Sample 9-2

離散ウェーブレット変換

ブロック DCT のフィルタバンク実装

画像処理特論

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動作確認: MATLAB R2023a

Discrete wavelet transform

Filter bank implementation of block DCT

Advanced Topics in Image Processing

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Verified: MATLAB R2023a

準備

(Preparation)

```
close all
```

ブロックサイズの設定

(Settings of block size)

```
% # of channels
nChs = 4;
```

入力信号の生成

(Generation of input)

```
% Input signal
u = rand(1,3*nChs);
u = [0 u zeros(1,nChs-1)];
```

並列フィルタバンク実装

(Parallel filter bank implementation)

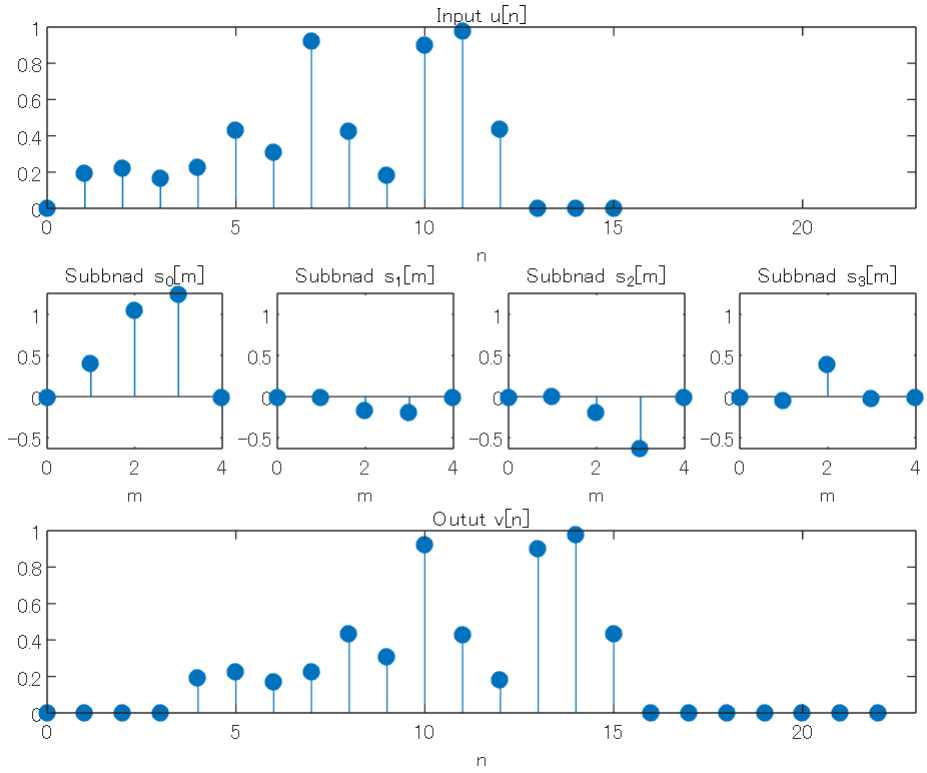
```
% DCT matrix
P =  dctmtx(nChs);

% IDCT matrix
Q = P.';
```

```
% Analysis process
s = cell(nChs,1);
for iCh = 1:nChs
    hk = fliplr(P(iCh,:));
    % Decimation
    s{iCh} = downsample(conv(hk,u),nChs);
end
s = cell2mat(s);

% Synthesis process
v = 0;
for iCh = 1:nChs
    fk = Q(:,iCh);
    % Interpolation
    vk = conv(fk,upsample(s(iCh,:),nChs));
    v = v + vk;
end
```

信号表示

(Signal display)

