

Sample 5-5

周波数解析

多変量畳み込み

画像処理特論

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動作確認: MATLAB R2020a

Fourier analysis

Multivariate convolution

Advanced Topics in Image Processing

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Verified: MATLAB R2020a

準備

(Preparation)

```
close all
```

サンプル画像 $\{u[n]\}_n$ の準備

(Preparation of sample image $\{u[n]\}_n$)

```
% Reading original image  
u = im2double(imread('cameraman.tif'));  
figure(1)  
imshow(u)  
title('Original')
```



線形シフト不変システムのインパルス応答 $\{h[n]\}_n$

(Impulse response of a linear shift-invariant system $\{h[\mathbf{n}]\}_n$)

```
% Impulse response h[n]
ftype = "gaussian";
h = rot90(fspecial(ftype),2);
```

線形シフト不変システムの出力応答 $\{v[\mathbf{n}]\}_n$

(The linear shift-invariant system response $\{v[\mathbf{n}]\}_n$)

畳み込み演算 (Convolution)

$$\{v[\mathbf{n}]\}_n = \{h[\mathbf{n}]\}_n * \{u[\mathbf{n}]\}_n = \sum_{\mathbf{k} \in \Omega \subset \mathbb{Z}^2} u[\mathbf{k}] \{h[\mathbf{n} - \mathbf{k}]\}_n$$

```
% Output v[n]
v = imfilter(u,h,'conv','full');
```

入力信号 $\{u[\mathbf{n}]\}_n$ のスペクトル

(Spectrum of input signal $\{u[\mathbf{n}]\}_n$)

$$U(e^{j\omega^T}) = \sum_{\mathbf{n} \in \mathbb{Z}^2} u[\mathbf{n}] e^{-j\omega^T \mathbf{n}}, \quad \omega \in \mathbb{R}^2$$

DFT(FFT)による DSFT の周波数サンプル計算 (Frequency sampling of DSFT by DFT (FFT))

$$U[\mathbf{k}] = U(e^{j\omega^T}) \Big|_{\omega=2\pi\mathbf{Q}^{-T}\mathbf{k}}, \quad \mathbf{k} \in \mathcal{N}(\mathbf{Q}^T)$$

関数 FFTN を利用. (The function FFTN is used.)

```
% Setting the number of frequency sample points in  $[0,2\pi)^2$ 
nPoints = pow2(nextpow2(size(u)+size(h)-1)); % The least power of two that matches the normal c

% Spectrum of u[n]
U = fftn(u,nPoints);
```

フィルタ $\{h[\mathbf{n}]\}_n$ の周波数応答

(Frequency response of filter $\{h[\mathbf{n}]\}_n$)

$$H(e^{j\omega^T}) = \sum_{\mathbf{n} \in \mathbb{Z}^2} h[\mathbf{n}] e^{-j\omega^T \mathbf{n}}, \quad \omega \in \mathbb{R}^2$$

DFT(FFT)による DSFT の周波数サンプル計算 (Frequency sampling of DSFT by DFT (FFT))

$$H[\mathbf{k}] = H(e^{j\omega^T}) \Big|_{\omega=2\pi\mathbf{Q}^{-T}\mathbf{k}}, \quad \mathbf{k} \in \mathcal{N}(\mathbf{Q}^T)$$

```
% Frequency response of h[n]
H = fftn(h,nPoints);
```

出力信号 $\{v[n]\}_n$ のスペクトル

(Spectrum of input signal $\{v[n]\}_n$)

$$V(e^{j\omega^T}) = \sum_{n \in \mathbb{Z}^2} v[n] e^{-j\omega^T n}, \quad \omega \in \mathbb{R}^2$$

DFT(FFT)による DSFT の周波数サンプル計算 (Frequency sampling of DSFT by DFT (FFT))

$$V[\mathbf{k}] = V(e^{j\omega^T}) \Big|_{\omega=2\pi\mathbf{Q}^{-T}\mathbf{k}}, \quad \mathbf{k} \in \mathcal{N}(\mathbf{Q}^T)$$

```
% Frequency response of v[n]
V = fftn(v,nPoints);
```

