Adams Methods Formulas

The following are formulas for some of the explicit Adams–Bashforth and implicit Adams–Moulton methods. In the tables, $f_n = f(t_n, y_n)$, $f_{n-1} = f(t_{n-1}, y_{n-1})$, etc., and, in each formula, ξ_n is some point between the smallest t-value in the formula and t_{n+1} (of course, ξ_n is generally different in different formulas).

Adams–Bashforth methods.

Order	Method	Local Error
1	$y_{n+1} = y_n + hf_n$ (Forward Euler)	$\frac{1}{2}h^2y''(\xi_n)$
2	$y_{n+1} = y_n + \frac{h}{2}[3f_n - f_{n-1}]$	$\frac{5}{12}h^3y^{(3)}(\xi_n)$
3	$y_{n+1} = y_n + \frac{h}{12}[23f_n - 16f_{n-1} + 5f_{n-2}]$	$\frac{3}{8}h^4y^{(4)}(\xi_n)$
4	$y_{n+1} = y_n + \frac{h}{24}[55f_n - 59f_{n-1} + 37f_{n-2} - 9f_{n-3}]$	$\frac{251}{720}h^5y^{(5)}(\xi_n)$

Adams–Moulton methods.

Order	Method	Local Error
1	$y_{n+1} = y_n + hf_{n+1}$ (Backward Euler)	$-\frac{1}{2}h^2y''(\xi_n)$
2	$y_{n+1} = y_n + \frac{h}{2}[f_{n+1} + f_n]$ (Trapezoidal)	$-\frac{1}{12}h^3y^{(3)}(\xi_n)$
3	$y_{n+1} = y_n + \frac{h}{12}[5f_{n+1} + 8f_n - f_{n-1}]$	$-\frac{1}{24}h^4y^{(4)}(\xi_n)$
4	$y_{n+1} = y_n + \frac{h}{24}[9f_{n+1} + 19f_n - 5f_{n-1} + f_{n-2}]$	$-\frac{19}{720}h^5y^{(5)}(\xi_n)$