Sample 7-5

幾何学処理

畳み込みの随伴作用素

画像処理特論

村松 正吾

動作確認: MATLAB R2023a

Geometric image processing

Adjoint of convolution

Advanced Topics in Image Processing

Shogo MURAMATSU

Verified: MATLAB R2023a

準備

(Preparation)

```
close all
```

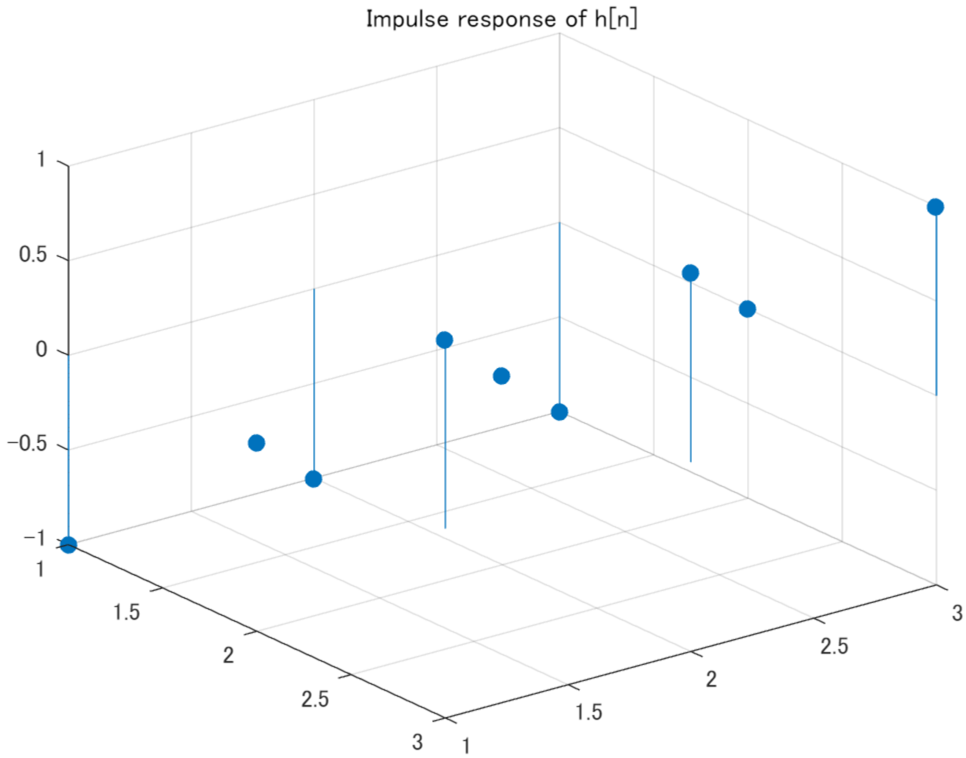
インパルス応答の生成

(Generation of impulse response)

```
ftype = "prewitt";
h = rot90(fspecial(ftype),2)
```

```
h = 3×3
    -1    -1    -1
     0     0     0
     1     1     1
```

```
figure(1)
stem3(h,'filled')
axis ij
title('Impulse response of h[n]')
```



二変量循環畳み込みの行列表現

(Matrix representation of bivariate circular convolution)

周期 $\mathbf{Q}$ の循環畳み込み演算 (Circular convolution with period $\mathbf{Q}$)

$$\{v[\mathbf{n}]\}_{\mathbf{n}} = \{h[\mathbf{n}]\}_{\mathbf{n}} \bigcirc \{u[\mathbf{n}]\}_{\mathbf{n}} = \sum_{\mathbf{k}\in\Omega\subset\mathbb{Z}^2} h[\mathbf{k}]\{u[((\mathbf{n}-\mathbf{k}))_{\mathbf{Q}}]\}_{\mathbf{n}}$$

```
% Input array size
N1 =6;
N2 =4;

% Find the matrix representation of the bivariate downsampling
N  = N1*N2;
T = [];
for idx = 1:N
    % Generating a standard basis vector
    e = zeros(N1,N2);
    e(idx) = 1;
    % Response to the standard basis vector
    t = imfilter(e,h,'conv','circ');
    T(:,idx) = t(:);
end
```

行列表現 (Matrix represntation)

