Sample 7-3

幾何学処理

拡大処理

画像処理特論

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動作確認: MATLAB R2020a

Geometric image processing

Interpolation

Advanced Topics in Image Processing

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Verified: MATLAB R2020a

準備

(Preparation)

```
close all
```

補間率の設定

(Setting of upsampling factor)

- M: 補間率 (upsampling factor)

```
% Upsampling factor
uFactor = 2;

% Upsampling phase
uPhase = 0;
```

最近傍補間フィルタのインパルス応答 (Impulse response of nearest-neighbor filter)

$$f[n] = \begin{cases} 1 & 0 \leq n \leq M-1 \\ 0 & \text{otherwise} \end{cases}$$

一次補間フィルタのインパルス応答 (Impulse response of linear interpolation filter)

$$f[n] = \begin{cases} \frac{1}{M}(M-|n|) & -M+1 \leq n \leq M-1 \\ 0 & \text{otherwise} \end{cases}$$

オフセットを考慮した場合 (When considering the offset)

$$f[n] = \begin{cases} \frac{1}{M}\left(M - \left|n - \frac{1}{2}\right|\right) & -M+1 \leq n \leq M \\ 0 & \text{otherwise} \end{cases}$$

ただし，非因果性に注意．(Note that the incausal property.)

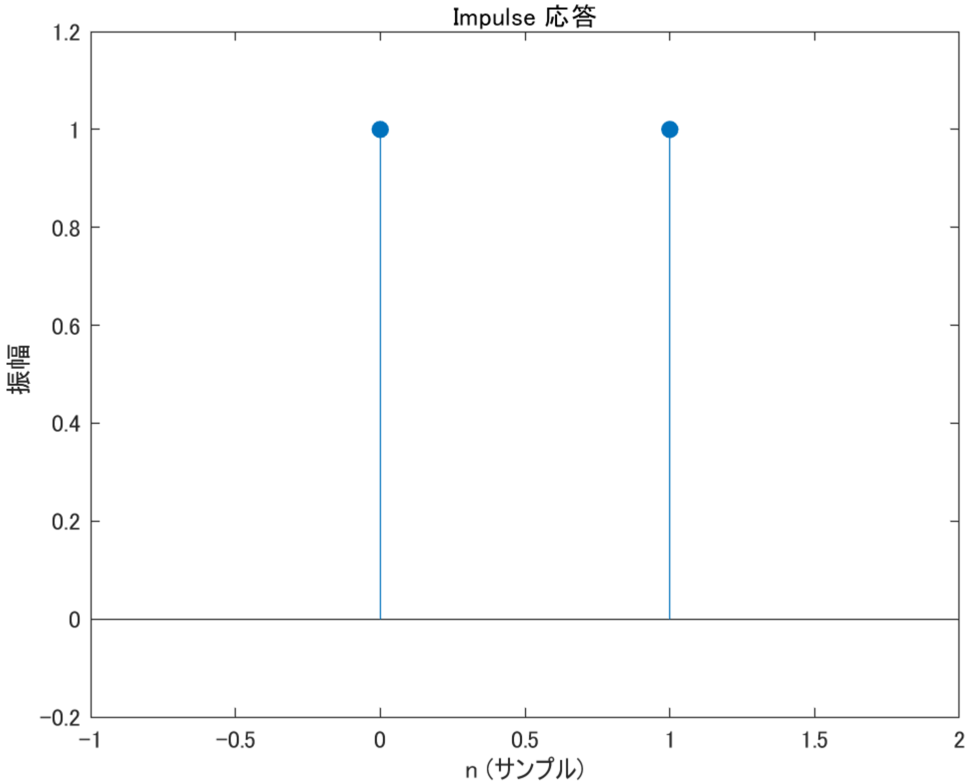
- $\{f[n]\}_n$: インパルス応答 (Impulse response)

```
% Filter seletion
ftype = "Nearest neighbor";
offset = false; % Set TRUE for even M, FALSE for odd M to make the handmade bilinear interpolat

% Impulse response of interpolation filter
if strcmp(ftype,'Nearest neighbor')
    f = ones(1,uFactor);
    offset = false;
elseif strcmp(ftype, 'Bilinear interpolation')
    if ~offset
         f = 1-abs((-uFactor+1):(uFactor-1))/uFactor;
    else
        f = 1-abs(((-uFactor+1):uFactor)-0.5)/uFactor;
    end
else
    error('Invalid ftype')
end
```

