Sample 11-5

画像ノイズ除去

事前分布

画像処理特論

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動作確認: MATLAB R2023a

Image denoising

Prior distribution

Advanced Topics in Image Processing

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Verified: MATLAB R2023a

準備

(Preparation)

```
clear
close all
import msip.download_img
msip.download_img
```

```
kodim01.png already exists in ./data/
kodim02.png already exists in ./data/
kodim03.png already exists in ./data/
kodim04.png already exists in ./data/
kodim05.png already exists in ./data/
kodim06.png already exists in ./data/
kodim07.png already exists in ./data/
kodim08.png already exists in ./data/
kodim09.png already exists in ./data/
kodim10.png already exists in ./data/
kodim11.png already exists in ./data/
kodim12.png already exists in ./data/
kodim13.png already exists in ./data/
kodim14.png already exists in ./data/
kodim15.png already exists in ./data/
kodim16.png already exists in ./data/
kodim17.png already exists in ./data/
kodim18.png already exists in ./data/
kodim19.png already exists in ./data/
kodim20.png already exists in ./data/
kodim21.png already exists in ./data/
kodim22.png already exists in ./data/
kodim23.png already exists in ./data/
kodim24.png already exists in ./data/
See Kodak Lossless True Color Image Suite
```

パラメータ設定

(Parameter settings)

- sgm: ノイズ標準偏差 σ_w (Standard deviation of noise)
- nlevels: ウェーブレット段数 (Wavelet levels)

```
% Parameter settings
nlevels = 3;
```

画像の読込

(Read image)

```
img = "kodim23";
u = im2double(imread("./data/" + img + ".png"));
if size(u,3) == 3
    u = rgb2gray(u);
end
```

分析処理

(Analysis process)

直交ウェーブレット変換 Symlet を利用．(Uses Symlet, which is an orthogonal wavelet transform.)

```
% Preperation of filters for wavelets
iswtb = license('checkout','wavelet_toolbox');
if iswtb % Functions in Wavelet Toolbox are used
    dwtmode('per')
    wname = "sym4";
    [h0,h1,f0,f1] = wfilters(wname);
    %save(['./data/' char(wname) '.mat'],'h0','h1','f0','f1')
else
    import msip.ezwavedec2
    import msip.ezwaverec2
    S = load('./data/sym4.mat');
    h0 = S.h0;
    h1 = S.h1;
    f0 = S.f0;
    f1 = S.f1;
    clear H  F
    % Analysis bivariate filters
    H.h00 = h0(:)*h0(:).';
    H.h01 = h0(:)*h1(:).';
    H.h10 = h1(:)*h0(:).';
    H.h11 = h1(:)*h1(:).';
    % Synthesis bivariate filters
    F.f00 = f0(:)*f0(:).';
    F.f01 = f0(:)*f1(:).';
```

```
    F.f10 = f1(:)*f0(:).';
    F.f11 = f1(:)*f1(:).';
end
```

```
**********************
**  DWT 拡張モード: 周期化  **
**********************
```

分析処理 (Analysis process)

```
if iswtb
    [coefs,scales] = wavedec2(u,nlevels,h0,h1);
    % Reconstruction to check PR
    r = waverec2(coefs,scales,f0,f1);
else
    [coefs,scales] = ezwavedec2(u,nlevels,H);
    % Reconstruction to check PR%
    r = ezwaverec2(coefs,scales,F);
end
assert(norm(u-r,"fro")^2/numel(u)<1e-18,'Perfect reconstruction is violated.')
```

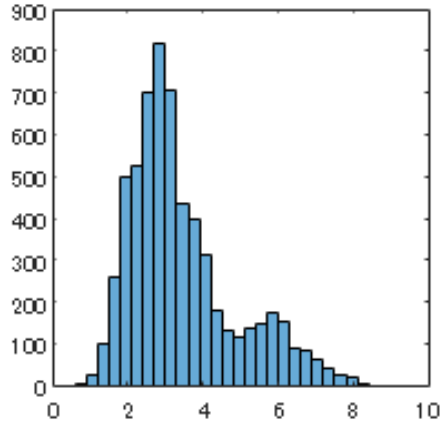
変換係数の抽出 (Extraction of coefficients)

