Sample 8-4

離散コサイン変換

基底画像

画像処理特論

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動作確認: MATLAB R2023a

Discrete cosine transform

Basis images

Advanced Topics in Image Processing

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Verified: MATLAB R2023a

準備

(Preparation)

```
close all
```

DCT 点数

(DCT points)

```
nPoints = 4;
```

DCT 基底ベクトルの抽出

(Extraction of DCT basis vectors)

$$[\mathbf{C}_M]_{k,n} = \sqrt{\frac{2}{M}}\alpha_k \cos\frac{k(n+1/2)\pi}{M}, \quad k,n = 0,1,\cdots,M-1$$

$$\alpha_k = \begin{cases} \dfrac{1}{\sqrt{2}} & k = 0 \\ 1 & k = 1,2,\cdots,M-1 \end{cases}$$

基底ベクトル (Basis vector)

$$\mathbf{b}_k = \mathbf{C}_M^{-1}\mathbf{e}_k, \quad k = 0,1,\cdots,M-1$$

$$\mathbf{e}_0 = (1 \quad 0 \quad \cdots \quad 0)^T$$

$\mathbf{e}_1 = (0 \;\; 1 \;\; \cdots \;\; 0)^T$

$\vdots$

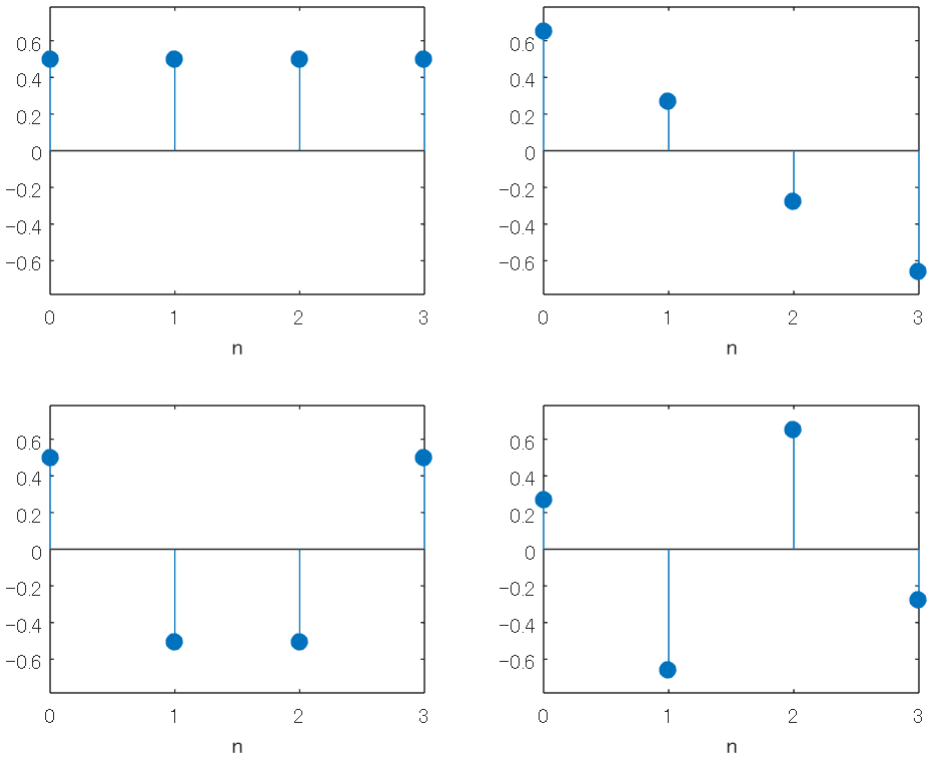
$\mathbf{e}_{M-1} = (0 \;\; 0 \;\; \cdots \;\; 1)^T$

```
B = zeros(nPoints,nPoints);
for idx = 1:nPoints
    e = zeros(nPoints,1);
    e(idx) = 1;
    B(:,idx) = idct(e);
end
```

基底ベクトルの表示

(Display the basis vectors)

```
figure(1)
for idx = 1:nPoints
    subplot(ceil(nPoints/2),2,idx);
    stem(0:nPoints-1,B(:,idx),'filled');
    ax = gca;
    ax.YLim = 1.2*[min(B(:)) max(B(:))];
    xlabel('n')
end
```



DCT 基底画像の抽出

(Extraction of DCT basis images)

$$\mathbf{B}_{k,\ell} = \mathbf{C}_M^{-1}\mathbf{E}_{k,\ell}\mathbf{C}_M^{-T},\ \ k,\ell = 0, 1, \cdots, M-1$$

