Sample 13-4

辞書学習

畳込み辞書学習

画像処理特論

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動作確認: MATLAB R2020a

Dictionary learning

Convolutional dictionary learning

Advanced Topics in Image Processing

Shogo MURAMATSU

Verified: MATLAB R2020a

準備

(Preparation)

```
clear
close all
import msip.download_img
msip.download_img
```

```
lena.png already exists in ./data/
baboon.png already exists in ./data/
goldhill.png already exists in ./data/
barbara.png already exists in ./data/
```

2 変量ラティス構造冗長フィルタバンク

(Bivariate lattice-structure oversampled filter banks)

例として，（偶対称チャネルと奇対称チャネルが等しい）偶数チャネル、偶数のポリフェーズ次数をもつタイプ I 非分離冗長重複変換(NSOLT)

(As an example, let us adopt a non-separable oversampled lapped transform (NSOLT) of type-I with the number of channels (the numbers of even and odd symmetric channels are identical to each other) and polyphase order (even):)

$$\mathbf{E}(z_{\mathrm{v}}, z_{\mathbf{h}}) = \left(\prod_{k_{\mathrm{h}}=1}^{N_{\mathrm{h}}/2} \mathbf{V}_{2k_{\mathrm{h}}}^{\{\mathrm{h}\}} \bar{\mathbf{Q}}(z_{\mathrm{h}}) \mathbf{V}_{2k_{\mathrm{h}}-1}^{\{\mathrm{h}\}} \mathbf{Q}(z_{\mathrm{h}})\right) \left(\prod_{k_{\mathrm{v}}=1}^{N_{\mathrm{v}}/2} \mathbf{V}_{2k_{\mathrm{v}}}^{\{\mathrm{v}\}} \bar{\mathbf{Q}}(z_{\mathrm{v}}) \mathbf{V}_{2k_{\mathrm{v}}-1}^{\{\mathrm{v}\}} \mathbf{Q}(z_{\mathrm{v}})\right) \mathbf{V}_0 \mathbf{E}_0,$$

$$\mathbf{R}(z_{\mathrm{v}}, z_{\mathbf{h}}) = \mathbf{E}^T(z_{\mathrm{v}}^{-1}, z_{\mathrm{h}}^{-1}),$$

を採用する．ただし，(where)

- $\mathbf{E}(z_\mathrm{v}, z_\mathrm{h})$: Type-I polyphase matrix of the analysis filter bank
- $\mathbf{R}(z_\mathrm{v}, z_\mathrm{h})$: Type-II polyphase matrix in the synthesis filter bank
- $z_d \in \mathbb{C}, d \in \{\mathrm{v}, \mathrm{h}\}$: The parameter of Z-transformation direction
- $N_d \in \mathbb{N}, d \in \{\mathrm{v}, \mathrm{h}\}$: Polyphase order in direction d (number of overlapping blocks)
- $\mathbf{V}_0 = \begin{pmatrix} \mathbf{W}_0 & \mathbf{O} \\ \mathbf{O} & \mathbf{U}_0 \end{pmatrix} \begin{pmatrix} \mathbf{I}_{M/2} \\ \mathbf{O} \\ \mathbf{I}_{M/2} \\ \mathbf{O} \end{pmatrix} \in \mathbb{R}^{P\times M}, \mathbf{V}_n^{\{d\}} = \begin{pmatrix} \mathbf{I}_{P/2} & \mathbf{O} \\ \mathbf{O} & \mathbf{U}_n^{\{d\}} \end{pmatrix} \in \mathbb{R}^{P\times P}, d \in \{\mathrm{v}, \mathrm{h}\}$, where $\mathbf{W}_0, \mathbf{U}_0, \mathbf{U}_n^{\{d\}} \in \mathbb{R}^{P/2\times P/2}$ are orthonromal matrices.
- $\mathbf{Q}(z) = \mathbf{B}_P \begin{pmatrix} \mathbf{I}_{P/2} & \mathbf{O} \\ \mathbf{O} & z^{-1}\mathbf{I}_{P/2} \end{pmatrix} \mathbf{B}_P,\ \bar{\mathbf{Q}}(z) = \mathbf{B}_P \begin{pmatrix} z\mathbf{I}_{P/2} & \mathbf{O} \\ \mathbf{O} & \mathbf{I}_{P/2} \end{pmatrix} \mathbf{B}_P,\ \mathbf{B}_P = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{I}_{P/2} & \mathbf{I}_{P/2} \\ \mathbf{I}_{P/2} & -\mathbf{I}_{P/2} \end{pmatrix}$