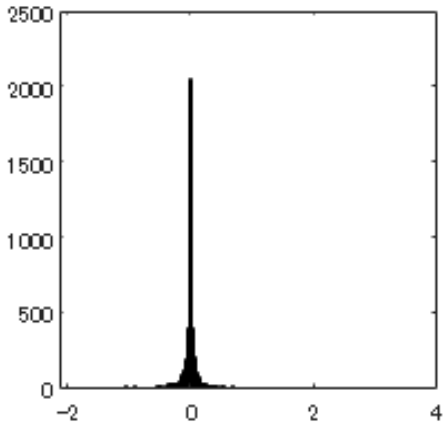
```
s = extractcoefs(coefs,scales);
```

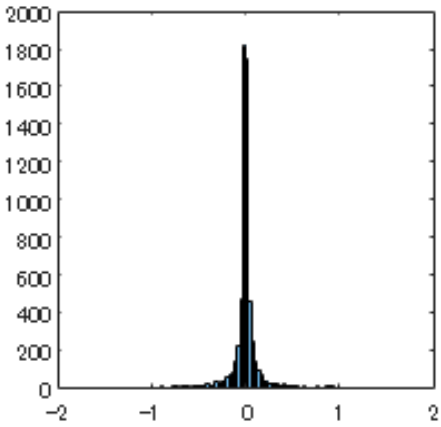
変換係数の分布

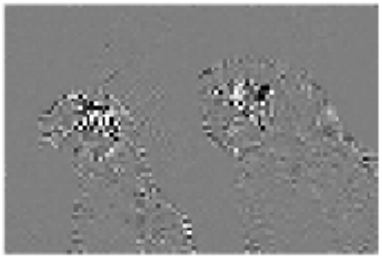
(Distribution of Coefs.)

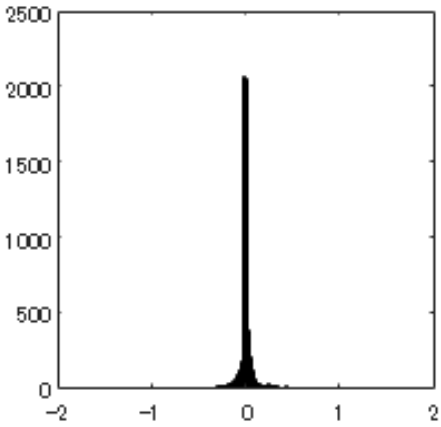
```
nchs = length(s);
for ich = 1:nchs
    figure
    subplot(1,2,1)
    if ich == 1
        imshow(s{ich}*pow2(-nlevels))
    else
        imshow(s{ich}+.5)
    end
    subplot(1,2,2)
    histogram(s{ich}(:))
    axis square
    drawnow
end
```

