qLearn Week 2
Intro to Quantum Information

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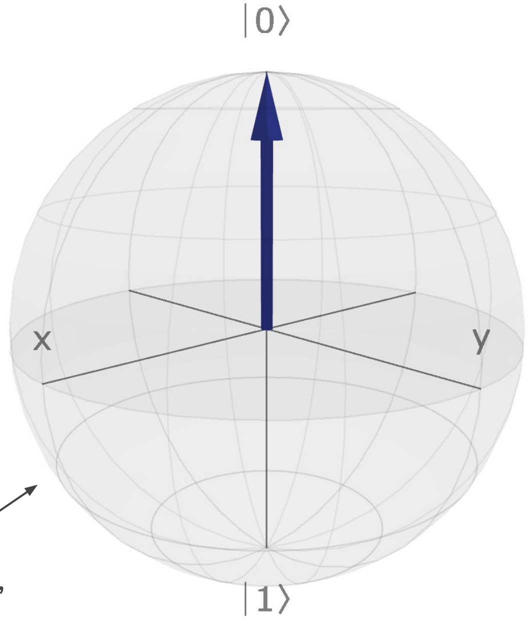
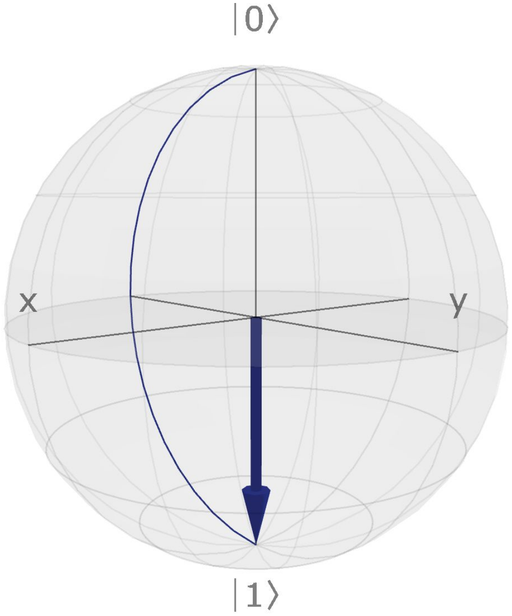
In Quantum Computers...

$$\text{qubit state } 0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \text{qubit state } 1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



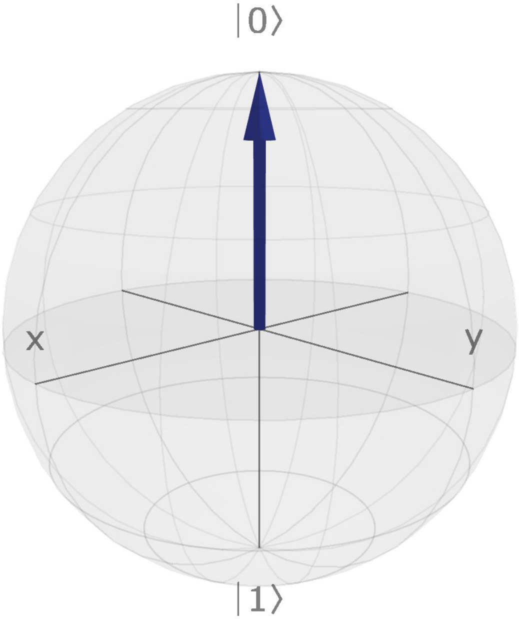
‘Bloch Sphere’

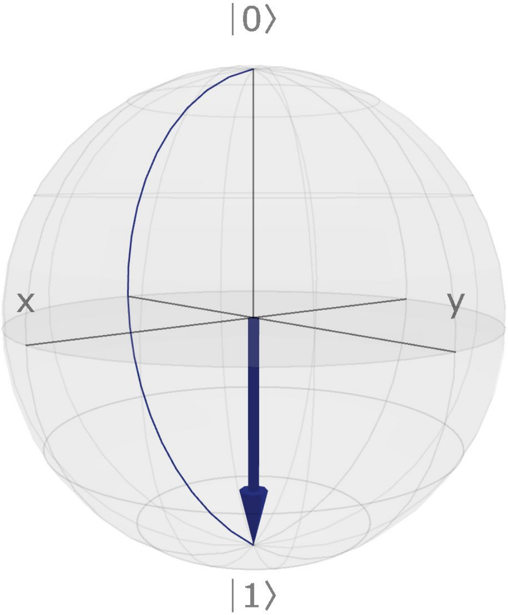


Using Simpler Terms… (Dirac/Braket Notation)

