# Sample 5-1

## 周波数解析

単変量畳み込み

画像処理特論

村松 正吾

動作確認: MATLAB R2020a

## Fourier analysis

Univariate convolution

Advanced Topics in Image Processing

Shogo MURAMATSU

Verified: MATLAB R2020a

### 準備

(Preparation)

```
close all
```

### 入力信号 $\{u[n]\}_n$

(Input signal $\{u[n]\}_n$)

```
% Input u[n]
u = [1 2 3];
```

### 線形シフト不変システムのインパルス応答 $\{h[n]\}_n$

(Impulse response of a linear shift-invariant system $\{h[n]\}_n$)

```
% Impulse response h[n]
h = [1 1 1]/3;
```

### 線形シフト不変システムの出力応答 $\{v[n]\}_n$

(The linear shift-invariant system response $\{v[n]\}_n$)

畳み込み演算 (Convolution)

$$\{v[n]\}_n = \{h[n]\}_n * \{u[n]\}_n = \sum_{k=-\infty}^{\infty} u[k]\{h[n-k]\}_n$$

```
% Output v[n]
v = conv(h,u);
```

## 入力信号 $\{u[n]\}_n$ のスペクトル

(Spectrum of input signal $\{u[n]\}_n$)

$$U(e^{\mathrm{j}\omega}) = \sum_{n=-\infty}^{\infty} u[n]e^{-\mathrm{j}\omega n},\ \omega \in \mathbb{R}$$

DFT(FFT)によるDTFTの周波数サンプル計算 (Frequency sampling of DTFT by DFT (FFT))

$$U[k] = U(e^{j\omega})\big|_{\omega=\frac{2\pi}{N}k},\ k \in \{0, 1, 2, \cdots, N-1\}$$

```
% Setting the number of frequency sample points in [0,2π)
nPoints = 128;

% Spectrum of u[n]
U = fft(u,nPoints);
```

## フィルタ $\{h[n]\}_n$ の周波数応答

(Frequency response of filter$\{h[n]\}_n$)

$$H(e^{\mathrm{j}\omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-\mathrm{j}\omega n},\ \omega \in \mathbb{R}$$

DFT(FFT)によるDTFTの周波数サンプル計算 (Frequency sampling of DTFT by DFT (FFT))

$$H[k] = H(e^{j\omega})\big|_{\omega=\frac{2\pi}{N}k},\ k \in \{0, 1, 2, \cdots, N-1\}$$

```
% Frequency response of h[n]
H = fft(h,nPoints);
```

## 出力信号 $\{v[n]\}_n$ のスペクトル

(Spectrum of input signal $\{v[n]\}_n$)

$$V(e^{\mathrm{j}\omega}) = \sum_{n=-\infty}^{\infty} v[n]e^{-\mathrm{j}\omega n},\ \omega \in \mathbb{R}$$

DFT(FFT)によるDTFTの周波数サンプル計算 (Frequency sampling of DTFT by DFT (FFT))

$$V[k] = V(e^{\mathrm{j}\omega})\big|_{\omega=\frac{2\pi}{N}k},\ k \in \{0, 1, 2, \cdots, N-1\}$$

```
% Frequency response of v[n]
```

```
V = fft(v,nPoints);
```

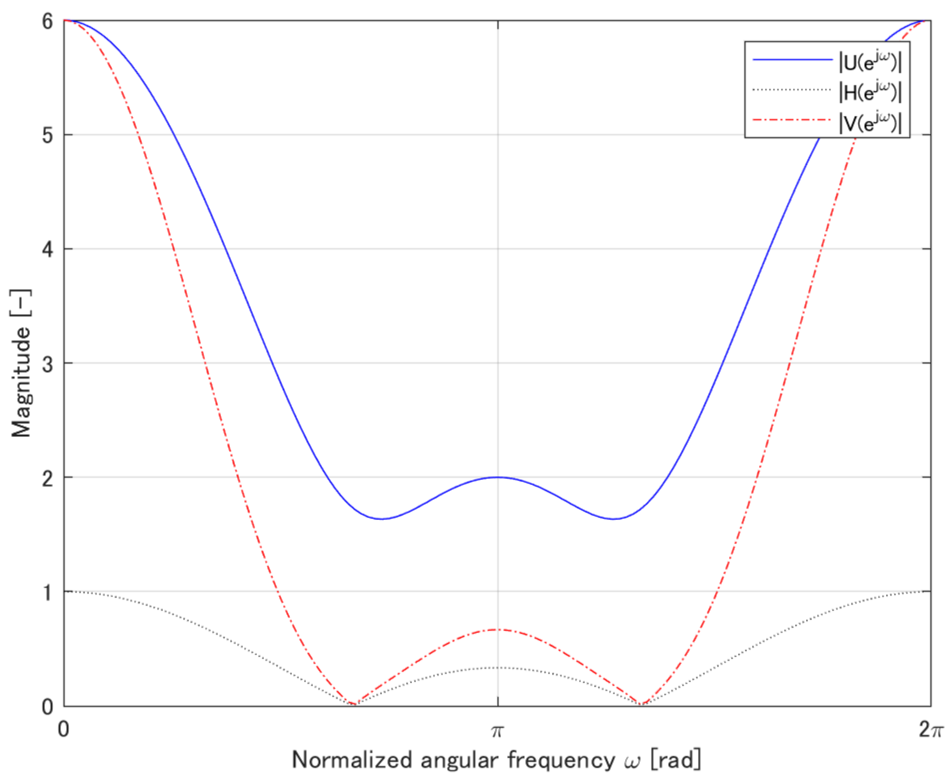
### スペクトルと周波数応答の表示

(Display of spectra and frequency response)

