

1. (5 points per part) Consider  $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ .
  - a. Determine the Jacobi and Gauss–Seidel iteration matrices  $T_J = -D^{-1}(L + U)$  and  $T_{GS} = -(L + D)^{-1}U$  in terms of  $a$ ,  $b$ ,  $c$ , and  $d$ .
  - b. Determine the spectral radii  $\rho(T_J)$  and  $\rho(T_{GS})$  in terms of  $a$ ,  $b$ ,  $c$ , and  $d$ .
  - c. From these spectral radii, what can you say about the convergence or divergence of the Jacobi and Gauss–Seidel iterates? (When do they converge or diverge? Which converges or diverges faster?)
  - d. Suppose that  $A = \begin{pmatrix} 10 & 9 \\ 9 & 10 \end{pmatrix}$ . Compute  $\rho(T_J)$  and  $\rho(T_{GS})$  for this  $A$  and use their values to determine approximately how many Jacobi and Gauss–Seidel iterations are needed to reduce the error in their iterates by a factor of 10.