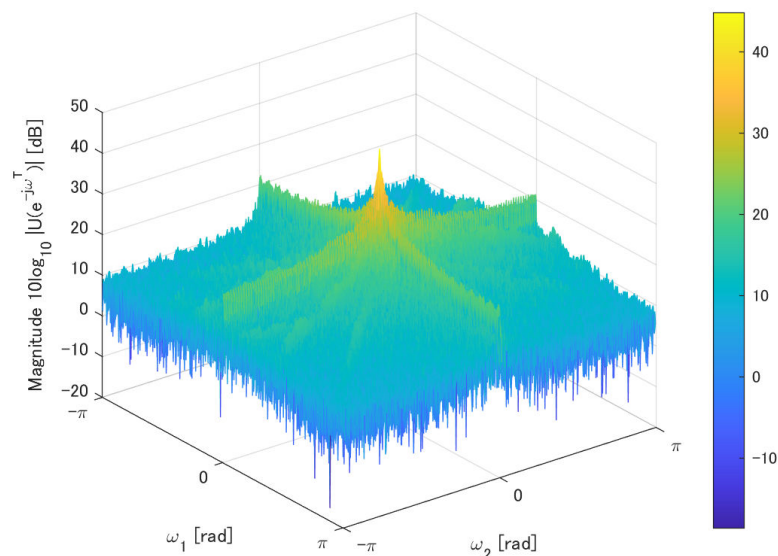
スペクトルと周波数応答の表示

(Display of spectra and frequency response)

$$V(e^{j\omega^T}) = H(e^{j\omega^T})U(e^{j\omega^T})$$

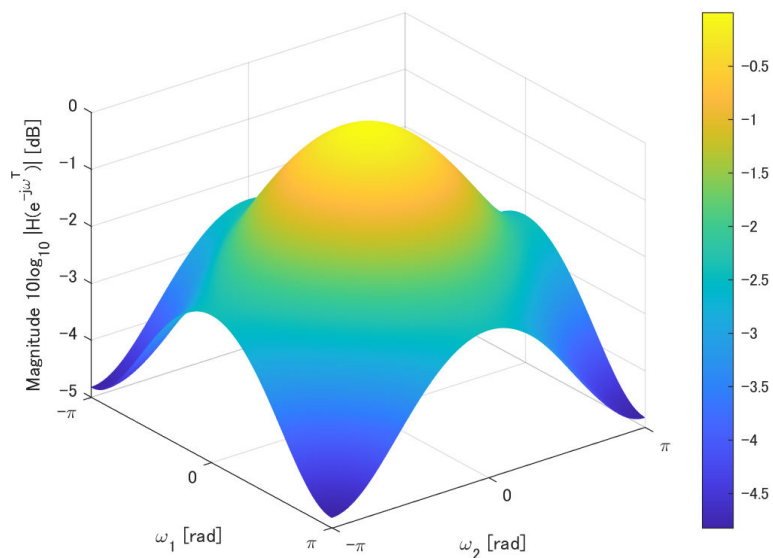
```
% Frequency sample points
[w1,w2] = meshgrid(-pi:2*pi/nPoints(1):pi-2*pi/nPoints(1), -pi:2*pi/nPoints(2):pi-2*pi/nPoints(2));

% Display of spectrum U
figure(2)
mesh(w1,w2,10*log10(abs(fftshift(U))))
ax = gca;
xlabel('\omega_2 [rad]')
ylabel('\omega_1 [rad]')
zlabel('Magnitude 10log_{10} |U(e^{-j\omega^T})| [dB]')
axis ij
ax.XLim = [-pi pi];
ax.XTick = [ -pi 0 pi ];
ax.XTickLabel = { '-\pi', '0', '\pi'};
ax.YLim = [-pi pi];
ax.YTick = [ -pi 0 pi ];
ax.YTickLabel = { '-\pi', '0', '\pi'};
colorbar(ax)
```



