Sample 8-6

離散コサイン変換

KLT との関係

画像処理特論

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動作確認: MATLAB R2020a

Discrete cosine transform

Relation to KLT

Advanced Topics in Image Processing

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Verified: MATLAB R2020a

準備

(Preparation)

```
close all
```

次元 M の設定

(Setting of dimension M)

```
nPoints = 4;
```

原画像の読込と表示

(Read and display an image)

```
V = rgb2gray(imread('data/barbaraFaceRgb.tif'));
figure(1)
imshow(V)
title('Original picture')
```

Original picture

M 次元ベクトル集合の抽出

(Extraction of a set of M-D vectors)

M 点の水平方向に連続する画素値をベクトルとして抽出。(Extracts the values of successive horizontal pixels as an M-D vector.)

```
nPixels = numel(V);
setOfX = reshape(V.', nPoints, nPixels/nPoints);
```

変換前の散布図

(Scatter plot before transform)

標本分散共分散行列と相関係数 (Sample covariance matrix and correlation coefficient)

```
Sxx = cov(double(setOfX.'))
```

```
Sxx = 4×4
10^3 ×
    2.2547    1.8973    1.7039    1.8259
    1.8973    2.2303    1.8933    1.6928
    1.7039    1.8933    2.2357    1.8829
    1.8259    1.6928    1.8829    2.2109
```

```
Rxx = corrcoef(double(setOfX.'))
```

```
Rxx = 4×4
    1.0000    0.8461    0.7589    0.8178
    0.8461    1.0000    0.8479    0.7623
    0.7589    0.8479    1.0000    0.8469
    0.8178    0.7623    0.8469    1.0000
```

カルーネンレーベ(K-L)変換

(Karhunen-Loève transform)

分散共分散行列の固有値分解(Eigenvalue decomposition of the variance-covariance matrix)

- $\mathbf{\Sigma}_{\mathrm{XX}} = \mathbf{\Phi}\mathbf{\Lambda}\mathbf{\Phi}^T$

```
[Phi,Lambda] = eig(Sxx);
```

固有値のソート (Sorting the eigen valuess)

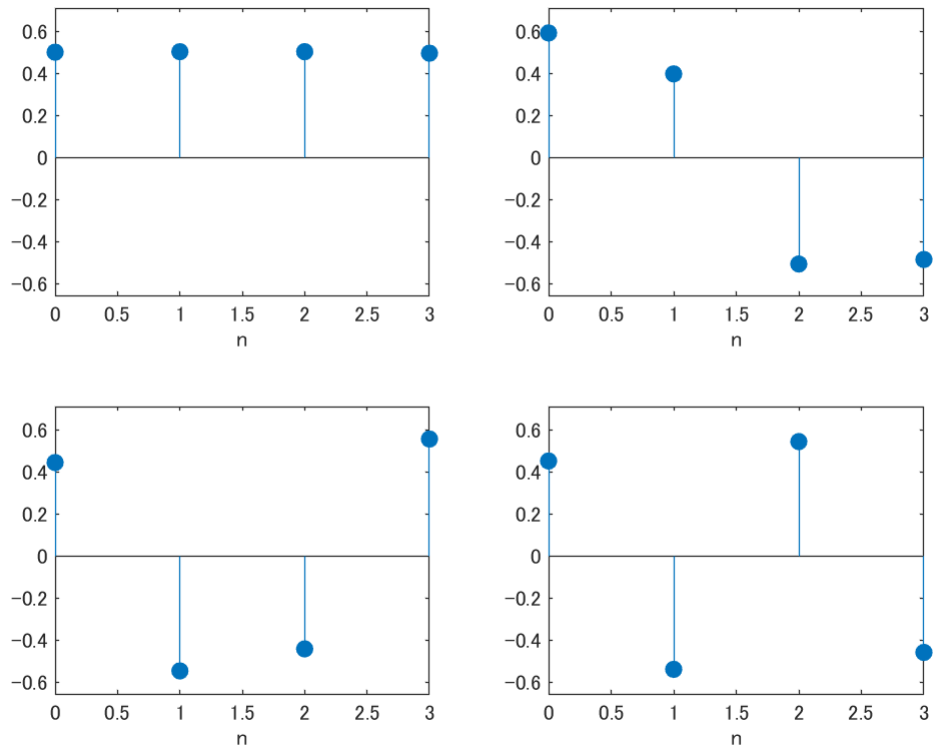
```
[~,I] = sort(diag(Lambda));
```

固有ベクトルを並び換え (Reordering eigenvectors)

```
Phi = Phi(:,nPoints-I+1);
```

基底ベクトルの表示 (Display the basis vectors)

```
figure(2)
for idx = 1:nPoints
    subplot(ceil(nPoints/2),2,idx);
    stem(0:nPoints-1,Phi(:,idx),'filled');
    ax = gca;
    ax.YLim = 1.2*[min(Phi(:)) max(Phi(:))];
    xlabel('n')
end
```



