Sample 5-1

周波数解析

単変量畳み込み

画像処理特論

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動作確認: MATLAB R2023a

Fourier analysis

Univariate convolution

Advanced Topics in Image Processing

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Verified: MATLAB R2023a

準備

(Preparation)

```
close all
```

入力信号 $\{u[n]\}_n$

(Input signal $\{u[n]\}_n$)

```
% Input u[n]
u = [1 2 3];
```

線形シフト不変システムのインパルス応答 $\{h[n]\}_n$

(Impulse response of a linear shift-invariant system $\{h[n]\}_n$)

```
% Impulse response h[n]
h = [1 1 1]/3;
```

線形シフト不変システムの出力応答$\{v[n]\}_n$

(The linear shift-invariant system response $\{v[n]\}_n$)

畳み込み演算 (Convolution)

$$\{v[n]\}_n = \{h[n]\}_n * \{u[n]\}_n = \sum_{k=-\infty}^{\infty} u[k]\{h[n-k]\}_n$$

```
% Output v[n]
v = conv(h,u);
```

入力信号 $\{u[n]\}_n$ のスペクトル

(Spectrum of input signal $\{u[n]\}_n$)

$$U(e^{\jmath\omega}) = \sum_{n=-\infty}^{\infty} u[n]e^{-\jmath\omega n},\ \ \omega \in \mathbb{R}$$

DFT(FFT)による DTFT の周波数サンプル計算 (Frequency sampling of DTFT by DFT (FFT))

$$U[k] = U(e^{j\omega})|_{\omega=\frac{2\pi}{N}k},\ \ k \in \{0, 1, 2, \cdots, N-1\}$$

```
% Setting the number of frequency sample points in [0,2π)
nPoints = 128;

% Spectrum of u[n]
U = fft(u,nPoints);
```

フィルタ $\{h[n]\}_n$ の周波数応答

(Frequency response of filter $\{h[n]\}_n$)

$$H(e^{\jmath\omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-\jmath\omega n},\ \ \omega \in \mathbb{R}$$

DFT(FFT)による DTFT の周波数サンプル計算 (Frequency sampling of DTFT by DFT (FFT))

$$H[k] = H(e^{j\omega})|_{\omega=\frac{2\pi}{N}k},\ \ k \in \{0, 1, 2, \cdots, N-1\}$$

```
% Frequency response of h[n]
H = fft(h,nPoints);
```

出力信号 $\{v[n]\}_n$ のスペクトル

(Spectrum of input signal $\{v[n]\}_n$)

$$V(e^{\jmath\omega}) = \sum_{n=-\infty}^{\infty} v[n]e^{-\jmath\omega n},\ \ \omega \in \mathbb{R}$$

DFT(FFT)による DTFT の周波数サンプル計算 (Frequency sampling of DTFT by DFT (FFT))

$$V[k] = V(e^{\jmath\omega})|_{\omega=\frac{2\pi}{N}k},\ \ k \in \{0, 1, 2, \cdots, N-1\}$$

```
% Frequency response of v[n]
```

```
V = fft(v,nPoints);
```

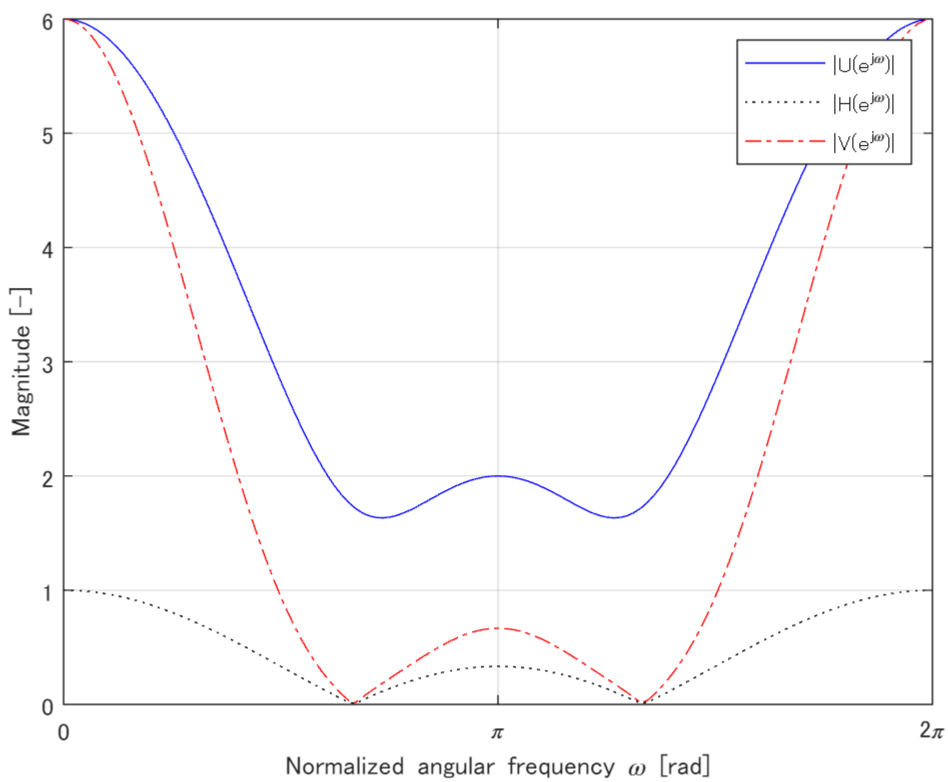
スペクトルと周波数応答の表示

(Display of spectra and frequency response)

