-1} \nu^{a-1}e^{-b\nu}}{(\Gamma(\nu))^n}\theta^{a+n\nu-1}\exp\left\{-\theta\left(\beta+\sum_{i=1}^n x_i\right)\right\}$$

The full conditionals:

$$\pi(\theta|\nu,\boldsymbol{x}) \propto \theta^{a+n\nu-1}\exp\left\{-\theta\left(\beta+\sum_{i=1}^n x_i\right)\right\}$$

thus, $\theta|\nu,\boldsymbol{x} \sim Gamma(n\nu, \beta+\sum x_i)$.

$$\pi(\nu|\theta,\boldsymbol{x}) \propto \theta^{n\nu}\frac{\left(\prod_{i=1}^n x_i\right)^{\nu-1} \nu^{a-1}e^{-b\nu}}{(\Gamma(\nu))^n}$$