MA 512 **Homework 1** **January 19, 2017**

1. In class, we considered the ODE $x'' + x = 0$, which can be thought of as modeling the displacement of a mechanical oscillator with no driving force, no drag, and initial displacement and velocity 1. By adding a drag term Dx', where D is a drag coefficient, this becomes an ODE modeling a simple *damped oscillator* :

$$x'' + Dx' + x = 0. \tag{$\star$}$$

Suppose we augment ($\star$) with initial conditions $x(0) = x'(0) = 1$. Rewrite the resulting initial-value problem as a first-order system in the way described in class. That is, define $y_1 = x$, $y_2 = x'$, and $y = (y_1, y_2)^T$; then rewrite the IVP in the form $y' = f(t, y)$, $y(0) = y_0$.

2. The ODE $x'' = ax' - b(x')^3 - kx$ was derived by J. W. Strutt (better known as Lord Rayleigh) to model the motion of the reed in a clarinet. Suppose the reed begins vibrating from rest with initial displacement $x(0) = 1$. Rewrite the resulting initial-value problem as a first-order system in the way described in class.

3. The purpose of this exercise is to get you started using MATLAB and to provide some experience with phase-plane "quiver" plots that show the vector fields associated with ODEs in two dependent variables. You don't have to do any actual MATLAB coding or to hand anything in. The instructions are

1. Download the M-file `PhasePlaneQuiverPlots.m` from the course website.
2. Start MATLAB in the directory (folder) where you stored the file.
3. In the MATLAB command window, type "`edit PhasePlaneQuiverPlots.m`".
4. In the editing window, you'll see three sections of code, separated by "`%%`", each of which produces a quiver plot for one of the ODEs we've considered. Click on one of the sections to highlight it.
5. Run the highlighted section by typing `command-return` on a Mac or `control-return` in Windows. You should see the quiver plot in the Figure 1 window.
6. Do the same thing for the ODEs in the other sections of code.

I encourage you to experiment with different values of the ODE parameters, the regions plotted, etc. In all cases, think about what the quiver plots tell you about the behavior of solutions of the ODEs.