です。

【References】

- MATLAB SaivDr Package: https://github.com/msiplab/SaivDr
- S. Muramatsu, K. Furuya and N. Yuki, "Multidimensional Nonseparable Oversampled Lapped Transforms: Theory and Design," in IEEE Transactions on Signal Processing, vol. 65, no. 5, pp. 1251-1264, 1 March1, 2017, doi: 10.1109/TSP.2016.2633240.
- S. Muramatsu, T. Kobayashi, M. Hiki and H. Kikuchi, "Boundary Operation of 2-D Nonseparable Linear-Phase Paraunitary Filter Banks," in IEEE Transactions on Image Processing, vol. 21, no. 4, pp. 2314-2318, April 2012, doi: 10.1109/TIP.2011.2181527.
- S. Muramatsu, M. Ishii and Z. Chen, "Efficient parameter optimization for example-based design of nonseparable oversampled lapped transform," 2016 IEEE International Conference on Image Processing (ICIP), Phoenix, AZ, 2016, pp. 3618-3622, doi: 10.1109/ICIP.2016.7533034.
- Furuya, K., Hara, S., Seino, K., & Muramatsu, S. (2016). Boundary operation of 2D non-separable oversampled lapped transforms. *APSIPA Transactions on Signal and Information Processing, 5*, E9. doi:10.1017/ATSIP.2016.3.

2 次元画像の階層的分析

(Hierachical decomposition for 2-D Grayscale image)

$R_M^P(\tau)$ をツリーレベル τの階層構造フィルタバンクの冗長度とすると、

(Let $R_M^P(\tau)$ be the redundancy of τ-level tree-structured filter bank, then we have the relation)

$$R_M^P(\tau) = \begin{cases} (P-1)\tau + 1, & M = 1, \\ \dfrac{P-1}{M-1} - \dfrac{P-M}{(M-1)M^\tau}, & M \geq 2. \end{cases}$$

となる.

```
% Decimation factor (Strides)
decFactor = [2 2]; % [My Mx]

% Number of channels ( sum(nChannels) >= prod(decFactors) )
nChannels = [1 1] * 4% [Ps Pa] (Ps=Pa)
```

```
nChannels = 1×2
     4     4
```

```
% Number of tree levels
nLevels = 3
```

```
nLevels = 3
```

```

% Redundancy
P = sum(nChannels);
M = prod(decFactor);
redundancy = ...
    (prod(decFactor)==1)*((P-1)*nLevels+1) + ...
    (prod(decFactor)>1)*((P-1)/(M-1)-(P-M)/((M-1)*M^nLevels))
```

```
redundancy = 2.3125
```

```

% Polyphase Order
ppOrder = [1 1] *4
```

```
ppOrder = 1×2
     4     4
```

```

% Sparsity ratio
sparsityRatio = 1/16;

% Number of patchs per image
nSubImgs = 128;

% No DC-leakage
noDcLeakage = true
```

```
noDcLeakage = logical
   1
```

Setting of dictionary update step

```
% Number of iterations
nIters = 8;

% Standard deviation of initial angles
stdInitAng = pi/6;

% Patch size for training
szPatchTrn = [64 64]; % > [ (Ny+1)My (Nx+1)Mx ]

% Mini batch size
```

```
miniBatchSize = 32;

% Number of Epochs (1 Epoch = nSubImgs/miniBachSize iterlations)
maxEpochs = 32;

% Number of iterations
maxIters = nSubImgs/miniBatchSize * maxEpochs
```

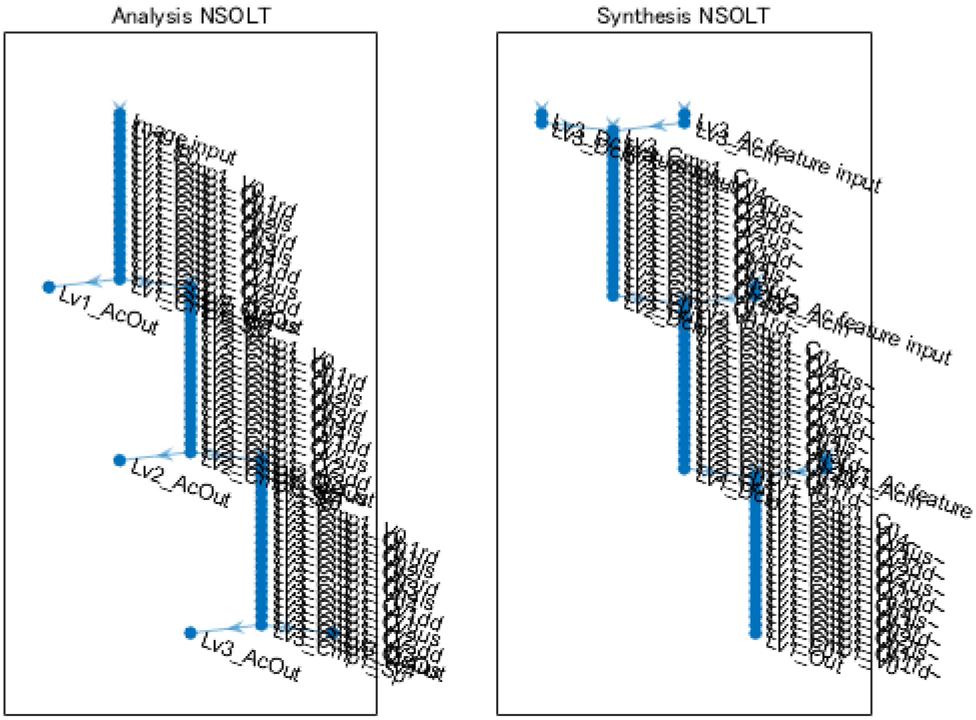
```
maxIters = 128
```

```

% Training options
opts = trainingOptions('sgdm', ... % Stochastic gradient descent w/ momentum
    ...'Momentum', 0.9000,...
    ...'InitialLearnRate',0.0100,...
    ...'LearnRateScheduleSettings','none',...
    'L2Regularization',0.0,...1.0000e-04,... % Set zero since parameters are rotaion angles.
    ...'GradientThresholdMethod','l2norm',...
    ...'GradientThreshold',Inf,...
    'MaxEpochs',maxEpochs,...30,...
    'MiniBatchSize',miniBatchSize,...128,...
    'Verbose',1,...
    'VerboseFrequency',32,...50,...
    ...'ValidationData',[],...
    ...'ValidationFrequency',50,...
    ...'ValidationPatience',Inf,...
    ...'Shuffle','once',...
    ...'CheckpointPath','',...
    ...'ExecutionEnvironment','auto',...
    ...'WorkerLoad',[],...
    ...'OutputFcn',[],...
    'Plots','none',...'training-progress',...
    ...'SequenceLength','longest',...
    ...'SequencePaddingValue',0,...
    ...'SequencePaddingDirection','right',...
    ...'DispatchInBackground',0,...
    'ResetInputNormalization',0);...1
```

