Sample 5-5

周波数解析

多変量畳み込み

画像処理特論

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動作確認: MATLAB R2020a

Fourier analysis

Multivariate convolution

Advanced Topics in Image Processing

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Verified: MATLAB R2020a

準備

(Preparation)

```
close all
```

サンプル画像 $\{u[\mathbf{n}]\}_\mathbf{n}$ の準備

(Preparation of sample image $\{u[\mathbf{n}]\}_\mathbf{n}$)

```
% Reading original image
u = im2double(imread('cameraman.tif'));
figure(1)
imshow(u)
title('Original')
```

線形シフト不変システムのインパルス応答 $\{h[\mathbf{n}]\}_\mathbf{n}$

(Impulse response of a linear shift-invariant system $\{h[\mathbf{n}]\}_\mathbf{n}$

```
% Impulse response h[n]
ftype = "gaussian";
h = rot90(fspecial(ftype),2);
```

線形シフト不変システムの出力応答 $\{v[\mathbf{n}]\}_\mathbf{n}$

(The linear shift-invariant system response $\{v[\mathbf{n}]\}_\mathbf{n}$)

畳み込み演算 (Convolution)

$$\{v[\mathbf{n}]\}_\mathbf{n} = \{h[\mathbf{n}]\}_\mathbf{n} * \{u[\mathbf{n}]\}_\mathbf{n} = \sum_{\mathbf{k}\in\Omega\subset\mathbb{Z}^2} u[\mathbf{k}]\{h[\mathbf{n}-\mathbf{k}]\}_\mathbf{n}$$

```
% Output v[n]
v = imfilter(u,h,'conv','full');
```

入力信号 $\{u[\mathbf{n}]\}_\mathbf{n}$ のスペクトル

(Spectrum of input signal $\{u[\mathbf{n}]\}_\mathbf{n}$)

$$U\left(e^{\jmath\boldsymbol{\omega}^T}\right) = \sum_{\mathbf{n}\in\mathbb{Z}^2} u[\mathbf{n}]e^{-\jmath\boldsymbol{\omega}^T n},\ \boldsymbol{\omega}\in\mathbb{R}^2$$

DFT(FFT)によるDSFTの周波数サンプル計算 (Frequency sampling of DSFT by DFT (FFT))

$$U[\mathbf{k}] = U\left(e^{j\boldsymbol{\omega}^T}\right)|_{\boldsymbol{\omega}=2\pi\mathbf{Q}^{-T}\mathbf{k}},\ \mathbf{k}\in\mathcal{N}(\mathbf{Q}^T)$$

関数FFTNを利用．(The function FFTN is used.)

```
% Setting the number of frequency sample points in [0,2π)^2
nPoints = pow2(nextpow2(size(u)+size(h)-1)); % The least power of two that matches the normal c

% Spectrum of u[n]
U = fftn(u,nPoints);
```

フィルタ $\{h[\mathbf{n}]\}_\mathbf{n}$ の周波数応答

(Frequency response of filter $\{h[\mathbf{n}]\}_\mathbf{n}$)

$$H\left(e^{\jmath\boldsymbol{\omega}^T}\right) = \sum_{\mathbf{n}\in\mathbb{Z}^2} h[\mathbf{n}]e^{-\jmath\boldsymbol{\omega}^T n},\ \boldsymbol{\omega}\in\mathbb{R}^2$$

DFT(FFT)によるDSFTの周波数サンプル計算 (Frequency sampling of DSFT by DFT (FFT))

$$H[\mathbf{k}] = H\left(e^{j\boldsymbol{\omega}^T}\right)|_{\boldsymbol{\omega}=2\pi\mathbf{Q}^{-T}\mathbf{k}},\ \mathbf{k}\in\mathcal{N}(\mathbf{Q}^T)$$

```
% Frequency response of h[n]
```

```
H = fftn(h,nPoints);
```

出力信号 $\{v[\mathbf{n}]\}_{\mathbf{n}}$ のスペクトル

(Spectrum of input signal $\{v[\mathbf{n}]\}_{\mathbf{n}}$)

$$V\left(e^{\jmath\boldsymbol{\omega}^T}\right) = \sum_{\mathbf{n}\in\mathbb{Z}^2} v[\mathbf{n}]e^{-\jmath\boldsymbol{\omega}^T n}, \ \boldsymbol{\omega}\in\mathbb{R}^2$$