- $\mathbf{T}$

```
% Matrix representation of the bivariate downsampling
T
```

```
T = 24×24
     0    -1     0     0     0     1     0    -1     0     0     0     1     0 ···
     1     0    -1     0     0     0     1     0    -1     0     0     0     0
     0     1     0    -1     0     0     0     1     0    -1     0     0     0
     0     0     1     0    -1     0     0     0     1     0    -1     0     0
     0     0     0     1     0    -1     0     0     0     1     0    -1     0
    -1     0     0     0     1     0    -1     0     0     0     1     0     0
     0    -1     0     0     0     1     0    -1     0     0     0     1     0
     1     0    -1     0     0     0     1     0    -1     0     0     0     1
     0     1     0    -1     0     0     0     1     0    -1     0     0     0
     0     0     1     0    -1     0     0     0     1     0    -1     0     0
     ⋮
```

二変量循環畳み込みの随伴作用素

(Adjoint operator of bivariate circular convolution)

エルミート転置 (Herimitian transposition)

- $\mathbf{T}^H$

```
% Adjoint matrix of the bivariate circular convolution
T'
```

```
ans = 24×24
     0     1     0     0     0    -1     0     1     0     0     0    -1     0 ···
    -1     0     1     0     0     0    -1     0     1     0     0     0     0
     0    -1     0     1     0     0     0    -1     0     1     0     0     0
     0     0    -1     0     1     0     0     0    -1     0     1     0     0
     0     0     0    -1     0     1     0     0     0    -1     0     1     0
     1     0     0     0    -1     0     1     0     0     0    -1     0     0
     0     1     0     0     0    -1     0     1     0     0     0    -1     0
    -1     0     1     0     0     0    -1     0     1     0     0     0    -1
     0    -1     0     1     0     0     0    -1     0     1     0     0     0
     0     0    -1     0     1     0     0     0    -1     0     1     0     0
     ⋮
```

随伴作用素(Adjoint operator)

$$T^*(\{v[\mathbf{m}]\}_{\mathbf{m}}) = \mathrm{vec}_{\Omega_\mathrm{u}}^{-1} \circ \mathbf{T}^H \mathrm{vec}_{\Omega_\mathrm{v}}(\{v[\mathbf{m}]\}_{\mathbf{m}})$$

```
% Adjoint operator T*
adjOp = @(x) reshape(T'*x(:),[N1 N2]);
```

内積の保存の確認

(Confirmation of the preservation of the inner product)

入力配列の生成 (Generation of an input array)

- $\{u[\mathbf{n}]\}_{\mathbf{n}}$

```
% Generation of an input array u
arrayU = randn(N1,N2);
```

循環畳み込みの出力 (Output of the circular convolution)

- $\{v[\mathbf{m}]\}_{\mathbf{m}} = T(\{u[\mathbf{n}]\}_{\mathbf{n}})$

```
% Circular convolution (v=Tu)
arrayV = imfilter(arrayU,h,'conv','circ');
```

任意の出力領域配列生成(Generation of an arbitrary array in output range)

```
% Array generation in the same domain with arrayV
arrayY = randn(size(arrayV),'like',arrayV);
```

内積 (Inner product)

$\alpha = \langle \mathbf{y}, \mathbf{v} \rangle = \langle \mathbf{y}, \mathbf{Tu} \rangle$

```
% Inner product <y,v>=<y,Tu>
innprodA = dot(arrayY(:),arrayV(:))
```

```
innprodA = -4.9493
```

循環畳み込みの随伴作用素 (The adjoint operator of circular convolution)

$\mathbf{r} = \mathbf{T}^H \mathbf{v}$

```
% Adjoint operation of circular convolution (r=T'v)
arrayR = adjOp(arrayY)
```

```
arrayR = 6×4
   -3.2545   -0.9530   -2.8635   -2.6745
    1.1585   -0.0261    0.4010   -2.8281
   -0.2289   -0.7080    1.2294    3.1791
    1.4331    2.1850    1.4794    2.2401
    3.4834    1.6610    1.6341   -0.5047
   -2.5917   -2.1589   -1.8803    0.5879
```

$\beta = \langle \mathbf{r}, \mathbf{u} \rangle = \langle \mathbf{T}^H \mathbf{y}, \mathbf{u} \rangle$

```
% Inner product <r,u>=<T'v,u>
innprodB = dot(arrayR(:),arrayU(:));

% Verify the preservation of the inner product
err = abs(innprodA - innprodB);
```

```
disp(['|<y,Tu> - <T''y,u>| = ' num2str(err)])
```

```
|<y,Tu> - <T'y,u>| = 1.7764e-15
```