層構造の構築

(Construction of layers)

```
import msip.*
analysislgraph = fcn_creatensoltlgraph2d(...
    'InputSize',szPatchTrn,...
    'NumberOfChannels',nChannels,...
    'DecimationFactor',decFactor,...
    'PolyPhaseOrder',ppOrder,...
    'NumberOfLevels',nLevels,...
    'NumberOfVanishingMoments',noDcLeakage,...
    'Mode','Analyzer');
synthesislgraph = fcn_creatensoltlgraph2d(...
    'InputSize',szPatchTrn,...
    'NumberOfChannels',nChannels,...
    'DecimationFactor',decFactor,...
    'PolyPhaseOrder',ppOrder,...
    'NumberOfLevels',nLevels,...
    'NumberOfVanishingMoments',noDcLeakage,...
    'Mode','Synthesizer');

figure(3)
subplot(1,2,1)
plot(analysislgraph)
title('Analysis NSOLT')
subplot(1,2,2)
plot(synthesislgraph)
title('Synthesis NSOLT')
```



Analysis NSOLT

Synthesis NSOLT

```
% Construction of deep learning network.
synthesisnet = dlnetwork(synthesislgraph);

% Initialize
nLearnables = height(synthesisnet.Learnables);
for iLearnable = 1:nLearnables
    if synthesisnet.Learnables.Parameter(iLearnable)=="Angles"
        layerName = synthesisnet.Learnables.Layer(iLearnable);
        synthesisnet.Learnables.Value(iLearnable) = ...
            cellfun(@(x) x+stdInitAng*randn(size(x)), ...
            synthesisnet.Learnables.Value(iLearnable),'UniformOutput',false);
    end
end
```

```
% Copy the synthesizer's parameters to the analyzer
synthesislgraph = layerGraph(synthesisnet);
analysislgraph = fcn_cpparamssyn2ana(analysislgraph,synthesislgraph);
analysisnet = dlnetwork(analysislgraph);
```

随伴関係（完全再構成）の確認

(Confirmation of the adjoint relation (perfect reconstruction))

NSOLT はパーセバルタイト性を満たすことに注意．(Note that NSOLT satisfy the Parseval tight property.)

