# Sample 11-6

## 画像ノイズ除去

近接勾配法

画像処理特論

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動作確認: MATLAB R2023a

## Image denoising

Proximal gradient

Advanced Topics in Image Processing

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Verified: MATLAB R2023a

### 準備

(Preparation)

```
clear
close all
import msip.download_img
msip.download_img
```

```
kodim01.png already exists in ./data/
kodim02.png already exists in ./data/
kodim03.png already exists in ./data/
kodim04.png already exists in ./data/
kodim05.png already exists in ./data/
kodim06.png already exists in ./data/
kodim07.png already exists in ./data/
kodim08.png already exists in ./data/
kodim09.png already exists in ./data/
kodim10.png already exists in ./data/
kodim11.png already exists in ./data/
kodim12.png already exists in ./data/
kodim13.png already exists in ./data/
kodim14.png already exists in ./data/
kodim15.png already exists in ./data/
kodim16.png already exists in ./data/
kodim17.png already exists in ./data/
kodim18.png already exists in ./data/
kodim19.png already exists in ./data/
kodim20.png already exists in ./data/
kodim21.png already exists in ./data/
kodim22.png already exists in ./data/
kodim23.png already exists in ./data/
kodim24.png already exists in ./data/
See Kodak Lossless True Color Image Suite
```

# 問題設定

(Problem settings)

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}} \frac{1}{2}\|\mathbf{v} - \mathbf{D}\mathbf{s}\|_2^2 + \lambda\|\mathbf{s}\|_1$$

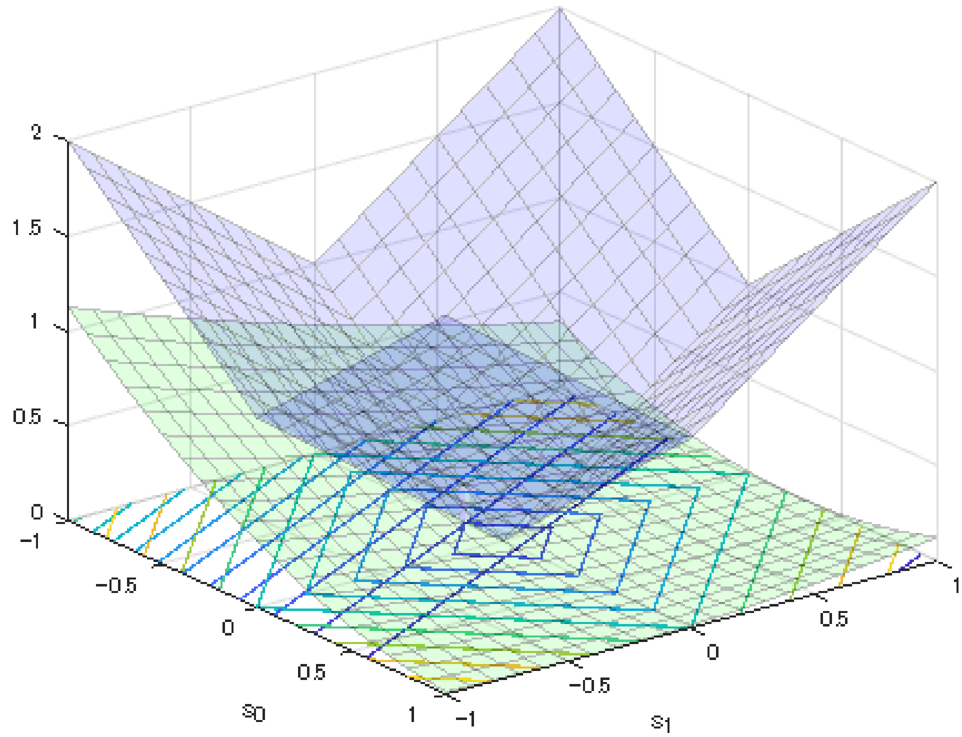
- $\mathbf{D} = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \end{pmatrix}: \quad \mathbb{R}^2 \to \mathbb{R}^1$
- $\mathbf{v} = \frac{1}{2} \in \mathbb{R}^1$
- $\lambda \in [0, \infty)$
- $\mathbf{s} \in \mathbb{R}^2$

```
D = [2 1]/3;
v = 0.5;
```

