Sample 3-1

平滑化／先鋭化処理

内積とノルム

画像処理特論

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動作確認: MATLAB R2020a

Image smoothing/sharpening

Inner product and norm

Advanced Topics in Image Processing

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Verified: MATLAB R2020a

準備

(Preparation)

```
close all
```

N 次元ベクトルの内積

(Inner product of N-dimensional vectors)

$$\langle \mathbf{u}, \mathbf{v} \rangle = \sum_{i=0}^{N-1} u_i v_i$$

ただし， u_i, v_iはベクトル $\mathbf{u}, \mathbf{v} \in \mathbb{R}^N$ の i-番目要素 $[\mathbf{u}]_i, [\mathbf{v}]_i$。

(where u_i, v_i are the i-th element of vector $\mathbf{u}, \mathbf{v} \in \mathbb{R}^N$, i.e., $[\mathbf{u}]_i, [\mathbf{v}]_i$, respectively.)

```
% Generate an two-dimensional vector u
u1 = -1;
u2 = 1;
u = [u1, u2].'
```

```
u = 2×1
    -1
     1
```

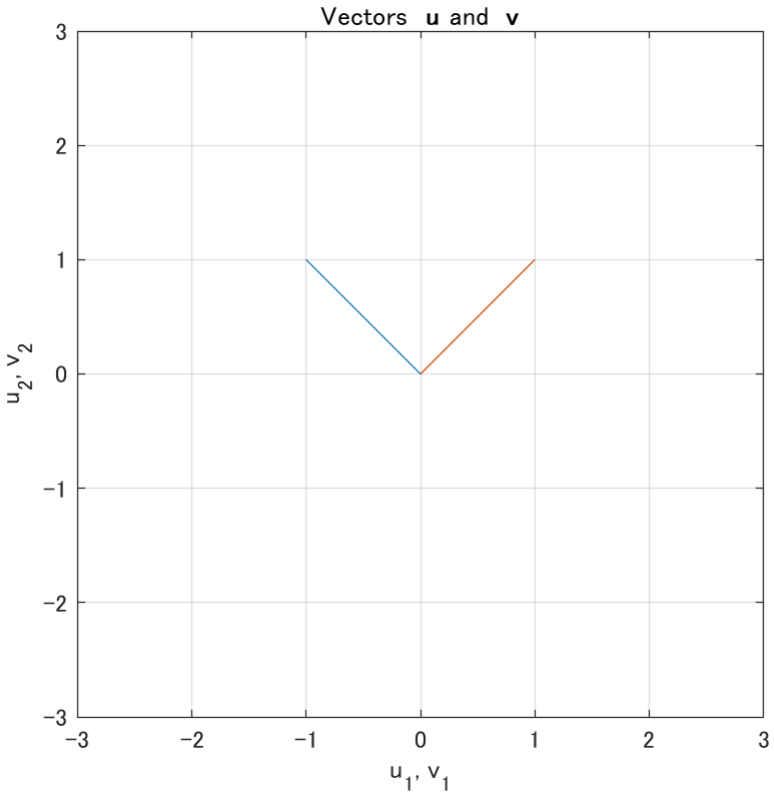
```
% Generate an two-dimensional vector v
v1 = 1;
v2 = 1;
v = [v1, v2].'
```

```
v = 2×1
     1
     1
```

```
% Inner product of vectors u and v
innerprod = dot(u,v);
disp(['<u,v> = ' num2str(innerprod)])
```

```
<u,v> = 0
```

```
% Plot vector v with the contour plot of lp-norm
figure(1)
plotv([u v],'-')
title('Vectors {\bf u} and {\bf v}')
xlabel('u_1, v_1')
ylabel('u_2, v_2')
colormap('default')
axis equal
axis([-3 3 -3 3])
grid on
```



ベクトルとしてみた$N_1 \times N_2$配列の 内積

(Inner product of $N_1 \times N_2$ arrays as vector)

$$\langle \mathbf{u}, \mathbf{v} \rangle = \sum_{j=0}^{N_2-1} \sum_{i=0}^{N_1-1} u_{i,j} v_{i,j}$$

ただし， $u_{i,j}, v_{i,j}$はベクトル $\mathbf{u}, \mathbf{v} \in \mathbb{R}^{N_1 \times N_2}$ の i, j-番目要素 $[\mathbf{u}]_{i,j}, [\mathbf{v}]_{i,j}$。

(where $u_{i,j}, v_{i,j}$ are the i, j-th element of vector $\mathbf{u}, \mathbf{v} \in \mathbb{R}^N$, i.e., $[\mathbf{u}]_{i,j}, [\mathbf{v}]_{i,j}$, respectively.)

```
% Array dimension
ndim1 = 2;
ndim2 = 2;
% Generate a N1xN2 arrya v with normally distributed random numbers
u = randn(ndim1,ndim2)
```