$$V(e^{j\omega}) = H(e^{j\omega})U(e^{j\omega})$$

```
% Frequency sample points
w = 0:2*pi/nPoints:2*pi-1/nPoints;

% Display of spectra and frequency response
figure(1)
ax = gca;
plot(ax,w,abs(U),'b',w,abs(H),'k:',w,abs(V),'r-.')
grid(ax,'on')
xlabel(ax,'Normalized angular frequency \omega [rad]')
ylabel(ax,'Magnitude [-]')
legend(ax,'|U(e^{j\omega})|','|H(e^{j\omega})|','|V(e^{j\omega})|')
ax.XLim = [0 2*pi];
ax.XTick = [0 pi 2*pi];
ax.XTickLabel = {'0', '\pi', '2\pi' };
```



線形シフト不変システムの解析

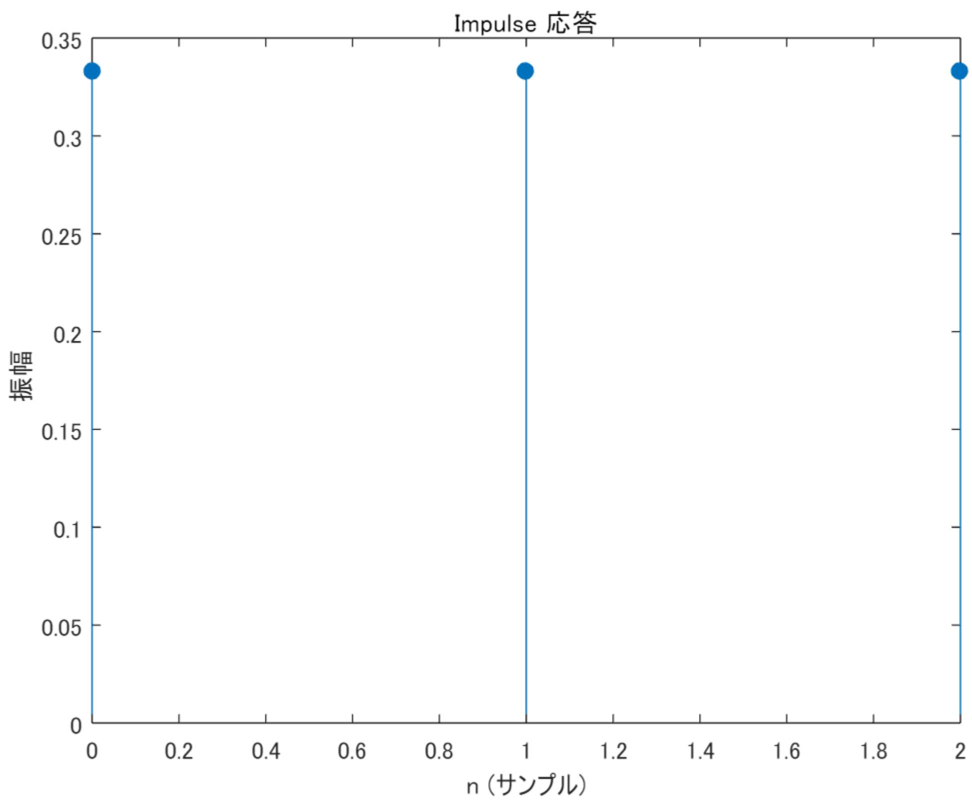
(Analysis of linear shift-invariant systems)

インパルス応答

(Impulse respone)

