Sample 9-3

離散ウェーブレット変換

重複変換のフィルタバンク実装

画像処理特論

村松 正吾

動作確認: MATLAB R2020a

Discrete wavelet transform

Filter bank implementation of lapped transforms

Advanced Topics in Image Processing

Shogo MURAMATSU

Verified: MATLAB R2020a

準備

(Preparation)

```
close all
```

入力信号の生成

(Generation of input)

```
% Input signal
u = [ 0 3 1 3 1 5 3 0 ]; % Set to even length
```

並列フィルタバンク実装

(Parallel filter bank implementation)

分析フィルタバンクをデシメータで、合成フィルタバンクをインタポレータで実装 (Analysis filter banks are implemented with decimetors and synthesis filter banks are implemented with interpolators.)

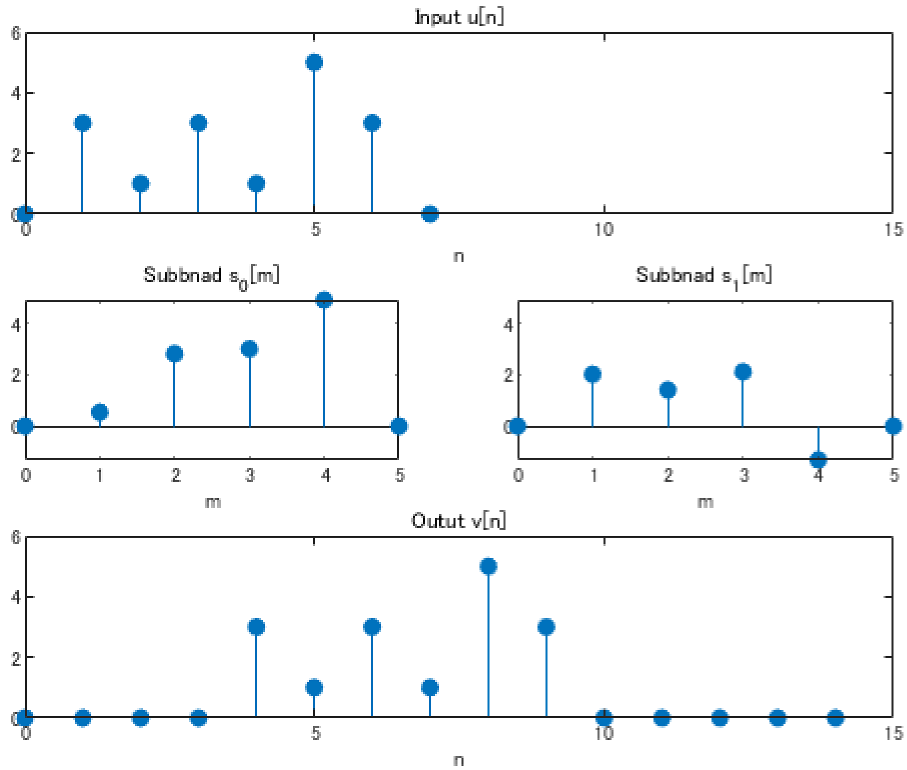
```
% # of channels
nChs = 2;

% Analysis filters (Daubiches 2 wavelet filter)
h0 = [   (1-sqrt(3)) (3-sqrt(3))  (3+sqrt(3)) (1+sqrt(3)) ]/(4*sqrt(2));
h1 = [  -(1+sqrt(3)) (3+sqrt(3)) -(3-sqrt(3)) (1-sqrt(3)) ]/(4*sqrt(2));

% Synthesis filters (Daubiches 2 wavelet filter)
f0 = fliplr(h0);
f1 = fliplr(h1);
```

```
% Or the function WFILTERS from Wavelet Toolbox as
% [h0,h1,f0,f1] = wfilters('db2');

% Analysis process
s0 = downsample(conv(h0,u),nChs);
s1 = downsample(conv(h1,u),nChs);

% Synthesis process
v0 = conv(f0,upsample(s0,nChs));
v1 = conv(f1,upsample(s1,nChs));
v = v0 + v1;
```