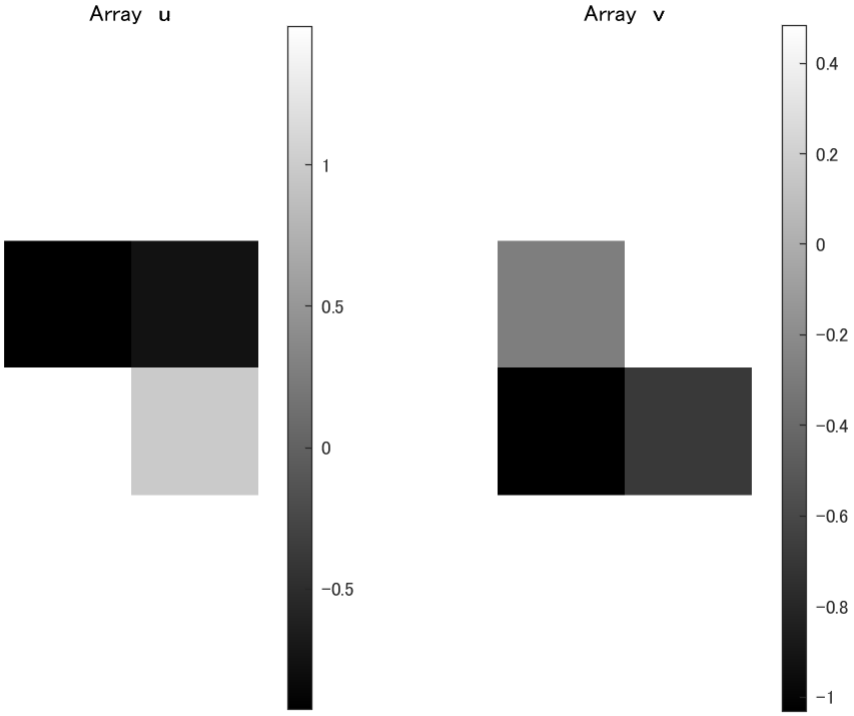
```
u = 2×2
   -0.9274   -0.7496
    1.4915    0.9853
```

```
v = randn(ndim1,ndim2)
```

```
v = 2×2
   -0.2839    0.4842
   -1.0306   -0.6902
```

```
% Visualization of array u
figure(2)
subplot(1,2,1)
imagesc(u)
title('Array {\bf u}')
colormap('gray')
colorbar
axis equal
axis off

% Visualization of array v
subplot(1,2,2)
imagesc(v)
title('Array {\bf v}')
colormap('gray')
colorbar
axis equal
axis off
```




```
% Inner product of arrays u and v as vectors
innerprod = dot(u(:),v(:));
disp(['<u,v> = ' num2str(innerprod)])
```

```
<u,v> = -2.3169
```

N 次元ベクトルの ℓ_p-ノルム

(ℓ_p-norm of a N-dimensional vector)

$$\|\mathbf{v}\|_p = \left(\sum_{i=0}^{N-1} |v_i|^p \right)^{\frac{1}{p}}$$

ただし， v_iはベクトル $\mathbf{v} \in \mathbb{R}^N$ の i-番目要素 $[\mathbf{v}]_i$。

(where v_i stands for the i-th element of vector $\mathbf{v} \in \mathbb{R}^N$, i.e., $[\mathbf{v}]_i$.)

```
% Generate an two-dimensional vector v
v1 = 1;
v2 = 1;
v = [v1, v2].'
```

```
v = 2×1
     1
```

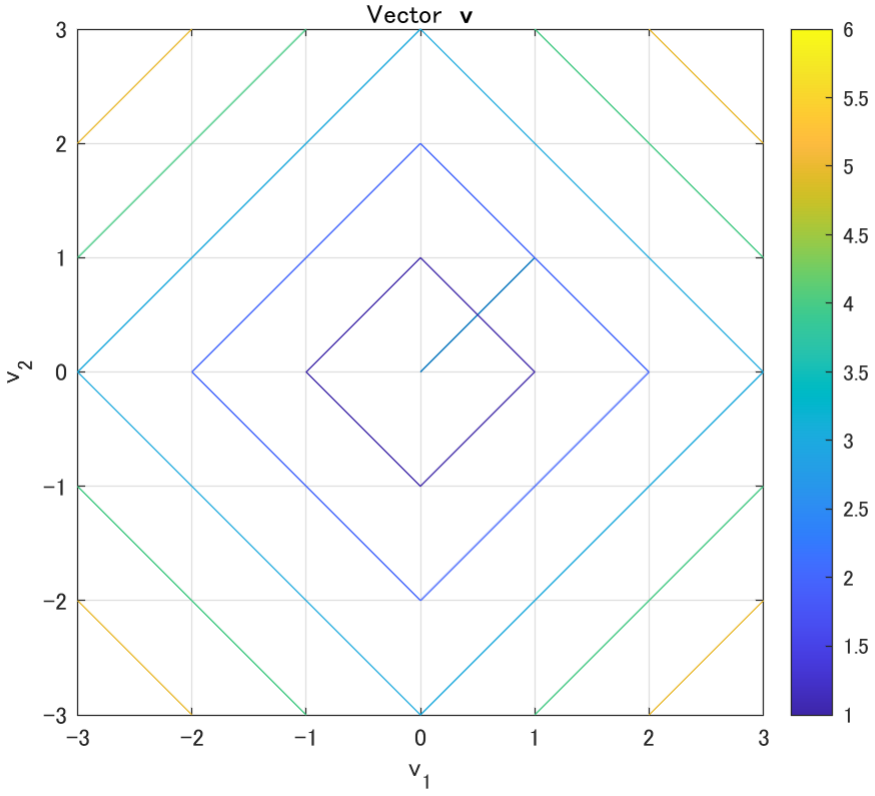
```
1
```

```
% Setting of parameter p
p = 1;

% lp-norm of v
lpnorm = norm(v,p);
disp(['||v||_' num2str(p) ' = ' num2str(lpnorm)])
```

```
||v||_1 = 2
```

```
% Plot vector v with the contour plot of lp-norm
figure(3)
plotv(v,'-')
title('Vector {\bf v}')
xlabel('v_1')
ylabel('v_2')
colormap('default')
axis equal
axis([-3 3 -3 3])
grid on
hold on
% Contour plot of lp-norm
fcontour(@(v1,v2) vecnorm([v1(:) v2(:)].',p),[-3 3 -3 3])
colorbar
hold off
```