$V(e^{j\omega}) = H(e^{j\omega})U(e^{j\omega})$

```
% Frequency sample points
w = 0:2*pi/nPoints:2*pi-1/nPoints;

% Display of spectra and frequency response
figure(1)
ax = gca;
plot(ax,w,abs(U),'b',w,abs(H),'k:',w,abs(V),'r-.')
grid(ax,'on')
xlabel(ax,'Normalized angular frequency \omega [rad]')
ylabel(ax,'Magnitude [-]')
legend(ax,'|U(e^{j\omega})|','|H(e^{j\omega})|','|V(e^{j\omega})|')
ax.XLim = [0 2*pi];
ax.XTick = [0 pi 2*pi];
ax.XTickLabel = {'0', '\pi', '2\pi' };
```




### 線形シフト不変システムの解析

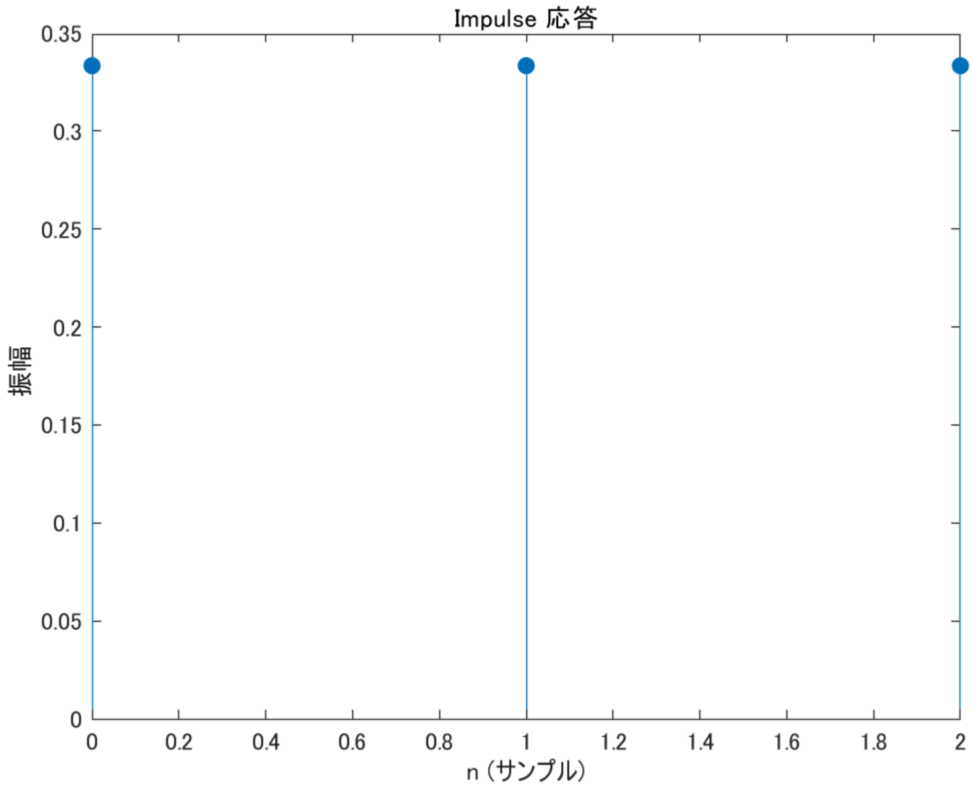
(Analysis of linear shift-invariant systems)

#### インパルス応答

(Impulse respone)