信号表示

(Signal display)

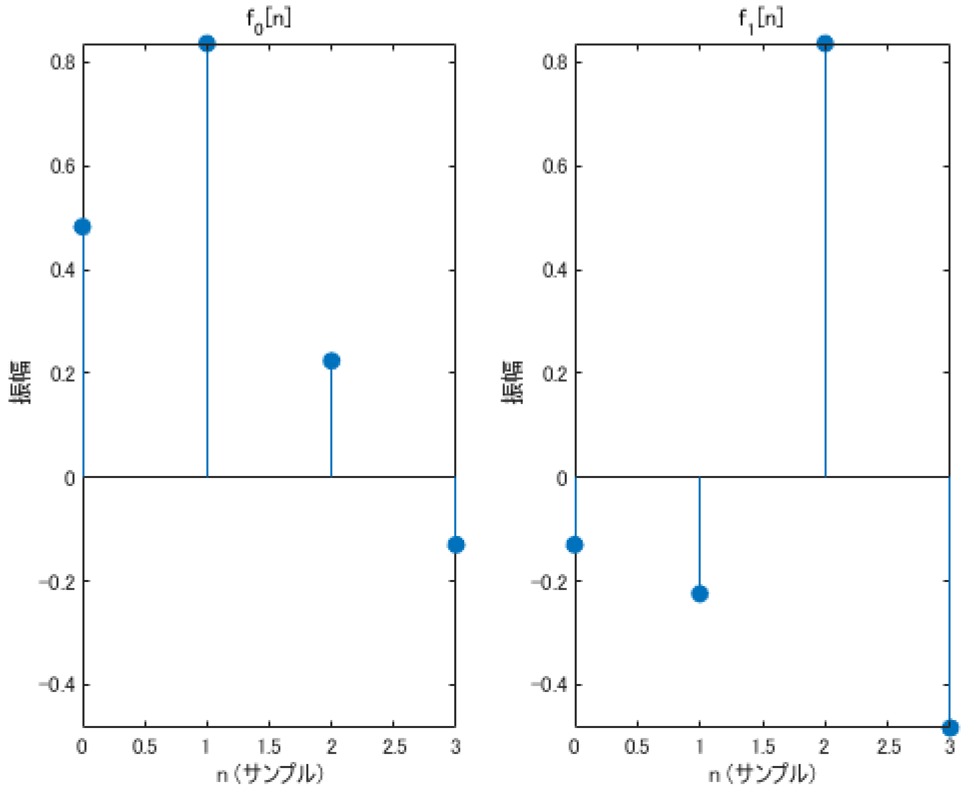
```
figure(1)
% Input
subplot(3,2,[1,2])
stem(0:length(u)-1,u,'filled')
title('Input u[n]')
xlabel('n')
ax = gca;
ax.XLim =[ 0 length(v)];

% Subband 0
subplot(3,2,3)
stem(0:length(s0)-1,s0,'filled')
title('Subbnad s_0[m]')
xlabel('m')
ax = gca;
ax.YLim =[ min([s0(:);s1(:)]) max([s0(:);s1(:)]) ];

% Subband 1
subplot(3,2,4)
stem(0:length(s1)-1,s1,'filled')
title('Subbnad s_1[m]')
xlabel('m')
ax = gca;
ax.YLim =[ min([s0(:);s1(:)]) max([s0(:);s1(:)]) ];

% Output
subplot(3,2,[5,6])
stem(0:length(v)-1,v,'filled')
title('Outut v[n]')
xlabel('n')
ax = gca;
ax.XLim =[ 0 length(v)];
```



インパルス応答（局所基底ベクトル）

(Impluse responses of synthesis filters; local basis vectors)