# 関数プロット

(Function plot)

```
% Function settings
f = @(s0,s1) 0.5*(v-(D(1)*s0+D(2)*s1)).^2;
g = @(s0,s1) (abs(s0)+abs(s1));
% Variable settings
s0 = linspace(-1,1,21);
s1 = linspace(-1,1,21);
[S0,S1] = ndgrid(s0,s1);
F = f(S0,S1);
G = g(S0,S1);
% Surfc plot of the fidelity
figure
hf = surfc(s0,s1,F);
hf(1).FaceAlpha = 0.125;
hf(1).FaceColor = 'green';
hf(1).EdgeAlpha = 0.25;
hf(2).LineWidth = 1;
set(gca,'YDir','reverse');
hold on
% Surfc plot of the regularizer
hg = surfc(s0,s1,G);
hg(1).FaceAlpha = 0.125;
hg(1).FaceColor = 'blue';
hg(1).EdgeAlpha = 0.25;
hg(2).LineWidth = 1;
xlabel('s_1')
ylabel('s_0')
hold off
```




## パラメータ設定

(Parameter settings)

```
lambda = 0.2;
gamma = 0.4;
niters = 20;
```

# $\ell_1$ -ノルム正則化最小自乗法による近似

($\ell_1$ -norm-regularized least square method)

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}} \frac{1}{2}\|\mathbf{v} - \mathbf{D}\mathbf{s}\|_2^2 + \lambda\|\mathbf{s}\|_1$$

近接勾配法に帰着させる．(Reduced to a proximal gradient method)

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}\in V} f(\mathbf{x}) + g(\mathbf{x})$$

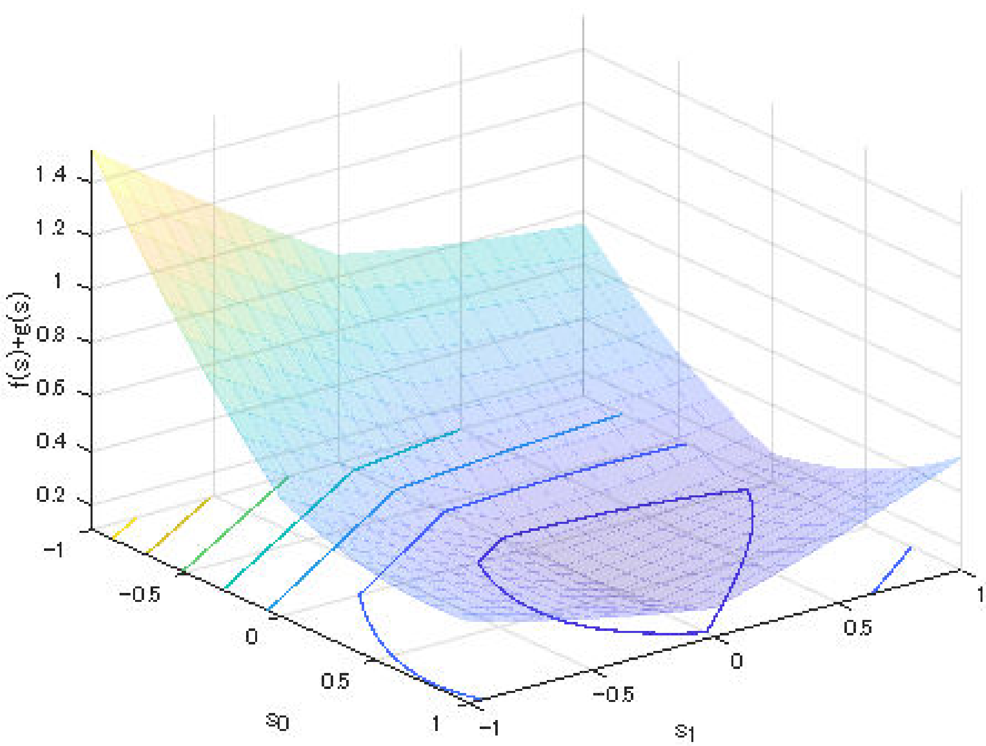
- $f(\cdot), g(\cdot) \in \Gamma_0(\mathbb{R}^L)$: Convex functions
- $f(\cdot)$ is differentiable ( $\beta$-Lipschitz continuous)
- $\Gamma_0(\mathbb{R}^L)$ : Set of proper semi-lower-continuous convex functions