DFT(FFT)によるDSFTの周波数サンプル計算 (Frequency sampling of DSFT by DFT (FFT))

$$V[\mathbf{k}] = V\left(e^{j\boldsymbol{\omega}^T}\right)\Big|_{\boldsymbol{\omega}=2\pi\mathbf{Q}^{-T}\mathbf{k}}, \ \mathbf{k}\in\mathcal{N}(\mathbf{Q}^T)$$

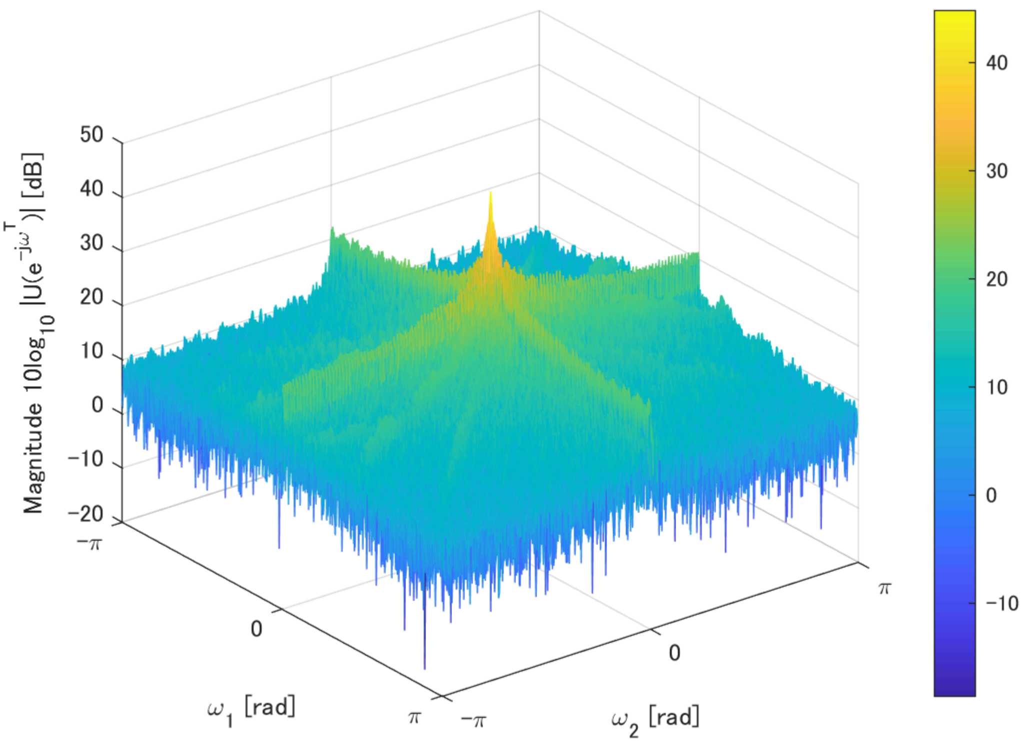
```
% Frequency response of v[n]
V = fftn(v,nPoints);
```

スペクトルと周波数応答の表示

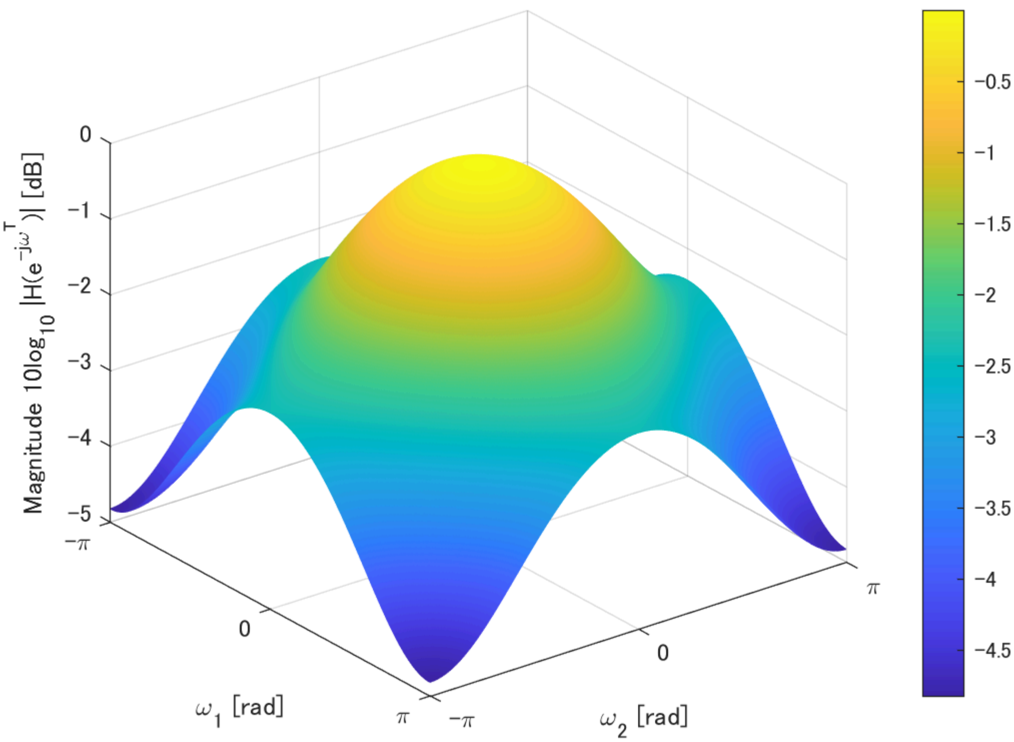
(Display of spectra and frequency response)