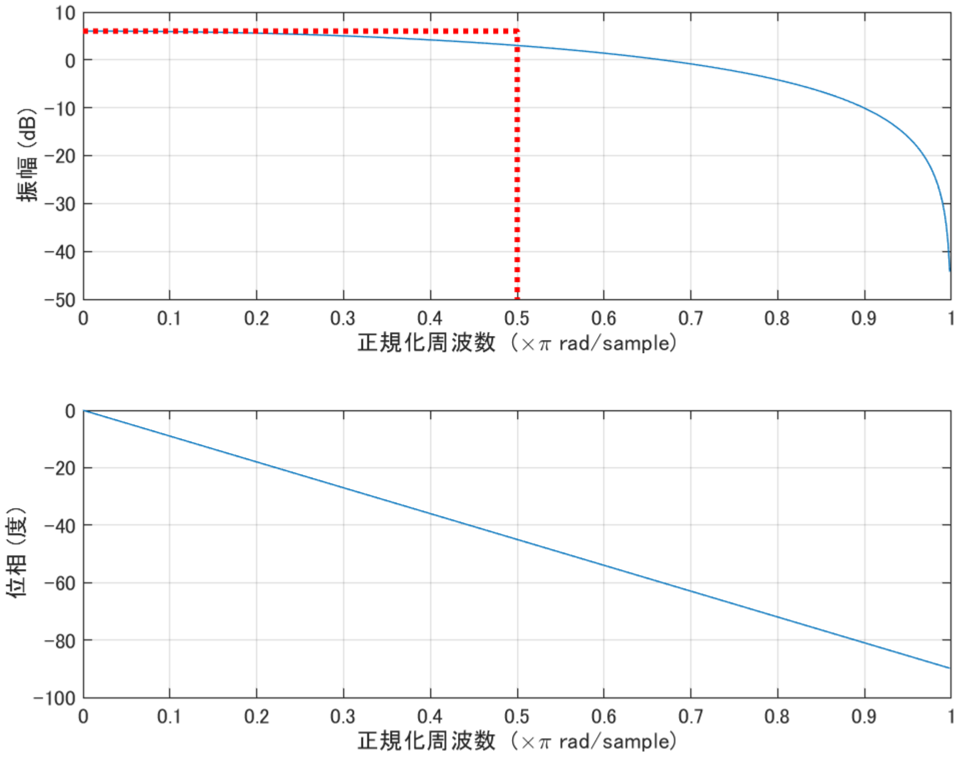
フィルタ特性の表示

(Display of filter characteristics)

```
% Impulse response
figure(1)
impz(f)
ax = gca;
ax.XLim = [-1 length(f)];
ax.YLim = [-0.2 1.2];
```



```
% Frequency response
figure(2)
freqz(f)
ax = gca;
hold on
line([0 1/uFactor 1/uFactor],[20*log10(uFactor) 20*log10(uFactor) ax.YLim(1)],...
    'LineStyle',':','LineWidth',2,'Color','red');
hold off
```



画像への適用

(Application to images)

$$v[\mathbf{m}] = \sum_{\mathbf{k}\in\mathbb{Z}^2} u[\mathbf{M}\mathbf{k}]\,f[\mathbf{m}-\mathbf{M}\mathbf{k}]$$

最近傍補間フィルタのインパルス応答 (Impulse response of nearest-neighbor filter)

$$f[\mathbf{n}] = \begin{cases} 1 & \mathbf{n} \in \mathcal{N}(\mathbf{M}) \\ 0 & \text{otherwise} \end{cases}$$

双一次補間フィルタのインパルス応答 (Impulse response of bilinear interpolation filter)

$$f[\mathbf{n}] = \begin{cases} \mathrm{prod}(\mathbf{1}-\mathrm{abs}(\mathbf{M}^{-1}\mathbf{n})) & \mathbf{n} \in \{\mathbf{M}\mathbf{x}\in\mathbb{Z}^2 \mid \mathbf{x}\in(-1,1)^2\} \\ 0 & \text{otherwise} \end{cases}$$