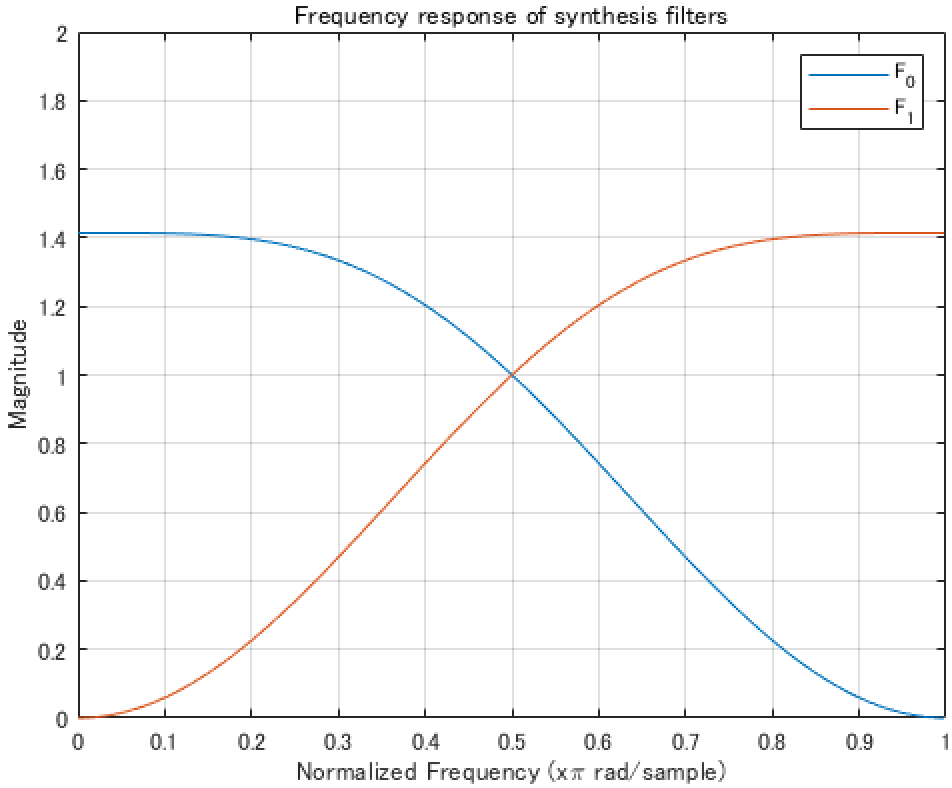
```
figure(2)
% Low-pass filter
subplot(1,2,1)
impz(f0)
title('f_0[n]')
ax = gca;
ax.YLim =[ min([f0(:);f1(:)]) max([f0(:);f1(:)]) ];

% High-pass filter
subplot(1,2,2)
impz(f1)
title('f_1[n]')
ax = gca;
ax.YLim =[ min([f0(:);f1(:)]) max([f0(:);f1(:)]) ];
```



周波数応答

(Frequency responses)

```
figure(3)
fftPoints = 512;
F = zeros(fftPoints,nChs);
% Low-pass filter
[F(:,1),W] = freqz(f0,1,fftPoints);
% High-pass filter
F(:,2)     = freqz(f1,1,fftPoints);
plot(W/pi, abs(F)) %20*log10(abs(F)))
axis([0 1 0 ceil(sqrt(nChs))]) %-70 10])
xlabel('Normalized Frequency (x\pi rad/sample)')
ylabel('Magnitude') % (dB)')
title('Frequency response of synthesis filters')
legend({ 'F_0', 'F_1'})
grid on
```



ポリフェーズ行列実装

(Polyphase matrix implemenation)

フィルタバンクをポリフェーズフィルタに分解して、ポリフェーズ行列として実装 (Decompose the filter bank into polyphase filters and implement them in a polyphase matrix.)