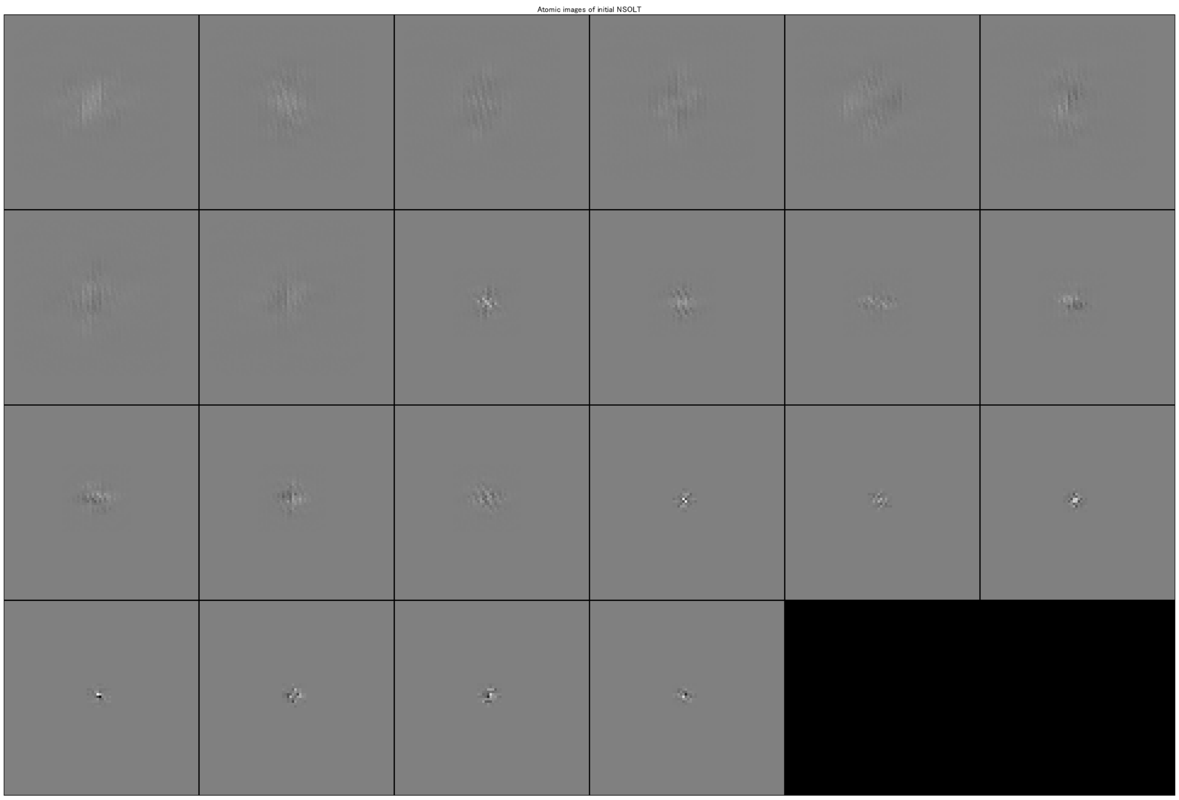
```
nOutputs = nLevels+1;
x = rand(szPatchTrn,'single');
s = cell(1,nOutputs);
dlx = dlarray(x,'SSCB'); % Deep learning array (SSCB: Spatial,Spatial,Channel,Batch)
[s{1:nOutputs}] = analysisnet.predict(dlx);
dly = synthesisnet.predict(s{:});
display("MSE: " + num2str(mse(dlx,dly)))
```

```
"MSE: 4.5506e-11"
```

要素画像の初期状態

(Initial state of the atomic images)

```
figure(4)
atomicimshow(synthesisnet)
title('Atomic images of initial NSOLT')
```



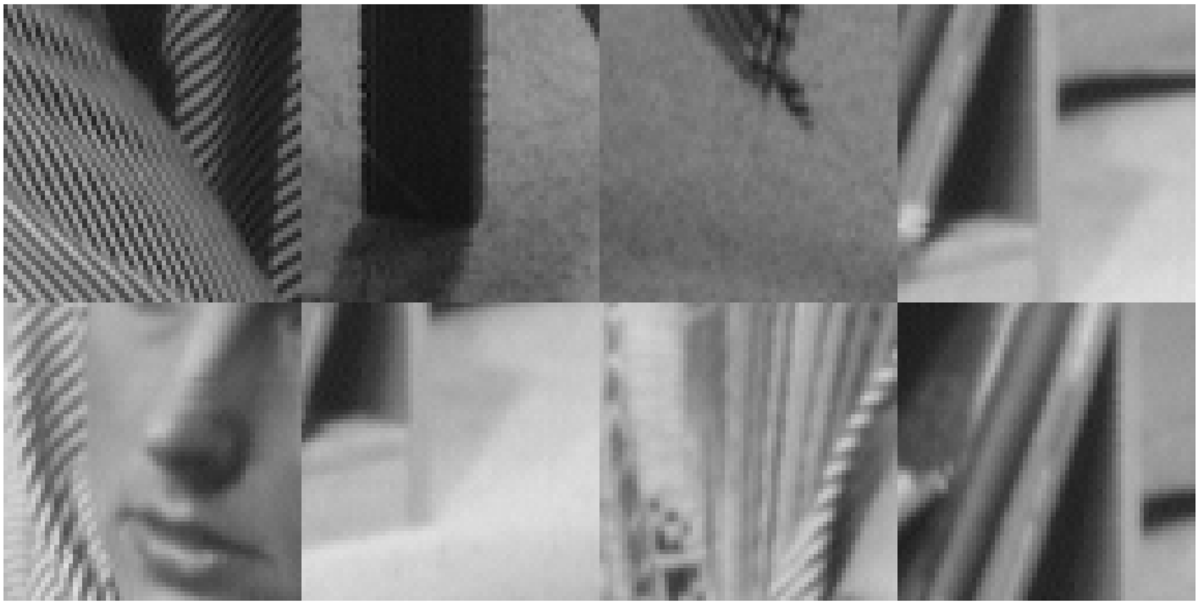
訓練画像の準備

(Preparation of traning image)