オフセットを考慮した場合

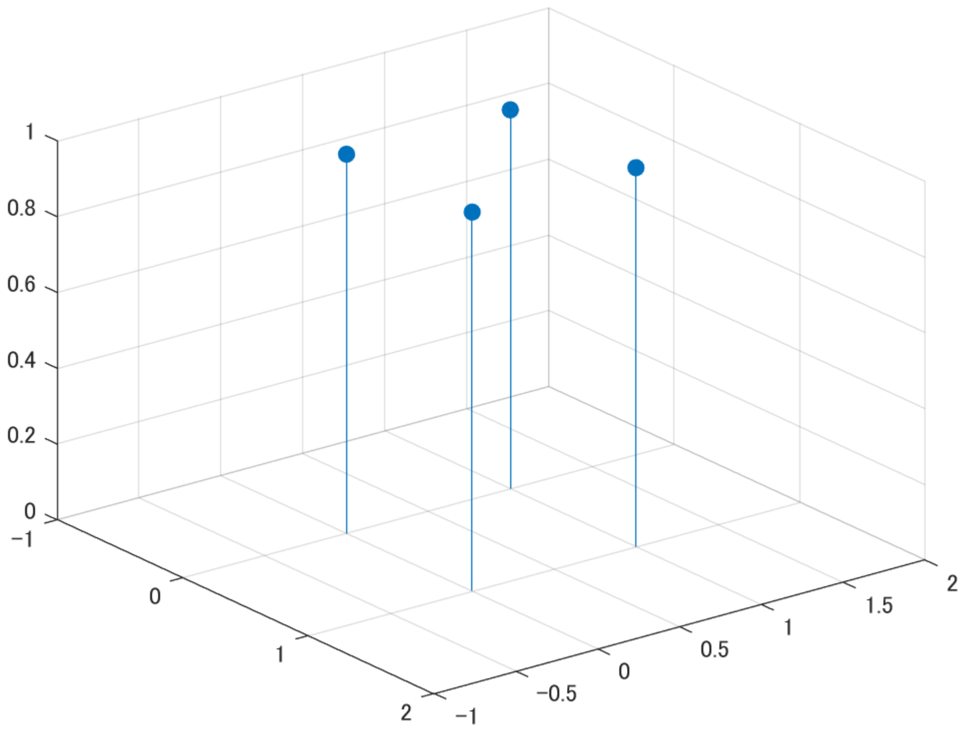
$$f[\mathbf{n}] = \begin{cases} \mathrm{prod}\left(\mathbf{1}-\mathrm{abs}\left(\mathbf{M}^{-1}\mathbf{n}-\frac{1}{2}\mathbf{1}\right)\right) & \mathbf{n} \in \{\mathbf{M}\mathbf{x}\in\mathbb{Z}^2 \mid \mathbf{x}\in(-1,1]^2\} \\ 0 & \text{otherwise} \end{cases}$$

ただし， $\mathrm{prod}(\cdot)$ は要素の積．非因果性に注意．(where $\mathrm{prod}(\cdot)$ denotes the product of the array elements. Note that the incausal property.)

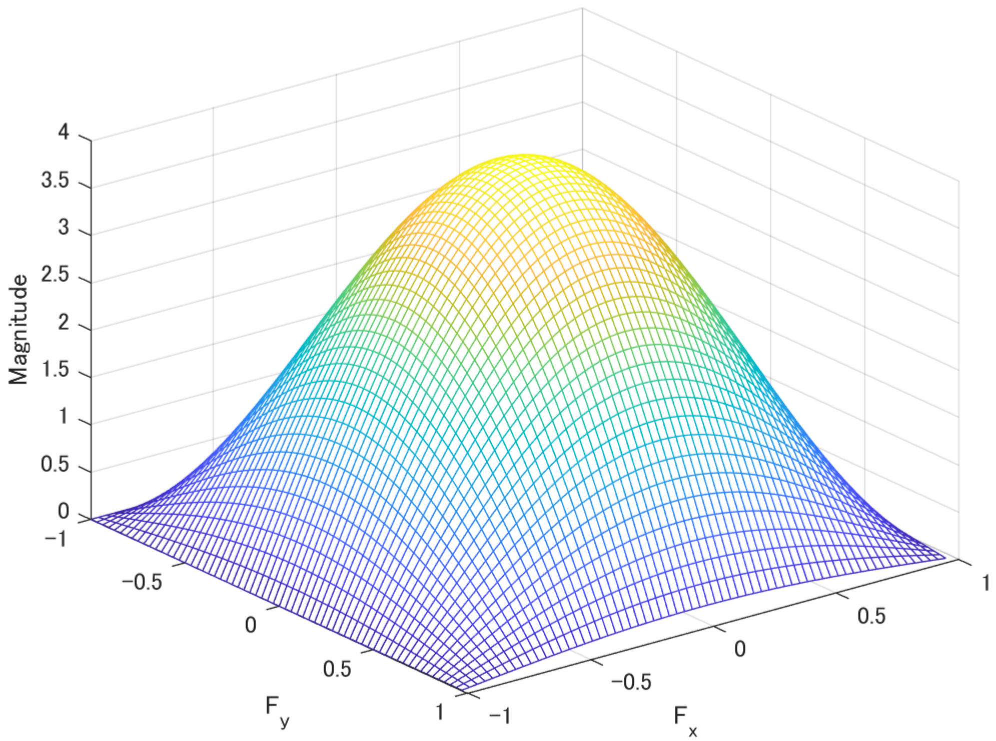
Note that if $\mathbf{M} = \mathrm{diag}(M, M) \Rightarrow \mathrm{prod}(\mathbf{1} - \mathrm{abs}(\mathbf{M}^{-1}\mathbf{n} - \alpha\mathbf{1})) = \frac{1}{M^2}(M - |n_1 - \alpha|)(M - |n_2 - \alpha|)$,

