Sample 12-1

画像復元

ウィーナーフィルタ

画像処理特論

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動作確認: MATLAB R2023a

Image restoration

Wiener filter

Advanced Topics in Image Processing

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Verified: MATLAB R2023a

準備

(Preparation)

```
clear
close all
import msip.download_img
msip.download_img
```

```
kodim01.png already exists in ./data/
kodim02.png already exists in ./data/
kodim03.png already exists in ./data/
kodim04.png already exists in ./data/
kodim05.png already exists in ./data/
kodim06.png already exists in ./data/
kodim07.png already exists in ./data/
kodim08.png already exists in ./data/
kodim09.png already exists in ./data/
kodim10.png already exists in ./data/
kodim11.png already exists in ./data/
kodim12.png already exists in ./data/
kodim13.png already exists in ./data/
kodim14.png already exists in ./data/
kodim15.png already exists in ./data/
kodim16.png already exists in ./data/
kodim17.png already exists in ./data/
kodim18.png already exists in ./data/
kodim19.png already exists in ./data/
kodim20.png already exists in ./data/
kodim21.png already exists in ./data/
kodim22.png already exists in ./data/
kodim23.png already exists in ./data/
kodim24.png already exists in ./data/
See Kodak Lossless True Color Image Suite
```

パラメータ設定

(Parameter settings)

- sgm: ノイズ標準偏差 σ_w (Standard deviation of noise)

```
sgmuint8 = 10;
sgmw = sgmuint8/255;
```

画像の読込

(Read image)

```
u = rgb2gray(im2double(imread('./data/kodim23.png')));
sgmu = std(u(:));
meanu = mean(u(:));
```

観測画像

(Observation image)

カメラの動きによって生じるボケ画像を生成．(Generate a blurred image that might result from camera motion.)

- $\mathbf{v} = \mathbf{P}\mathbf{u} + \mathbf{w}$
- $\mathbf{w} \sim \mathrm{Norm}\left(\mathbf{w}|\boldsymbol{\mu}_w = \mathbf{0}, \sigma_w^2\mathbf{I}\right)$

```
% Definition of measurment process
psf = fspecial('motion',21,11);
measureproc = @(x) imfilter(x,psf,'conv','circular');
% Simulation of AWGN
v = imnoise(measureproc(u),'gaussian',0,sgmw^2);
```

ウィーナーフィルタ

(Wiener filter)

画像 $\mathbf{u}$に対する仮定 (Assumptions on the original image $\mathbf{u}$)

- $\mathbf{u} \sim \mathrm{Norm}\left(\mathbf{u}|\boldsymbol{\mu}_u = \mathbf{0}, \sigma_u^2\mathbf{I}\right)$

問題設定 (Problem settings):

$$\hat{\mathbf{u}} = \arg\min_{\mathbf{u}} \frac{1}{2\sigma_w^2}\|\mathbf{v} - \mathbf{P}\mathbf{u}\|_2^2 + \frac{1}{2\sigma_u^2}\|\mathbf{u}\|_2^2$$

解 (Solution):

$$\hat{\mathbf{u}} = \left(\mathbf{P}^T\mathbf{P} + \frac{\sigma_w^2}{\sigma_u^2}\mathbf{I} \right)^{-1} \mathbf{P}^T\mathbf{v} \overset{\text{DFT}}{\longleftrightarrow} \widehat{\mathrm{U}}[\mathbf{k}] = \frac{\overline{P[\mathbf{k}]}}{|P[\mathbf{k}]|^2 + \sigma_w^2/\sigma_u^2}\mathrm{V}[\mathbf{k}]$$

ただし，循環畳み込み行列 $\mathbf{P}$ に対して，(where, for the circular convolution matrix $\mathbf{P}$,)

$$\mathbf{P}^T \overset{\text{DFT}}{\longleftrightarrow} \overline{P[\mathbf{k}]}$$

$$\mathbf{P}^T\mathbf{P} \overset{\text{DFT}}{\longleftrightarrow} \overline{P[\mathbf{k}]}P[\mathbf{k}] = |P[\mathbf{k}]|^2$$

```
% Determine the DFT points
nPoints = 2.^nextpow2(size(u))
```

```
nPoints = 1×2
         512        1024
```

```
% Ratio of variances between noise w and signal u (ideal estimation)
nsr = sgmw^2/sgmu^2;
% DFT(OTF) of PSF, where the phase response is adjusted to zero
P = psf2otf(psf,nPoints);
% DFT of observation v
V = fftn(v,nPoints);
% DFT of Wiener filter r
R = conj(P)./(abs(P).^2+nsr);
% IDFT of filtered spectrum
y0 = ifftn(R.*V);
y0 = y0(1:size(u,1),1:size(u,2));
```

逆フィルタ

(Inverse filter)

$$\check{\mathbf{u}} = (\mathbf{P}^T\mathbf{P})^{-1}\mathbf{P}^T\mathbf{v} = \mathbf{P}^{-1}\mathbf{v} \overset{\text{DFT}}{\longleftrightarrow} \check{\mathrm{U}}[\mathbf{k}] = \frac{\overline{P[\mathbf{k}]}}{|P[\mathbf{k}]|^2}\mathrm{V}[\mathbf{k}] = \frac{1}{P[\mathbf{k}]}\mathrm{V}[\mathbf{k}]$$

