MA 510 / CS 522 HW #10
Due Thursday December 8^{th}
Last Homework!

1. Use Von Neumann's Fourier method to determine the stability of the scheme for the following partial differential equation:

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0, \quad u(0,t) = 1 \text{ for } t > 0, \quad u(x,0) = 0 \text{ for } x > 0$$

   where $a > 0$. Analyze stability for a finite difference scheme where the spatial derivative is centered (second order accurate) at the previous or known time step and the time derivative is first order accurate. Comment on stability and reliability of the method (possibly with regards to choice of time and spatial step with regards to the parameter a).

2. Approximate the solution to the following partial differential equation:

$$\frac{\partial u}{\partial t} = \frac{1}{16}\frac{\partial^2 u}{\partial x^2} \quad u(0,t) = u(1,t) = 0 \text{ for } t > 0, \quad u(x,0) = 2\sin(2\pi x) \text{ for } 0 \leq x \leq 1$$

   Note that the exact solution is $u(x,t) = 2\exp\left(\frac{-\pi^2 t}{4}\right)\sin(2\pi x)$. For both methods described below, please provide a hand written summary of your scheme along with the code. Solve the equation from $t = 0$ to $t = 1$ using an appropriate number of spatial steps and time steps. Create a plot where you show several solution curves on the same plot. Also report on the error for different choices of time step, spatial step, etc. Show that the method is convergent by showing error decreases as spatial and/or time steps are decreased (can hold one fixed and decrease the other or decrease both by the same factor).

   (a) Write a code to approximate the solution using an explicit method as discussed in class where the time derivative is first order accurate and the spatial derivative is centered second order accurate using previous/known values. Show how the stability of the method (i.e. the validity of the solutions) changes based on the stability criteria we discussed in class.

   (b) Write a code to approximate the solution using an implicit method as discussed in class where the time derivative is first order accurate and the spatial derivative is centered and second order accurate using new/unknown values. Detail your matrix setup. You can use backslash in Matlab to solve the linear system.