Sample 6-1

標本化

Sinc関数

画像処理特論

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動作確認: MATLAB R2020a

Sampling

Sinc function

Advanced Topics in Image Processing

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Verified: MATLAB R2020a

準備

(Preparation)

```
close all
```

標本化周期の設定

(Setting the sampling period)

- Δ_{t}: 標本化周期 (Sampling period)

```
% Sampling period
deltat = 1
```

```
deltat = 1
```

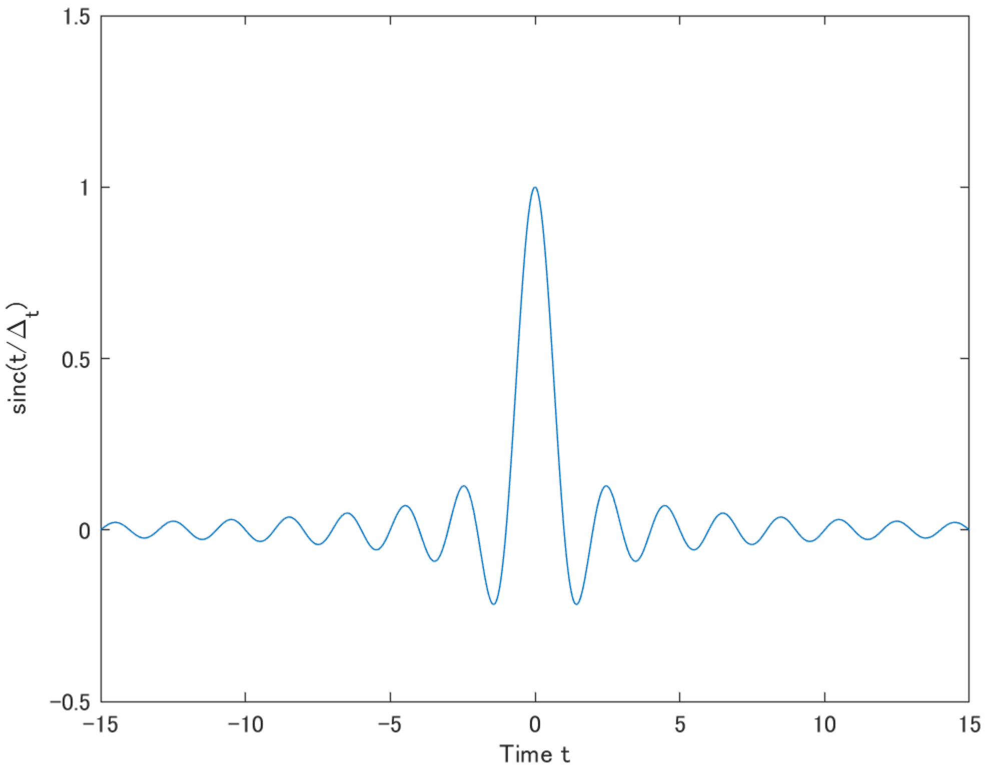
Sinc関数

(Sinc function)

$$\mathrm{sinc}(\Delta_{\mathrm{t}}^{-1}t) := \frac{\sin(\pi\Delta_{\mathrm{t}}^{-1}t)}{\pi\Delta_{\mathrm{t}}^{-1}t}$$

```
% Sinc function
if ~license('test','signal_toolbox')
    sinc = @(x) (x==0) + (x~=0).*(sin(pi*x)./(pi*x));
end
figure(1)
fplot(@(x) sinc(x/deltat),[-15 15])
xlabel('Time t')
```

```
ylabel('sinc(t/\Delta_t)')
axis([-15 15 -0.5 1.5])
```



入力信号の設定

(Setting the input signal)

$$u(t) = \sum_{k\in\mathbb{Z}} c[k]\mathrm{sinc}(\Delta_{\mathrm{t}}^{-1}t - k)$$

