Sample 10-6

冗長変換

ℓ₁-ノルム最小化

画像処理特論

村松 正吾

動作確認: MATLAB R2020a

Redundant transforms

 ℓ_1 -norm minimization

Advanced Topics in Image Processing

Shogo MURAMATSU

Verified: MATLAB R2020a

準備

(Preparation)

```
close all
```

非線形近似の設定

(Settings of non-linear approximation)

```
% # of Coefs.
K = 32;
```

入力信号の生成

(Generation of input sequence)

```
% # of input samples
nSamples = 128;

% Random process in AR(1) model
rng('default');
w = 0.1*randn(nSamples,1);
w(floor(end/2)) = 1;
u = filter(1,[1 -0.95],w);
```

合成辞書

(Synthesis dictionary)

```
% Synthesis filters
```

```
f0 = [ 1 1 ]/2;
f1 = [ -1 1 ]/2;
% (Circular) convolution matrix
nF = max(length(f0),length(f1));
X = [zeros(nF-1,nSamples-nF+1) eye(nF-1); eye(nSamples)]; % Circular extension matrix
C = [zeros(nSamples,nF-1) eye(nSamples) zeros(nSamples,nF-1)]; % Clipping matrix
% Atoms in (circular) convolution matrix
d0 = C*convmtx(f0.',nSamples+nF-1)*X;
d1 = C*convmtx(f1.',nSamples+1)*X;
```

辞書 (Dictionary) D

```
% Dictionary D (Global matrix representation of synthesis filter bank)
D = zeros(nSamples,2*nSamples);
D(:,1:2:end) = d0;
D(:,2:2:end) = d1;
disp(D)
```

1 列から 32 列

0.5000	-0.5000	0	0	0	0	0	0	0	0	0
0.5000	0.5000	0.5000	-0.5000	0	0	0	0	0	0	0
0	0	0.5000	0.5000	0.5000	-0.5000	0	0	0	0	0
0	0	0	0	0.5000	0.5000	0.5000	-0.5000	0	0	0
0	0	0	0	0	0	0.5000	0.5000	0.5000	-0.5000	0
0	0	0	0	0	0	0	0	0.5000	0.5000	0.5000
0	0	0	0	0	0	0	0	0	0	0.5000
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

•	•	•	•	•	•	•	•	•	•	
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
U										
0	0	0	0	0	0	0	0	0	0	0
										•
0	0	0	0	0	0	0	0	0	0	0
0	α		0							0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
										•
0	0	0	0	0	0	0	0	0	0	0
a	0	0	0	0		0	0	0	0	0
0	0				0	0			0	0
0	0	0	0	0	0	0	0	0	0	0
										-
0	0	0	0	0	0	0	0	0	0	0
										0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
										-
0	0	0	0	0	0	0	0	0	0	0
										0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
										U
0	0	0	0	0	0	0	0	0	0	0
										_
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
										U
0	0	0	0	0	0	0	0	0	0	0
										0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
										U
0	0	0	0	0	0	0	0	0	0	0
										•
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
										V
0	0	0	0	0	0	0	0	0	0	0
										•
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
										U
0	0	0	0	0	0	0	0	0	0	0
										-
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
										U
0	0	0	0	0	0	0	0	0	0	0
										-
0	0	0	0	0	0	0	0	0	0	0
a	0	0	0	0	0	0	0	0	0	0
0			0			Ø				Ø
0	0	0	0	0	0	0	0	0	0	0
										-
0	0	0	0	0	0	0	0	0	0	0
a	0	0	0	0	0	0	0	0	0	0
0										0
0	0	0	0	0	0	0	0	0	0	0
										•
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
										U
0	0	0	0	0	0	0	0	0	0	0
^	^		^	•	^	•	^	•	^	
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
						· ·				O
0	0	0	0	0	0	0	0	0	0	0
		0	0	0			0			α
0	0	U		U	0	0		0	0	0 0 0
0	0	0	0	0	0	0	0	0	0	0
	0	C		0	Δ.			0	0	
0	0	0	0	0	0	0	0	0	0	О
0	0	0	0	0	0	0	0	0	0	а
										0 0 0 0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	Ω
		Ø		Ø	Ø	Ø	0	Ø		Ø
0	0	0	0	0	0	0	0	0	0	0
		•	•	-	•	-	•	-	•	-
0	0	0	0	0	0	0	0	0	0	0
								0		0
0	0	0	0	0	0	0	0	0	0	0 0 0 0
0	0	0	0	0	0	0	0	0	0	a
		•	•		•	•	•	•		-
0	0	0	0	0	0	0	0	0	0	0
0		0	0	ρ	0	0	0	0		Ω
	0	v	v	0	v	V		v	0	
0	0	0	0	0	0	0	0	0	0	0
				•						-
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0		0	0	0	0	Ω
		v			0					О
0	0	0	0	0	0	0	0	0	0	0
		^				0		0		0 0 0 0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
										Ð
0	0	0	0	0	0	0	0	0	0	0
										^
0	0	0	0	0	0	0	0	0	0	0