ベクトルとしてみた$N_1 \times N_2$配列の ℓ_p-ノルム

(ℓ_p-norm of a $N_1 \times N_2$ array as a vector)

$$\|\mathbf{v}\|_p = \left(\sum_{j=0}^{N_2-1} \sum_{i=0}^{N_1-1} |v_{i,j}|^p \right)^{\frac{1}{p}}$$

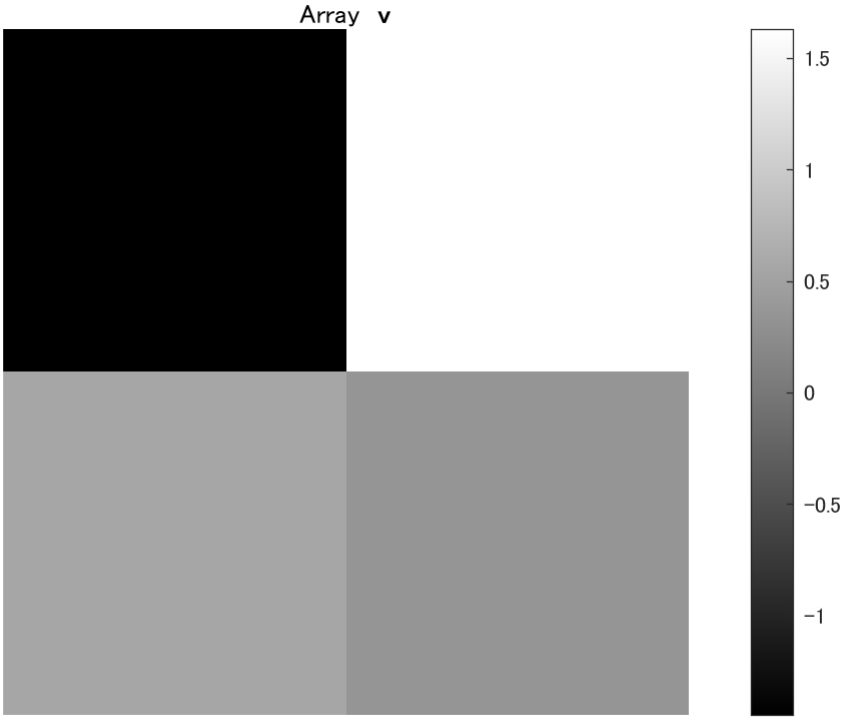
ただし， $v_{i,j}$はベクトルとしてみた配列 $\mathbf{v} \in \mathbb{R}^{N_1 \times N_2}$ の i, j-番目要素 $[\mathbf{v}]_{i,j}$。

(where $v_{i,j}$ stands for the i, j-th element of array $\mathbf{v}$ as a vector, i.e., $[\mathbf{v}]_{i,j}$.)

```
% Array dimension
ndim1 = 2;
ndim2 = 2;
% Generate a N1xN2 arrya v with normally distributed random numbers
v = randn(ndim1,ndim2)
```

```
v = 2×2
   -1.4436    1.6318
    0.5620    0.3565
```

```
% Visualization of array v
figure(4)
imagesc(v)
title('Array {\bf v}')
colormap('gray')
colorbar
axis equal
axis off
```




```
% Setting of parameter p
p = 1;

% lp-(element-wise) norm of v
lpnorm = norm(v(:),p);
disp(['||v||_' num2str(p) ' = ' num2str(lpnorm)])
```

```
||v||_1 = 3.9939
```