シャノンの標本化定理は帯域制限信号 (Shannon's sampling theorem is for the bandwidth-limiting signal)

$$u \in \mathrm{span}\{\mathrm{sinc}(\Delta_{\mathrm{t}}^{-1} \cdot - n)\}_n.$$

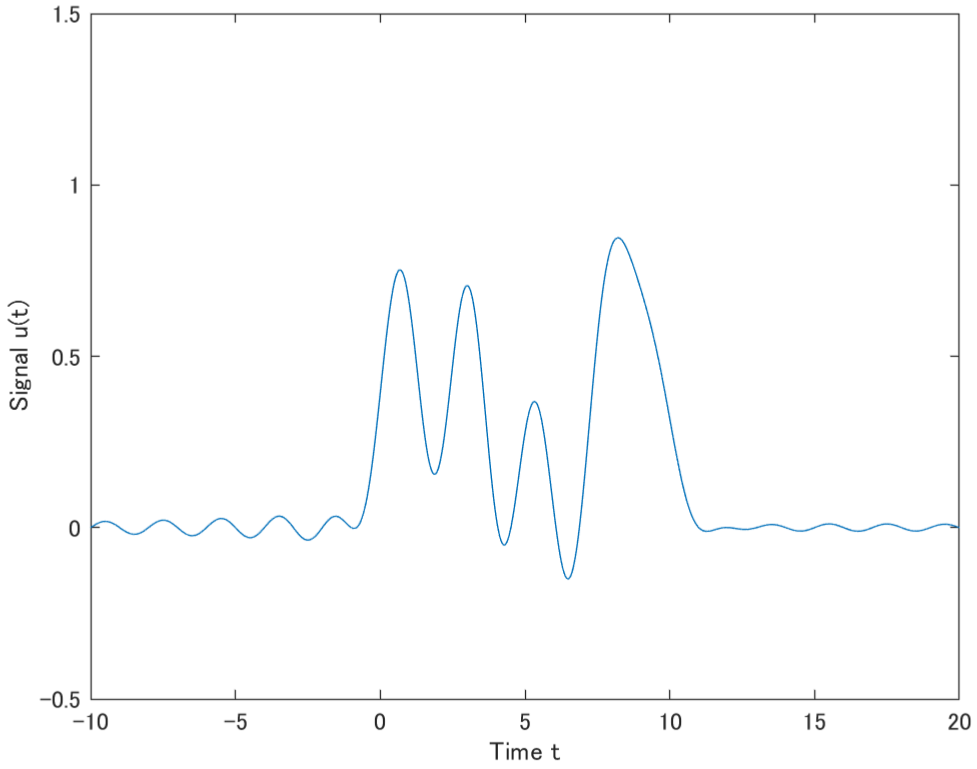
を対象とする。

```
% Configuration
ts = -10;
te = 20;
k = 0:floor(10/deltat);
td = linspace(ts,te,(te-ts)*floor(1e3/deltat));
[Ts,K] = ndgrid(td,k);

% Generation of signal
c = rand(size(k));
u = sinc(Ts/deltat - K)*c(:);

% Plot the signal
```

```
figure(2)
plot(td,u)
xlabel('Time t')
ylabel('Signal u(t)')
axis([ts te -0.5 1.5])
```



Sinc関数による標本化

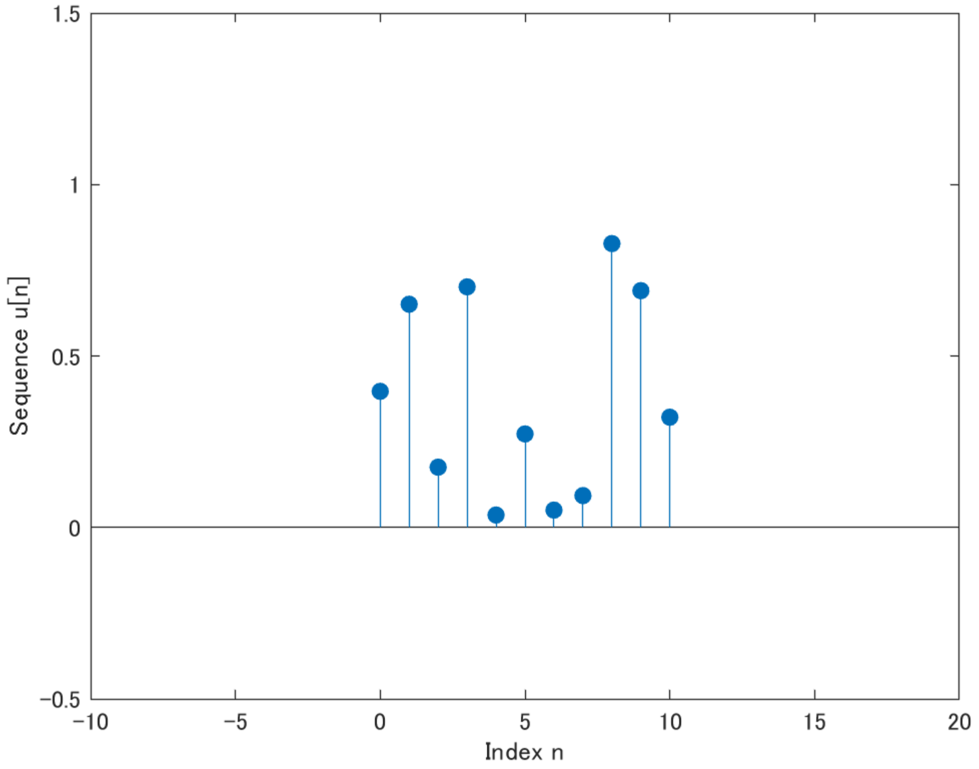
(Sampling with sinc function)