0	0	0 0	0	0 0	0	0 0	0	0 0	0	0	
0 0	0 0	0									
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0 0	0 0	0 0	0 0	0 0	0	0 0	0 0	0 0	0 0	0 0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
33 列から	64 列										
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0	0 0	0 0									
0 0.5000	-0.5000	0	0	0	0	0	0	0	0	0	
0.5000	0.5000	0.5000	-0.5000	0	0	0	0	0	0	0	
0	0	0.5000	0.5000	0.5000	-0.5000	0	0	0	0	0	
0	0	0	0	0.5000	0.5000	0.5000	-0.5000	0	0	0	
0	0	0	0	0	0	0.5000	0.5000	0.5000	-0.5000	0	
0	0	0	0	0	0	0	0	0.5000	0.5000	0.5000	-
0	0	0	0	0	0	0	0	0	0	0.5000	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0 0	0	0	0	0 0	0	0	
0 0	0 0	0 0	0 0	0	0 0	0 0	0 0	0	0 0	0 0	
0	0	0	0	0	0	0	0	0	0	0	
0	0	0	0	0	0	0	0	0	0	0	
	0	0	0	0	0	0	0	0	0	0	
0		-	-								
0 0	0	0	0	0	0	0	0	0	0	0	
		0 0	0 0	0 0	0 0	0 0	0 0	0 0	0	0 0	

ℓ_1 -ノルム最小化による非線形近似

(Non-linear approximation with ℓ_1 -norm minimization)

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s} \in \mathbb{R}^L} \|\mathbf{s}\|_1 \text{ s.t. } \mathbf{v} = \mathbf{D}\mathbf{s}$$

凸緩和法による分析処理と係数選択 (Analysis process and coefficient selection by a convex-relaxation)

• 基底追跡法 (Basis Pursuit; BP)

線形計画問題に帰着させる. (Reduced to a linear programming problem)

$$\begin{split} \widehat{\mathbf{x}} &= \arg\min_{\mathbf{x} \in \mathbb{R}^{2L}} \mathbf{1}^T \mathbf{x} \text{ s.t. } \mathbf{v} = \mathbf{D}(\mathbf{I} - \mathbf{I}) \mathbf{x} \cap \mathbf{x} \in [0, \infty)^{2L} \\ & t = t \in \mathbf{U}, \text{ (where)} \\ & \mathbf{s} = \mathbf{s}_+ - \mathbf{s}_- \in \mathbb{R}^L \\ & \mathbf{x} = \begin{pmatrix} \mathbf{s}_+ \\ \mathbf{s}_- \end{pmatrix} \in \mathbb{R}^{2L} \end{split}$$

```
% Initialization
M = size(D,2);
f = ones(2*M,1);
lu = zeros(2*M,1);
% Linear programming
z = linprog(f,[],[],[D -D],u,lu,[]);
```

最適解が見つかりました。

```
s = z(1:M) - z(M+1:end);
```

係数選択 (Coefficient selection)

```
s = s(:);
[~,ix] = sort(abs(s),'descend');
s(ix(K+1:end)) = 0;
```

近似結果 (Approximation result)

```
v = D*s;
```

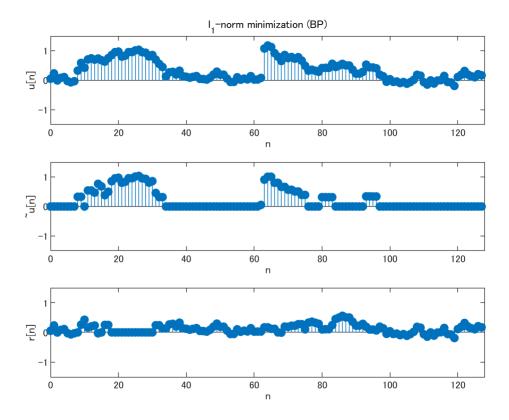
近似誤差 (Residual)

```
r = u - v;
```

グラフ描画

(Graph plot)

```
figure(1)
% Input
subplot(3,1,1)
stem(0:nSamples-1,u,'filled')
axis([0 nSamples -1.5 1.5])
xlabel('n')
ylabel('u[n]')
title('l_1-norm minimization (BP)')
% NLA
subplot(3,1,2)
stem(0:nSamples-1,v,'filled')
axis([0 nSamples -1.5 1.5])
xlabel('n')
ylabel('~u[n]')
% Residual
subplot(3,1,3)
stem(0:nSamples-1,r,'filled')
axis([0 nSamples -1.5 1.5])
xlabel('n')
ylabel('r[n]')
```



MSE 評価 (MSE evaluation)

mse = 0.035184

© Copyright, Shogo MURAMATSU, All rights reserved.