画像データストアからランダムにパッチを抽出

(Randomly extracting patches from the image data store)

```
imds = imageDatastore("./data/barbara.png","ReadFcn",@(x) im2single(imread(x)));
patchds = randomPatchExtractionDatastore(imds,imds,szPatchTrn,'PatchesPerImage',nSubImgs);
figure(5)
minibatch = preview(patchds);
responses = minibatch.ResponseImage;
montage(responses,'Size',[2 4]);
```



畳み込み辞書学習

(Convolutional dictionary learning)

問題設定(Problem setting):

$$\{\hat{\boldsymbol{\theta}}, \{\hat{\mathbf{s}}_n\}\} = \arg\min_{\{\boldsymbol{\theta},\{\mathbf{s}_n\}\}} \frac{1}{2S}\sum_{n=1}^{S} \|\mathbf{v}_n - \mathbf{D}_{\boldsymbol{\theta}}\hat{\mathbf{s}}_n\|_2^2, \quad \text{s.t. } \forall n, \|\mathbf{s}_n\|_0 \leq K,$$

ただし， $\mathbf{D}_{\boldsymbol{\theta}}$は設計パラメータベクトル $\boldsymbol{\theta}$をもつ畳み込み辞書.

(where $\mathbf{D}_{\boldsymbol{\theta}}$ is a convolutional dictionary with the design parameter vector $\boldsymbol{\theta}$.)

アルゴリズム(Algorithm):

スパース近似ステップと辞書更新ステップを繰返す.

(Iterate the sparse approximation step and the dictionary update step.)

- Sparse approximation step

$$\hat{\mathbf{s}}_n = \arg\min_{\mathbf{s}_n} \frac{1}{2}\|\mathbf{v}_n - \hat{\mathbf{D}}\mathbf{s}_n\|_2^2 \quad \text{s.t.} \ \ \|\mathbf{s}_n\|_0 \leq K$$

- Dictionary update step

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \frac{1}{2S}\sum_{n=1}^{S} \|\mathbf{v}_n - \mathbf{D}_{\boldsymbol{\theta}}\hat{\mathbf{s}}_n\|_2^2$$

$$\hat{\mathbf{D}} = \mathbf{D}_{\hat{\boldsymbol{\theta}}}$$

採用するスパース近似と辞書更新の手法(Adopted methods for the sparse approximation step and dictioary update step):

- Sparse approximation：Iterative hard thresholding
- Dictionary update： Stochastic gradient descent w/ momentum

```
% Check if IHT works for dlarray
%x = dlarray(randn(szPatchTrn,'single'),'SSCB');
%[y,coefs{1:nOutputs}] = iht(x,analysisnet,synthesisnet,sparsityRatio);
```