$$u[n] = \Delta_{\mathrm{t}}^{-1} \int_{-\infty}^{\infty} u(t)\mathrm{sinc}(\Delta_{\mathrm{t}}^{-1} t - n)dt$$

以下では有限なサポート領域および台形則で積分を近似していることに注意 (Note that the following approximates the integral with a finite support region and a trapezoidal law.)

```
% Trapezoidal numerical integration
s = (1/deltat)*trapz(td,u.*sinc(Ts/deltat-K));

% Plot samples
figure(2)
stem(deltat*k,s,'filled')
xlabel('Index n')
ylabel('Sequence u[n]')
axis([ts te -.5 1.5])
```

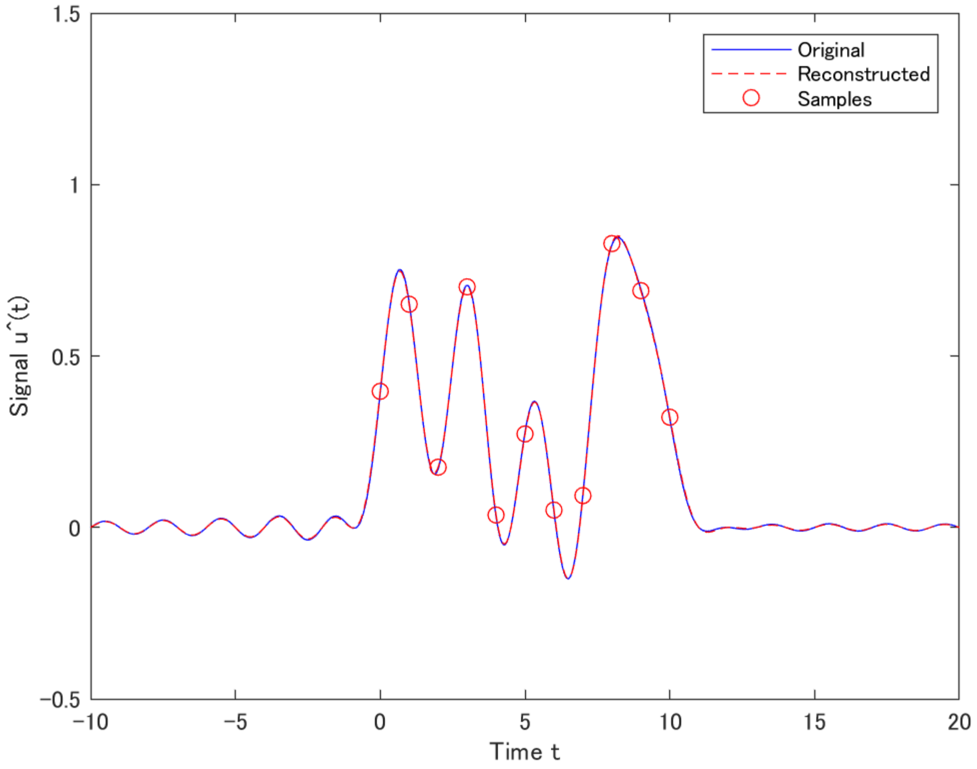


信号再構成
(Signal reconstruction)

$$\widehat{u}(t) = \sum_{k\in\mathbb{Z}} u[k]\mathrm{sinc}(\Delta_{\mathrm{t}}^{-1}t - k)$$

```
% Reconstruct the signal from the samples
uhat = sinc(Ts/deltat - K)*s(:);

% Plot the signal
figure(3)
plot(td,u,'b',td,uhat,'--r',deltat*k,s,'or')
xlabel('Time t')
ylabel('Signal u\^(t)')
axis([ts te -0.5 1.5])
legend('Original','Reconstructed','Samples')
```


Original
Reconstructed
Samples
Signal u^(t)
Time t
1.5
1
0.5
0
−0.5
−10
−5
0
5
10
15
20