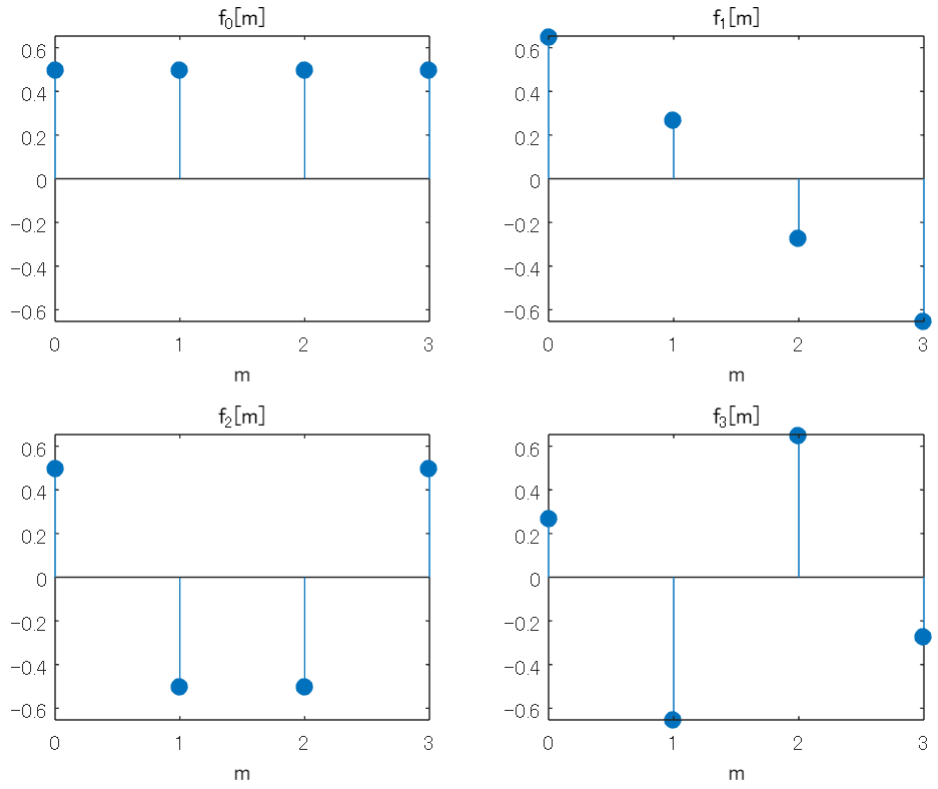
```
figure(1)
subplot(3,nChs,1:nChs)
stem(0:length(u)-1,u,'filled')
title('Input u[n]')
xlabel('n')
ax = gca;
ax.XLim =[ 0 length(v)];

for iCh = 1:nChs
    subplot(3,nChs,nChs+iCh)
    stem(0:size(s,2)-1,s(iCh,:),'filled')
    title(['Subbnad s_' num2str(iCh-1) '[m]'])
    xlabel('m')
    ax = gca;
    ax.YLim =[ min(s(:)) max(s(:)) ];
end

subplot(3,nChs,2*nChs+(1:nChs))
stem(0:length(v)-1,v,'filled')
title('Outut v[n]')
xlabel('n')
ax = gca;
ax.XLim =[ 0 length(v)];
```



合成フィルタのインパルス応答（基底ベクトル）

(Impluse responses of synthesis filters; basis vectors)

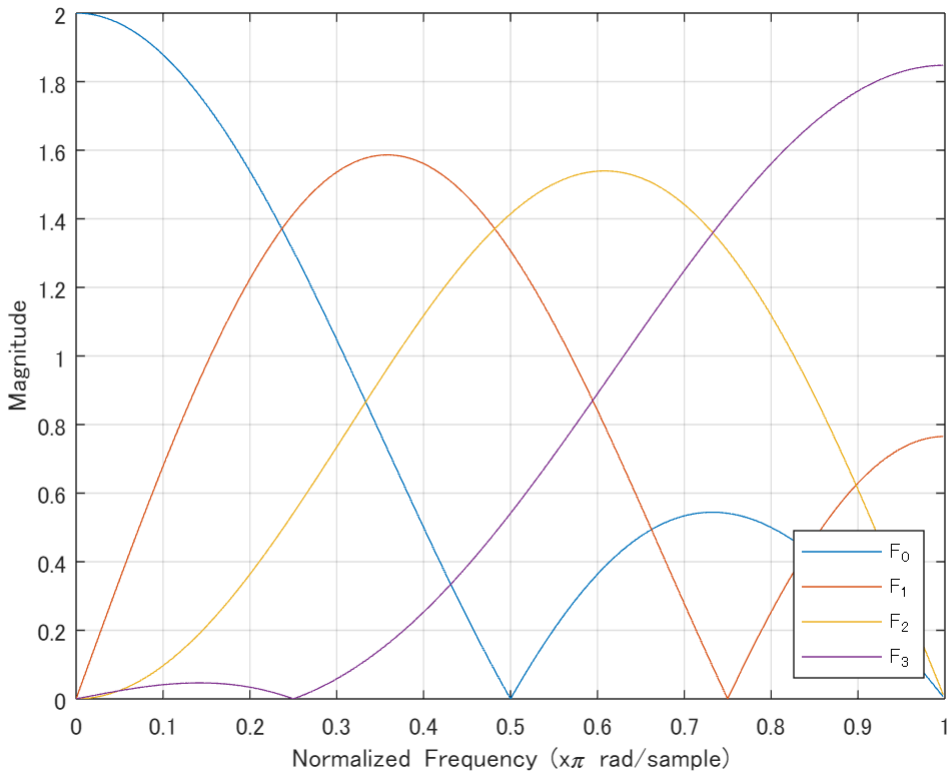
```
figure(2)
for iCh = 1:nChs
    fk = Q(:,iCh);
    subplot(nChs/2,2,iCh)
    stem(0:length(fk)-1,fk,'filled')
    title([ 'f_' num2str(iCh-1) '[m]' ])
    xlabel('m')
    ax = gca;
    ax.YLim =[ min(Q(:)) max(Q(:)) ];
end
```