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$





Computational (and other) Basis

$$\langle 0| = (1 \quad 0), \quad \langle 1| = (0 \quad 1).$$

'Bra', as opposed to 'ket'

$$\langle 0||1\rangle = \langle 0|1\rangle = (1 \quad 0) \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$
$$= 1 \cdot 0 + 0 \cdot 1$$
$$= 0.$$

'Braket' Expression

orthogonal

When vectors (or states in our case) are **orthogonal**, they form a **basis** (any vector in the space can be expressed as a unique linear combination of the basis vectors)
This particular basis is called the **computational basis**, ie. the most commonly used

$$\sqrt{\langle 1|1\rangle} = \sqrt{(0 \quad 1) \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix}}$$
$$= \sqrt{0 \cdot 0 + 1 \cdot 1}$$
$$= 1.$$

Calculating the length of a qubit state vector

Normalized
(has length 1)

When a basis consists of two normalized, orthogonal vectors, is it called an **orthonormal basis**

All quantum bases will be orthonormal

Notation in Quantum Information

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

'Probability amplitudes': Carry information about the relative strengths of 0 and 1 in the state

$$\text{Prob}(\text{measure and observe } |0\rangle) = |\alpha|^2 = \alpha\alpha^*$$

$$\text{Prob}(\text{measure and observe } |1\rangle) = |\beta|^2 = \beta\beta^*$$

$$\alpha\alpha^* + \beta\beta^* = 1$$

Think about why they equal 1

Example

We call i the 'imaginary unit', it creates complex numbers, just think of it as the square root of -1

