MA 510 / CS 522 HW #3
Due Thursday September 22^{nd} In Class

1. Write a code for Newton's Method to find the root of the following function:

$$f(x) = x^3 - x - 3$$

The code should be set up as follows:

- Initialize function and derivative, initialize starting guess x_o, set maximum number of iterations to 100
- Use a tolerance, TOL=10^{-6}, and a stopping criterion as follows:

$$\frac{|x_n - x_{n-1}|}{|x_n|} < TOL$$

- Use/modify your fixed point code from the last homework assignment (HW # 2)

Through the derivation of this method, we emphasized that the convergence of the Newton method is heavily dependent on the starting guess.

(a) What is the zero/root of this function? Recall, you can use the fzero command in MATLAB to tell you where the zero of the function is.

(b) Let $x_o = 0$.

i. Does the method converge?

ii. Graph the iteration vs the estimate of the root at the n^{th} iteration for 100 iterations. That is, graph the points (n,x_n). You need to have a vector of all the x_n values and a vector of the iterations. Use a scatter plot. What do you notice? Why does this occur? Doing a little research, what is this called?

(c) Let $x_o = 2$. How many iterations does it take for the method to converge?

2. Approximate the root of $f(x) = \cos x^2 - x$ in $[0, \pi/2]$ to within $TOL = 10^{-7}$, i.e. $\frac{|x_n - x_{n-1}|}{|x_n|} < TOL = 10^{-7}$, with $x_0 = 1.5$ using:

(a) Newton's method (modify code from # 1)

(b) Secant method, $x_o = 1.5$, $x_1 = 1.4$ (modify code from # 2)

(c) Fixed point iterations (modify code from HW # 2)

At each step in each method, compute the error $err_n = |x_n - x|$. Plot in log-log scale each error versus the previous error (i.e. (err_{n-1}, err_n)). The slope of the curve should give you the order of convergence of the method for this problem. Explain why that is true, and tell me the order of convergence for each of the three methods. Include the plots with your assignment when you turn it in. (When you plot in Matlab, the figure will have options. Click on Tools, then on basic fitting, click on show equations, click on linear plot fit, then click on close. You should now have a log-log plot of the error that has a linear equation on the upper left hand corner of your plot.)

3. Suppose that x is a zero or root **with multiplicity** $m > 1$.

(a) Prove that the standard Newton Iteration method, given below, converges linearly to the root $x = \alpha$.

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}$$

(b) Prove that the Accelerated Newton-Rhapson Iteration method, given below, will produce a sequence $\{x_n\}_{n=1}^{\infty}$ that converges quadratically to α.

$$x_n = x_{n-1} - \frac{mf(x_{n-1})}{f'(x_{n-1})}$$