Sample 5-2

周波数解析

単変量循環畳み込み

画像処理特論

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動作確認: MATLAB R2023a

Fourier analysis

Univariate circular convolution

Advanced Topics in Image Processing

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Verified: MATLAB R2023a

準備

(Preparation)

```
close all
```

入力信号 $\{u[n]\}_n$

(Input signal $\{u[n]\}_n$)

```
% Input x[n]
u = [1 2 3];
```

線形シフト不変システムのインパルス応答 $\{h[n]\}_n$

(Impulse response of a linear shift-invariant system $\{h[n]\}_n$)

```
% Impulse response h[n]
h = [1 1 1]/3;
```

周期 N の循環畳み込みの出力応答 $\{v[n]\}_n$

(Output response $\{v[n]\}_n$ of circular convolution with period N)

循環畳み込み演算 (Circular convolution)

$$\{v[n]\}_n = \{h[n]\}_n \bigcirc \{u[n]\}_n = \sum_{k=-\infty}^{\infty} u[k]\{h[((n-k))_N]\}_n$$

```
% Setting the period N
nZeroPadding =0;
nPeriod = max(length(u),length(h)) + nZeroPadding;

% Output v[n]
v = cconv(h,u,nPeriod);
```

畳み込み演算との比較

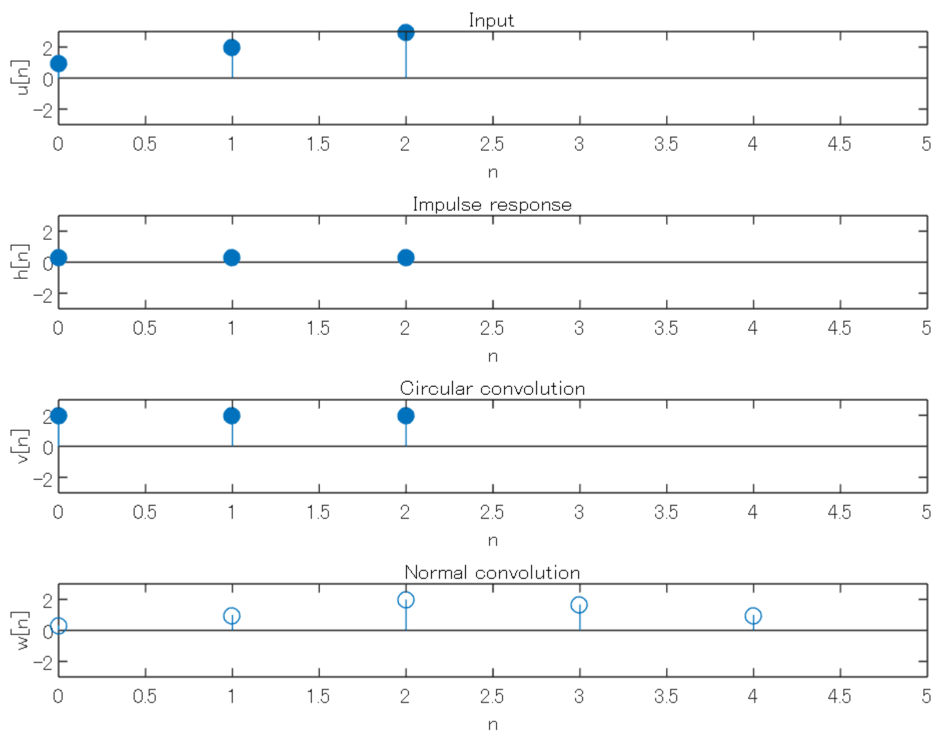
(Comparison with convolution)

```
% Normal convolution
w = conv(h,u);

% Lengths of u, h, x and v
nu = length(u);
nh = length(h);
nv = length(v);
nw = length(w);

% Display sequences
figure(1)
nx = max(length(v),length(w));
amax = max(max(abs(u)),max(abs(v)));
% u[n]
subplot(4,1,1)
stem(0:nu-1,u,'filled')
axis([0 nx -amax amax])
xlabel('n')
ylabel('u[n]')
title('Input')
% h[n]
subplot(4,1,2)
stem(0:nh-1,h,'filled')
axis([0 nx -amax amax])
xlabel('n')
ylabel('h[n]')
title('Impulse response')
% v[n]
subplot(4,1,3)
stem(0:nv-1,v,'filled')
axis([0 nx -amax amax])
xlabel('n')
ylabel('v[n]')
title('Circular convolution')
% w[n]
subplot(4,1,4)
stem(0:nw-1,w)
axis([0 nx -amax amax])
xlabel('n')
```

```
ylabel('w[n]')
title('Normal convolution')
```



通常の畳み込みと循環畳み込みが一致する条件

(The condition that normal convolution and circular convolution match)

$L_v = L_h + L_u - 1 \leq N$

ただし、(where)

