Sample 12-3

画像復元

合成モデル

画像処理特論

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動作確認: MATLAB R2020a

Image restoration

Synthesis model

Advanced Topics in Image Processing

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Verified: MATLAB R2020a

準備

(Preparation)

```
clear
close all
import msip.download_img
msip.download_img
```

```
kodim01.png already exists in ./data/
kodim02.png already exists in ./data/
kodim03.png already exists in ./data/
kodim04.png already exists in ./data/
kodim05.png already exists in ./data/
kodim06.png already exists in ./data/
kodim07.png already exists in ./data/
kodim08.png already exists in ./data/
kodim09.png already exists in ./data/
kodim10.png already exists in ./data/
kodim11.png already exists in ./data/
kodim12.png already exists in ./data/
kodim13.png already exists in ./data/
kodim14.png already exists in ./data/
kodim15.png already exists in ./data/
kodim16.png already exists in ./data/
kodim17.png already exists in ./data/
kodim18.png already exists in ./data/
kodim19.png already exists in ./data/
kodim20.png already exists in ./data/
kodim21.png already exists in ./data/
kodim22.png already exists in ./data/
kodim23.png already exists in ./data/
kodim24.png already exists in ./data/
See Kodak Lossless True Color Image Suite
```

パラメータ設定

(Parameter settings)

- sgm: ノイズ標準偏差 σ_w (Standard deviation of noise)
- nlevels: ウェーブレット段数 (Wavelet levels)

```
% Parameter settings
isaprxleft = true;
lambda = 10^-0.1
```

```
lambda = 0.7943
```

```
gamma = 10^0.3
```

```
gamma = 1.9953
```

```
sgmuint8 = 10;
sgm = sgmuint8/255;
nlevels = 3;
niters = 80;
```

画像の読込

(Read image)

```
u = rgb2gray(im2double(imread('./data/kodim01.png')));
```

観測画像

(Observation image)

- $\mathbf{v} = \mathbf{Pu} + \mathbf{w}$
- $\mathbf{u} = \mathbf{Ds}$
- $\mathbf{w} \sim \mathrm{Norm}\left(\mathbf{w}|\boldsymbol{\mu}_w = \mathbf{0}, \sigma_w^2\mathbf{I}\right)$

```
% Definition of measurment process
psf = fspecial('motion',21,11);
measureproc = @(x) imfilter(x,psf,'conv','circular');
% Adjoint process of the measurment process
measureadjp = @(x) imfilter(x,psf,'corr','circular');
% Simulation of AWGN
v = imnoise(measureproc(u),'gaussian',0,sgm^2);
```

非間引きハール DWT

(Undecimated Haar DWT)

```
import msip.udhaarwtdec2
import msip.udhaarwtrec2
```

完全再構成の確認 (Checki the perfect reconstruction)

非間引きハール DWT はパーセバルタイト性 (The undecimated DWT satisfies the Parseval tight property,)

$$\mathbf{D}\mathbf{D}^T = \mathbf{I}$$

を満たすため， $\mathbf{D}$ の転置システムは完全再構成分析システムとなり得る．(and thus Its transposition system can be a PR analysis system.)

```
[coefs,scales] = udhaarwtdec2(v,nlevels);
r = udhaarwtrec2(coefs,scales);
assert(norm(v-r,"fro")^2/numel(v)<1e-18,'Perfect reconstruction is violated.')
```

合成辞書と転置辞書の定義 (Definition of synthesis dictionary and its adjoint)

```
% Definiton of dictionay and its adjoint
adjdic = @(x) udhaarwtdec2(x,nlevels); % D
syndic = @(x) udhaarwtrec2(x,scales);  % D.'
```

近接勾配法

(Proximal gradient method)

問題設定 (Problem setting)

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}} \frac{1}{2}\|\mathbf{v} - \mathbf{P}\mathbf{D}\mathbf{s}\|_2^2 + \lambda\|\mathbf{s}\|_1$$

アルゴリズム (Algorithm)

1. Initialization: $\mathbf{s}^{(0)}$, $t \leftarrow 0$
2. Proximal gradient descent: $\mathbf{s}^{(t+1)} \leftarrow \mathrm{prox}_{\gamma g}\left(\mathbf{s}^{(t)} - \gamma\nabla_{\mathbf{s}} f(\mathbf{s}^{(t)})\right)$
3. If a stopping critera is satisfied then finish, otherwise $t \rightarrow t+1$ and go to Step 2.