合成フィルタの周波数応答（基底ベクトル）

(Frequency responses of synthesis filters; basis vectors)

```
figure(3)
fftPoints = 512;
F = zeros(fftPoints,nChs);
legends = cell(nChs,1);
for iCh = 1:nChs
    fk = Q(:,iCh);
    if iCh == 1
        [F(:,iCh),W] = freqz(fk,1,fftPoints);
    else
        F(:,iCh) = freqz(fk,1,fftPoints);
    end
    legends{iCh} = [ 'F_' num2str(iCh-1) ];
end
plot(W/pi,abs(F)) %20*log10(abs(F)))
axis([0 1 0 ceil(sqrt(nChs))]) %-70 10])
xlabel('Normalized Frequency (x\pi rad/sample)')
ylabel('Magnitude') % (dB)')
legend(legends,'Location','southeast')
grid on
```



ポリフェーズ行列実装

(Polyphase matrix implemenation)

分析合成処理 (Analysis and synthesis process)

$$\mathbf{E}(z) = \begin{pmatrix} h_0[0] & h_0[1] & \cdots & h_0[M-1] \\ h_1[0] & h_1[1] & \cdots & h_1[M-1] \\ \vdots & \vdots & \ddots & \vdots \\ h_{M-1}[0] & h_{M-1}[1] & \cdots & h_{M-1}[M-1] \end{pmatrix}$$

$$\mathbf{R}(z) = \begin{pmatrix} f_0[M-1] & f_1[M-1] & \cdots & f_{M-1}[M-1] \\ f_0[M-2] & f_1[M-2] & \cdots & f_{M-1}[M-2] \\ \vdots & \vdots & \ddots & \vdots \\ f_0[0] & f_1[0] & \cdots & f_{M-1}[0] \end{pmatrix}$$

```
% Input Signal
u = [zeros(1,nChs-1) u 0 ]; % Adjust delay for downsampling
disp(u)
```

```
         0         0         0         0    0.1948    0.2259    0.1707    0.2277    0.4357    0.3111    0.9234    0
```

```
% Serial/Parallel conversion
```

```
x = cell(nChs,1);
idx = 1;
for phase = 0:nChs-1
    x{idx} = downsample(u,nChs,mod(nChs-1-phase,nChs));
    idx = idx + 1;
end
x = cell2mat(x);
disp(x)
```

```
         0    0.2277    0.4302    0.4389         0
         0    0.1707    0.9234    0.9797         0
         0    0.2259    0.3111    0.9049         0
         0    0.1948    0.4357    0.1848         0
```

```
% Analysis process w/ the polyphase matrix
E = fliplr(P); % E = PJ
s = E*x;
disp(s)
```

```
         0    0.4095    1.0502    1.2542         0
         0   -0.0066   -0.1621   -0.1862         0
         0    0.0129   -0.1843   -0.6305         0
         0   -0.0450    0.4015   -0.0198         0
```

```
% Synthesis process w/ the polyphase matrix
R = flipud(Q); % R = JQ
y = R*s;
disp(y)
```

```
         0    0.2277    0.4302    0.4389         0
         0    0.1707    0.9234    0.9797         0
         0    0.2259    0.3111    0.9049         0
         0    0.1948    0.4357    0.1848         0
```

```
% Parallel/Serial conversion
v = 0;
idx = 1;
for phase = nChs-1:-1:0
    v = v + upsample(y(idx,:),nChs,phase);
    idx = idx + 1;
end
disp(v)
```

```
         0         0         0         0    0.1948    0.2259    0.1707    0.2277    0.4357    0.3111    0.9234    0
```