$$\mathbf{E}(z)=\begin{pmatrix} h_0[0] & h_0[1] \\ h_1[0] & h_1[1] \end{pmatrix}+\begin{pmatrix} h_0[2] & h_0[3] \\ h_1[2] & h_1[3] \end{pmatrix}z^{-1}=\begin{pmatrix} h_0[0]+h_0[2]z^{-1} & h_0[1]+h_0[3]z^{-1} \\ h_1[0]+h_1[2]z^{-1} & h_1[1]+h_1[3]z^{-1} \end{pmatrix}$$

$$\mathbf{R}(z)=\begin{pmatrix} f_0[1] & f_1[1] \\ f_0[0] & f_1[0] \end{pmatrix}+\begin{pmatrix} f_0[3] & f_1[3] \\ f_0[2] & f_1[2] \end{pmatrix}z^{-1}=\begin{pmatrix} f_0[1]+f_0[3]z^{-1} & f_1[1]+f_1[3]z^{-1} \\ f_0[0]+f_0[2]z^{-1} & f_1[0]+f_1[2]z^{-1} \end{pmatrix}$$

```
% Type-I polyphase filters of analyzer
e00 = h0(1:nChs:end);
e01 = h0(2:nChs:end);
e10 = h1(1:nChs:end);
e11 = h1(2:nChs:end);

% Type-II polyphase filters of synthesizer
r00 = f0(2:nChs:end);
r10 = f0(1:nChs:end);
r01 = f1(2:nChs:end);
r11 = f1(1:nChs:end);
```

分析合成処理 (Analysis and synthesis process)

```
% Input Signal
uadj = [zeros(1,nChs-1) u 0]; % Adjust delay for downsampling
disp(uadj)
```

```
     0     0     3     1     3     1     5     3     0     0
```

```
% Serial/Pallalel conversion
phase = 0;
u0 = downsample(uadj,nChs,mod(nChs-1-phase,nChs));
phase = 1;
u1 = downsample(uadj,nChs,mod(nChs-1-phase,nChs));
x = [ u0 ;
      u1 ];
disp(x)
```

```
     0     1     1     3     0
     0     3     3     5     0
```

（補足）z-変換の定義 (Definition of z-transform)

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

畳み込みとの関係 (Relation to convolution)

$$y[n] = h[n] * x[n] \leftrightarrow Y(z) = H(z)X(z)$$

```
% Analysis process w/ the polyphase matrix
% s = E*x
s0 = conv(e00,u0) + conv(e01,u1);
s1 = conv(e10,u0) + conv(e11,u1);
disp([s0;s1])
```

```
         0    0.5430    2.8284    3.0179    4.9244         0
         0    2.0266    1.4142    2.1213   -1.3195         0
```

```
% Synthesis process w/ the polyphase matrix
% v = R*s
v0 = conv(r00,s0) + conv(r01,s1);
v1 = conv(r10,s0) + conv(r11,s1);
disp([v0;v1])
```

```
         0    0.0000    1.0000    1.0000    3.0000    0.0000         0
         0    0.0000    3.0000    3.0000    5.0000    0.0000         0
```

```
% Parallel/Serial conversion
v = upsample(v0,nChs,1) + upsample(v1,nChs,0);
disp(v)
```

```
         0         0    0.0000    0.0000    3.0000    1.0000    3.0000    1.0000    5.0000    3.0000    0.0000    0.
```

ポリフェーズ行列実装（PPMATRIX オブジェクト利用）

(Polyphase matrix implemenation with PPMATRIX object)

ポリフェーズ行列演算が簡便となるよう以下のクラスを定義した。インスタンス化の際、3 番目の添え字が遅延を示す 3 次元配列を渡す。(In order to make polyphase matrix operations easier, the following classe is defined. At instantiation, plase pass a three-dimensional array where the third subscript indicates a delay.)