$$V\left(e^{\jmath\boldsymbol{\omega}^T}\right) = H\left(e^{\jmath\boldsymbol{\omega}^T}\right)U\left(e^{\jmath\boldsymbol{\omega}^T}\right)$$

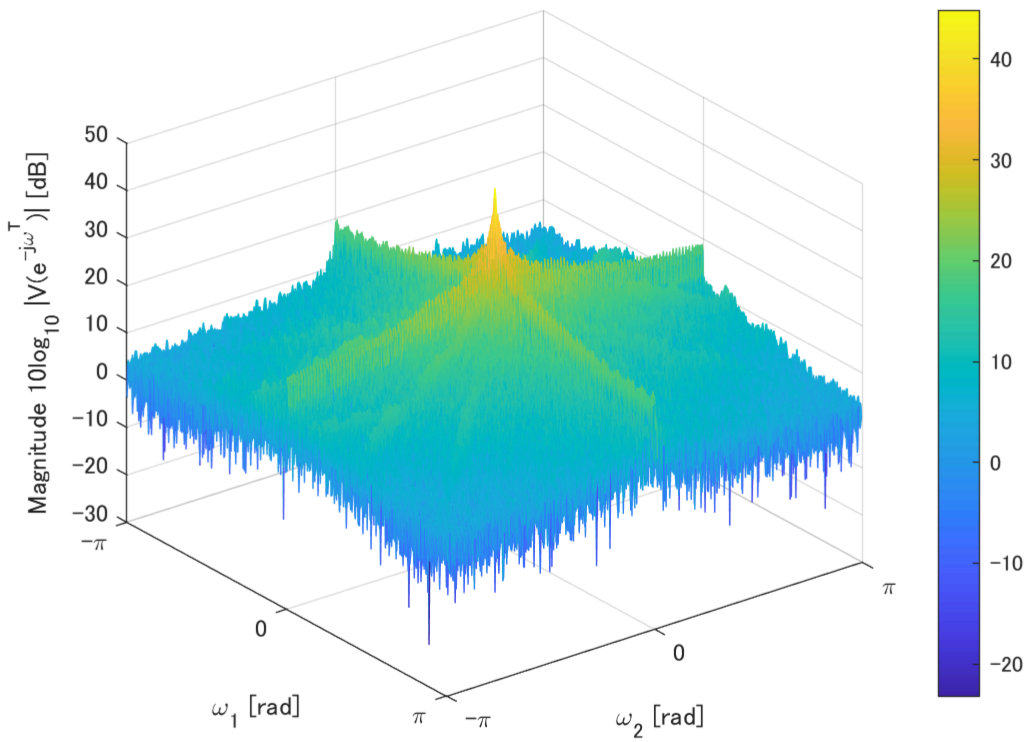
```
% Frequency sample points
[w1,w2] = meshgrid(-pi:2*pi/nPoints(1):pi-2*pi/nPoints(1),-pi:2*pi/nPoints(2):pi-2*pi/nPoints(2

% Display of spectrum U
figure(2)
mesh(w1,w2,10*log10(abs(fftshift(U))))
ax = gca;
xlabel('\omega_2 [rad]')
ylabel('\omega_1 [rad]')
zlabel('Magnitude 10log_{10} |U(e^{-j\omega^T})| [dB]')
axis ij
ax.XLim = [-pi pi];
ax.XTick = [ -pi 0 pi ];
ax.XTickLabel = { '-\pi', '0', '\pi'};
ax.YLim = [-pi pi];
ax.YTick = [ -pi 0 pi ];
ax.YTickLabel = { '-\pi', '0', '\pi'};
colorbar(ax)
```




```
% Display of Freq. response H
figure(3)
mesh(w1,w2,10*log10(abs(fftshift(H))))
ax = gca;
xlabel('\omega_2 [rad]')
ylabel('\omega_1 [rad]')
zlabel('Magnitude 10log_{10} |H(e^{-j\omega^T})| [dB]')
axis ij
ax.XLim = [-pi pi];
ax.XTick = [ -pi 0 pi ];
ax.XTickLabel = { '-\pi', '0', '\pi'};
ax.YLim = [-pi pi];
ax.YTick = [ -pi 0 pi ];
ax.YTickLabel = { '-\pi', '0', '\pi'};
colorbar(ax)
```




```
% Display of Freq. response V
figure(4)
mesh(w1,w2,10*log10(abs(fftshift(V))))
ax = gca;
xlabel('\omega_2 [rad]')
ylabel('\omega_1 [rad]')
zlabel('Magnitude 10log_{10} |V(e^{-j\omega^T})| [dB]')
axis ij
ax.XLim = [-pi pi];
ax.XTick = [ -pi 0 pi ];
ax.XTickLabel = { '-\pi', '0', '\pi'};
ax.YLim = [-pi pi];
ax.YTick = [ -pi 0 pi ];
ax.YTickLabel = { '-\pi', '0', '\pi'};
colorbar(ax)
```