繰返し計算

(Iterative calculation of alternative steps)

```
import msip.*
%profile on
for iIter = 1:nIters

    % Sparse approximation (Applied to produce an object of TransformedDatastore)
    coefimgds = transform(patchds, @(x) iht4inputimage(x,analysisnet,synthesisnet,sparsityRati

    % Synthesis dictionary update
    trainlgraph = synthesislgraph.replaceLayer('Lv1_Out',...
        regressionLayer('Name','Lv1_Out'));
    trainednet = trainNetwork(coefimgds,trainlgraph,opts);

    % Analysis dictionary update (Copy parameters from synthesizer to analyzer)
    trainedlgraph = layerGraph(trainednet);
    analysislgraph = fcn_cpparamssyn2ana(analysislgraph,trainedlgraph);
    analysisnet = dlnetwork(analysislgraph);

    % Check the adjoint relation (perfect reconstruction)
    checkadjointrelation(analysislgraph,trainedlgraph,nLevels,szPatchTrn);

    % Replace layer
    synthesislgraph = trainedlgraph.replaceLayer('Lv1_Out',...
        nsoltIdentityLayer('Name','Lv1_Out'));
    synthesisnet = dlnetwork(synthesislgraph);

end
```

```
単一の GPU で学習中。
|=====================================================================================================
|    エポック    |    反復    |       経過時間        |    ミニバッチ　ＲＭＳＥ    |    ミニバッチ損失    |  基本学
|                |            |     （ｈｈ：ｍｍ：ｓｓ）  |                            |                      |
|=====================================================================================================
|            １  |         １ |     ００：００：２５  |                ４．６２    |           １０．７   |  ０．
|            ８  |       ３２ |     ００：０２：３７  |                ３．９５    |            ７．８    |  ０．
|          １６  |       ６４ |     ００：０５：０６  |                ４．５５    |           １０．４   |  ０．
|          ２４  |       ９６ |     ００：０７：３５  |                ４．３４    |            ９．４    |  ０．
|          ３２  |     １２８ |     ００：１０：０４  |                ３．５４    |            ６．３    |  ０．
|=====================================================================================================
   "MSE: 1.8799e-14"
単一の GPU で学習中。
|=====================================================================================================
|    エポック    |    反復    |       経過時間        |    ミニバッチ　ＲＭＳＥ    |    ミニバッチ損失    |  基本学
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|            １  |         １ |     ００：００：２５  |                ３．６１    |            ６．５    |  ０．
|            ８  |       ３２ |     ００：０２：４１  |                ３．５３    |            ６．２    |  ０．
|          １６  |       ６４ |     ００：０５：１０  |                ３．５７    |            ６．４    |  ０．
|          ２４  |       ９６ |     ００：０７：４０  |                ４．０４    |            ８．２    |  ０．
|          ３２  |     １２８ |     ００：１０：０９  |                ３．６０    |            ６．５    |  ０．
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単一の GPU で学習中。
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|    エポック    |    反復    |       経過時間        |    ミニバッチ　ＲＭＳＥ    |    ミニバッチ損失    |  基本学
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|            １  |         １ |     ００：００：２５  |                ３．６７    |            ６．７    |  ０．
|            ８  |       ３２ |     ００：０２：４１  |                ３．１２    |            ４．９    |  ０．
|          １６  |       ６４ |     ００：０５：１１  |                ３．１４    |            ４．９    |  ０．
|          ２４  |       ９６ |     ００：０７：４１  |                ３．９０    |            ７．６    |  ０．
|          ３２  |     １２８ |     ００：１０：１０  |                ３．１９    |            ５．１    |  ０．
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   "MSE: 1.8801e-14"
単一の GPU で学習中。
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|    エポック    |    反復    |       経過時間        |    ミニバッチ　ＲＭＳＥ    |    ミニバッチ損失    |  基本学
|                |            |     （ｈｈ：ｍｍ：ｓｓ）  |                            |                      |
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|            １  |         １ |     ００：００：２５  |                ３．３４    |            ５．６    |  ０．
|            ８  |       ３２ |     ００：０２：４１  |                ３．５０    |            ６．１    |  ０．
|          １６  |       ６４ |     ００：０５：１０  |                ３．８０    |            ７．２    |  ０．
