Intro to Quantum Algorithms (pt.2)

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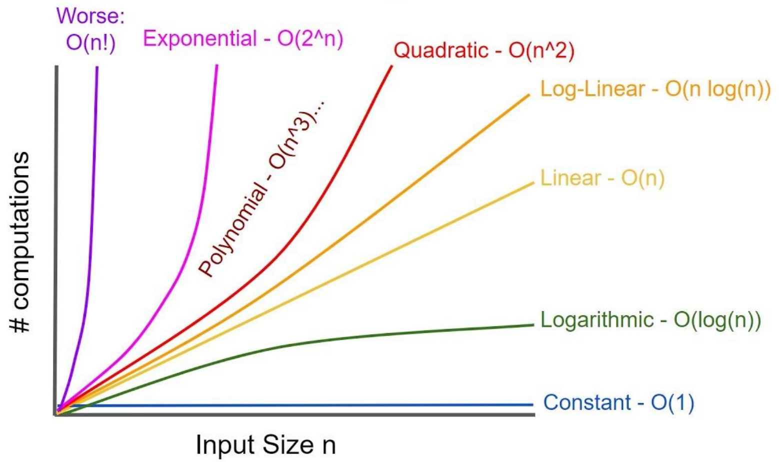
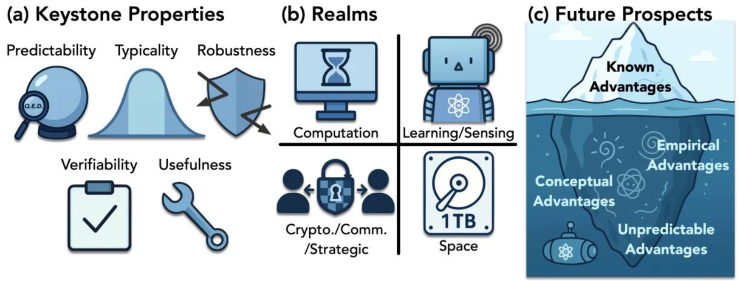
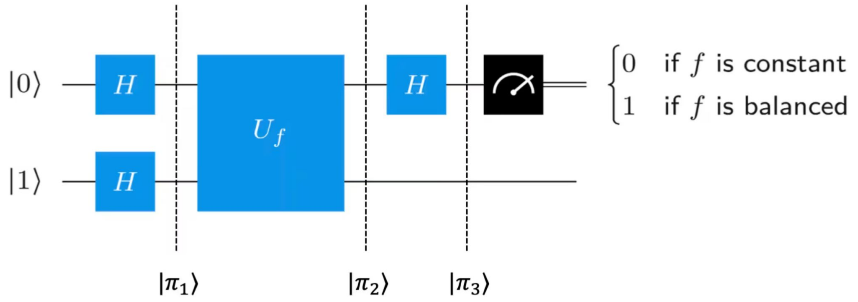
Previously...







$$|\pi_1\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$|\pi_2\rangle = \frac{(-1)^{f(0)}|0\rangle + (-1)^{f(1)}|1\rangle}{\sqrt{2}} + \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$|\pi_3\rangle :$$

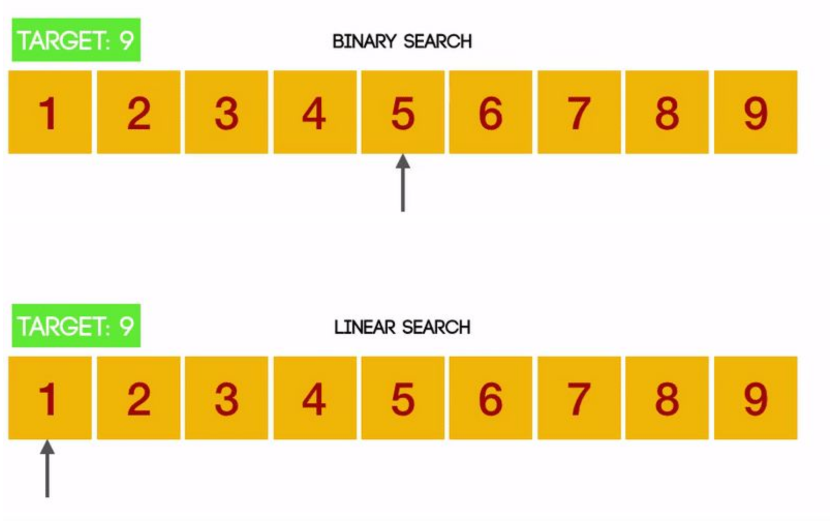
$$\text{If f constant, first qubit: } \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \text{ when measured: } |0\rangle$$

$$\text{If f balanced, first qubit: } \frac{|0\rangle - |1\rangle}{\sqrt{2}}, \text{ when measured: } |1\rangle$$