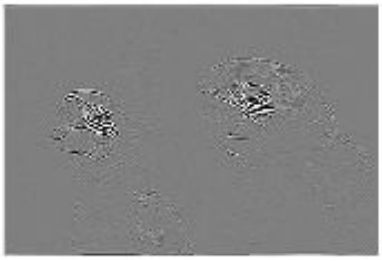

900
800
700
600
500
400
300
200
100
0
0
2
4
6
8
10

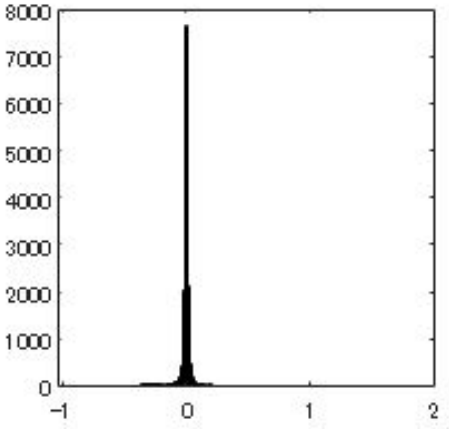

2500
2000
1500
1000
500
0
−2
0
2
4

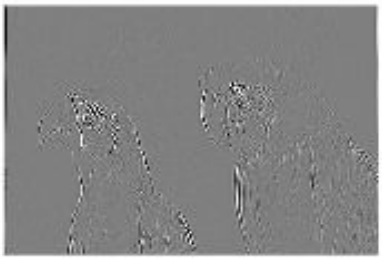

2000
1800
1600
1400
1200
1000
800
600
400
200
0
−2
−1
0
1
2

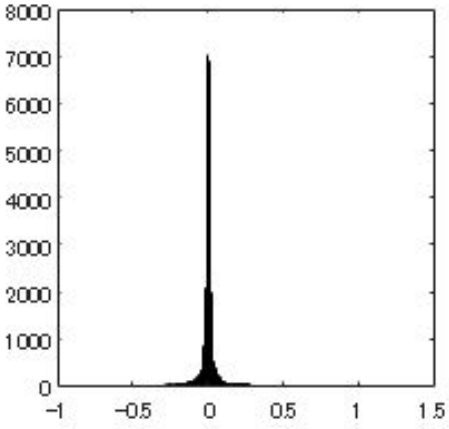


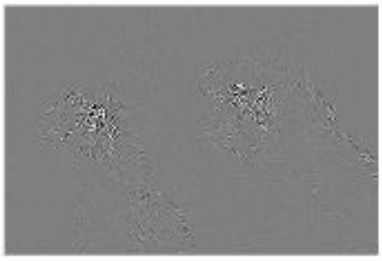

2500
2000
1500
1000
500
0
−2
−1
0
1
2

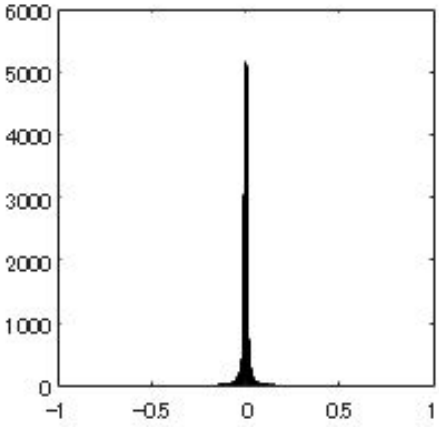


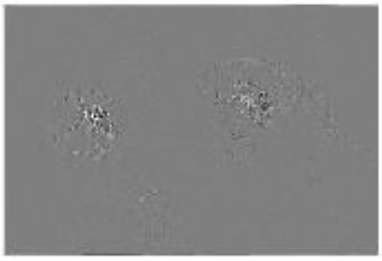

8000
7000
6000
5000
4000
3000
2000
1000
0
−1
0
1
2

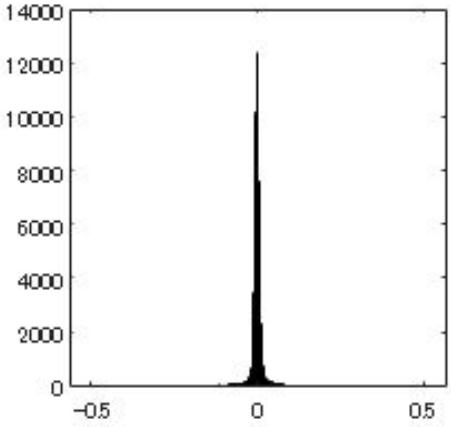


