MA 510 / CS 522 HW #2
Due Thursday September 15^{th} by 5:30 pm

1. If a polynomial, $f(x)$, has an odd number of real zeros in the interval $[a, b]$ and each of the zeros is of odd multiplicity, then $f(a)f(b) < 0$ and the bisection method will converge to one of the zeros. If $a < 1$ and $b > 3$ are selected such that $x_n = (a_n + b_n)/2$ is not equal to any of the zeros of $f(x)$ for any $n \geq 1$, then the bisection method will never converge to which zero(s)? Why?

2. With what order of convergence does the sequence $x_n = 10^{-2^n}$ approach 0.

3. Show that the following sequence converges linearly to $p = 0$. How large must n be before we have $|p_n - p| \leq 5 \times 10^{-2}$?

$$p_n = \frac{1}{n}, \qquad n \geq 1 \tag{1}$$

4. Write a program using the fixed point iteration method to approximate $\sqrt[3]{25}$ within 10^{-4}. Notice that $\sqrt[3]{25}$ is a root of the function $f(x) = x^3 - 25$. We just need to turn this root-finding problem into a fixed point problem.

   (a) Let $g_1(x) = 25/x^2$. Notice that $\sqrt[3]{25}$ is a fixed point of g_1. Try your fixed point algorithm with $x_o = 2$ on this function. Set maximum number of iterations to 100 and output the value of $|\alpha - x_n|$ at $n = 100$ iterations (where α is the fixed point). This does not converge. Why? Might be helpful to include a graph for the answer.

   (b) There is a better fixed-point problem to use in finding $\sqrt[3]{25}$. Show that $g_2(x) = 0.5(x + \frac{25}{x^2})$ is also a fixed-point problem for $\sqrt[3]{25}$. Use g_2 to estimate $\sqrt[3]{25}$ as above. How many steps are required?