MA 512 **Homework 3** **February 2, 2017**

1. Recall the IVP for the damped mechanical oscillator:

$$x'' + Dx' + x = 0, \quad x(0) = x'(0) = 1.$$

As in Homework 1, this can be recast as a first-order system $y' = f(t, y)$, $y(0) = y_0$, by setting

$$y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \equiv \begin{pmatrix} x \\ x' \end{pmatrix}, \quad f(t, y) = \begin{pmatrix} y_2 \\ -y_1 - Dy_2 \end{pmatrix}, \quad y_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

Use MATLAB's `ode45` to solve this IVP from $t = 0$ to $t = 20$ with $D = .1$, 1, 10, 100. Use `odeset` to set the `RelTol` and `AbsTol` error tolerances to 10^{-8}. Print out (or write up) and hand in a table showing the values of D in the first column and the corresponding numbers of steps and f-evaluations in the second and third columns. (Recall that `ode45` uses six f-evaluations per step.)

We'll discuss in next week's class why the numbers of steps and f-evaluations behave as they do as D changes. Meanwhile, I encourage you to think about it.

For insight into how the solution behaves as D changes, here are some lines you might want to put at the end of your code. (No need to hand in the plots.)

```
figure(1); plot(tvals,yvals);
figure(2); plot(yvals(:,1),yvals(:,2));
axis([-1.5 1.5 -1.5 1.5]); axis square;
```

In these, `tvals` and `yvals` denote the output of `ode45`: `tvals` is a vector of time values with length equal to the number of steps taken (say *nsteps*), and `yvals` is an *nsteps*$\times$2 matrix, each row of which gives the components of the solution at the corresponding time value. The first line plots the two solution components (the displacement and velocity of the oscillator) vs. time in figure 1. The second line creates a phase-plane plot of the two solution components (displacement vs. velocity) in figure 2. The third line just makes the phase-plane plot a little prettier.

2. Consider again the predator-prey model ODE

$$\begin{aligned} r' &= 2r - \alpha r f, \\ f' &= -f + \alpha r f. \end{aligned}$$

You may recall that the phase-plane quiver plot for this ODE produced in problem 3 of Homework 1 suggested the possibility that solutions are *periodic*, i.e., return to their initial values after some finite time. Verifying that solutions are periodic would be of interest to modelers because it would indicate that, while the predator and prey populations might fluctuate in time, they would continue to exist indefinitely without going extinct.

Use `ode45` to approximately solve this ODE from $t = 0$ to $t = 20$ with $r(0) = 8$ and $f(0) = 3$. Do not use an `options` argument, so that `ode45` uses the default error tolerances `RelTol`$= 10^{-3}$ and `AbsTol`$= 10^{-6}$. Plot the resulting solution in the phase plane, i.e., the (r, f)-plane. You should see that the solution looks roughly periodic but not clearly exactly periodic. Print out and hand in this plot.

Use an options structure to tighten the tolerances until the periodicity can be clearly seen on the plot. Print out and hand in this plot.

Now that you've verified that the solution is periodic, it's tempting to conclude that the predator-prey populations will exist for all time. Not so fast! Inspection of the plot reveals that one of the populations drops below one during part of the orbit and so must surely be regarded as becoming extinct. Modify your options structure and create an "event" function to detect the first time when one of the solution components drops below one. Have the function terminate the integration when this event occurs. Note and hand in the time of termination and which population becomes extinct, along with the phase-plane plot.