ただし，(where)

- $\nabla_{\mathbf{s}} f(\mathbf{s}) = \mathbf{D}^T\mathbf{P}^T(\mathbf{P}\mathbf{D}\mathbf{s} - \mathbf{v})$
- $\mathrm{prox}_{\gamma\lambda\|\cdot\|_1}(\mathbf{s}) = \mathcal{T}_{\gamma\lambda}(\mathbf{s}) = \mathrm{sign}(\mathbf{s}) \odot \max(\mathrm{abs}(\mathbf{s}) - \gamma\lambda\mathbf{1}, \mathbf{0})$

ソフト閾値処理 (Soft-thresholding)

```
softthresh = @(x,t) sign(x).*max(abs(x)-t,0);
```

初期化 (Initialization)

```
[coefs,scales] = udhaarwtdec2(v,nlevels);
sp = coefs;
```

近接勾配降下 (Proximal gradient descent)

- $\gamma < 2/\beta$: Step size
- β: Lipschitz constant of ∇f, where $\beta = (\sigma_{\max}(\mathbf{PD}))^2$

```
beta = max(abs(fftn(psf,2.^nextpow2(size(v)))),[],'all');
assert(gamma < 2/beta,'Step size condition is violated.')
if isaprxleft
    mask = ones(size(coefs));
    mask(1:prod(scales(1,:))) = 0;
    lambda = lambda * mask;
end
for idx=0:niters-1
    % Proximal gradient descent
    sg = adjdic(measureadjp(measureproc(syndic(sp))-v));
    sc = softthresh(sp-gamma*sg,gamma*lambda);
    % Update
    sp = sc;
end
```

復元画像

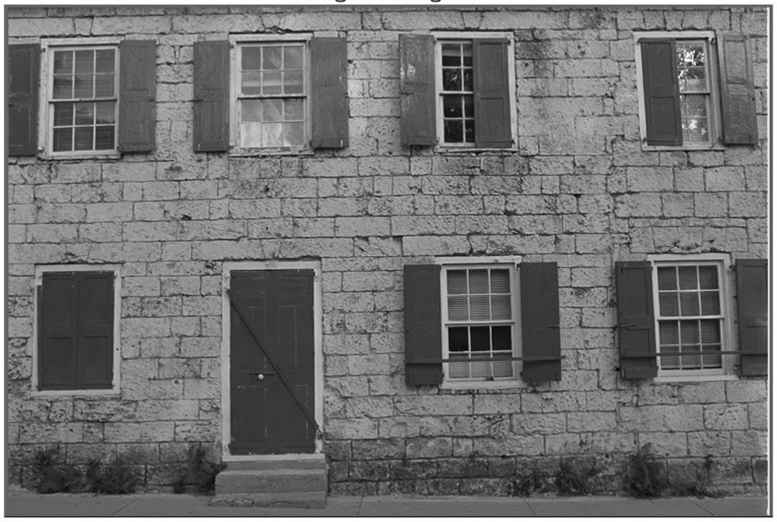
(Restored image)

```
r = syndic(sc);
```

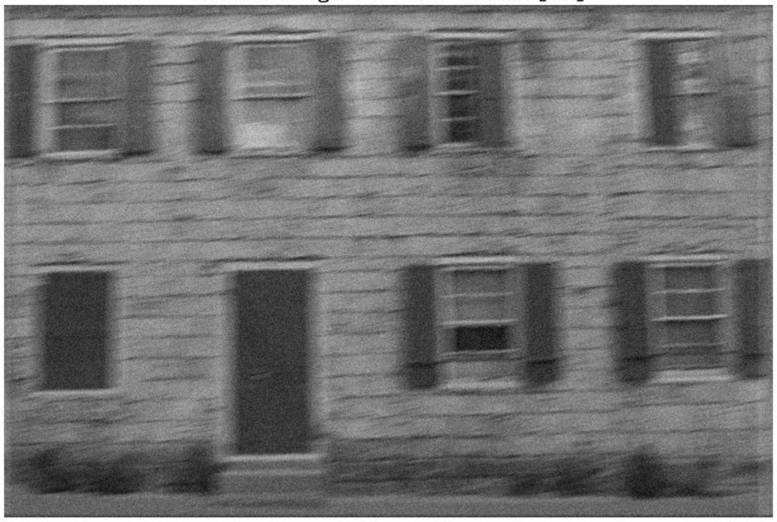
画像表示

(Image show)

```
figure(1)
imshow(u);
title('Original image u')
```


Original image u

```
figure(2)
imshow(v)
title(sprintf('Blurred image v : PSNR = %5.2f [dB]',psnr(u,v)))
```


Blurred image v : PSNR = 20.07 [dB]

```
figure(3)
imshow(r)
title(sprintf('Restored image r w/ ISTA : PSNR = %5.2f [dB]',psnr(u,r)))
```

Restored image r w/ ISTA:PSNR = 21.74 [dB]