$$|\psi\rangle = \frac{1}{2}|0\rangle - \frac{\sqrt{3}i}{2}|1\rangle$$

$$|\frac{1}{2}^2| + |\frac{\sqrt{3}i}{2}^2| = 1$$

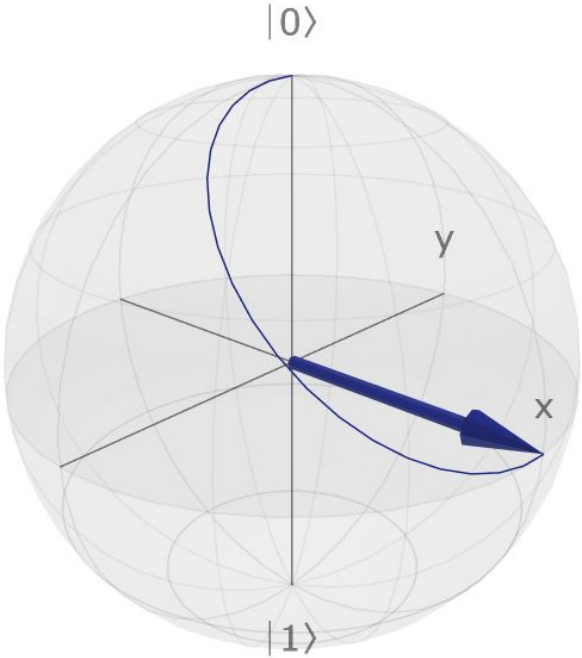
State is Normalized

$$-\frac{\sqrt{3}i}{2} \cdot \frac{\sqrt{3}i}{2} = \frac{3}{4}$$

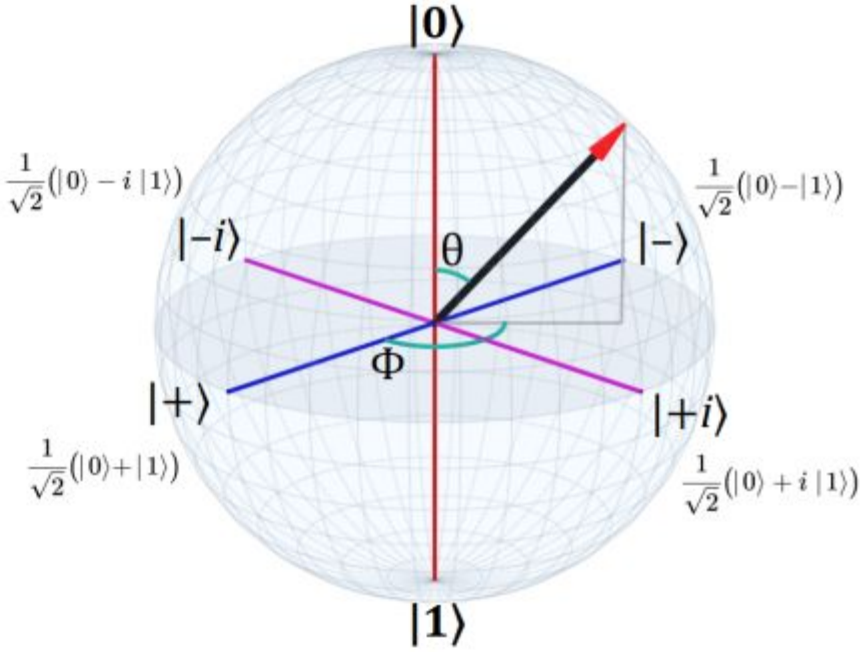
Probability of Observing 1 is $\frac{3}{4}$

Quantum Superposition

The Idea of Superposition



The ‘Superposition States’

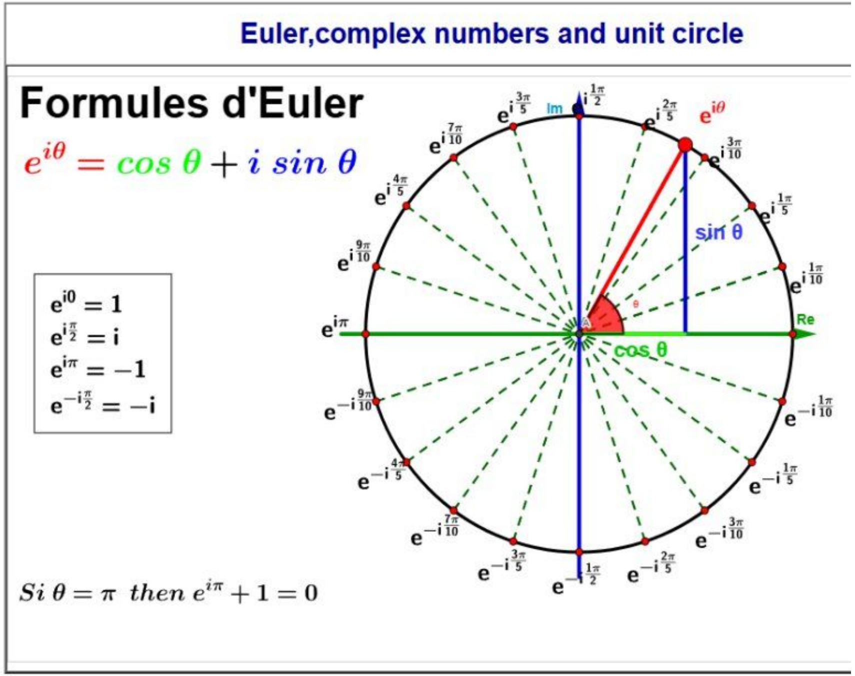


$$|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

Mom… It’s just a Phase!

Using Phases in Quantum Computation



Quantum Amplitude Amplification (Board)

Quantum Computation

How do we Transform Vectors?

MATRICES!!!

What sends a 2D vector to another 2D vector?
Multiplication by a 2x2 Matrix, U

$$|\psi'\rangle = U|\psi\rangle.$$

But remember, we need to preserve the length of the quantum state

$$UU^\dagger = I, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Special class of matrices that preserves the length of vectors: **unitary matrices**

All quantum operations are **unitary matrices**

Exercise: Find the matrix to put a state into superposition

$$\text{Superposition state } |+\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\text{How would we get that from } |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}?$$

$$\begin{pmatrix} ? & ? \\ ? & ? \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = H$$

Hadamard Gate: Our superposition operation; how we put a state into and out of superposition (notice anything?)

See you Next Week!