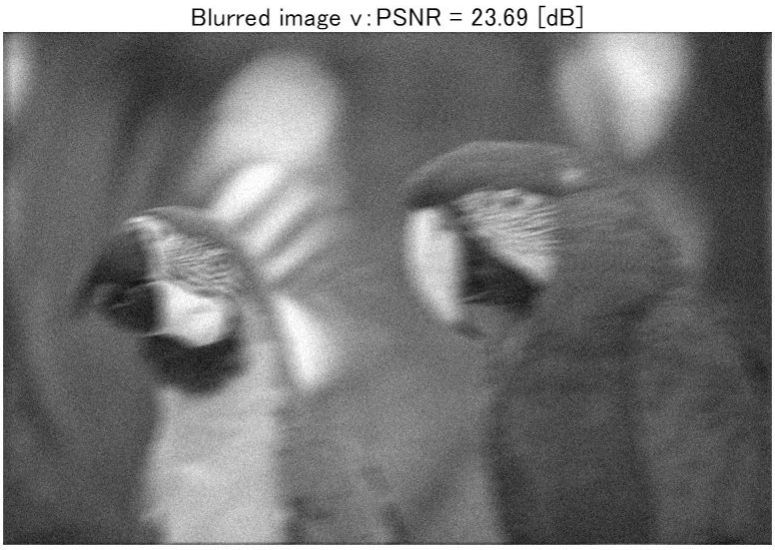
```
% IDFT of filtered spectrum
y1 = ifftn(V./P);
y1 = y1(1:size(u,1),1:size(u,2));
```

画像表示

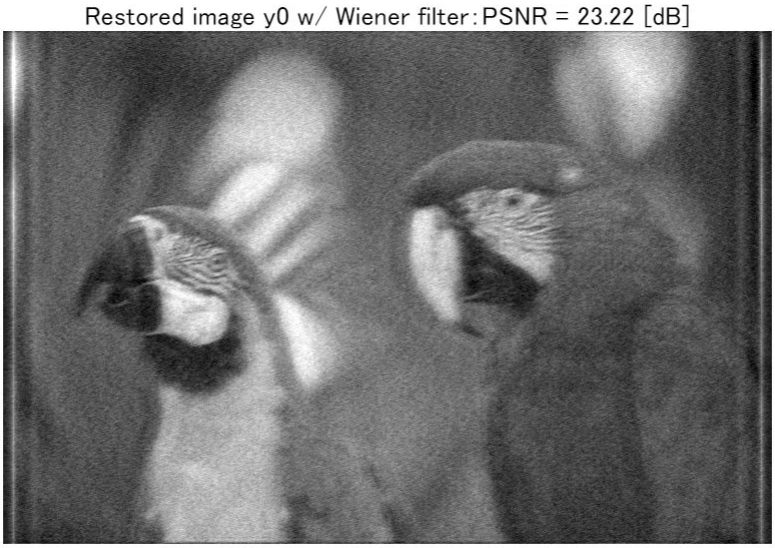
(Image show)

```
figure(1)
imshow(u);
title('Original image u')
```

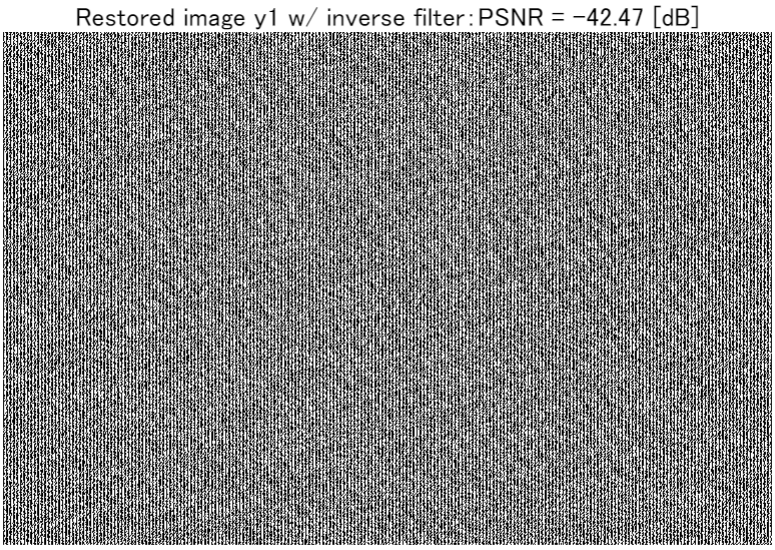

```
figure(2)
imshow(v)
title(sprintf('Blurred image v : PSNR = %5.2f [dB]',psnr(u,v)))
```




```
figure(3)
imshow(y0)
title(sprintf('Restored image y0 w/ Wiener filter : PSNR = %5.2f [dB]',psnr(u,y0)))
```




```
figure(4)
imshow(y1)
title(sprintf('Restored image y1 w/ inverse filter : PSNR = %5.2f [dB]',psnr(u,y1)))
```



ウィーナーフィルタ関数

(Wiener filter function)

- DECONVWNR

```
y2 = deconvwnr(v,psf,nsr);
figure(5)
```

```
imshow(y2)
title(sprintf('Restored image y2 w/ DECONVWNR : PSNR = %5.2f [dB]',psnr(u,y2)))
```