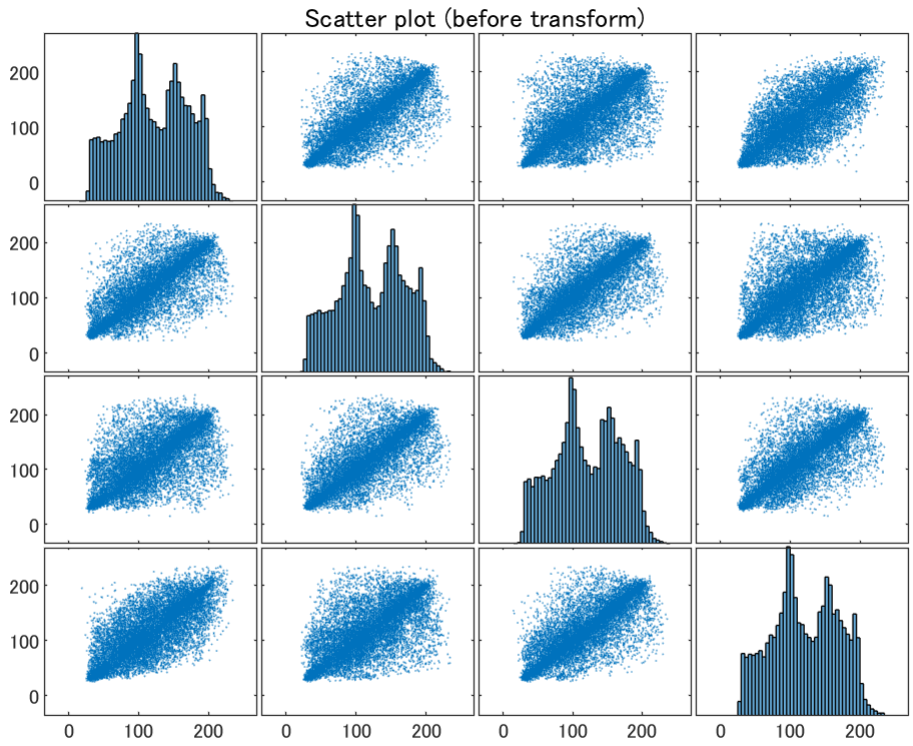
K-L 変換 (K-L transform)

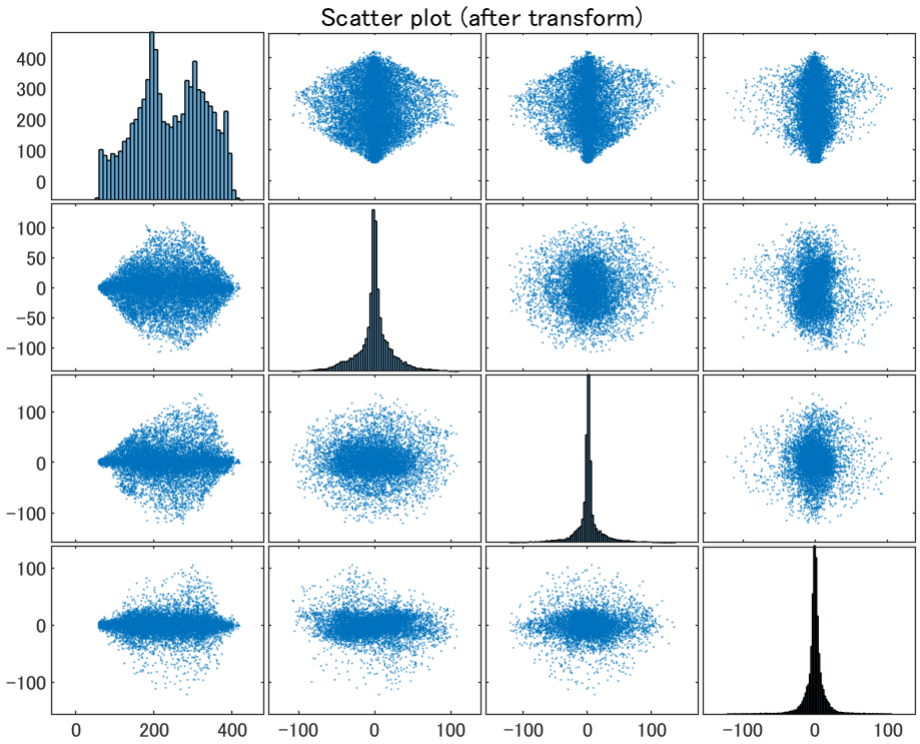
```
T = Phi.';
setOfY = T * double(setOfX);
```