Deutsch-Jozsa Algorithm

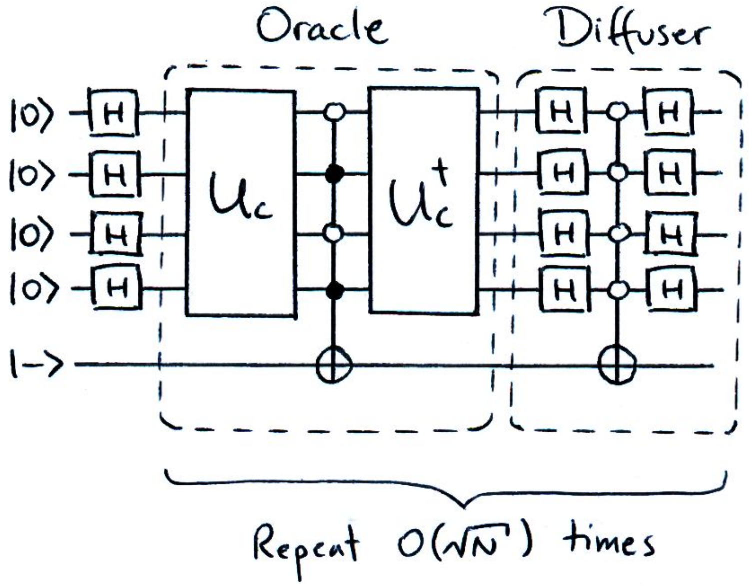
New Problem: Searching a List

- Suppose we have an unstructured list of n items
- Linear Search: O(n)
- Quicksort + Binary Search: O(nlogn)



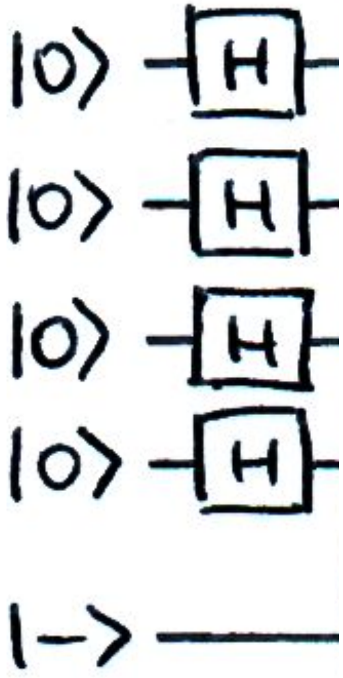
Grover’s Algorithm

- Uses an oracle (from last week!) to find item
- O(sqrt(n)) complexity -> offers real complexity over classical algorithms



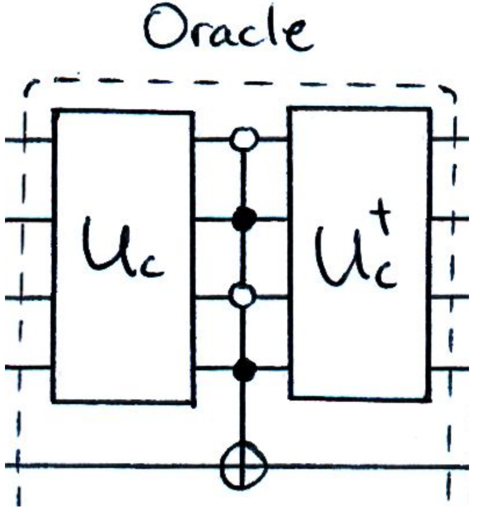
Step 1: Hadamard Transform

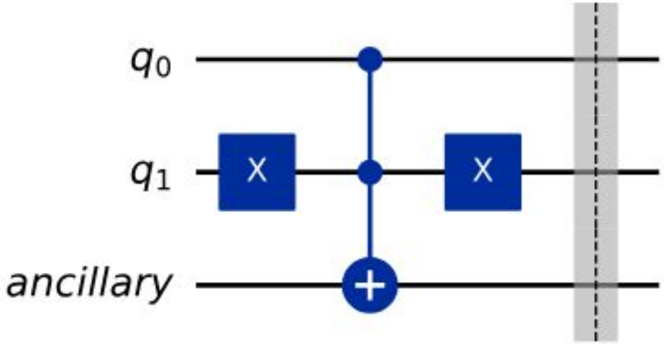
- Simply applying Hadamard gates so we can use our superposition principles to search all solutions
- Note bottom qubit, we call this an ‘auxiliary’ qubit, can anyone guess what it’s used for? (hint, the next step will be applying the oracle)
- Win a prize: what gates do we apply to get to the ‘minus’ state?



