Chapter 10 #2

x $AVG_{18} = 100$ $SD_{18} = 15$

y $AVG_{35} = 100$ $SD_{35} = 16$ $r \approx 0.8$

Estimate AVG score at age 35 y for an individual who scored 115 at age 18 x

3 steps

$$y = mx + b$$

① slope $= m = r \times \frac{SD_y}{SD_x} = 0.8 \times \frac{15}{15} = 0.8$

$$y = 0.8x + b$$

② Intercept : b (use averages & solve for b)

$$100 = 0.8(100) + b$$

$$b = 20$$

③ Regression line: $y = 0.8x + 20$ ← regression line

plug in $x = 115$ to get estimated y

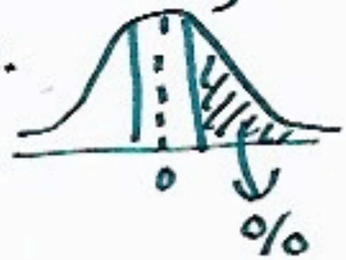
$$y = 0.8(115) + 20 = 112$$

So the avg score at age 35 for an individual who scored 115 at age 18 is 112.

$$\text{RMS ERROR} = \sqrt{1-0.6^2} \times 15 = 12$$

$$\text{"New" SD} = 12$$

- Lastly: we are calculating the percentage who scored over 80 on final given they scored 80 on midterm: i.e.



$$Z = \frac{\text{Value} - (\text{"new" AVG})}{\text{"new" SD}}$$

$$= \frac{80 - 65.8}{12} \approx 1.2$$

$$\text{AREA} = \frac{100-77}{2}$$

$$= 11.5\%$$

Ch.17 1, 4, 5, 7, 10, 13

(1) $\boxed{1, 6, 7, 9, 9, 10}$ 100 draws

(a) How small can the sum be?

A: The sum of 100 of the smallest # in box: $\underbrace{1+1+\cdots}_{100 \text{ times}} = 100$

(b) The sum is between 650 and 750 w/ a prob of what?

$AVG_{Box} = 7$ we need $SD_{Box} \Rightarrow SE$

#	Dev	Dev^2
1	-6	36
6	-1	1
7	0	0
9	2	4
9	2	4
10	3	9

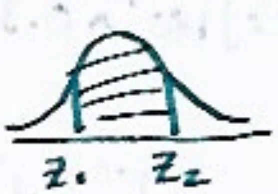
$$SD = \sqrt{\frac{36+1+0+4+4+9}{6}} = \sqrt{9} = 3$$

$AVG_{Box} = 7 \Rightarrow$ Expected Value $EV = n_{draws} \times AVG_{Box} \ldots$

$SD_{Box} = 3 \Rightarrow SE = \sqrt{n_{draws}} \times SD = \sqrt{100} \times 3 = 30$

$\Rightarrow EV = 100 \times 7 = 700$

We use EV and SE now & calculate just like before



$$z_1 = \frac{\# - EV}{SE} = \frac{650-700}{30} = -1.7$$

$$z_2 = \frac{750-700}{30} = 1.7 \qquad T(1.7)$$

Easy way to compute SD:

If you have a box model w/ only two numbers:

$$\left(\begin{matrix}\text{big}\\\#\end{matrix} - \begin{matrix}\text{small}\\\#\end{matrix}\right)\sqrt{\begin{matrix}\text{frac}\\\text{big\#}\end{matrix} \times \begin{matrix}\text{frac}\\\text{sm.\#}\end{matrix}}$$

Try to turn a box model in to a box of 0's (failures) or 1's (successes)

Example

#4 Here the "success" is getting a 1 $\boxed{\cdot}$ so we can make our box model of 0's and 1's: $\boxed{1\ 0\ 0\ 0\ 0\ 0}$ 180 draws

Then we use the shortcut SD formula.

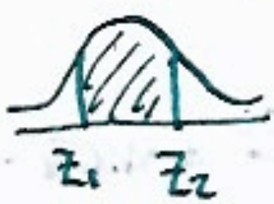
$$SD = (1-0)\sqrt{\tfrac{1}{6} \times \tfrac{5}{6}} = \sqrt{\tfrac{5}{36}}$$

About what % should get counts betw. 15-45?