$\{\mathbf{Mx} \in \mathbb{Z}^2 \ | \ \mathbf{x} \in (-1, 1)^2\} = \{-M + 1, -M + 2, \cdots, M - 1\}^2$ and
$\{\mathbf{Mx} \in \mathbb{Z}^2 \ | \ \mathbf{x} \in (-1, 1]^2\} = \{-M + 1, -M + 2, \cdots, M\}^2$.

```
% Reading an image
u = imread('cameraman.tif');

% Generating an interpolation filter
if strcmp(ftype,'Nearest neighbor')
    [n1,n2] = ndgrid(0:uFactor-1);
    f = ones(uFactor,uFactor);
elseif strcmp(ftype,'Bilinear interpolation')
    if ~offset
        [n1,n2] = ndgrid(-uFactor+1:uFactor-1);
        f = (1-abs(n1)/uFactor).*(1-abs(n2)/uFactor);
    else
        [n1,n2] = ndgrid(-uFactor+1:uFactor);
        f = (1-abs(n1-0.5)/uFactor).*(1-abs(n2-0.5)/uFactor);
    end
else
    error('Invalid ftype')
end
figure(3)
stem3(n2,n1,f,'filled')
axis ij
ax = gca;
ax.XLim = ax.XLim + [-1 1];
ax.YLim = ax.YLim + [-1 1];
```



```
figure(4)
freqz2(f)
axis ij
```



```
% Bivariate upsampling function
upsample2 = @(x,n,phase) ...
    shiftdim(upsample(...
    shiftdim(upsample(x,...
    n(1),phase(1)),1),...
    n(2),phase(2)),1);

% Interpolation with upsampling and filtering
x = padarray(u,[1 1],'replicate','both');
w = imfilter(upsample2(x,uFactor*[1 1],uPhase*[1 1]),f,'conv');
s = ceil(uFactor/2);
v = w(s+1:s+uFactor*size(u,1),s+1:s+uFactor*size(u,2));

% Interpolation with IMRESIZE
if strcmp(ftype,'Nearest neighbor')
    y = imresize(u,uFactor,'nearest');
elseif strcmp(ftype, 'Bilinear interpolation')
    y = imresize(u,uFactor,'bilinear');
else
    error('Invalid ftype')
end
```

画像表示

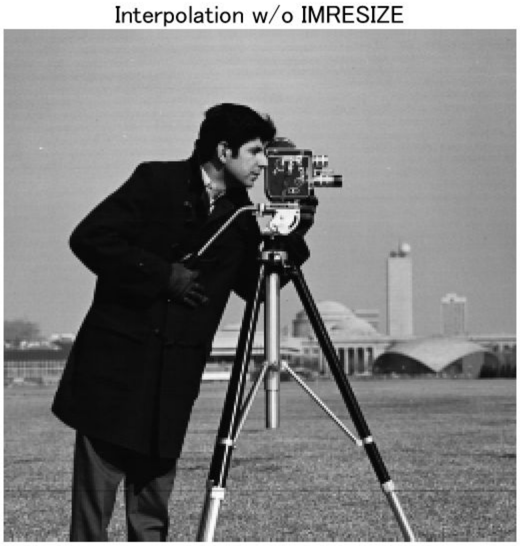
(Display image)

原画像 (Original)

```
figure(5)
imshow(u)
title('Original')
```

拡大画像 (Interpolated image)

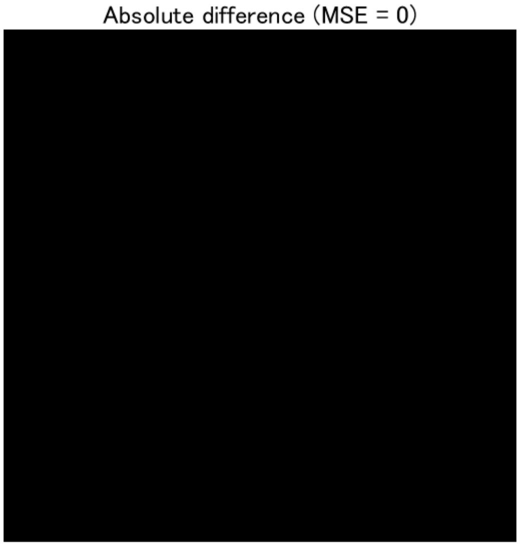
```
% Definition of MSE
mymse = @(x,y) sum((x-y).^2,'all')/numel(x);

% Display results
figure(6)
imshow(v)
title('Interpolation w/o IMRESIZE')
```



```
figure(7)
```

```
imshow(y)
title('Interpolation w/ IMRESIZE')
```

```
figure(8)
imshow(imabsdiff(v,y))
title(['Absolute difference (MSE = ' num2str(mymse(v,y)) ')'])
```


Absolute difference (MSE = 0)