Sample 4-2

線形シフト不変システム

相関と畳み込み

画像処理特論

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動作確認: MATLAB R2023a

Linear shift-invariant systems

Correlation and convolution

Advanced Topics in Image Processing

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Verified: MATLAB R2023a

準備

(Preparation)

```
close all
```

入力信号 $\{u[n]\}_n$

(Input signal $\{u[n]\}_n$)

```
% Input x[n]
u = [1 2 3];
```

フィルタカーネル $\{w[n]\}_n$

(Filter kernel $\{w[n]\}_n$)

```
% Filter kernel w[n]
w = [-1 0 1];
```

相互相関 $\{x[n]\}_n$

(Cross-correlation $\{x[n]\}_n$)

相互相関 (Cross-correlation)

$$\{x[n]\}_n = \sum_{k=-\infty}^{\infty} u[k]\{w[n+k]\}_n$$

```
% Output x[n]
x = xcorr(u,w);
ndiff = length(u)-length(w);
% Extract the significant subsequence
if ndiff <= 0
    x = x(1:end+ndiff)
else
    x = x(ndiff+1:end)
end
```

```
x = 1×5
    1.0000    2.0000    2.0000   -2.0000   -3.0000
```

線形シフト不変システムの出力応答 $\{v[n]\}_n$

(The linear shift-invariant system response $\{v[n]\}_n$)

畳み込み演算 (Convolution)

$$\{v[n]\}_n = \{h[n]\}_n * \{u[n]\}_n = \sum_{k=-\infty}^{\infty} u[k]\{h[n-k]\}_n$$

フィルタカーネル $\{w[n]\}_n$の反転 (Reversing the filter kernel $\{w[n]\}_n$.)

$\{h[n]\}_n = \{w[-n]\}_n$

```
% Flip the elements in w[n]
h = flip(w)
```

```
h = 1×3
     1     0    -1
```

$\{h[n]\}_n$をインパルス応答とした畳み込み演算. (A convolutional operation with $\{h[n]\}_n$ as the impulse response.)

```
% Output y[n]
v = conv(h,u)
```

```
v = 1×5
     1     2     2    -2    -3
```

入出力のプロット

(Plot of the input and output)

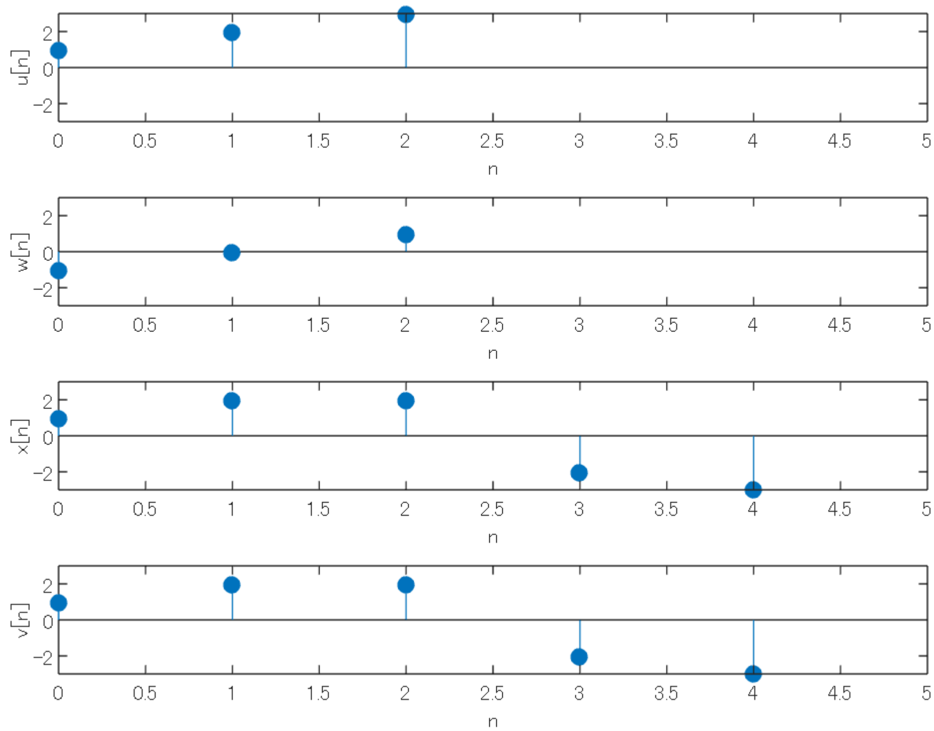
```
% Lengths of u, h, x and v
nu = length(u);
nw = length(w);
nx = length(x);
nv = length(v);
```

```
figure(1)
amax = max(max(abs(u)),max(abs(x)));
%% u[n]
subplot(4,1,1)
stem(0:nu-1,u,'filled')
axis([0 nx -amax amax])
xlabel('n')
ylabel('u[n]')

%% w[n]
subplot(4,1,2)
stem(0:nw-1,w,'filled')
axis([0 nx -amax amax])
xlabel('n')
ylabel('w[n]')

%% x[n]
subplot(4,1,3)
stem(0:nx-1,x,'filled')
axis([0 nx -amax amax])
xlabel('n')
ylabel('x[n]')

%% v[n]
subplot(4,1,4)
stem(0:nv-1,v,'filled')
axis([0 nx -amax amax])
xlabel('n')
ylabel('v[n]')
```



