Sample 12-6

画像復元

交互方向乗数法 (ADMM)

画像処理特論

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動作確認: MATLAB R2023a

Image restoration

Alternating Direction Method of Multipliers (ADMM)

Advanced Topics in Image Processing

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Verified: MATLAB R2023a

準備

(Preparation)

```
clear
close all
```

パラメータ設定

(Parameter settings)

```
lambda = 10^-2;
rho = 1;
niters = 100;
```

画像の読込

(Read image)

```
T = 0.3*im2double(imread('text.png'));
B = 0.7*imresize(checkerboard,size(T));
```

観測画像

(Observation image)

```
M = T + B;
figure(1)
imshow(M)
```


```
imwrite(M,'admmm.png')
```

交互方向乗数法

(Alternating direction method of multipliers)

問題設定 (Problem settings):

$$\min_{\mathbf{u},\mathbf{y}} f(\mathbf{u}) + g(\mathbf{y}), \quad \mathbf{y} = \mathbf{G}\mathbf{u}$$

- $f(\cdot), g(\cdot)$: Convex functions
- $\mathbf{G}$: Full rank matrix

アルゴリズム (Algorithm):

1. Initialization: $\mathbf{d}^{(0)}, \mathbf{y}^{(0)}$, $t \leftarrow 0$
2. $\mathbf{u}^{(t+1)} \leftarrow \arg\min_{\mathbf{x}} f(\mathbf{x}) + \frac{\rho}{2}\|\mathbf{y}^{(t)} - \mathbf{G}\mathbf{x} - \mathbf{d}^{(t)}\|_2^2$
3. $\mathbf{y}^{(t+1)} \leftarrow \mathrm{prox}_{\rho^{-1}g}(\mathbf{G}\mathbf{u}^{(t+1)} + \mathbf{d}^{(t)})$
4. $\mathbf{d}^{(t+1)} \leftarrow \mathbf{d}^{(t)} + \mathbf{G}\mathbf{u}^{(t+1)} - \mathbf{y}^{(t+1)}$
5. If a stopping critera is satisfied then finish, otherwise $t \rightarrow t+1$ and go to Step 2.

ただし，(where)

$\rho > 0$

【Example】Robust principal component analysis (Robust PCA)

$\mathbf{M} = \mathbf{L} + \mathbf{S}$

問題設定(Problem settings)

$$\min_{\mathbf{L},\mathbf{S}} \|\mathbf{L}\|_* + \lambda\|\mathbf{S}\|_1, \qquad \text{s.t.} \quad \mathbf{M} = \mathbf{L} + \mathbf{S}$$

$\Updownarrow$

$$\min_{\mathbf{u},\mathbf{y}} f(\mathbf{u}) + g(\mathbf{y}), \quad \mathbf{y} = \mathbf{G}\mathbf{u}$$

ただし，(where)

- $\|\cdot\|_*$: Nuclear norm (sum of singular values $\{\sigma_k\}$)
- $\|\cdot\|_1$: ℓ_1-norm as a vector (sum of absolute of elements)
- $f(\mathbf{u}) = 0$
- $g(\mathbf{y}) = \|\boldsymbol{\ell}\|_* + \|\mathbf{s}\|_1 + \iota_{\{\mathbf{m}\}}(\boldsymbol{\ell} + \mathbf{s})$
- $\mathbf{y} = \begin{pmatrix} \boldsymbol{\ell} \\ \mathbf{s} \\ \boldsymbol{\ell} + \mathbf{s} \end{pmatrix}, \mathbf{u} = \begin{pmatrix} \boldsymbol{\ell} \\ \mathbf{s} \end{pmatrix}, \mathbf{G} = \begin{pmatrix} \mathbf{I} & \mathbf{O} \\ \mathbf{O} & \mathbf{I} \\ \mathbf{I} & \mathbf{I} \end{pmatrix}$
- $\boldsymbol{\ell} := \mathrm{vec}(\mathbf{L})$, $\mathbf{s} := \mathrm{vec}(\mathbf{S})$

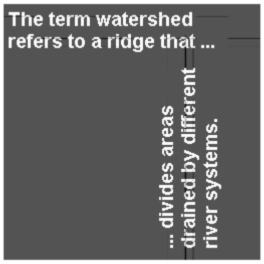
アルゴリズム (Algorithm)

```
% Step1: Initialization
imgY1 = M;
imgY2 = zeros(size(M),'like',M);
imgY3 = M;
imgD1 = zeros(size(M),'like',M);
imgD2 = zeros(size(M),'like',M);
imgD3 = zeros(size(M),'like',M);

for idx=1:niters
    % Set state variables
    Y1 = imgY1;
    Y2 = imgY2;
    Y3 = imgY3;
    D1 = imgD1;
    D2 = imgD2;
    D3 = imgD3;

    % Step 2: Update L S
    L = (2*(Y1-D1) - (Y2-D2) + (Y3-D3))/3;
    S = (Y1-D1) + (Y3-D3) - 2*L;

    % Step 3-1: Update Y1
    Y1 = proxnuclearnorm(L + D1, 1/rho);
    % Step 3-2: Update Y2
    Y2 = softthresholding(S + D2, lambda/rho);
    % Step 3-3: Update Y3
    Y3 = M;

    % Step 4: Update dual variables
    D1 = D1 + L - Y1;
    D2 = D2 + S - Y2;
    D3 = D3 + L + S - Y3;
```

```
    % Update state variables
    imgY1 = Y1;
    imgY2 = Y2;
    imgY3 = Y3;
    imgD1 = D1;
    imgD2 = D2;
    imgD3 = D3;
end
```

```
figure(2)
imshow(L)
```

```
figure(3)
imshow(S,[])
```




```
function y = softthresholding(x,lambda)
    y = sign(x).*max(abs(x)-lambda,0);
end
```

```
function y = proxnuclearnorm(x,lambda)
    [U,S,V] = svd(x,'econ');
```

```
    T = softthresholding(S,lambda);
    S(1:size(T,1),1:size(T,2)) = T;
    y = U*S*V';
end
```