Sample 13-1

辞書学習

カルーネン-レーベ変換（主成分分析）

画像処理特論

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動作確認: MATLAB R2020a

Dictionary learning

Karhunen–Loève transform (principle component analysis)

Advanced Topics in Image Processing

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Verified: MATLAB R2020a

準備

(Preparation)

```
clear
close all
import msip.download_img
msip.download_img
```

```
lena.png already exists in ./data/
baboon.png already exists in ./data/
goldhill.png already exists in ./data/
barbara.png already exists in ./data/
```

パラメータ設定

(Parameter settings)

- ブロックサイズ (Block size)

```
szBlk = [ 8 8 ];
```

画像の読込

(Read image)

- $\mathbf{u} \in \mathbb{R}^N$

```
u = rgb2gray(im2double(imread('./data/lena.png')));
meanu = mean(u(:));
```

カルーネン-レーベ変換（主成分分析）

(Karhunen–Loève transform (principle component analysis))

問題設定 (Problem setting):

$$\widehat{\mathbf{\Phi}} = \arg \max_{\mathbf{\Phi} \in \mathbb{R}^{M \times M}} \mathrm{tr}\left(\mathbf{\Phi}_{:,1:p}^T \widehat{\mathbf{\Sigma}}_y \mathbf{\Phi}_{:,1:p}\right), \mathrm{s.t.} \mathbf{\Phi}^T \mathbf{\Phi} = \mathbf{I}_M, \forall p \in \{1, 2, \cdots, M\}$$

ただし，$\widehat{\mathbf{\Sigma}}_y$は 観測ベクトル $\{\mathbf{y}_n\}_n$ （零平均を仮定）の標本分散共分散行列 (where, $\widehat{\mathbf{\Sigma}}_y$ is the sample covariance matrix of the observation vectors $\{\mathbf{y}_n\}_n$ (assumed to have zero mean:)

$$\widehat{\mathbf{\Sigma}}_y = \frac{1}{S-1} \sum_{n=1}^{S} \mathbf{y}_n \mathbf{y}_n^T$$

解 (Solution):

固有値分解 (Eigendecomposition)

$$\widehat{\mathbf{\Phi}}^T \widehat{\mathbf{\Sigma}}_y \widehat{\mathbf{\Phi}} = \mathbf{\Lambda}$$

ただし，$\mathbf{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_M)$．$\lambda_1 \geq \lambda_2 \geq \cdots \lambda_M$ は $\widehat{\mathbf{\Sigma}}_y$の固有値．(where, $\mathbf{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_M)$．$\lambda_1 \geq \lambda_2 \geq \cdots \lambda_M$ are the eigenvalues of $\widehat{\mathbf{\Sigma}}_y$.)

画像 $\mathbf{u}$からのデータ行列 $\mathbf{Y}$ の生成 (Generation of data matrix $\mathbf{Y}$ of image $\mathbf{u}$)

```
Y = im2col(u,szBlk,'distinct');
Y = Y - mean(Y,2);
```

標本分散共分散行列 $\widehat{\mathbf{\Sigma}}_y$の計算 (Calculation of sample covariance matrix $\widehat{\mathbf{\Sigma}}_y$)

```
SigmaY = cov(Y.');
```

標本分散共分散行列 $\widehat{\mathbf{\Sigma}}_y$の固有値分解 (Eigendecomposition of sample covariance matrix $\widehat{\mathbf{\Sigma}}_y$)

```
[Phi,Lambda] = eig(SigmaY);
```

固有値 λ の大きさの降順に列ベクトルをソート (Sorting column vectors in the descending order of the eigenvalues λ)

```
[~,idx] = sort(diag(Lambda),'descend');
Phi = Phi(:,idx);
```

固有ベクトルを基底画像に変換 (Reshape the eigenvectures into basis images)

```
nBases = prod(szBlk);
basisImages = zeros(szBlk(1),szBlk(2),nBases);
for iBasis = 1:nBases
    basisImages(:,:,iBasis) = reshape(Phi(:,iBasis),szBlk(1),szBlk(2));
```

```
end
```

画像表示

(Image show)

```
figure(1)
imshow(u);
title('Original image u')
```


```
figure(2)
montage(imresize(basisImages,8,'nearest')+.5,'BorderSize',[2 2])
title('Basis images of KLT (PCA)')
```