平均自乗誤差(MSE)による評価

(Evaluation in terms of the mean squared errors (MSE))

$$\mathrm{MSE}(\{x[n]\}_n, \{v[n]\}_n) := \frac{1}{|\Omega|}\sum_{n\in\Omega}|x[n]-v[n]|^2,$$

ただし， Ωは添え字集合， $|\Omega|$ は添え字の数．(where Ω denotes the index set and $|\Omega|$ means the cardinality.)

```
% Comparison between x and v
mymse = @(x,y) mean((double(x)-double(y)).^2,'all');
mymse(x,v)
```

```
ans = 4.1908e-32
```

2 変量フィルタリングと畳み込み

(Bivariate filtering and convolution)

インパルス信号 (Impulse signal)

$$\delta[\mathbf{n}] = \begin{cases} 1 & \mathbf{n} = \mathbf{0} \\ 0 & \text{otherwise} \end{cases}$$

```
% Bivariate impulse signal
D = 1
```

```
D = 1
```

フィルタカーネル $\{f[\mathbf{n}]\}_\mathbf{n}$の設定 (Setting of filter kernel $\{f[\mathbf{n}]\}_\mathbf{n}$)

```
% Definition of filter kernel f[n]
f = reshape(1:9,[3 3])
```

```
f = 3×3
     1     4     7
     2     5     8
     3     6     9
```

関数 IMFILTER のインパルス応答はフィルタカーネル$\{f[\mathbf{n}]\}_\mathbf{n}$の各軸反転となる.

(The impulse response of function IMFILTER is an inversion of each axis of the filter kernel $\{f[\mathbf{n}]\}_\mathbf{n}$.)

'full' オプションはクリッピングをせずに出力する．(The 'full' option outputs without clipping.)

```
% Impulse response of IMFILTER
imfilter(D,f,'full')
```

```
ans = 3×3
     9     6     3
     8     5     2
     7     4     1
```

フィルタカーネル$\{f[\mathbf{n}]\}_\mathbf{n}$を各軸反転 (Flip the filter kernel $\{f[\mathbf{n}]\}_\mathbf{n}$ on each axis.)

```
% Flipping filer kernel f[n]
h = rot90(f,2)
```

```
h = 3×3
     9     6     3
     8     5     2
     7     4     1
```

関数 IMFILTER の'conv'オプションは 2 番目の引数をインパルス応答$\{h[\mathbf{n}]\}_\mathbf{n}$として畳み込みを行う.

(The 'conv' option of function IMFILTER performs convolution with the second argument as an impulse response.)

```
% IMFILTER with the options 'conv' and 'full'
imfilter(D,h,'conv','full')
```

```
ans = 3×3
     9     6     3
     8     5     2
     7     4     1
```

オプション'full'のみの結果と同じことが確かめられる．(It can be verified that the result is the same as for the option 'full' only.)

