Sample 11-4

画像ノイズ除去

勾配降下法

画像処理特論

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動作確認: MATLAB R2023a

Image denoising

Gradient descent

Advanced Topics in Image Processing

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Verified: MATLAB R2023a

準備

(Preparation)

```
clear
close all
import msip.download_img
msip.download_img
```

```
kodim01.png already exists in ./data/
kodim02.png already exists in ./data/
kodim03.png already exists in ./data/
kodim04.png already exists in ./data/
kodim05.png already exists in ./data/
kodim06.png already exists in ./data/
kodim07.png already exists in ./data/
kodim08.png already exists in ./data/
kodim09.png already exists in ./data/
kodim10.png already exists in ./data/
kodim11.png already exists in ./data/
kodim12.png already exists in ./data/
kodim13.png already exists in ./data/
kodim14.png already exists in ./data/
kodim15.png already exists in ./data/
kodim16.png already exists in ./data/
kodim17.png already exists in ./data/
kodim18.png already exists in ./data/
kodim19.png already exists in ./data/
kodim20.png already exists in ./data/
kodim21.png already exists in ./data/
kodim22.png already exists in ./data/
kodim23.png already exists in ./data/
kodim24.png already exists in ./data/
See Kodak Lossless True Color Image Suite
```

問題設定

(Problem settings)

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}} \frac{1}{2}\|\mathbf{v} - \mathbf{D}\mathbf{s}\|_2^2 + \frac{\lambda}{2}\|\mathbf{s}\|_2^2$$

- $\mathbf{D} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}: \quad \mathbb{R}^2 \to \mathbb{R}^1$
- $\mathbf{v} = \frac{1}{2} \in \mathbb{R}^1$
- $\lambda \in [0, \infty)$
- $\mathbf{s} \in \mathbb{R}^2$

```
D = [2 1]/3;
v = 0.5;
```

パラメータ設定

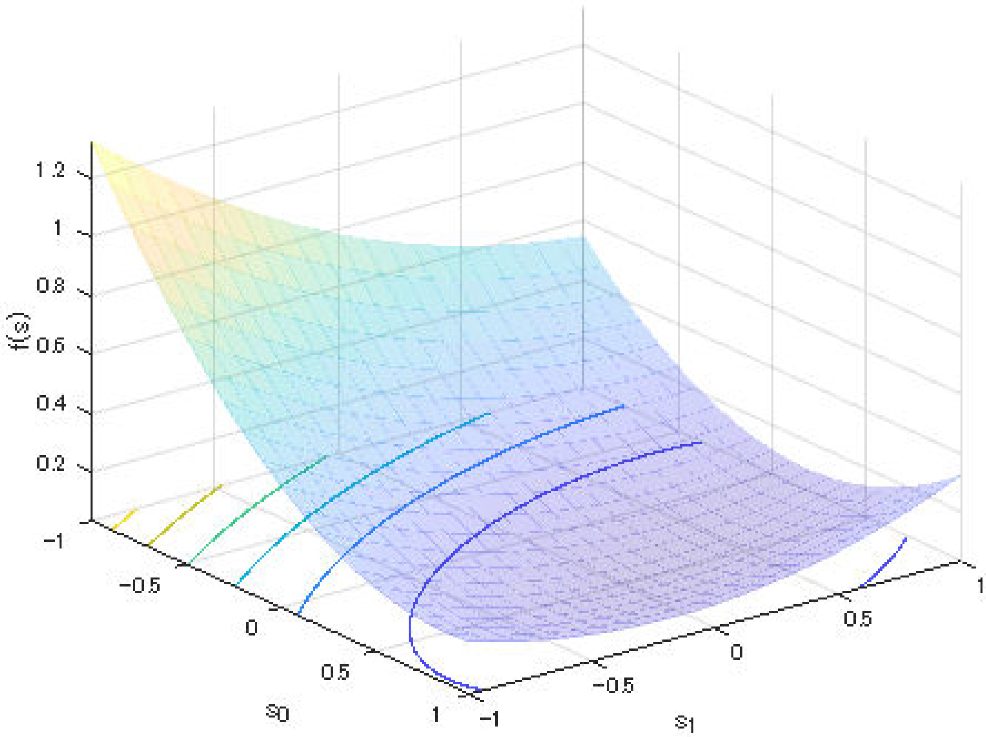
(Parameter settings)

```
lambda = 0.2;
gamma = 0.4;
niters = 20;
```