変換前後の散布図

(Scatter plots before and after transform)

```
if nPoints == 2
    figure(3)
    scatter(setOfX(1,:),setOfX(2,:),'.')
    axis square
    xlabel('x_0')
    ylabel('x_1')
    title('Scatter plot (before transform)')
    figure(4)
    scatter(setOfY(1,:),setOfY(2,:),'.')
    axis square
    xlabel('y_0')
    ylabel('y_1')
    title('Scatter plot (after transform)')
else
    figure(3)
    plotmatrix(setOfX.','.')
    title('Scatter plot (before transform)')
    figure(4)
    plotmatrix(setOfY.','.')
    title('Scatter plot (after transform)')
end
```





標本分散共分散行列と相関係数 (Sample covariance matrix and correlation coefficient)

```
Syy = cov(double(setOfY.'))
```

```
Syy = 4×4
10^3 ×
    7.6812    0.0000   -0.0000    0.0000
    0.0000    0.5687   -0.0000    0.0000
   -0.0000   -0.0000    0.5031   -0.0000
    0.0000    0.0000   -0.0000    0.1786
```

```
Ryy = corrcoef(double(setOfY.'))
```

```
Ryy = 4×4
    1.0000    0.0000   -0.0000    0.0000
    0.0000    1.0000   -0.0000    0.0000
   -0.0000   -0.0000    1.0000   -0.0000
    0.0000    0.0000   -0.0000    1.0000
```

変換後の分散共分散行列と相関係数の非対角成分が 0 となり、無相関となる。(The non-diagonal components of the sample covariance matrix and correlation coefficient after the transform is zero. That is, the coefficients become uncorrelated.)

AR(1)モデルの KLT

(KLT of AR(1) model)

- AR(1): the 1-st order autoregressive model

```
% Correlation coefficient |ρ|<1
rho = 0.999;

% Covariance matrix
sigma = 1;
Sxx = sigma^2*toeplitz(power(rho,0:nPoints-1))
```

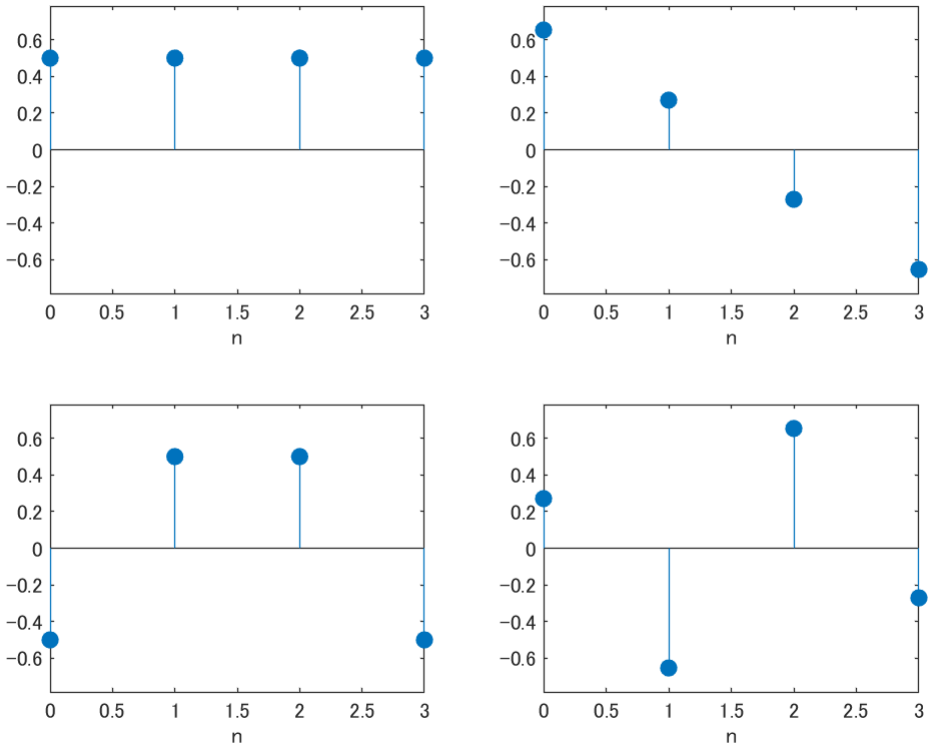
```
Sxx = 4×4
    1.0000    0.9990    0.9980    0.9970
    0.9990    1.0000    0.9990    0.9980
    0.9980    0.9990    1.0000    0.9990
    0.9970    0.9980    0.9990    1.0000
```