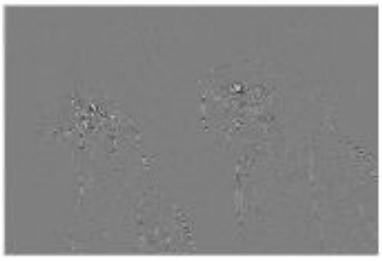

8000
7000
6000
5000
4000
3000
2000
1000
0
−1
−0.5
0
0.5
1
1.5

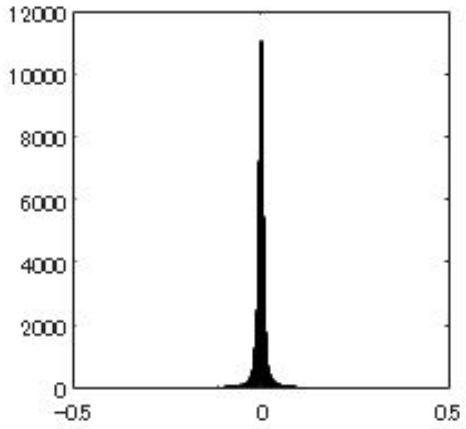


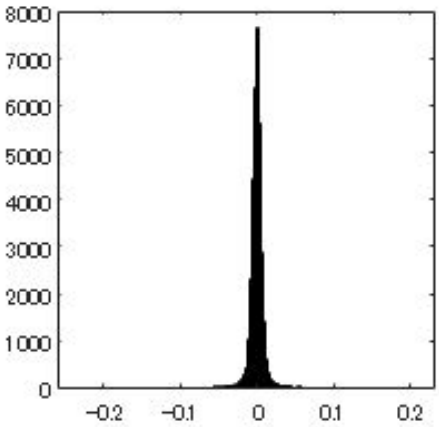

6000
5000
4000
3000
2000
1000
0
−1
−0.5
0
0.5
1




14000
12000
10000
8000
6000
4000
2000
0
−0.5
0
0.5




12000
10000
8000
6000
4000
2000
0
−0.5
0
0.5


8000
7000
6000
5000
4000
3000
2000
1000
0
−0.2
−0.1
0
0.1
0.2

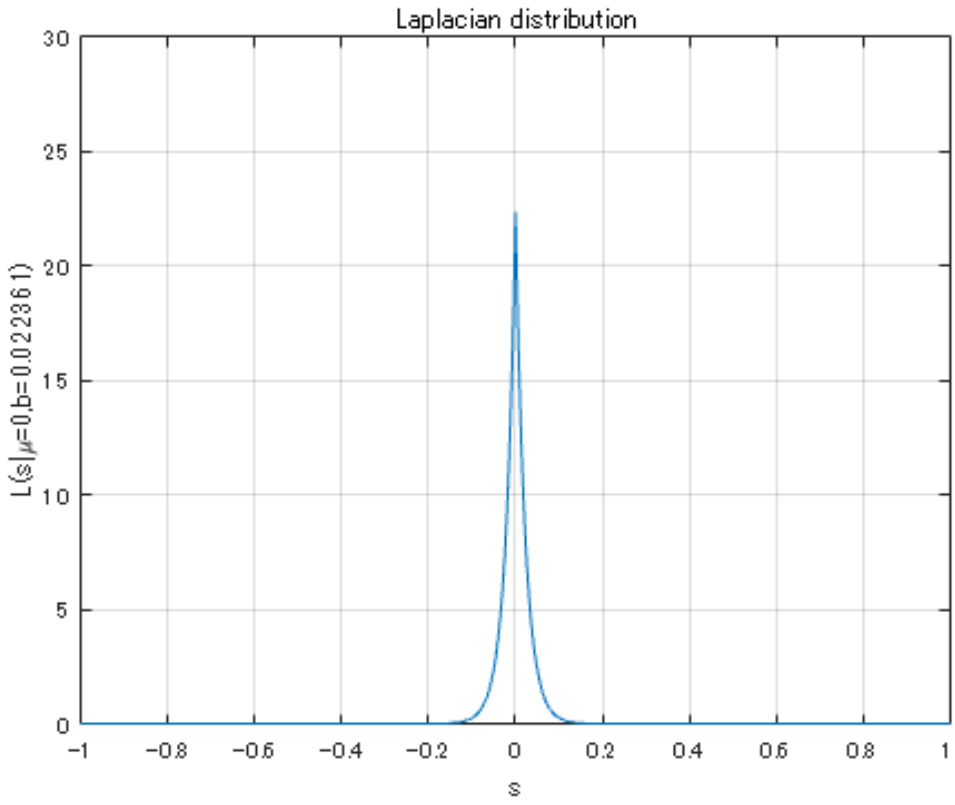
ラプラス分布

(Laplace distribution)

変換係数はラプラス分布に従う乱数と仮定．(Coefs. are assumed to be random numbers drawn from a Laplace distribution.)