|          ２４  |       ９６ |     ００：０７：４０  |                ３．４７    |            ６．０    |  ０．
|          ３２  |     １２８ |     ００：１０：１０  |                ２．９０    |            ４．２    |  ０．
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単一の GPU で学習中。
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|            １  |         １ |     ００：００：２６  |                ３．５３    |            ６．２    |  ０．
|            ８  |       ３２ |     ００：０２：４１  |                ３．５８    |            ６．４    |  ０．
|          １６  |       ６４ |     ００：０５：１１  |                ３．３１    |            ５．５    |  ０．
|          ２４  |       ９６ |     ００：０７：４０  |                ３．５４    |            ６．３    |  ０．
|          ３２  |     １２８ |     ００：１０：１０  |                ３．３１    |            ５．５    |  ０．
|=====================================================================================================
   "MSE: 1.8712e-14"
```

```
単一の GPU で学習中。
|========================================================================================================
|     エポック     |     反復     |          経過時間          |       ミニバッチ  ＲＭＳＥ       |       ミニバッチ損失       |     基本学
|                  |              |       （ｈｈ：ｍｍ：ｓｓ）       |                            |                        |
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|              １ |            １ |          ００：００：２５ |                   ３．００ |                 ４．５ |     ０．０
|              ８ |           ３２ |          ００：０２：４２ |                   ３．０５ |                 ４．７ |     ０．０
|             １６ |           ６４ |          ００：０５：１１ |                   ３．１８ |                 ５．１ |     ０．０
|             ２４ |           ９６ |          ００：０７：３８ |                   ２．５８ |                 ３．３ |     ０．０
|             ３２ |          １２８ |          ００：１０：０７ |                   ３．０７ |                 ４．７ |     ０．０
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単一の GPU で学習中。
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|                  |              |       （ｈｈ：ｍｍ：ｓｓ）       |                            |                        |
|========================================================================================================
|              １ |            １ |          ００：００：２６ |                   ３．７１ |                 ６．９ |     ０．０
|              ８ |           ３２ |          ００：０２：４１ |                   ３．２０ |                 ５．１ |     ０．０
|             １６ |           ６４ |          ００：０５：１４ |                   ３．３３ |                 ５．６ |     ０．０
|             ２４ |           ９６ |          ００：０７：４６ |                   ３．６９ |                 ６．８ |     ０．０
|             ３２ |          １２８ |          ００：１０：１６ |                   ３．６３ |                 ６．６ |     ０．０
|========================================================================================================
    "MSE: 1.8608e-14"
単一の GPU で学習中。
|========================================================================================================
|     エポック     |     反復     |          経過時間          |       ミニバッチ  ＲＭＳＥ       |       ミニバッチ損失       |     基本学
|                  |              |       （ｈｈ：ｍｍ：ｓｓ）       |                            |                        |
|========================================================================================================
|              １ |            １ |          ００：００：２６ |                   ２．７３ |                 ３．７ |     ０．０
|              ８ |           ３２ |          ００：０２：４１ |                   ３．３１ |                 ５．５ |     ０．０
|             １６ |           ６４ |          ００：０５：１０ |                   ２．９９ |                 ４．５ |     ０．０
|             ２４ |           ９６ |          ００：０７：４０ |                   ３．１６ |                 ５．０ |     ０．０
|             ３２ |          １２８ |          ００：１０：０９ |                   ３．１７ |                 ５．０ |     ０．０
|========================================================================================================
    "MSE: 1.8274e-14"
```
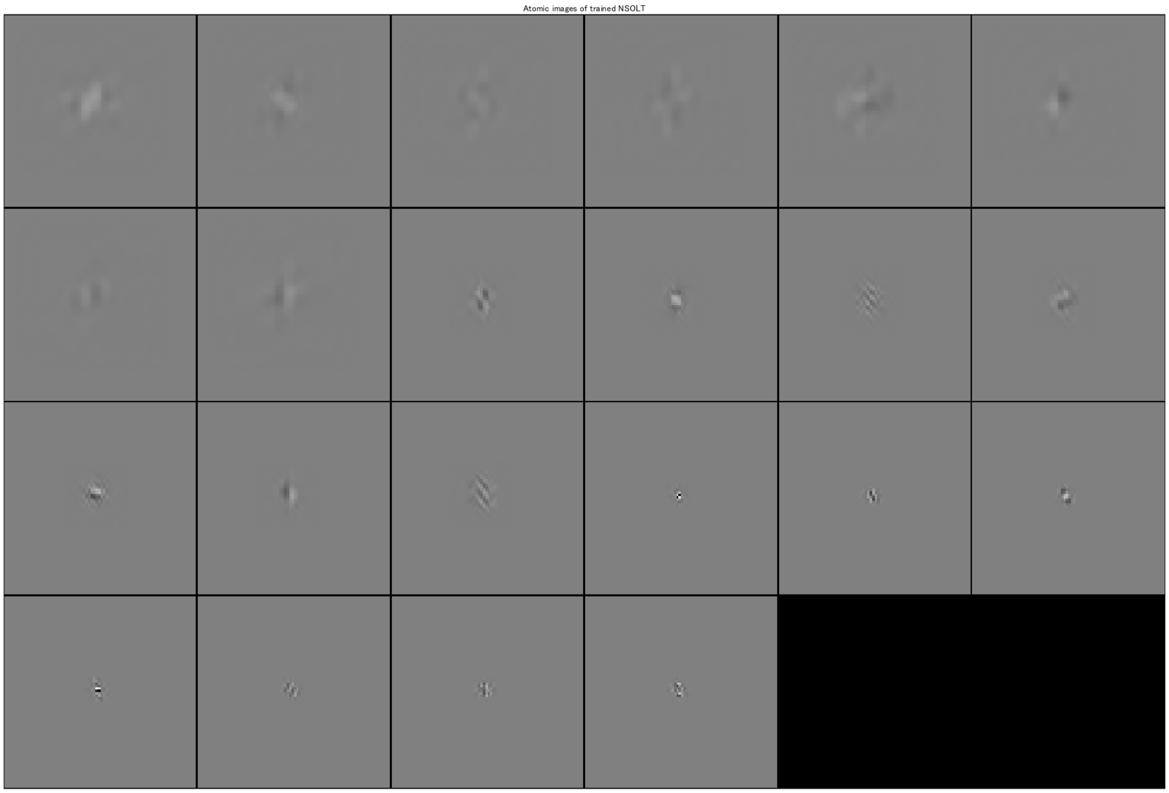

```
%profile off
%profile viewer
```