【Example】

- $f(\mathbf{s}) = \frac{1}{2}\|\mathbf{v} - \mathbf{D}\mathbf{s}\|_2^2$
- $g(\mathbf{s}) = \lambda\|\mathbf{s}\|_1$

関数プロット (Function plot)

```
% Function setting
fg = @(s0,s1) 0.5*(v-(D(1)*s0+D(2)*s1)).^2 + lambda*(abs(s0)+abs(s1));
% Surfc plot of cost function f+g
figure
J = fg(S0,S1);
hf = surfc(s0,s1,J);
hf(1).FaceAlpha = 0.25;
hf(1).EdgeAlpha = 0.25;
hf(1).EdgeColor = 'interp';
hf(2).LineWidth = 1;
set(gca,'YDir','reverse')
ylabel('s_0')
xlabel('s_1')
zlabel('f(s)+g(s)')
hold on
```



## 近接勾配法

(Proximal gradient method)

1. Initialization: $\mathbf{x}^{(0)}$, $t \leftarrow 0$
2. Proximal gradient descent: $\mathbf{x}^{(t+1)} \leftarrow \mathrm{prox}_{\gamma g}\left(\mathbf{x}^{(t)} - \gamma \nabla_{\mathbf{x}} f(\mathbf{x}^{(t)})\right)$
3. If a stopping critera is satisfied then finish, otherwise $t \rightarrow t+1$ and go to Step 2.

【Example】

- $\nabla_{\mathbf{s}} f(\mathbf{s}) = \mathbf{D}^T(\mathbf{D}\mathbf{s} - \mathbf{v})$
- $\mathrm{prox}_{\gamma\lambda\|\cdot\|_1}(\mathbf{s}) = \mathcal{T}_{\gamma\lambda}(\mathbf{s}) = \mathrm{sign}(\mathbf{s}) \odot \max(\mathrm{abs}(\mathbf{s}) - \gamma\lambda\mathbf{1}, \mathbf{0})$

ソフト閾値処理 (Soft-thresholding)

```
softthresh = @(x,t) sign(x).*max(abs(x)-t,0);
```

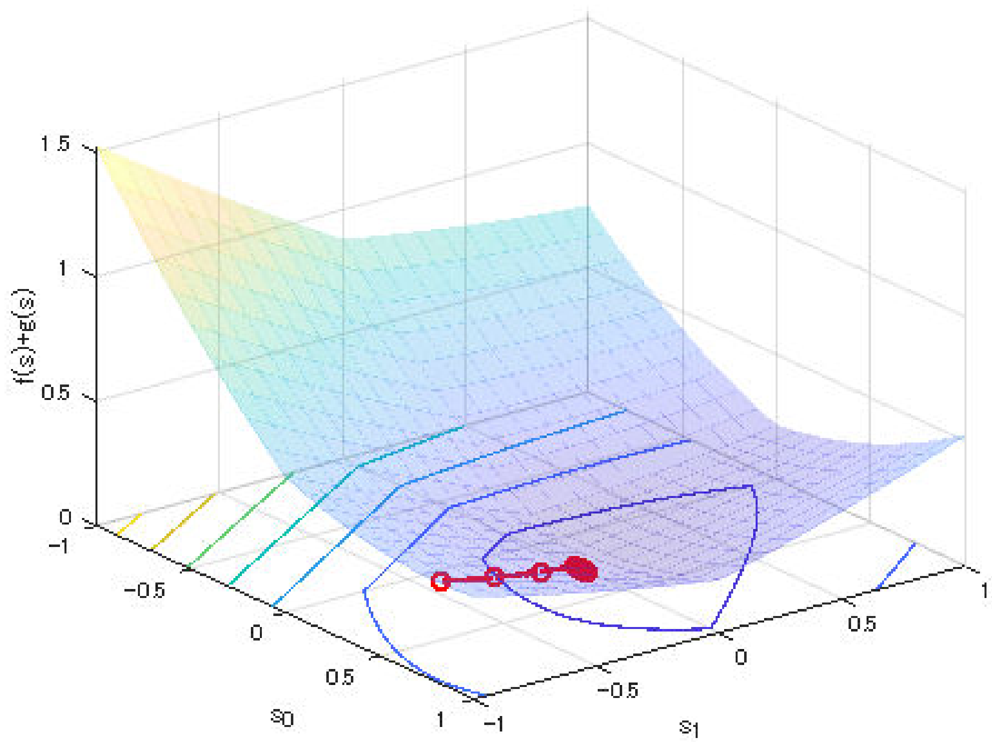
初期化 (Initialization)

```
sp = 2*rand(2,1)-1; % in [-1,1]^2
```

