Sample 11-3

画像ノイズ除去

正規方程式

画像処理特論

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動作確認: MATLAB R2020a

Image denoising

Normal equation

Advanced Topics in Image Processing

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Verified: MATLAB R2020a

準備

(Preparation)

```
close all
```

問題設定

(Problem setting)

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}} \frac{1}{2}\|\mathbf{v} - \mathbf{D}\mathbf{s}\|_2^2 + \frac{\lambda}{2}\|\mathbf{s}\|_2^2$$

- $\mathbf{D} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}: \quad \mathbb{R}^2 \to \mathbb{R}^1$
- $\mathbf{v} = \frac{1}{2} \in \mathbb{R}^1$
- $\lambda \in [0, \infty)$
- $\mathbf{s} \in \mathbb{R}^2$

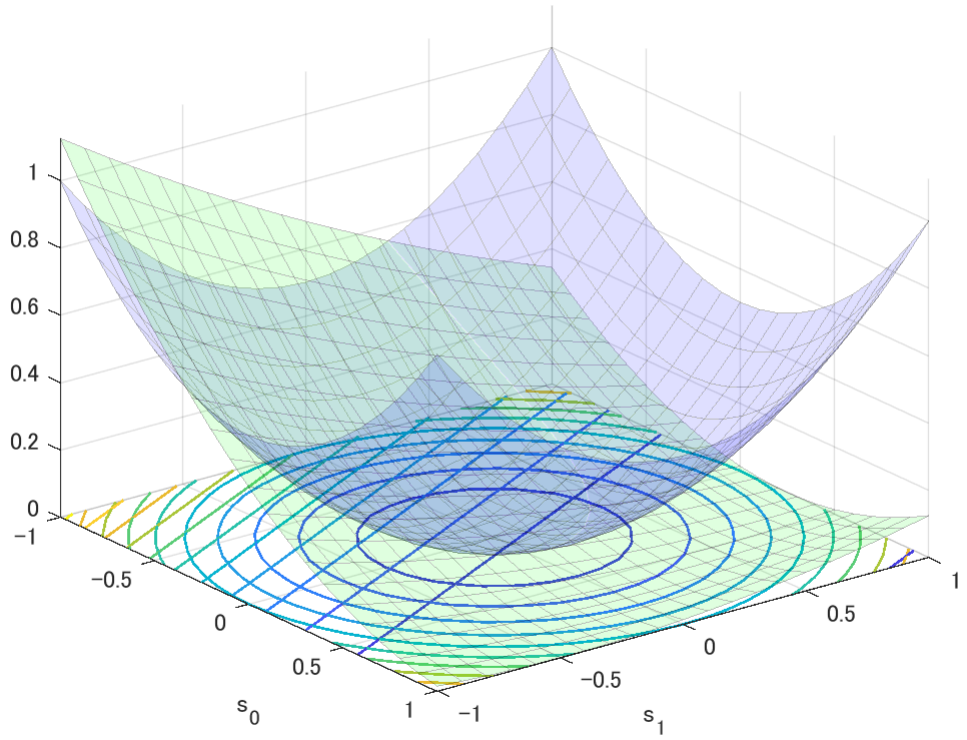
```
D = [2 1]/3;
v = 0.5;
```

関数プロット

(Function plot)

```
% Function settings
f = @(s0,s1) 0.5*(v-(D(1)*s0+D(2)*s1)).^2; % Fidelity term
```

```
r = @(s0,s1) 0.5*(s0.^2+s1.^2); % Regularizer
% Variable settings
s0 = linspace(-1,1,21);
s1 = linspace(-1,1,21);
[S0,S1] = ndgrid(s0,s1);
% Evaluation
F = f(S0,S1);
R = r(S0,S1);
% Surfc plot of the fidelity
figure(1)
hf = surfc(s0,s1,F);
hf(1).FaceAlpha = 0.125;
hf(1).FaceColor = 'green';
hf(1).EdgeAlpha = 0.25;
hf(2).LineWidth = 1;
set(gca,'YDir','reverse');
hold on
% Surfc plot of the regularizer
hg = surfc(s0,s1,R);
hg(1).FaceAlpha = 0.125;
hg(1).FaceColor = 'blue';
hg(1).EdgeAlpha = 0.25;
hg(2).LineWidth = 1;
xlabel('s_1')
ylabel('s_0')
```



正規方程式とその解

(Normal Equation and its Solution)

正規方程式 (Normal equation)

$$(\mathbf{D}^T\mathbf{D} + \lambda\mathbf{I})\mathbf{s} = \mathbf{D}^T\mathbf{v}$$

解 (Solution)

$$\hat{\mathbf{s}} = (\mathbf{D}^T\mathbf{D} + \lambda\mathbf{I})^{-1}\mathbf{D}^T\mathbf{v}$$

```
% Evaluation values for λ
lmdset = logspace(-1,1,3);
idx = 1;
s = zeros(size(D,2),length((lmdset)));
for lambda = lmdset
    s(:,idx) = (D.'*D+lambda*eye(2))\(D.'*v);
    ht = text(s(2,idx)+.1,s(1,idx)-.2,['\lambda=' num2str(lambda)]);
    ht.FontSize = 12;
    idx = idx+1;
end
```

解のプロット (Solution plot)

```
hp = plot(s(2,:),s(1,:));
hp.Marker = 'o';
hp.MarkerSize = 6;
hp.MarkerEdgeColor = 'r';
hp.MarkerFaceColor = 'r';
hp.Color = 'r';
hp.LineWidth = 2;
hp.LineStyle = ':';
hp.Visible = true;
hold off
```