- $\mathbf{s} \sim \mathrm{Lap}(\mathbf{s}|\boldsymbol{\mu} = \mathbf{0}, b)$
- $\mathrm{Lap}(\mathbf{s}|\boldsymbol{\mu} = \mathbf{0}, b) = \frac{1}{2b}\exp\left(-\frac{\|\mathbf{s} - \boldsymbol{\mu}\|_1}{b}\right)$

```
% Laplacian parameters
mu = 0;
sgm2 = 0.001;
b = sqrt(sgm2/2);
% Laplacian distribution
spdf = @(x) 1/(2*b)*exp(-abs(x-mu)/b);
figure
h = fplot(spdf);
xlabel('s')
ylabel(['L(s|\mu=0,b=' num2str(b) ')'])
title('Laplacian distribution')
grid on
axis([-1 1 0 30])
```



観測画像

(Observation image)

パラメータ設定 (Parameter settings)

```
sgmuint8 = 30;
sgm = sgmuint8/255;
```

ノイズ付加 (Add noise)

```
v = imnoise(u,'gaussian',0,sgm^2);
```

分析処理 (Analysis process)

```
if iswtb
    [coefs,scales] = wavedec2(v,nlevels,h0,h1);
else
    [coefs,scales] = ezwavedec2(v,nlevels,H);
end
```

ウェーブレット縮退処理

(Wavelet shrinkage)

問題設定 (Problem settings)

- $\hat{\mathbf{s}} = \arg\min_{\mathbf{s}} \frac{1}{2}\|\mathbf{v} - \mathbf{D}\mathbf{s}\|_2^2 + \lambda\|\mathbf{s}\|_1$
- $\mathbf{D}\mathbf{D}^T = \mathbf{D}^T\mathbf{D} = \mathbf{I}$ (Orthonormal)

パラメータ設定 (Parameter settings)

```
isbayesshrink = false;
if ~isbayesshrink
    isaprxleft = true;
    lambda = 10^-0.6
end
```

```
lambda = 0.2512
```

ソフト閾値処理 (Soft-thresholding)

$$\hat{\mathbf{s}} = \mathcal{T}_\lambda(\mathbf{s}) = \mathrm{sign}(\mathbf{s}) \odot \max(\mathrm{abs}(\mathbf{s}) - \lambda\mathbf{1}, \mathbf{0})$$

```
% サブバンド適応ソフト縮退処理
if isbayesshrink
    import msip.bayesshrink
    coefs = bayesshrink(coefs,scales);
else
    if isaprxleft
        mask = ones(size(coefs));
        mask(1:prod(scales(1,:))) = 0;
```

```
        lambda = lambda * mask;
    end
    softshrink = @(x) sign(x).*max(abs(x)-lambda,0);
    coefs = softshrink(coefs);
end
```

合成処理 (Synthesis process)

```
if iswtb
    r = waverec2(coefs,scales,f0,f1);
else
    r = ezwaverec2(coefs,scales,F);
end
```

画像表示

(Image show)

```
figure
imshow(u)
title('Original image u')
```


```
figure
imshow(v)
title(sprintf('Noisy image v : PSNR = %5.2f [dB]',psnr(u,v)))
```


```
figure
imshow(r)
title(sprintf('Denoised image r : PSNR = %5.2f [dB]',psnr(u,r)))
```

ウェーブレット画像ノイズ除去関数

(Wavelet image denoising function)

