MA 510 / CS 522 HW #5
Due in class Thursday 10/6

1. Consider the problem of finding a quadratic polynomial $p(x)$ for which

$$p(x_o) = y_o, \quad p'(x_1) = y_1', \quad p(x_2) = y_2$$

   with $x_o \neq x_2$ and $\{y_o, y_1', y_2\}$ the given data points. Assuming that the nodes x_o, x_1, x_2 are real, what conditions must be satisfied for such a $p(x)$ to exist and be unique? Note: This is an example of Hermite-Birkhoff interpolation.

2. Assume that $f \in C^2[a,b]$ and $\mathcal{S}(x)$ is the unique cubic spline interpolant for $f(x)$ that passes through the points $\{x_k, f(x_k)\}_{k=1}^n$, letting $x_1 = a$ and $x_n = b$. Assuming that the endpoints satisfy the clamped boundary conditions $\mathcal{S}'(a) = f'(a)$ and $\mathcal{S}'(b) = f'(b)$.

   Prove that the following is true:

$$\int_a^b (\mathcal{S}''(x))^2 dx \leq \int_a^b (f''(x))^2 dx$$

3. Derive the boundary conditions for the clamped case in terms of M_i. That is, determine the form of the first and last row of the matrix $\tilde{A}$ and determine the first and law row of the right hand side vector $\vec{R}$.