反転インパルス応答による循環畳み込み

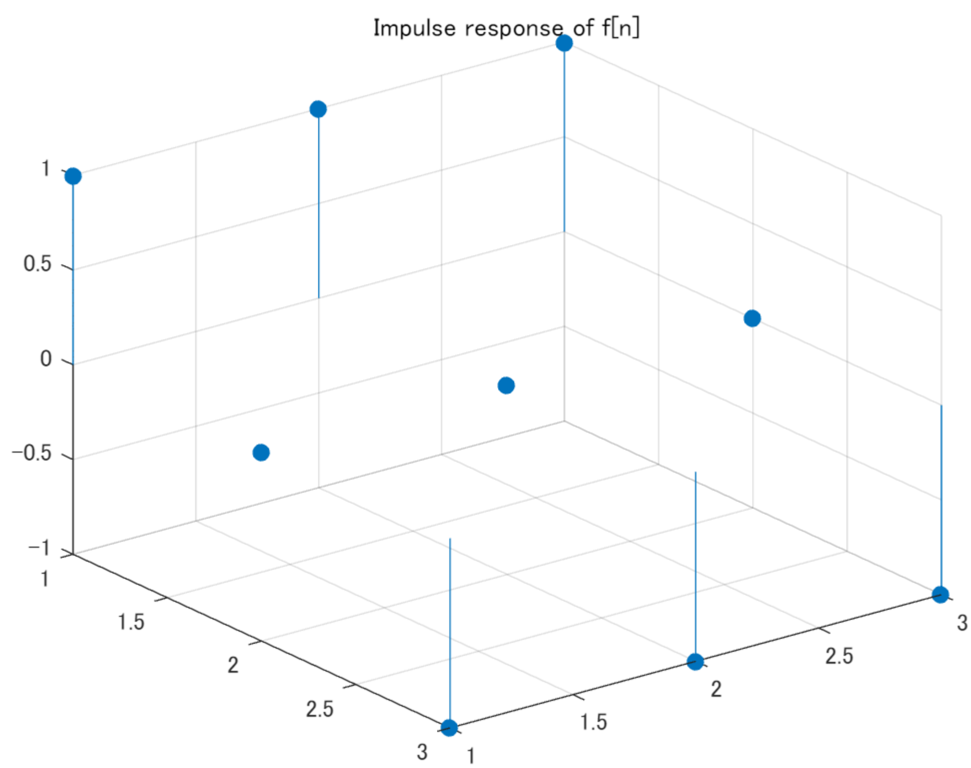
(Circular convolution with the reversal impulse response)

$$\{r[\mathbf{n}]\}_\mathbf{n} = \{\bar{h}[-\mathbf{n}]\}_\mathbf{n} \bigcirc \{y[\mathbf{n}]\}_\mathbf{n} = \sum_{\mathbf{k}\in\Omega\subset\mathbb{Z}^2} \bar{h}[-\mathbf{k}]\{y[((\mathbf{n}-\mathbf{k}))_\mathbf{Q}]\}_\mathbf{n}$$

```
% Revaersal impulse response
f = conj(rot90(h,2))
```

```
f = 3×3
     1     1     1
     0     0     0
    -1    -1    -1
```

```
figure(2)
stem3(f,'filled')
axis ij
title('Impulse response of f[n]')
```




```
% Circular convolution with impulse response f
arrayS = imfilter(arrayY,f,'conv','circ')
```

```
arrayS = 6×4
   -3.2545   -0.9530   -2.8635   -2.6745
    1.1585   -0.0261    0.4010   -2.8281
   -0.2289   -0.7080    1.2294    3.1791
    1.4331    2.1850    1.4794    2.2401
    3.4834    1.6610    1.6341   -0.5047
   -2.5917   -2.1589   -1.8803    0.5879
```

行列演算と IMFILTER の比較

```
% Definition of MSE
mymse = @(x,y) sum((x-y).^2,'all')/numel(x);

% Evaluation
disp(['MSE between matrix operation and IMFILTER: ' num2str(mymse(arrayR,arrayS))])
```

```
MSE between matrix operation and IMFILTER: 9.149e-32
```