- L_v: 出力の長さ (Output length)
- L_h: インパルス応答の長さ (Length of impulse response)
- L_u: 入力の長さ (Input length)

```
% Adjusting the lengths
mymse = @(x,y) mean((double(x)-double(y)).^2,'all');
if nv > nw
    wc = [w(:); zeros(nv-nw,1)];
    vc = v(:);
else
    vc = [v(:); zeros(nw-nv,1)];
    wc = w(:);
end
% Check the condition
if (nw <= nPeriod)
    msg = 'TRUE';
```

```
else
    msg = 'FALSE (Increse # of zeros for padding)';
end
fprintf('Condition for CCONV to match CONV: \n\t %s', msg)
```

```
Condition for CCONV to match CONV:
     FALSE (Increse # of zeros for padding)
```

```
% MSE
fprintf('MSE: %f', mymse(vc,wc))
```

```
MSE: 1.511111
```

入力信号 $\{u[n]\}_n$ の DFT

(DFT of input signal $\{u[n]\}_n$)

$$U[k] = \sum_{n=0}^{N-1} u[n]W_N^{nk}, k \in \{0, 1, 2, \cdots, N-1\}$$

ただし，(where)

$$W_N = e^{-\mathrm{j}\frac{2\pi}{N}}$$

```
% DFT of u[n]
U = fft(u,nPeriod);
```

フィルタ $\{h[n]\}_n$ の DFT

(DFT of impulse response $\{h[n]\}_n$)

$$H[k] = \sum_{n=0}^{N-1} h[n]W_N^{nk}, k \in \{0, 1, 2, \cdots, N-1\}$$

```
% DFT of h[n]
H = fft(h,nPeriod);
```

出力信号 $\{v[n]\}_n$ の DFT

(DFT of output signal $\{v[n]\}_n$)

$$V[k] = \sum_{n=0}^{N-1} v[n]W_N^{nk}, k \in \{0, 1, 2, \cdots, N-1\}$$

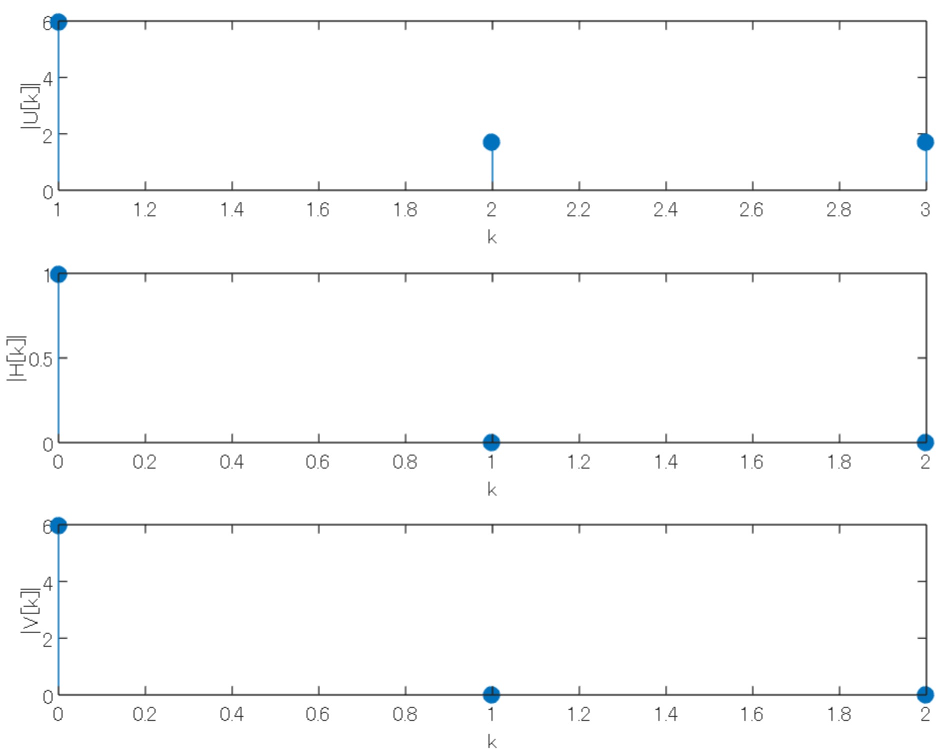
```
% Frequency response of v[n]
V = fft(v,nPeriod);
```

DFT の表示

(Display of DFTs)

```
% DFT index
k = 0:nPeriod-1;

% Display of DFTs
figure(2)
subplot(3,1,1)
stem(abs(U),'filled')
xlabel('k')
ylabel('|U[k]|')
subplot(3,1,2)
stem(k,abs(H),'filled')
xlabel('k')
ylabel('|H[k]|')
subplot(3,1,3)
stem(k,abs(V),'filled')
xlabel('k')
ylabel('|V[k]|')
```



DFT 積

(DFT product)

$V[k] = H[k]U[k],\ \ k \in \{0, 1, 2, \cdots, N-1\}$

循環畳み込みとの比較 (Comparison with circular convolution)

```
% IDFT of DFT product
y = ifft(H.*U);
% MSE with the cconv result 'v'
fprintf('MSE: %f', mymse(v,y))
```

```
MSE: 0.000000
```