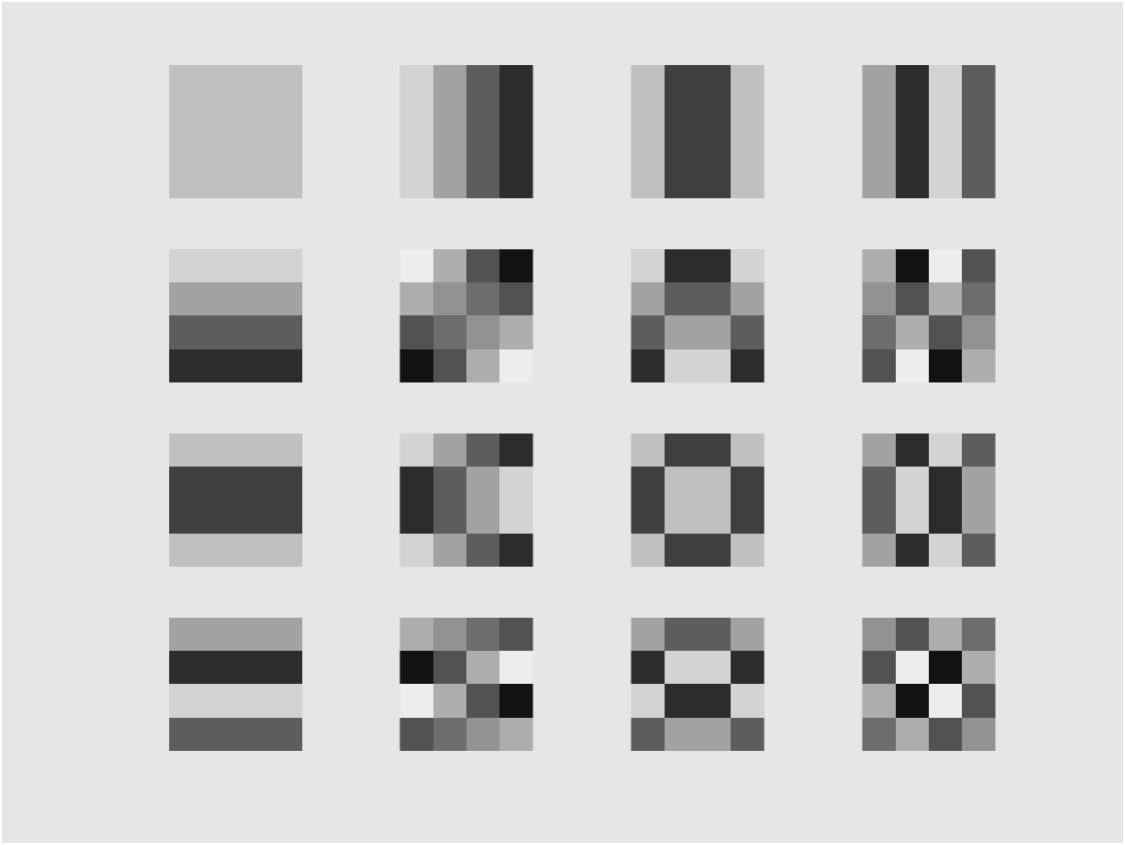
$$\mathbf{E}_{k,\ell} = \mathbf{e}_k\mathbf{e}_\ell^T$$

```
B = zeros(nPoints,nPoints,nPoints^2);
iBasis = 1;
for iRow=1:nPoints
    for iCol=1:nPoints
        E = zeros(nPoints,nPoints);
        E(iRow,iCol) = 1;
        B(:,:,iBasis) = idct2(E,nPoints,nPoints);
        iBasis = iBasis + 1;
    end
end
```

基底画像の表示

(Display the basis images)

```
hfig2 = figure(2);
hfig2.Color = 0.9*[1 1 1];
iBasis = 1;
for iRow=1:nPoints
    for iCol=1:nPoints
        b = B(:,:,iBasis);
        subplot(nPoints,nPoints,nPoints*(iRow-1)+iCol);
        imshow(b+.5)
        iBasis = iBasis + 1;
    end
end
```



非線形近似

(Non-linear approximation)

絶対値の大きな係数のみを残して再構成 (Reconst by leaving only the coefficients with large absolute values)

```
% Ratio of remaining coefficients
coefRatio = 5 ;

% Read image
X = im2double(imread('cameraman.tif'));

% Block DCT
U = blockproc(X,nPoints*[1 1],@(x) dct2(x.data));

% Non-linear approximation (Shrinking small coefficients to zero)
nCoefs = round(coefRatio/100*numel(U));
[~,I] = mink(U(:),numel(U)-nCoefs,'ComparisonMethod','abs');
U(I) = 0;
disp(['Sparsity: ' num2str(nnz(U(:))) '/' num2str(numel(U(:))) ])
```

```
Sparsity: 3277/65536
```

```
% Block IDCT
R = blockproc(U,nPoints*[1 1],@(x) idct2(x.data));
```

非線形近似の結果表示

(Display non-linear approximatiom result)

```
figure(3)
subplot(1,2,1)
imshow(X)
title('Original')
subplot(1,2,2)
imshow(R)
title(['NLA result w/ ' num2str(coefRatio) ' % Coefs. (PSNR: ' num2str(psnr(X,R)) '
dB)'])
```