The count is like the sum of 180 draws from our 0,1 box model. ~~EV = 180×1 = 180~~

$AVG_{Box} = 1/6 \Rightarrow EV = 180 \times \frac{1}{6} = 30$

$SD \approx 0.37 \Rightarrow SE = \sqrt{180} \times 0.37 \approx 4.9$



$z_1 = \dfrac{15-30}{4.9} = -3.1$

$T(3.1)$

$z_2 = \dfrac{45-30}{4.9} = 3.1$

Ch. 16

1 3 6 4 7 9

(1)

1000 ~~4000~~ tix

|$\boxed{0}$ $\boxed{1}$|

10,000 drawn w/ replacement

4000 are $\boxed{0}$

6000 are $\boxed{1}$

The more draws you have the closer your ~~#~~ prop of tickets will match to the prop of the box model. (ii)

(3) Both are wrong. Luck is irrelevant and for roulette each turn is independent.

(4) Think of this as a box of tickets like #1

|1 2 3 4 5 6|

The more rolls/"draws" the closer you'll be to the prop inside your box.

(a) In this case more draws gets you closer to 1/6 being $\boxed{\cdot}$, 16%. So we would have better chance w/ fewer rolls.

b) In this case more draws $\approx 16\%$ $>15\%$, leads to a win which is better

c) Again, same as (b)

(d) 60 rolls because the more rolls you have there is a small chance to be exactly $16\frac{2}{3}\%$.

(7) Box Model $\boxed{1\ 2\ 3\ 4\ 5}$ 25 draws

a) The score will be like the sum of 25 draws from a box model because we can think of the box as being ~~0~~ -1 for incorrect and 4 for correct $\boxed{\cancel{0}\ \cancel{0}\ \cancel{0}\ \cancel{0}\ 4}$ (-1 -1 -1 -1) so we just add upp the 4's in the end.

(9)(ii) More draws gets you closer to the prop. in the box model, and since we're told theres more red to begin w/ thats ideal.

Review Exercize #7/8

| 1 2 2 3 3 | Four draws made at random.

a) P(That a 2 drawn at least once) W/Replacement

The phrase "at least" $= 1 - P(\text{NONE are two's})$

A: $\underbrace{1-\left(\frac{3}{5}\right)^4}$ ≈ 0.87 or 87%

because $P(\text{None are 2's}) = \underbrace{\left(\frac{3}{5}\right)\times\left(\frac{3}{5}\right)\times\left(\frac{3}{5}\right)\times\left(\frac{3}{5}\right)}_{\text{by Replacement}}$

b) Now w/out replacement: P(A 2 is drawn at least once)

We only draw 4/5 cards and 2 of the cards in box are two's so a 2 is guaranteed to be drawn no matter what: 100% prob.

c) P(that a 1 is drawn at least once) W/ repl.

$P(x \geq 1) = 1 - P(x < 1)$ i.e. $1 - P(\text{None are one})$

$$= 1 - \left(\frac{4}{5}\right)^4$$

~~AVG LSAT Score = 162 & SD = 6~~ (LSAT & First year score)

x	$AVG_{LSAT} = 162$	$SD_{LSAT} = 6$	
y	$AVG_{FY} = 68$	$SD_{FY} = 10$	$r = 0.6$

a) About what % of the students had first-year scores over 75?

b) Of the st. who scored 165 on LSAT, about what % had FY scores over 75?

a)

$$Z = \frac{\text{Value} - \text{AVG}}{\text{SD}} = \frac{75 - 68}{10} = 0.7$$

Right tail $\Rightarrow \dfrac{100 - T(0.7)}{2} \simeq 24\%$

b) Break down into two parts:

I) Prediction = New AVG

II) Z-score new AVG / new SD

1) Predict/estimate the FY score (y), given LSAT score (x) was a 165 (Remember 3 steps)

1) ~~slope~~ $y = mx + b$

$$\text{slope} = m = \frac{r\,SD_y}{SD_x}$$

so what is y what is x? y is pred, x is given

$$= \frac{r \times 10}{6} = \frac{0.6 \times 10}{6} = 1$$

2) Intercept: $y = x + b$ use given avg to solve for b

$68 = 162 + b \Rightarrow b = 68 - 162 \Rightarrow b = -94$

③ Plug in & predict:

$$y = x - 94$$

$$y = (165) - 94$$

$$y = 71 = \text{NEW AVG}$$

II) $Z_{new} = \dfrac{\text{Value} - \text{NewAVG}}{SD_{NEW}} \leftarrow ?$ RMS ERROR $= \sqrt{1-r^2} \times SD_y$

$= \sqrt{0.64} \times 10$

$= 8$

$$= \frac{75-71}{8} = 0.5$$

$$\frac{100 - T(0.5)}{2} \approx \boxed{31\%}$$