MA 512 **Homework 2** **January 26, 2017**

These exercises are intended (a) to illustrate how errors resulting from methods of different orders behave as step-sizes are reduced and (b) to demonstrate the advantages of higher-order methods in obtaining high-accuracy solutions.

1. Using Matlab or the language/environment of your choice, apply the first-order forward Euler method to the initial-value problem

$$y' = y + e^t \cos t, \qquad y(0) = 0 \tag{$\star$}$$

over the interval $[0, \pi]$. (The exact solution is $y(t) = e^t \sin t$.) Have the method use a constant stepsize $h = \pi/N$ for $N = 10$, 100, 1000, 10000. Create a table that shows the N-values in the first column, the stepsizes in the second column, and the maximum errors $\max\limits_{0 \le n \le N} |y_n - y(t_n)|$ in the third column. You should see that the maximum errors are roughly $\mathcal{O}(h)$, as expected with a first-order method.

2. A step of the classical 4th-order Runge-Kutta method is given by

$$\begin{aligned}
V_1 &= f(t_n, y_n) \\
V_2 &= f(t_n + \frac{h}{2}, y_n + \frac{h}{2} V_1) \\
V_3 &= f(t_n + \frac{h}{2}, y_n + \frac{h}{2} V_2) \\
V_4 &= f(t_n + h, y_n + h V_3) \\
y_{n+1} &= y_n + \frac{h}{6} \left(V_1 + 2V_2 + 2V_3 + V_4 \right)
\end{aligned}$$

As in the first exercise, apply this method to the IVP ($\star$) using a constant stepsize $h = \pi/N$ for $N = 1$, 10, 100. Create a table as in the first exercise. You should see that the maximum errors are roughly $\mathcal{O}(h^4)$, as expected with a 4th-order method.