線形シフト不変システムの解析

(Analysis of linear shift-invariant systems)

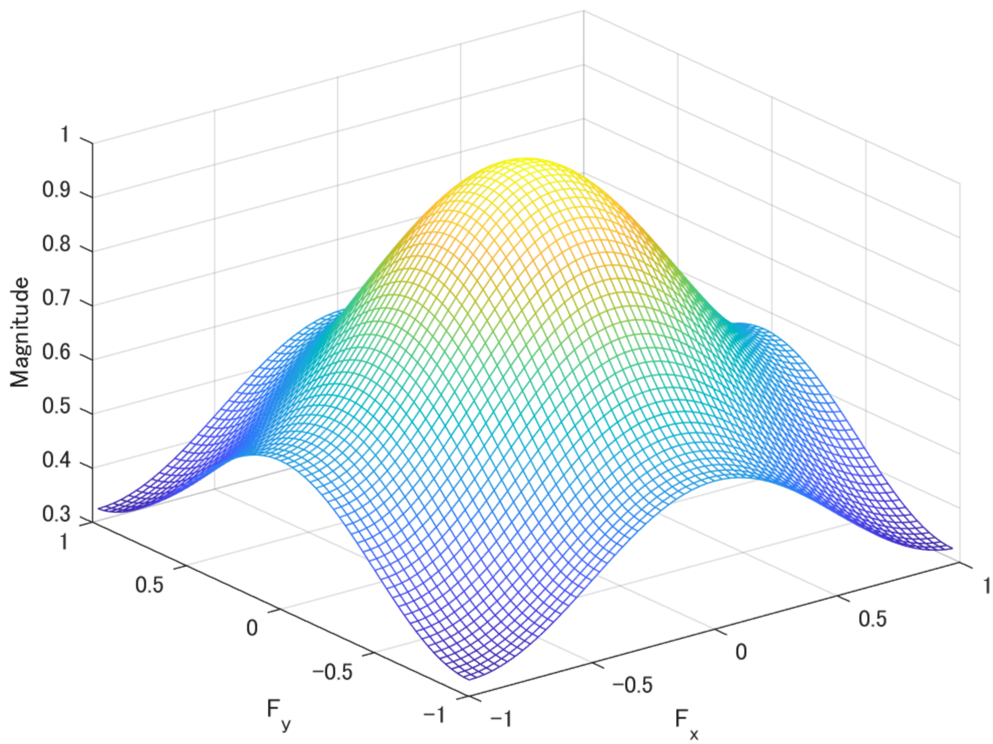
周波数応答(Frequency response)の表示

$$H\left(e^{j\boldsymbol{\omega}^T}\right) = H(\mathbf{z})\big|_{\mathbf{z}=e^{j\boldsymbol{\omega}^T}},\ \boldsymbol{\omega} \in \mathbb{R}^2$$

- 振幅応答 (Magnitude response): $\left|H\left(e^{j\boldsymbol{\omega}^T}\right)\right|$
- 位相応答 (Phase response): $\angle H\left(e^{j\boldsymbol{\omega}^T}\right)$

2変量周波数振幅応答の表示関数FREQZ2を利用．(Using FREQZ2, a display function for bivariate frequency magnitude responses.)

```
figure(5)
freqz2(h)
```


Magnitude
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
1
0.5
0
−0.5
−1
F
y
−1
−0.5
0
0.5
1
F
x