関数プロット

(Function plot)

```
% Function setting
f = @(x0,x1) 0.5*(v-(D(1)*x0+D(2)*x1)).^2 + lambda*0.5*(x0.^2+x1.^2);
% Variable settings
s0 = linspace(-1,1,21);
s1 = linspace(-1,1,21);
% Surfc plot of cost function f()
figure
[S0,S1] = ndgrid(s0,s1);
J = f(S0,S1);
hf = surfc(s0,s1,J);
hf(1).FaceAlpha = 0.25;
hf(1).EdgeAlpha = 0.25;
hf(1).EdgeColor = 'interp';
hf(2).LineWidth = 1;
set(gca,'YDir','reverse')
ylabel('s_0')
xlabel('s_1')
zlabel('f(s)')
hold on
```



勾配降下法

(Gradient descent)

1. Initialization: $\mathbf{x}^{(0)}$, $t \leftarrow 0$
2. Gradient descent: $\mathbf{x}^{(t+1)} \leftarrow \mathbf{x}^{(t)} - \gamma \nabla_{\mathbf{x}} f(\mathbf{x}^{(t)})$
3. If a stopping critera is satisfied then finish, otherwise $t \rightarrow t+1$ and go to Step 2.

【Example】

- $f(\mathbf{s}) = \frac{1}{2}\|\mathbf{v} - \mathbf{D}\mathbf{s}\|_2^2 + \frac{\lambda}{2}\|\mathbf{s}\|_2^2$
- $\nabla_{\mathbf{s}} f(\mathbf{s}) = \mathbf{D}^T(\mathbf{D}\mathbf{s} - \mathbf{v}) + \lambda\mathbf{s}$

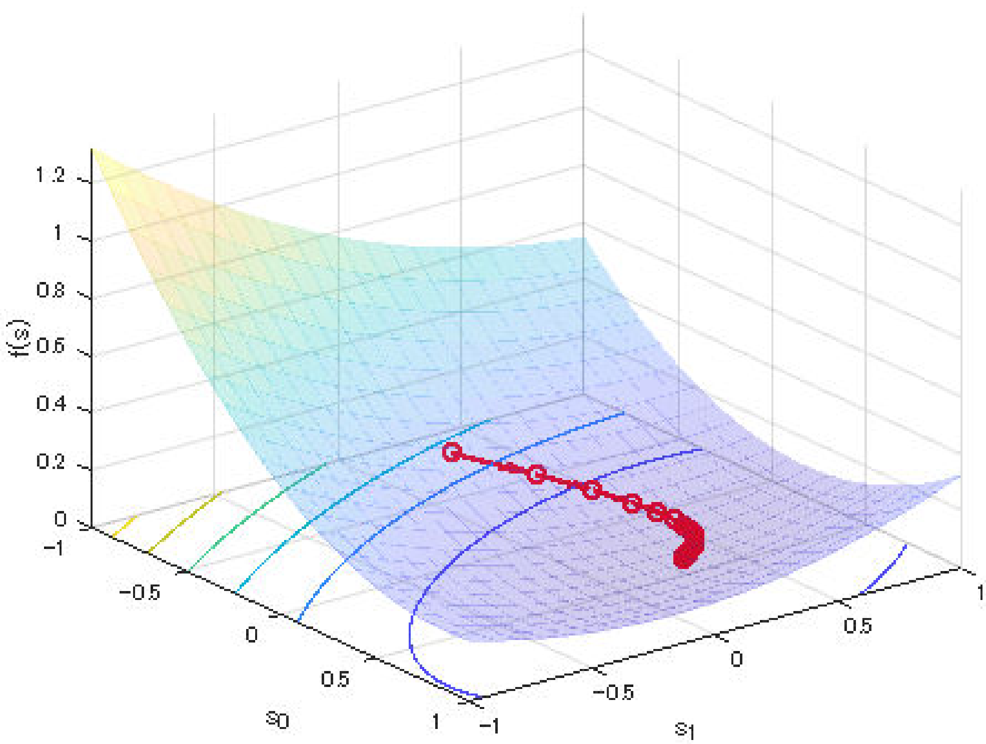
初期化 (Initialization)

```
sp = 2*rand(2,1)-1; % in [-1,1]^2
```