関数 IMFILTER の'conv'と'full'オプションは関数 CONV2 と同等の機能をもつ．(The 'conv' and 'full' options of the function IMFILTER have the same functions as those of the function CONV2.)

```
conv2(D,h)
```

```
ans = 3×3
     9     6     3
     8     5     2
     7     4     1
```

画像フィルタリングの例

(Example of image filtering)

サンプル画像の読み込み (Reading a sample image)

```
% Reading image 'cameraman.tif' as double type.
I = im2double(imread('cameraman.tif'));
figure(2)
subplot(2,2,1)
imshow(I)
title('(a) Original')
```

水平 Prewitt カーネルの生成

(Generate a horizontal Prewitt kernel)

```
% Generate the horizontal Prewitt kernel
f = fspecial('prewitt').'
```

```
f = 3×3
     1     0    -1
     1     0    -1
     1     0    -1
```

オプションなし IMFILTER 実行

(IMFILTER without any option)

```
% IMFILTER w/o any option
J = imfilter(I,f);
subplot(2,2,2)
imshow(J+.5)
title('(b) Prewitt w/o any option ')
```

'corr'オプション付き IMFILTER 実行

(IMFILTER without the option 'corr')

```
% IMFILTER w the option 'corr' (Correlation mode)
K = imfilter(I,f,'corr');
subplot(2,2,3)
imshow(K+.5)
title(sprintf('(c) Prewitt w "corr" (MSE w (b): %4.2f)',mymse(J,K)))
```

フィルタカーネル $\{f[\mathbf{n}]\}_\mathbf{n}$ を各軸反転 (Flip the filter kernel $\{f[\mathbf{n}]\}_\mathbf{n}$ on each axis.)

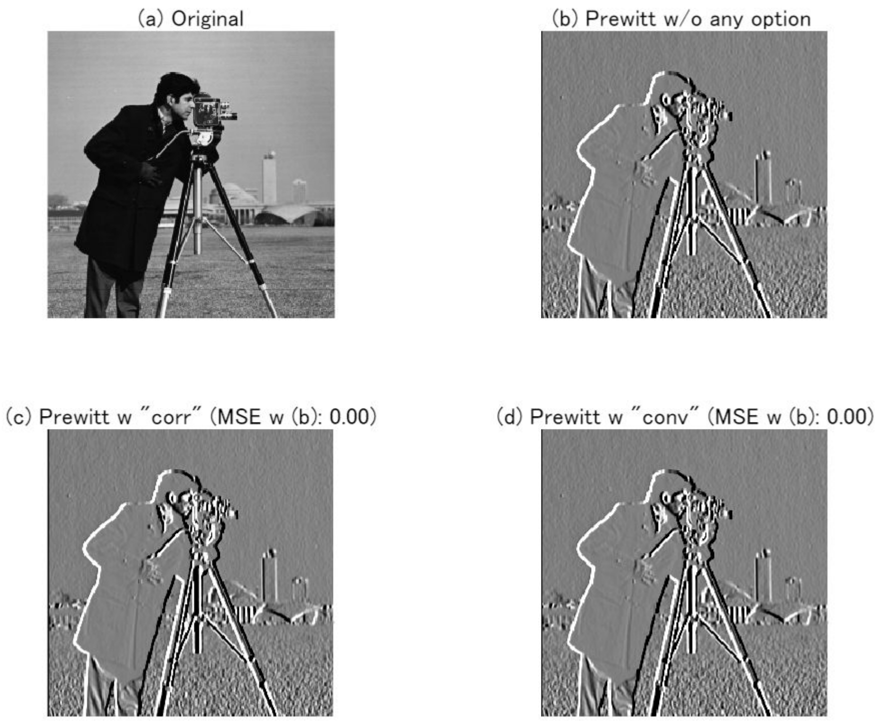
```
% Flipping filer kernel f[n]
h = rot90(f,2)
```

```
h = 3×3
    -1     0     1
    -1     0     1
    -1     0     1
```

'conv'オプション付き IMFILTER 実行

(IMFILTER without the option 'conv')

```
% IMFILTER w the option 'conv' (Convolution mode)
L = imfilter(I,h,'conv');
subplot(2,2,4)
imshow(L+.5)
title(sprintf('(d) Prewitt w "conv" (MSE w (b): %4.2f)',mymse(J,L)))
```



(b),(c),(d)の結果はすべて同じ．(The results in (b), (c), and (d) are all the same.)