近接勾配降下 (Proximal gradient descent)

```
beta = D*D';
assert(gamma < 2/beta,'Step size condition is violated.')
for idx=0:niters-1
    % Preious state
    s(1,1) = sp(1); % s0
    s(2,1) = sp(2); % s1
    % Proximal gradient descent
    sc = softthresh(sp-gamma*D'*(D*sp-v),gamma*lambda);
    % sc = sign(c).*max(abs(c)-gamma*lambda,0);
    % Current state
    s(1,2) = sc(1); % s0
    s(2,2) = sc(2); % s1
    % Quiver plot
    xp = s(2,1);
    yp = s(1,1);
    xn = s(2,2);
    yn = s(1,2);
    hp = quiver(xp,yp,xn-xp,yn-yp);
    hp.Marker = 'o';
    hp.ShowArrowHead = 'on';
    hp.MaxHeadSize = 120;
    hp.MarkerSize = 6;
    hp.MarkerEdgeColor = 'r';
    hp.Color = 'r';
    hp.LineWidth = 2;
```

```
    % Update
    sp = sc;
end
hold off
```




## パラメータ設定

(Parameter settings)

- sgm: ノイズ標準偏差 $\sigma_w$ (Standard deviation of noise)
- nlevels: ウェーブレット段数 (Wavelet levels)

```
% Parameter settings
isaprxleft = true;
lambda = 10^-1
```

```
lambda = 0.1000
```

```
gamma = 10^-0.1
```

```
gamma = 0.7943
```

```
sgmuint8 = 10;
sgm = sgmuint8/255;
nlevels = 3;
niters = 80;
```

## 画像の読込

(Read image)

```
u = im2double(imread('./data/kodim23.png'));
if size(u,3) == 3
    u = rgb2gray(u);
end
```

## 観測画像

(Observation image)

- $\mathbf{v} = \mathbf{u} + \mathbf{w}$
- $\mathbf{u} = \mathbf{D}\mathbf{s}$
- $\mathbf{s} \sim \mathrm{Lap}(\mathbf{s}|\boldsymbol{\mu} = \mathbf{0}, b)$
- $\mathbf{w} \sim \mathrm{Norm}\left(\mathbf{w}|\boldsymbol{\mu}_w = \mathbf{0}, \sigma_w^2\mathbf{I}\right)$

```
v = imnoise(u,'gaussian',0,sgm^2);
```

### 非間引きハール DWT

(Undecimated Haar DWT)

```
import msip.udhaarwtdec2
import msip.udhaarwtrec2
```

**完全再構成の確認 (Checki the perfect reconstruction)**

非間引きハール DWT はパーセバルタイト性 (The undecimated DWT satisfies the Parseval tight property,)

$\mathbf{D}\mathbf{D}^T = \mathbf{I}$

を満たすため，$\mathbf{D}$ の転置システムは完全再構成分析システムとなり得る．(and thus Its transposition system can be a PR analysis system.)

```
[coefs,scales] = udhaarwtdec2(v,nlevels);
r = udhaarwtrec2(coefs,scales);
assert(norm(v-r,"fro")^2/numel(v)<1e-18,'Perfect reconstruction is violated.')
```