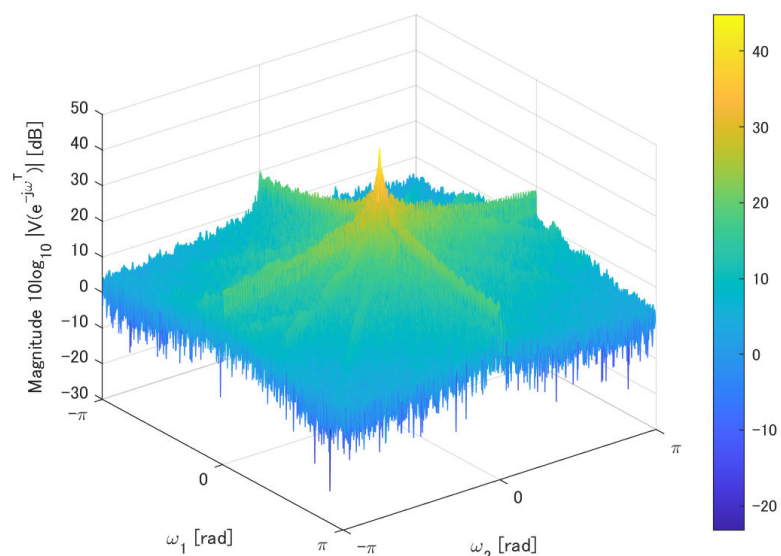
% Display of Freq. response H

```
figure(3)
mesh(w1,w2,10*log10(abs(fftshift(H))))
ax = gca;
xlabel('\omega_2 [rad]')
ylabel('\omega_1 [rad]')
zlabel('Magnitude 10log_{10} |H(e^{-j\omega}^T)| [dB]')
axis ij
ax.XLim = [-pi pi];
ax.XTick = [ -pi 0 pi ];
ax.XTickLabel = { '-\pi', '0', '\pi'};
ax.YLim = [-pi pi];
ax.YTick = [ -pi 0 pi ];
ax.YTickLabel = { '-\pi', '0', '\pi'};
colorbar(ax)
```



% Display of Freq. response V

```
figure(4)
mesh(w1,w2,10*log10(abs(fftshift(V))))
ax = gca;
xlabel('\omega_2 [rad]')
ylabel('\omega_1 [rad]')
zlabel('Magnitude 10log_{10} |V(e^{-j\omega}^T)| [dB]')
axis ij
ax.XLim = [-pi pi];
ax.XTick = [ -pi 0 pi ];
ax.XTickLabel = { '-\pi', '0', '\pi'};
ax.YLim = [-pi pi];
ax.YTick = [ -pi 0 pi ];
ax.YTickLabel = { '-\pi', '0', '\pi'};
colorbar(ax)
```



線形シフト不変システムの解析

(Analysis of linear shift-invariant systems)

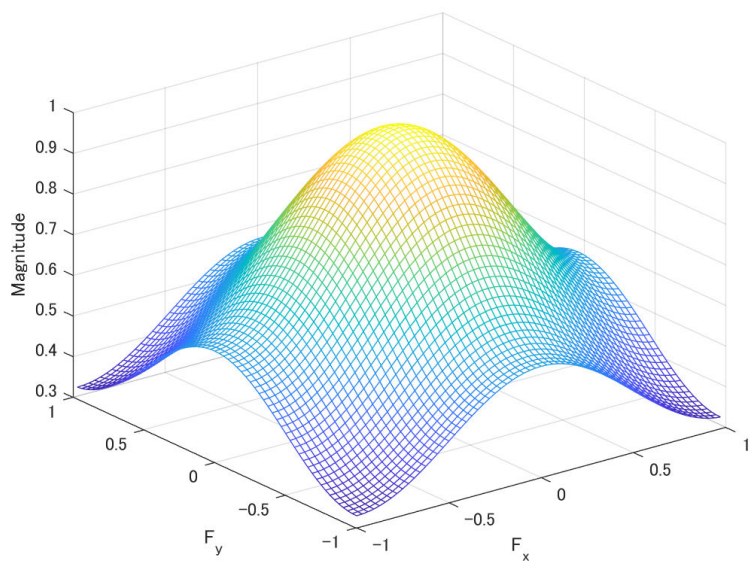
周波数応答(Frequency response)の表示

$$H(e^{j\omega^T}) = H(z)|_{z=e^{j\omega^T}}, \quad \omega \in \mathbb{R}^2$$

- 振幅応答 (Magnitude response): $|H(e^{j\omega^T})|$
- 位相応答 (Phase response): $\angle H(e^{j\omega^T})$

2 変量周波数振幅応答の表示関数 `FREQZ2` を利用. (Using `FREQZ2`, a display function for bivariate frequency magnitude responses.)

```
figure(5)
freqz2(h)
```



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