参考資料 (Reference)

```
if iswtb
    help wthcoef2
    help wdenoise2
end
```

```
 wthcoef2 - 2-D wavelet coefficient thresholding
    This MATLAB function returns the horizontal, vertical, or diagonal
    coefficients obtained from the wavelet decomposition structure [C,S] by
    soft or hard thresholding defined in vectors N and T.

    構文
      NC = wthcoef2('type',C,S,N,T,SORH)
      NC = wthcoef2('type',C,S,N)
      NC = wthcoef2('a',C,S)
      NC = wthcoef2('t',C,S,N,T,SORH)

    入力引数
      'type' - Type of coefficients
        'h' | 'v' | 'd'
      C - Wavelet decomposition vector
        real-valued vector
      S - Bookkeeping matrix
        integer-valued matrix
      N - Threshold vector
```

```
      1 ≤ N(i) ≤ size(S,1)-2
    T - Threshold vector
      nonnegative vectors
    SORH - Soft or hard threshold
      's' | 'h'

  出力引数
    NC - Wavelet coefficient threshold
      real-valued vector

  例
    Calculate Coefficients Obtained From Wavelet Decomposition Structure

  参考 wthcoef, wavedec2, wthresh

  R2006a より前に Wavelet Toolbox で導入
  wthcoef2 のドキュメンテーション
wdenoise2 - Wavelet image denoising
  This MATLAB function denoises the grayscale or RGB image IM using an
  empirical Bayesian method.

  構文
    IMDEN = wdenoise2(IM)
    IMDEN = wdenoise2(IM,LEVEL)
    [IMDEN,DENOISEDCFS] = wdenoise2(___)

    [IMDEN,DENOISEDCFS,ORIGCFS] = wdenoise2(___)
    [IMDEN,DENOISEDCFS,ORIGCFS,S] = wdenoise2(___)
    [IMDEN,DENOISEDCFS,ORIGCFS,S,SHIFTS] = wdenoise2(___)

    [___] = wdenoise2(___,Name,Value)

    wdenoise2(___)

  入力引数
    IM - Input image
      real-valued 2-D matrix | real-valued 3-D array
    LEVEL - Wavelet decomposition level
      positive integer

  名前と値の引数
    Wavelet - Name of wavelet
      'bior4.4' (既定の設定) | character vector | string scalar
    DenoisingMethod - Denoising method
      'Bayes' (既定の設定) | 'FDR' | 'Minimax' | 'SURE' |
      'UniversalThreshold'
    ThresholdRule - Threshold rule
      'Hard' | 'Soft' | 'Mean' | 'Median'
    NoiseEstimate - Method of estimating variance of noise
      'LevelIndependent' (既定の設定) | 'LevelDependent'
    NoiseDirection - Wavelet subbands
      ["h","v","d"] (既定の設定) | string vector | scalar string
    CycleSpinning - Number of circular shifts
      0 (既定の設定) | nonnegative integer
    ColorSpace - Color space
      'PCA' (既定の設定) | 'Original'

  出力引数
    IMDEN - Denoised image
      real-valued matrix
    DENOISEDCFS - Scaling and denoised wavelet coefficients
      real-valued matrix
    ORIGCFS - Scaling and wavelet coefficients
      real-valued matrix
```

```
    S - Bookkeeping matrix
      integer-valued matrix
    SHIFTS - Image shifts
      integer-valued matrix

例
    Denoise Grayscale Image Using Default Settings
    Denoise Color Image Using Cycle Spinning
    Denoise Image Using Specific Subband

参考 wdenoise, wavedec2

R2019a の Wavelet Toolbox で導入
wdenoise2 のドキュメンテーション
```

変換係数の抽出

(Extraction of Coefs.)

```
function s = extractcoefs(coefs,scales)
nscales = size(scales,1)-1;
s = cell(3*(nscales-1)+1,1);
sidx = 1;
ndims = scales(1,:);
eidx = sidx + prod(ndims) - 1;
s{1} = reshape(coefs(sidx:eidx),ndims);
sidx = eidx + 1;
ich = 2;
for iscale = 2:nscales
    ndims = scales(iscale,:);
    for iband = 1:3
        eidx = sidx + prod(ndims) - 1;
        s{ich} = reshape(coefs(sidx:eidx),ndims);
        sidx = eidx + 1;
        ich = ich + 1;
    end
end
end
```