Sample 10-3

冗長変換

ムーア・ペンローズの一般逆行列

画像処理特論

村松 正吾

動作確認: MATLAB R2020a

Redundant transforms

Moore-Penrose inverse

Advanced Topics in Image Processing

Shogo MURAMATSU

Verified: MATLAB R2020a

準備

(Preparation)

```
close all
```

合成フィルタバンクの大域行列表現

(Global matrix representation of synthesis filter bank)

```
% # of inputs
nSamples = 4;

% Synthesis filters
f0 = [  1 1 ]/2;
f1 = [ -1 1 ]/2;

% (Circular) convolution matrix
nF = max(length(f0),length(f1));
X = [zeros(nF-1,nSamples-nF+1) eye(nF-1); eye(nSamples)]; % Circular extension matrix
C = [zeros(nSamples,nF-1) eye(nSamples) zeros(nSamples,nF-1)]; % Clipping matrix

% Atoms in (circular) convolution matrix
d0 = C*convmtx(f0.',nSamples+nF-1)*X;
d1 = C*convmtx(f1.',nSamples+1)*X;
```

辞書 (Dictionary) $\mathbf{D}$

```
% Dictionary D (Global matrix representation of synthesis filter bank)
D = zeros(nSamples,2*nSamples);
D(:,1:2:end) = d0;
D(:,2:2:end) = d1;
```

```
disp(D)
```

```
    0.5000   -0.5000         0         0         0         0    0.5000    0.5000
    0.5000    0.5000    0.5000   -0.5000         0         0         0         0
         0         0    0.5000    0.5000    0.5000   -0.5000         0         0
         0         0         0         0    0.5000    0.5000    0.5000   -0.5000
```

ムーア・ペンローズ一般逆行列

(Moore-Penrose's inverse)

$\mathbf{T} = \mathbf{D}^T(\mathbf{D}\mathbf{D}^T)^{-1} = \mathbf{D}^+$

```
T = pinv(D);
disp(T)
```

```
    0.5000    0.5000         0         0
   -0.5000    0.5000         0         0
         0    0.5000    0.5000         0
         0   -0.5000    0.5000         0
         0         0    0.5000    0.5000
         0         0   -0.5000    0.5000
    0.5000   -0.0000         0    0.5000
    0.5000   -0.0000         0   -0.5000
```

分析合成処理

(Analysis-synthesis process)

```
% Signal generation
u = rand(nSamples,1);
disp(u)
```

```
    0.1361
    0.8693
    0.5797
    0.5499
```

```
% Analysis process
s = T*u;
disp(s)
```

```
    0.5027
    0.3666
    0.7245
   -0.1448
    0.5648
   -0.0149
    0.3430
   -0.2069
```

```
% Energy of subband Coef. vector s
disp(['||s||_2^2 = ' num2str(norm(s,2).^2)])
```

```
||s||_2^2 = 1.4126
```

```
% Synthesis process
v = D*s;
```

```
disp(v)
```

```
    0.1361
    0.8693
    0.5797
    0.5499
```

```
% MSE evaluation
mymse = @(x,y) mean((x(:)-y(:)).^2);
disp(['MSE = ', num2str(mymse(u, v))]);
```

```
MSE = 1.5407e-32
```

他の一般逆行列

(Another generalized inverse)

```
% Coefficients of analysis filters
gamma = -0.5;
delta = 1 - gamma;

% Analysis filters
h0 = [ gamma  delta ];
h1 = [ gamma -delta ];

% (Circular) convolution matrix
nH = max(length(h0),length(h1));
X = [eye(nSamples) ; eye(nH-1) zeros(nH-1,nSamples-nH+1) ]; % Circular extension matrix
C = [zeros(nSamples,nH-1) eye(nSamples) zeros(nSamples,nH-1)]; % Clipping matrix

% Global matrix representation of analysis filter bank
t0 = C*convmtx(h0.',nSamples+1)*X;
t1 = C*convmtx(h1.',nSamples+1)*X;
T = zeros(2*nSamples,nSamples);
T(1:2:end,:) = t0;
T(2:2:end,:) = t1;
disp(T)
```

```
    1.5000   -0.5000         0         0
   -1.5000   -0.5000         0         0
         0    1.5000   -0.5000         0
         0   -1.5000   -0.5000         0
         0         0    1.5000   -0.5000
         0         0   -1.5000   -0.5000
   -0.5000         0         0    1.5000
   -0.5000         0         0   -1.5000
```

```
% Analysis process
s = T*u;
disp(s)
```

```
   -0.2305
   -0.6387
    1.0141
   -1.5938
    0.5946
   -1.1445
    0.7568
```

```
    -0.8928
```

```
% Energy of subband Coef. vector s
disp(['||s||_2^2 = ' num2str(norm(s,2).^2)])
```

```
||s||_2^2 = 7.0629
```

```
% Synthesis process
v = D*s;
disp(v)
```

```
    0.1361
    0.8693
    0.5797
    0.5499
```

```
% MSE evaluation
disp(['MSE = ', num2str(mymse(u, v))]);
```

```
MSE = 6.163e-33
```

γ に対するサブバンド係数のエネルギ変化

(Energy change of sub-band coefficient vector w.r.t. γ)

```
% Sweep gamma and evaluate energy of subband coefficient vectors
gammas = linspace(-1.0,2.0,32);
engs = zeros(length(gammas),1);
mses = zeros(length(gammas),1);
for idx = 1:length(gammas)
    % Analysis filters
    gamma = gammas(idx);
    delta = 1 - gamma;
    h0 = [ gamma  delta ];
    h1 = [ gamma -delta ];

    % (Circular) convolution matrix
    nH = max(length(h0),length(h1));
    X = [eye(nSamples) ; eye(nH-1) zeros(nH-1,nSamples-nH+1) ]; % Circular extension matrix
    C = [zeros(nSamples,nH-1) eye(nSamples) zeros(nSamples,nH-1)]; % Clipping matrix

    % Global matrix representation of analysis filter bank
    t0 = C*convmtx(h0.',nSamples+1)*X;
    t1 = C*convmtx(h1.',nSamples+1)*X;
    T = zeros(2*nSamples,nSamples);
    T(1:2:end,:) = t0;
    T(2:2:end,:) = t1;

    % Analysis process
    s = T*u;

    % Energy of subband Coef. vector s
    engs(idx) = norm(s,2).^2;

    % MSE evaluation
```

```
    v = D*s;
    mses(idx) = mymse(u, v);
end
```

グラフ描画 (Plot)

```
figure(1)
yyaxis left
plot(gammas,engs)
xlabel('\gamma')
ylabel('||{\bf s}||_2^2')
grid on
hold on
yyaxis right
plot(gammas,mses,':')
ylabel('MSE')
hold off
```