MA 512 **Homework 12** **April 27, 2017**

1. (20 points) The purpose of this exercise is to give you hands-on experience with the method of lines and also to show just how sensitive some problems are to their parameters.

The time-dependent Bratu problem in one space dimension is

$$\begin{aligned} &u_t = u_{xx} + \lambda\, e^u, && 0 < x < 1, 0 < t, \\ &u(x,0) = 0, && 0 < x < 1, \\ &u(0,t) = u(1,t) = 0, && 0 < t. \end{aligned}$$

It is sometimes used as a simplified model of ignition phenomena. You will see why.

Use the method of lines to solve this problem numerically over the t-interval $[0, 10]$, first with $\lambda = 3.51$ and then with $\lambda = 3.52$. Specifically, in each case, discretize in the spatial variable x on a mesh of 128 equally spaced interior points in $[0, 1]$. Then apply `ode15s` to solve the ODE initial-value problem resulting from this discretization over the t-interval $[0, 10]$. In applying `ode15s`, use an options structure created with `odeset` to print out statistics for the run and also to have `ode15s` use a routine that you supply for evaluating the Jacobian of the right-hand side of the ODE. Feel free to download and use the `mol_demo_1D.m` code as a basis for your code; you will have to modify it appropriately, of course.

You will note that when $\lambda = 3.51$, `ode15s` successfully determines the approximate solution for $0 \le t \le 10$; however, when $\lambda = 3.52$, it stops just short of $t = 10$ with a failure message.

For each value of λ, print out and hand in the run statistics. Also, print out and hand in a plot of the approximate solution at the last t-value returned by `ode15s` and a plot of the maximum absolute value of the approximate solution at each t-value returned by `ode15s`. For the latter plot, a suitable command is `plot(T,max(abs(U'))');`, where `T` and `U` are the t and u values returned by `ode15s`.