

## Sample 8-5

### 離散コサイン変換

DFT との関係

画像処理特論

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動作確認: MATLAB R2023a

### Discrete cosine transform

Relation to DFT

Advanced Topics in Image Processing

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Verified: MATLAB R2023a

### 準備

(Preparation)

```
close all
```

### DCT 行列

(DCT matrix)

$$[C_M]_{k,n} = \sqrt{\frac{2}{M}} \alpha_k \cos \frac{k(n+1/2)\pi}{M}, \quad k, n = 0, 1, \dots, M-1$$

$$\alpha_k = \begin{cases} \frac{1}{\sqrt{2}} & k = 0 \\ 1 & k = 1, 2, \dots, M-1 \end{cases}$$

```
% DCT points
```

```
nPoints = 4;
```

```
C = dctmtx(nPoints)
```

```
C = 4×4
```

```
0.5000    0.5000    0.5000    0.5000  
0.6533    0.2706   -0.2706   -0.6533  
0.5000   -0.5000   -0.5000    0.5000  
0.2706   -0.6533    0.6533   -0.2706
```

### OTDFT 行列

(OTDFT matrix)

OTDFT は DCT と深い関係にある。(OTDFT is closely related to DCT.)

OTDFT: Odd-time discrete Fourier transform (GDFT w/  $a = 0, b = 1/2$ )

- 一般化 DFT (GDFT: generalized DFT)

$$X_N^{(a,b)}[k] = \sum_{n=0}^{N-1} x[n] W_N^{(k+a)(n+b)}, \quad k = 0, 1, \dots, N-1$$

- OTDFT 行列 (OTDFT matrix)

$$[\mathbf{W}_N^{(0, \frac{1}{2})}]_{k,n} = e^{-j\frac{\pi}{N}k} e^{-j\frac{2\pi}{N}kn}$$

```
% DFT points
nPointsDft = 2 * nPoints;

% OTDFT matrix
k=0:nPointsDft-1;
Wdft = dftmtx(nPointsDft);
Lambda = diag(exp(-1j*pi/nPointsDft*k));
Wotdft = Lambda * Wdft
```

```
Wotdft = 8x8 complex
    1.0000 + 0.0000i    1.0000 + 0.0000i    1.0000 + 0.0000i    1.0000 + 0.0000i ...
    0.9239 - 0.3827i    0.3827 - 0.9239i   -0.3827 - 0.9239i   -0.9239 - 0.3827i
    0.7071 - 0.7071i   -0.7071 - 0.7071i   -0.7071 + 0.7071i    0.7071 + 0.7071i
    0.3827 - 0.9239i   -0.9239 + 0.3827i    0.9239 + 0.3827i   -0.3827 - 0.9239i
    0.0000 - 1.0000i   -0.0000 + 1.0000i    0.0000 - 1.0000i   -0.0000 + 1.0000i
   -0.3827 - 0.9239i    0.9239 + 0.3827i   -0.9239 + 0.3827i    0.3827 - 0.9239i
   -0.7071 - 0.7071i    0.7071 - 0.7071i    0.7071 + 0.7071i   -0.7071 + 0.7071i
   -0.9239 - 0.3827i   -0.3827 - 0.9239i    0.3827 - 0.9239i    0.9239 - 0.3827i
```

## OTDFT による DCT

(DCT through OTDFT)

OTDFT と DCT の関係 (Relation between OTDFT and DCT)

$$[\mathbf{C}_M]_{k,n} = \frac{\alpha_k}{\sqrt{2M}} \left[ \mathbf{W}_{2M}^{(0, \frac{1}{2})} \mathbf{E}_M \right]_{k,n}, \quad k, n = 0, 1, \dots, M-1$$

ただし、 $\mathbf{E}_M$  は対称拡張行列 (where  $\mathbf{E}_M$  is the symmetric extension matrix defined by )

$$\mathbf{E}_M = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 1 & \dots & 0 \\ 1 & 0 & \dots & 0 \end{pmatrix}.$$

である。

```
% Symmetric extension matrix
E = [ eye(nPoints) ; fliplr(eye(nPoints)) ]
```

```
E = 8x4
    1     0     0     0
    0     1     0     0
    0     0     1     0
    0     0     0     1
    0     0     0     1
    0     0     1     0
    0     1     0     0
    1     0     0     0
```

```
% DCT matrix through OTDFT
D = 1/sqrt(2*nPoints)*...
    diag([1/sqrt(2) ones(1,nPoints-1) ])*...
    Wotdft(1:nPoints,:)*E
```

```
D = 4x4 complex
    0.5000 + 0.0000i    0.5000 + 0.0000i    0.5000 + 0.0000i    0.5000 + 0.0000i
    0.6533 + 0.0000i    0.2706 + 0.0000i   -0.2706 + 0.0000i   -0.6533 + 0.0000i
    0.5000 + 0.0000i   -0.5000 - 0.0000i   -0.5000 - 0.0000i    0.5000 + 0.0000i
    0.2706 + 0.0000i   -0.6533 - 0.0000i    0.6533 + 0.0000i   -0.2706 - 0.0000i
```

誤差の評価 (Evaluation of error)

```
norm(D-C, 'Fro')
```

```
ans = 4.8552e-16
```

Wdft による変換(DFT)には、高速フーリエ変換(FFT)を適用できる。(The Fast Fourier Transform (FFT) can be applied to the transform with Wdft (DFT).)

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