訓練辞書の要素画像

(The atomic images of trained dictionary)

```
figure(6)
atomicimshow(synthesisnet)
title('Atomic images of trained NSOLT')
```



訓練ネットワークの保存

(Save the designed network)

```
import msip.*
synthesislgraph = layerGraph(synthesisnet);
analysislgraph = fcn_cpparamssyn2ana(analysislgraph,synthesislgraph);
analysisnet = dlnetwork(analysislgraph);
save(sprintf('./data/nsoltdictionary_%s',datetime('now','Format','yyyyMMddhhmmssSSS')),'analysi
```

繰返しハード閾値処理関数

(Function of iterative hard thresholding)

The input images of the patch pairs are replaced with sparse coefficients obtained by IHT, where normalization is omitted for the Parseval tight property of NSOLT ($\mathbf{D}\mathbf{D}^T = \mathbf{I}$).

$$\mathbf{s}^{(t+1)} \leftarrow \mathscr{H}_{T_K}\left(\mathbf{s}^{(t)} - \gamma\hat{\mathbf{D}}^T\left(\hat{\mathbf{D}}\mathbf{s}^{(t)} - \mathbf{v}\right)\right)$$

$$t \leftarrow t+1$$

where

$$[\mathscr{H}_{T_K}(\mathbf{x})]_i = \begin{cases} 0, & |[\mathbf{x}]_i| \leq T_K \\ [\mathbf{x}]_i, & |[\mathbf{x}]_i| > T_K \end{cases}$$

【Reference】

- T. Blumensath and M. E. Davies, "Normalized Iterative Hard Thresholding: Guaranteed Stability and Performance," in IEEE Journal of Selected Topics in Signal Processing, vol. 4, no. 2, pp. 298-309, April 2010, doi: 10.1109/JSTSP.2010.2042411.

```
function newdata = iht4inputimage(oldtbl,analyzer,synthesizer,sparsityRatio)
% IHT for InputImage in randomPatchExtractionDatastore
%
nInputs = length(synthesizer.InputNames);

% Apply IHT process for every input patch
restbl = removevars(oldtbl,'InputImage');
dlv = dlarray(cat(4,oldtbl.InputImage{:}),'SSCB');
[~,dlcoefs{1:nInputs}] = iht(dlv,analyzer,synthesizer,sparsityRatio);
coefs = cellfun(@(x) permute(num2cell(extractdata(x),1:3),[4 1 2 3]),dlcoefs,'UniformOutput',fa
%
nImgs = length(oldtbl.InputImage);
coefarray = cell(nImgs,nInputs);
for iImg = 1:nImgs
    for iInput = 1:nInputs
        coefarray{iImg,iInput} = coefs{iInput}{iImg};
    end
end
% Output as a cell in order to make multiple-input datastore
newdata = [ coefarray table2cell(restbl) ];
end
```

```
function [dly,varargout] = iht(dlx,analyzer,synthesizer,sparsityRatio)
% IHT Iterative hard thresholding
%
nInputs = length(synthesizer.InputNames);
szBatch = size(dlx,4);

% Iterative hard thresholding w/o normalization
% (A Parseval tight frame is assumed)
gamma = (1.-1e-3);
nIters = 10;
nCoefs = floor(sparsityRatio*numel(dlx(:,:,:,1)));
[dlcoefs{1:nInputs}] = analyzer.predict(dlarray(zeros(size(dlx),'like',dlx),'SSCB'));
% IHT
for iter=1:nIters
    % Gradient descent
    dly = synthesizer.predict(dlcoefs{1:nInputs});
    [grad{1:nInputs}] = analyzer.predict(dly-dlx);
    dlcoefs = cellfun(@(x,y) x-gamma*y,dlcoefs,grad,'UniformOutput',false);
    % Hard thresholding
    coefvecs = cellfun(@(x) extractdata(reshape(x,[],szBatch)),dlcoefs,'UniformOutput',false);
    srtdabscoefs = sort(abs(cell2mat(coefvecs.')),1,'descend');
    thk = reshape(srtdabscoefs(nCoefs,:),1,1,1,szBatch);
    dlcoefs = cellfun(@(x) (abs(x)>thk).*x,dlcoefs,'UniformOutput',false);
    % Monitoring
    %checkSparsity =...
    %nnz(srtdabscoefs>srtdabscoefs(nCoefs,:))/numel(dlx)<=sparsityRatio;
    %assert(checkSparsity)
    %fprintf("IHT(%d) MSE: %6.4f\n",iter,mse(dlx,dly));
end
varargout = dlcoefs;
end
```

```
function checkadjointrelation(analysislgraph,synthesislgraph,nLevels,szInput)
import msip.*
x = rand(szInput,'single');
% Assemble analyzer
analysislgraph4predict = analysislgraph;
for iLayer = 1:length(analysislgraph4predict.Layers)
    layer = analysislgraph4predict.Layers(iLayer);
    if contains(layer.Name,"Lv"+nLevels+"_DcOut") || ...
            ~isempty(regexp(layer.Name,'^Lv\d+_AcOut','once'))
        analysislgraph4predict = analysislgraph4predict.replaceLayer(layer.Name,...
            regressionLayer('Name',layer.Name));
    end
end
analysisnet4predict = assembleNetwork(analysislgraph4predict);

% Assemble synthesizer
synthesislgraph4predict = synthesislgraph;
synthesisnet4predict = assembleNetwork(synthesislgraph4predict);

% Analysis and synthesis process
[s{1:nLevels+1}] = analysisnet4predict.predict(x);
if isvector(s{end-1})
    s{end-1} = permute(s{end-1},[1,3,2]);
end
y = synthesisnet4predict.predict(s{:});

% Evaluation
display("MSE: " + num2str(mse(x,y)))
end
```