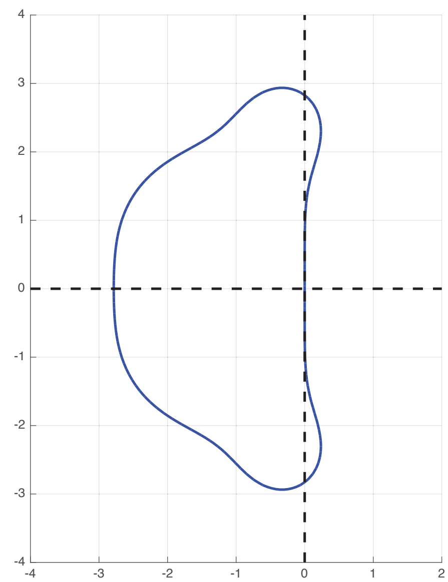
MA 512 **Homework 5** **February 23, 2017**

1. (10 points) The absolute-stability region for explicit 4th-order Runge–Kutta methods is shown at right.



a. Using the method or technique of your choice (MATLAB routine `fzero`, trial and error, etc.), approximate to three significant digits the point at which the negative real axis intersects the boundary of the absolute-stability region. (As shown in class, the absolute-stability region for explicit 4th-order Runge–Kutta methods is $S_A = \{z \in \mathbb{C} : \ |g(z)| \leq 1\}$, where $g(z) = 1 + z + \frac{1}{2}z^2 + \frac{1}{6}z^3 + \frac{1}{24}z^4$.)

b. Consider the following variation on our earlier stiff ODE IVP

$$y' = -1000[y - \sin(t)] + \cos(t), \quad y(0) = 0. \qquad (\star)$$

(The exact solution is $y(t) = \sin(t)$.) How small must the step-size be in order for an explicit 4th-order Runge–Kutta method to be absolutely stable on this problem? If we use the classical 4th-order Runge–Kutta method to approximately solve this IVP over $[0, \pi]$, how many function evaluations will be needed just to maintain absolute stability (with no consideration of accuracy)?

2. (10 points) Apply the trapezoidal method $y_{n+1} = y_n + \frac{h}{2}[f(t_n, y_n) + f(t_{n+1}, y_{n+1})]$ to approximately solve $(\star)$ over the interval $[0, \pi]$. Use constant stepsizes $h = \pi/N$ for $N = 5$, 50, 500. Create a table that shows the N-values in the first column, the stepsizes in the second column, and the maximum errors $\max\limits_{0 \leq n \leq N} |y_n - y(t_n)|$ in the third column. You should see that the maximum errors are roughly $\mathcal{O}(h^2)$, as expected with a second-order method.

Here's something to think about: Replace the ODE in $(\star)$ with the earlier stiff ODE $y' = -1000(y - t^2) + 2t$ and repeat this exercise. (The solution is $y(t) = t^2$.) Can you explain the errors that you see?