Step 2: Applying the oracle

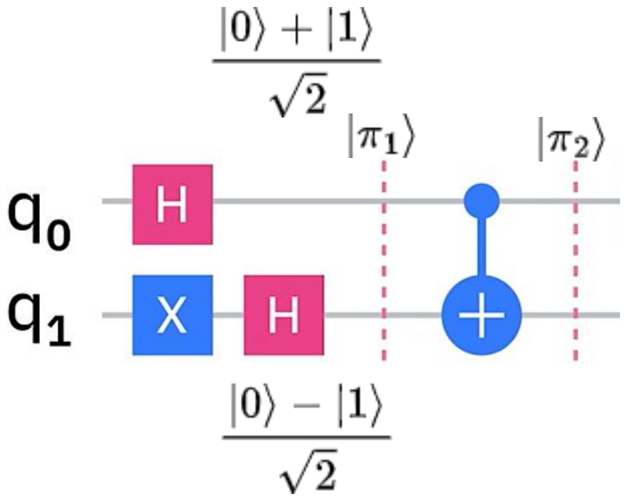
- We want to differentiate the correct answer from the rest, we can use an oracle for this!
- There are many different operators an oracle can apply to the solution, in Grover's we apply a **phase flip**
- Note: In our oracle lecture we used Uf to denote this 'subcircuit', we define it based on the solution we're looking for
- EX. Consider a list of 4 items. We want to find the index of the element equal to 2. In this case we're looking for the quantum state |01> relative to how we've drawn the circuit





Note: Phase Kickback

- We touched on this last week in Deutsch-Jozsa, but let's talk about it a bit more
- How does a controlled gate affect the phase of the control qubit??
- Notice the difference in phases before and after the CZ. What happens if the first qubit is 1?

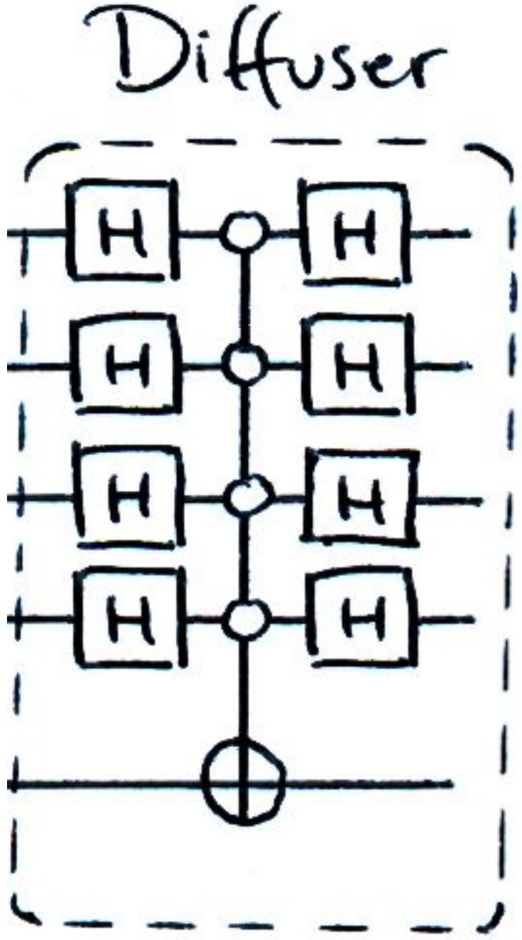


$$|\pi_1\rangle = \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$$

$$|\pi_2\rangle = \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle + |11\rangle)$$

Step 3: Diffusion Operator

- Uses a trick called **amplitude amplification** to isolate solution
  - Oracle applies phase flip to chosen solution
  - Invert all probability amplitudes about the mean
- This trick raises the probability that we will measure the wanted solution



Step 4: Repeat √n times and Measure

- Why repeat √n times?
  - Since all answers start with an equal probability, they all have a probability amplitude of 1/√n
  - Applying our search √n times cancels this out for our desired answer!
- We then simply apply measurements after to get our solution (ideally the one that was amplified)
  - Why ideally?