勾配降下 (Gradient descent)

```
for idx=0:niters-1
    % Preious state
    s(1,1) = sp(1); % s0
    s(2,1) = sp(2); % s1
```

```
    % Gradient descent
    sc = sp-gamma*(D'*(D*sp-v)+lambda*sp);
    % Current state
    s(1,2) = sc(1); % s0
    s(2,2) = sc(2); % s1
    % Quiver plot
    xp = s(2,1);
    yp = s(1,1);
    xn = s(2,2);
    yn = s(1,2);
    hp = quiver(xp,yp,xn-xp,yn-yp);
    hp.Marker = 'o';
    hp.ShowArrowHead = 'on';
    hp.MaxHeadSize = 120;
    hp.MarkerSize = 6;
    hp.MarkerEdgeColor = 'r';
    hp.Color = 'r';
    hp.LineWidth = 2;
    % Update
    sp = sc;
end
hold off
```



パラメータ設定

(Parameter settings)

- sgm: ノイズ標準偏差 σ_w (Standard deviation of noise)
- nlevels: ウェーブレット段数 (Wavelet levels)

```
% Parameter settings
isaprxleft = true;
lambda = 10^1
```

```
lambda = 10
```

```
gamma = 10^-1
```

```
gamma = 0.1000
```

```
sgmuint8 = 30;
sgm = sgmuint8/255;
nlevels = 3;
niters = 80;
```

画像の読込

(Read image)

```
u = im2double(imread('./data/kodim23.png'));
if size(u,3) == 3
    u = rgb2gray(u);
end
```

観測画像

(Observation image)

- $\mathbf{v} = \mathbf{u} + \mathbf{w}$
- $\mathbf{u} = \mathbf{D}\mathbf{s}$
- $\mathbf{s} \sim \mathrm{Norm}\left(\mathbf{s}|\boldsymbol{\mu} = \mathbf{0}, \sigma_s^2\mathbf{I}\right)$
- $\mathbf{w} \sim \mathrm{Norm}\left(\mathbf{w}|\boldsymbol{\mu}_w = \mathbf{0}, \sigma_w^2\mathbf{I}\right)$

```
v = imnoise(u,'gaussian',0,sgm^2);
```

非間引きハール DWT

(Undecimated Haar DWT)

```
import msip.udhaarwtdec2
import msip.udhaarwtrec2
```

完全再構成の確認 (Checki the perfect reconstruction)

非間引きハール DWT はパーセバルタイト性 (The undecimated DWT satisfies the Parseval tight property,)

$\mathbf{D}\mathbf{D}^T = \mathbf{I}$

を満たすため， $\mathbf{D}$ の転置システムは完全再構成分析システムとなり得る．(and thus Its transposition system can be a PR analysis system.)

```
[coefs,scales] = udhaarwtdec2(v,nlevels);
r = udhaarwtrec2(coefs,scales);
assert(norm(v-r,"fro")^2/numel(v)<1e-18,'Perfect reconstruction is violated.')
```

合成辞書と転置辞書の定義 (Definition of synthesis dictionary and its adjoint)

```
% Definiton of dictionay and its adjoint
adjdic = @(x) udhaarwtdec2(x,nlevels); % D
syndic = @(x) udhaarwtrec2(x,scales);  % D.'
```

勾配降下法

(Gradient descent method)

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- $\nabla_{\mathbf{s}} f(\mathbf{s}) = \mathbf{D}^T(\mathbf{D}\mathbf{s} - \mathbf{v}) + \lambda\mathbf{s}$

初期化 (Initialization)

```
sp = coefs;
```

勾配降下 (Gradient descent)

```
if isaprxleft
    mask = ones(size(coefs));
    mask(1:prod(scales(1,:))) = 0;
    lambda = lambda * mask;
end
for idx=0:niters-1
    % Gradient descent
    sc = sp-gamma*(adjdic(syndic(sp)-v)+lambda.*sp);
    % Update
    sp = sc;
end
```

ノイズ除去画像

(Denoised image)

```
r = syndic(sc);
```

画像表示

(Image show)

```
figure
imshow(u);
title('Original image u')
```

```
figure
imshow(v)
title(sprintf('Noisy image v : PSNR = %5.2f [dB]',psnr(u,v)))
```


```
figure
imshow(r)
title(sprintf('Denoised image r : PSNR = %5.2f [dB]',psnr(u,r)))
```

Denoised image r:PSNR = 27.66 [dB]