合成辞書と転置辞書の定義 (Definition of synthesis dictionary  and its adjoint)

```
% Definiton of dictionay and its adjoint
adjdic = @(x) udhaarwtdec2(x,nlevels); % D
syndic = @(x) udhaarwtrec2(x,scales);  % D.'
```

### 近接勾配法

(Proximal gradient method)

初期化 (Initialization)

```
[coefs,scales] = udhaarwtdec2(v,nlevels);
sp = coefs;
```

近接勾配降下 (Proximal gradient descent)

```
if isaprxleft
    mask = ones(size(coefs));
    mask(1:prod(scales(1,:))) = 0;
    lambda = lambda * mask;
end
for idx=0:niters-1
    % Proximal gradient descent
    sc = softthresh(sp-gamma*adjdic(syndic(sp)-v),gamma*lambda);
    % Update
    sp = sc;
end
```

## ノイズ除去画像

(Denoised image)

```
r = syndic(sc);
```

## 画像表示

(Image show)

```
figure
imshow(u);
title('Original image u')
```


```
figure
imshow(v)
title(sprintf('Noisy image v : PSNR = %5.2f [dB]',psnr(u,v)))
```


```
figure
imshow(r)
title(sprintf('Denoised image r：PSNR = %5.2f [dB]',psnr(u,r)))
```

Denoised image r:PSNR = 31.24 [dB]

## ウェーブレット画像ノイズ除去関数

(Wavelet image denoising function)

参考資料 (Reference)

```
iswtb = license('checkout','wavelet_toolbox');
if iswtb
    help wdenoise2
end
```

```
 wdenoise2 - Wavelet image denoising
    This MATLAB function denoises the grayscale or RGB image IM using an
    empirical Bayesian method.

    構文
      IMDEN = wdenoise2(IM)
      IMDEN = wdenoise2(IM,LEVEL)
      [IMDEN,DENOISEDCFS] = wdenoise2(___)

      [IMDEN,DENOISEDCFS,ORIGCFS] = wdenoise2(___)
      [IMDEN,DENOISEDCFS,ORIGCFS,S] = wdenoise2(___)
      [IMDEN,DENOISEDCFS,ORIGCFS,S,SHIFTS] = wdenoise2(___)

      [___] = wdenoise2(___,Name,Value)

      wdenoise2(___)

    入力引数
      IM - Input image
```

```
    real-valued 2-D matrix | real-valued 3-D array
  LEVEL - Wavelet decomposition level
    positive integer

名前と値の引数
  Wavelet - Name of wavelet
    'bior4.4' (既定の設定) | character vector | string scalar
  DenoisingMethod - Denoising method
    'Bayes' (既定の設定) | 'FDR' | 'Minimax' | 'SURE' |
    'UniversalThreshold'
  ThresholdRule - Threshold rule
    'Hard' | 'Soft' | 'Mean' | 'Median'
  NoiseEstimate - Method of estimating variance of noise
    'LevelIndependent' (既定の設定) | 'LevelDependent'
  NoiseDirection - Wavelet subbands
    ["h","v","d"] (既定の設定) | string vector | scalar string
  CycleSpinning - Number of circular shifts
    0 (既定の設定) | nonnegative integer
  ColorSpace - Color space
    'PCA' (既定の設定) | 'Original'

出力引数
  IMDEN - Denoised image
    real-valued matrix
  DENOISEDCFS - Scaling and denoised wavelet coefficients
    real-valued matrix
  ORIGCFS - Scaling and wavelet coefficients
    real-valued matrix
  S - Bookkeeping matrix
    integer-valued matrix
  SHIFTS - Image shifts
    integer-valued matrix

例
  Denoise Grayscale Image Using Default Settings
  Denoise Color Image Using Cycle Spinning
  Denoise Image Using Specific Subband

参考 wdenoise, wavedec2

R2019a の Wavelet Toolbox で導入
wdenoise2 のドキュメンテーション
```