カルーネンレーベ(K-L)変換 (Karuhen Loeve transform)

```
[Phi,Lambda] = eig(Sxx);
[~,I] = sort(diag(Lambda));
Phi = Phi(:,nPoints-I+1);
```

基底ベクトルの表示 (Display the basis vectors)

```
figure(5)
for idx = 1:nPoints
    subplot(ceil(nPoints/2),2,idx);
    stem(0:nPoints-1,Phi(:,idx),'filled');
    ax = gca;
    ax.YLim = 1.2*[min(Phi(:)) max(Phi(:))];
    xlabel('n')
end
```



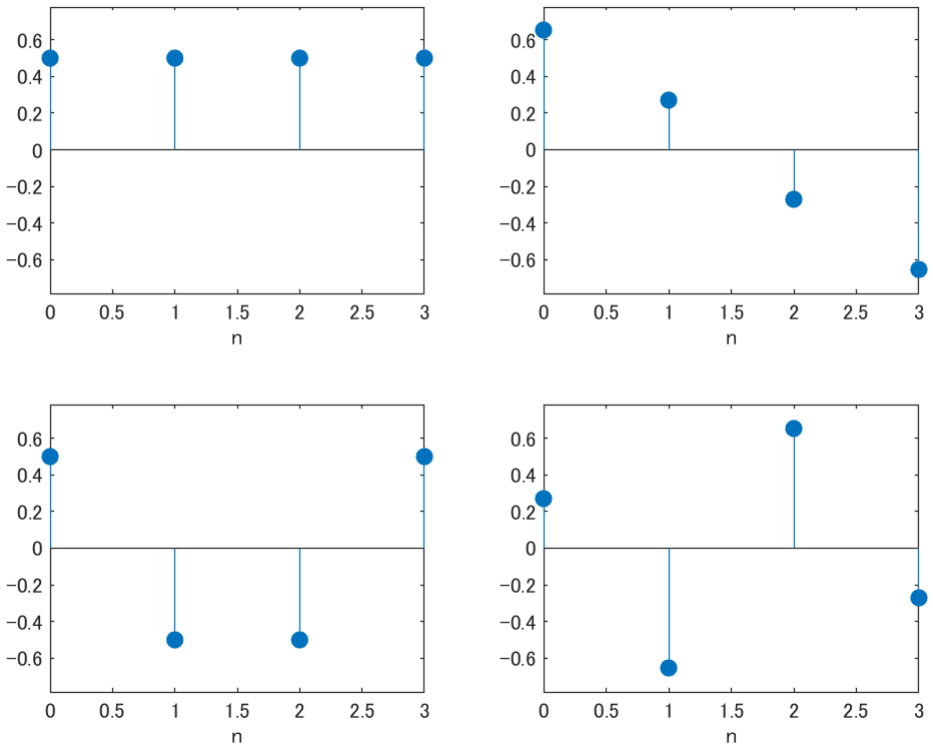
DCT 行列

(DCT matrix)

```
C = dctmtx(nPoints);
B = C.';
```

基底ベクトルの表示 (Display the basis vectors)

```
figure(6)
for idx = 1:nPoints
    subplot(ceil(nPoints/2),2,idx);
    stem(0:nPoints-1,B(:,idx),'filled');
    ax = gca;
    ax.YLim = 1.2*[min(B(:)) max(B(:))];
    xlabel('n')
end
```



相関係数$\rho \to 1$の AR(1)モデルに対する KLT 行列は極限で DCT 行列に収束する。符号の反転は無視してよい。
(The KLT matrix for the AR(1) model with correlation coefficient $\rho \to 1$converges to the DCT matrix in the limit. Flipping in signs can be ignored.)