- PPMATRIX： Polyphase Matrix Class (originally from saivdr.dictionary.utility.PolyPhaseMatrix1D in SaivDr package)

分析合成処理 (Analysis and synthesis process)

```
import msip.ppmatrix
% Polyphase matrix of analysis bank
e00 = reshape(e00,1,1,length(e00));
e01 = reshape(e01,1,1,length(e01));
e10 = reshape(e10,1,1,length(e10));
e11 = reshape(e11,1,1,length(e11));
E = ppmatrix(cat(1,cat(2,e00,e01),cat(2,e10,e11)))
```

```
E =
[
    -0.12941 + 0.83652*z^(-1),    0.22414 + 0.48296*z^(-1);
    -0.48296 - 0.22414*z^(-1),    0.83652 - 0.12941*z^(-1)
]
```

```
% Polyphase matrix of synthesis bank
r00 = reshape(r00,1,1,length(r00));
r10 = reshape(r10,1,1,length(r10));
r01 = reshape(r01,1,1,length(r01));
r11 = reshape(r11,1,1,length(r11));
R = ppmatrix(cat(1,cat(2,r00,r01),cat(2,r10,r11)))
```

```
R =
[
    0.83652 - 0.12941*z^(-1),    -0.22414 - 0.48296*z^(-1);
    0.48296 + 0.22414*z^(-1),    -0.12941 + 0.83652*z^(-1)
]
```

完全再構成条件の確認 (Confirmation of perfect reconstruction)

$$\mathbf{R}(z)\mathbf{E}(z) = (\mathbf{R}_0 + \mathbf{R}_1 z^{-1})(\mathbf{E}_0 + \mathbf{E}_1 z^{-1}) = \mathbf{R}_0\mathbf{E}_0 + (\mathbf{R}_1\mathbf{E}_0 + \mathbf{R}_0\mathbf{E}_1)z^{-1} + \mathbf{R}_1\mathbf{E}_1 z^{-2} = z^{-1}\mathbf{I}$$

```
disp(R*E)
```

```
[
    5.5511e-17 + 1*z^(-1) + 5.5511e-17*z^(-2),    0;
    0,    5.5511e-17 + 1*z^(-1) + 5.5511e-17*z^(-2)
]
```

```
% Define delaychain
clear delaychain
delaychain(1,1,1) = 1;
delaychain(2,1,2) = 1;
```

```
delaychain = ppmatrix(delaychain)
```

```
delaychain =
[
    1;
    z^(-1)
]
```

```
% Z-transform of input
u = ppmatrix(reshape(u,1,1,length(u)));
disp(u)
```

```
[
    3*z^(-1) + z^(-2) + 3*z^(-3) + z^(-4) + 5*z^(-5) + 3*z^(-6)
]
```

```
% Polyphase decomposition of input
x = downsample(delaychain*u,nChs);
disp(x)
```

```
[
    z^(-1) + z^(-2) + 3*z^(-3);
    3*z^(-1) + 3*z^(-2) + 5*z^(-3)
]
```

```
% Analysis process w/ the polyphase matrix
s = E*x;
disp(s)
```

```
[
    0.54302*z^(-1) + 2.8284*z^(-2) + 3.0179*z^(-3) + 4.9244*z^(-4);
    2.0266*z^(-1) + 1.4142*z^(-2) + 2.1213*z^(-3) - 1.3195*z^(-4)
]
```

```
% Synthesis process w/ the polyphase matrix
y = R*s;
disp(y)
```

```
[
    1.6653e-16*z^(-1) + z^(-2) + z^(-3) + 3*z^(-4) + 2.2204e-16*z^(-5);
    2.2204e-16*z^(-1) + 3*z^(-2) + 3*z^(-3) + 5*z^(-4) + 2.2204e-16*z^(-5)
]
```

```
% Parallel/Serial conversion
v = delaychain.'*upsample(y,nChs);
disp(v)
```

```
[
    2.2204e-16*z^(-2) + 1.6653e-16*z^(-3) + 3*z^(-4) + z^(-5) + 3*z^(-6) + z^(-7) + 5*z^(-8) + 3*z^(-9) + 2.2204e-16
]
```

```
disp(squeeze(double(v)).')
```

```
         0         0    0.0000    0.0000    3.0000    1.0000    3.0000    1.0000    5.0000    3.0000    0.0000    0
```