$h[n],\ n \in \mathbb{Z}$

```
figure(2)
impz(h)
```

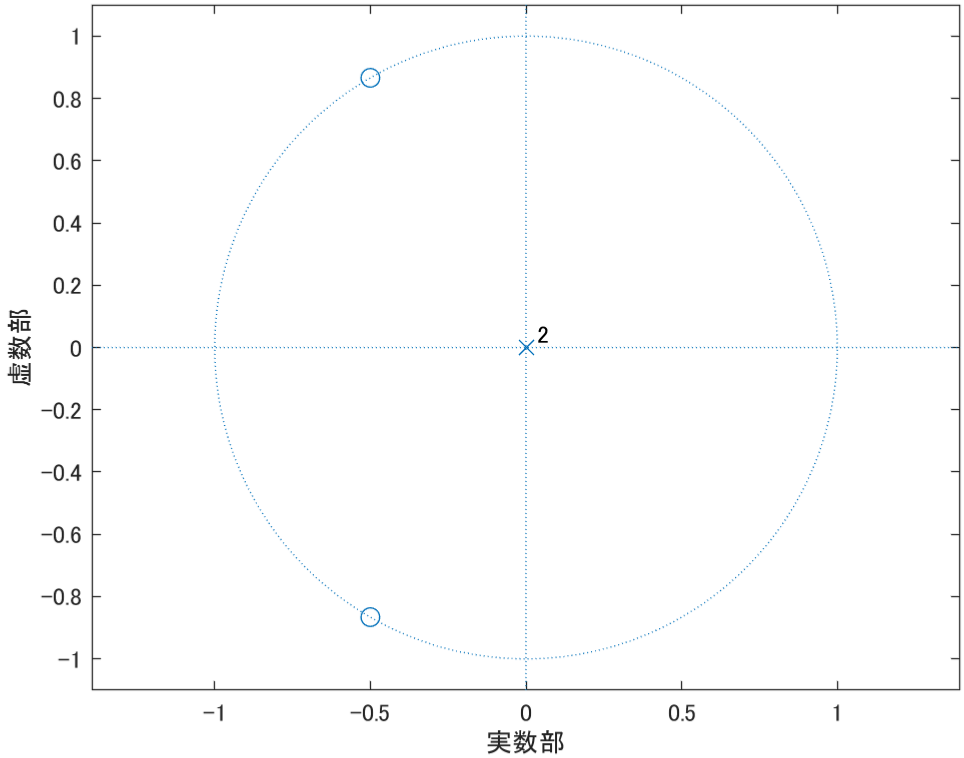



伝達関数

(Transfer function)

$$H(z) = \sum_{n=-\infty}^{\infty} h[n] z^{-n},\ z \in \mathbb{C}$$

```
figure(3)
zplane(h)
```



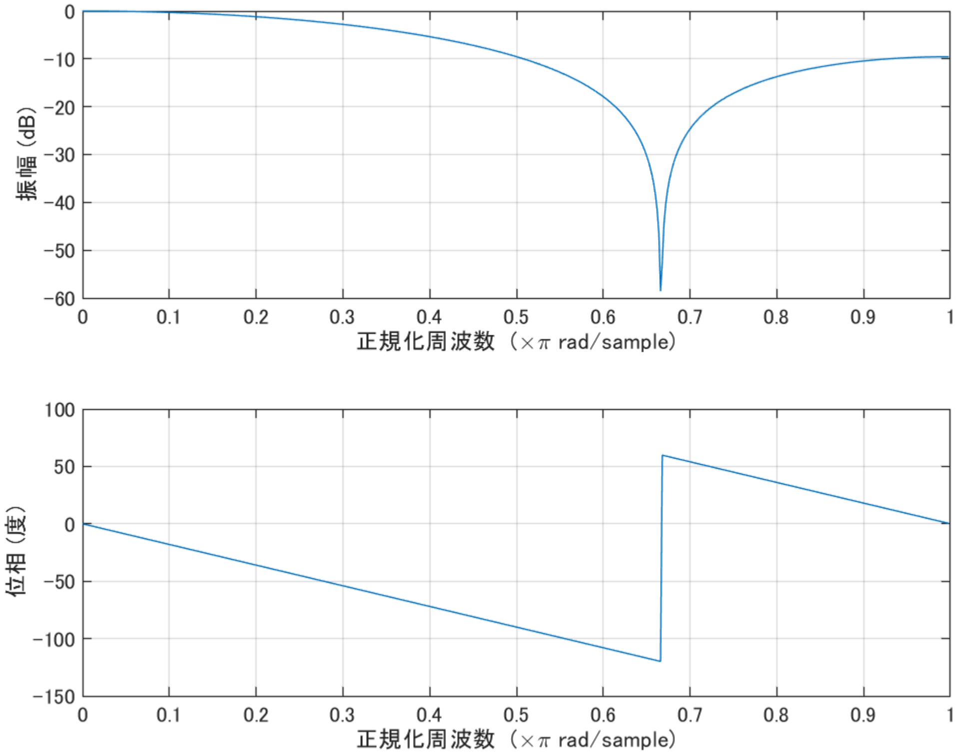

周波数応答

(Frequency response)

$H(e^{j\omega}) = H(z)|_{z=e^{j\omega}},\ \omega \in \mathbb{R}$

- 振幅応答 (Magnitude response): $|H(e^{j\omega})|$
- 位相応答 (Phase response): $\angle H(e^{j\omega})$

```
figure(4)
freqz(h)
```


振幅 (dB)
0
−10
−20
−30
−40
−50
−60
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
正規化周波数 (×π rad/sample)
位相 (度)
100
50
0
−50
−100
−150
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
正規化周波数 (×π rad/sample)