$h[n],\ n \in \mathbb{Z}$

```
figure(2)
impz(h)
```

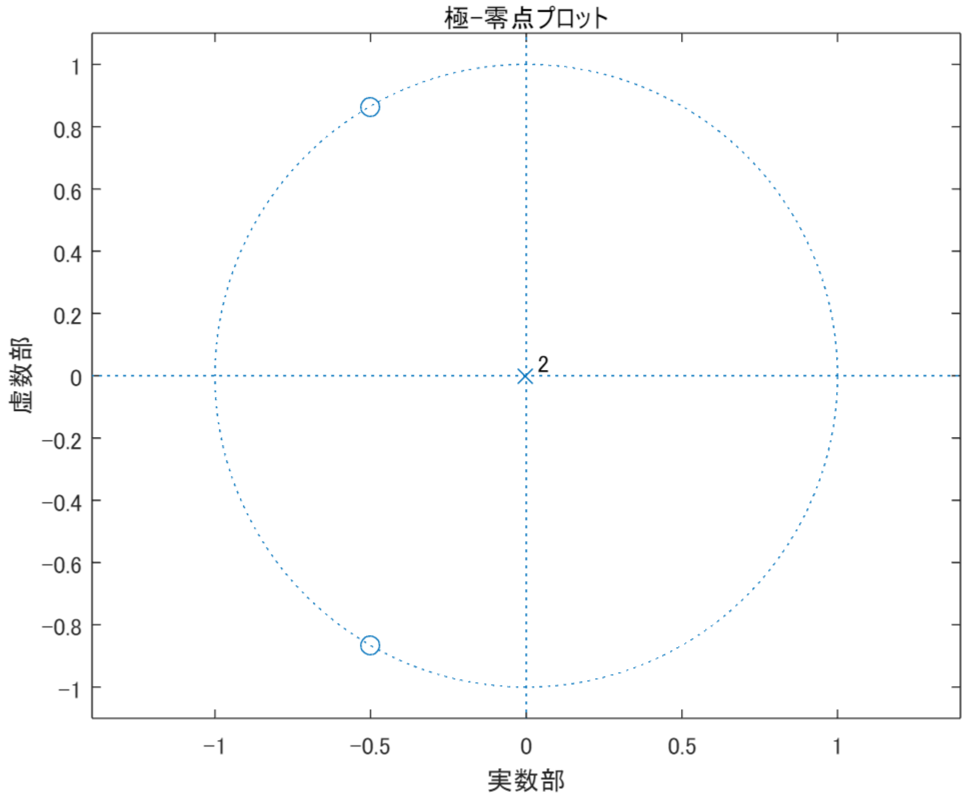


伝達関数

(Transfer function)

$$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n},\ z \in \mathbb{C}$$

```
figure(3)
zplane(h)
```



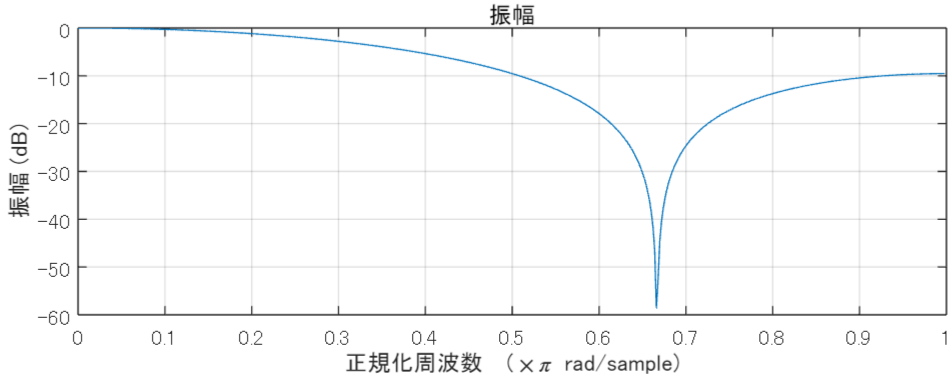
周波数応答

(Frequency response)

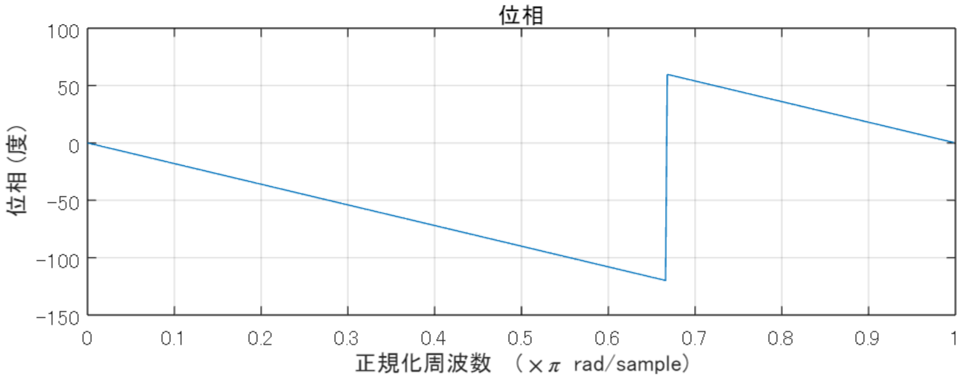
$H(e^{j\omega}) = H(z)|_{z=e^{j\omega}},\ \omega \in \mathbb{R}$

- 振幅応答 (Magnitude response): $|H(e^{j\omega})|$
- 位相応答 (Phase response): $\angle H(e^{j\omega})$

```
figure(4)
freqz(h)
```



振幅
0
−10
−20
−30
−40
−50
−60
振幅（dB）
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
正規化周波数 （×π rad/sample）



位相
100
50
0
−50
−100
−150
位相（度）
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
正規化周波数 （×π rad/sample）