Sample 13-5

辞書学習

学習辞書による画像復元

画像処理特論

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動作確認: MATLAB R2020a

Dictionary learning

Image restoration with learned dictionary

Advanced Topics in Image Processing

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Verified: MATLAB R2020a

準備

(Preparation)

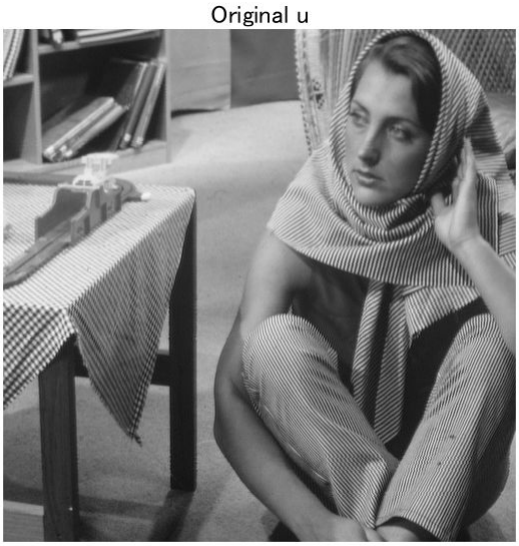
```
clear
close all
import msip.download_img
msip.download_img
```

```
lena.png already exists in ./data/
baboon.png already exists in ./data/
goldhill.png already exists in ./data/
barbara.png already exists in ./data/
```

原画像

(Original image)

```
% Original image
%u = im2single(rgb2gray(imread('./data/baboon.png')));
u = im2single(imread('./data/barbara.png'));
figure(1)
imshow(u)
title('Original u')
```



観測画像

(Observed image)

以下の観測の単一画像超解像を行う.

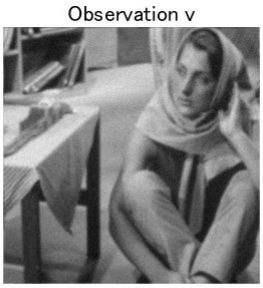
(Let us consider the single image super resolution problem of observation))

$$\mathbf{v} = \mathbf{P}\mathbf{u} + \mathbf{w}$$

$\mathbf{u}$ を原画像として，加法性白色ガウスノイズ$\mathbf{w}$ と低解像度観測過程$\mathbf{P}$が加わり劣化している．ここで，観測過程は低域通過フィルタとダウンサンプルからなる線形処理である.

(which originates $\mathbf{u}$ and is contaminated by additive white Gaussian noise $\mathbf{w}$ and downscale measurement proces, where $\mathbf{P}$ is a linear process consisting of low-pass filter and downsampling.)

```
% Measurement process P
sigmah = 2;
h = fspecial('gaussian',9,sigmah);
downsample2 = @(x,m) ipermute(downsample(permute(downsample(x,m),[2 1 3]),m),[2 1 3]);
linproc = @(x) downsample2(imfilter(x,h,'conv','circ'),2);
H = fftn(h,2.^nextpow2([size(u,1) size(u,2)]));
beta = max(abs(H(:)).^2); % λmax(PP.')

% Observation
sigmaw = 5/255;
v = imnoise(linproc(u),'gaussian',0,sigmaw^2);
figure(2)
imshow(v)
title('Observation v');
```



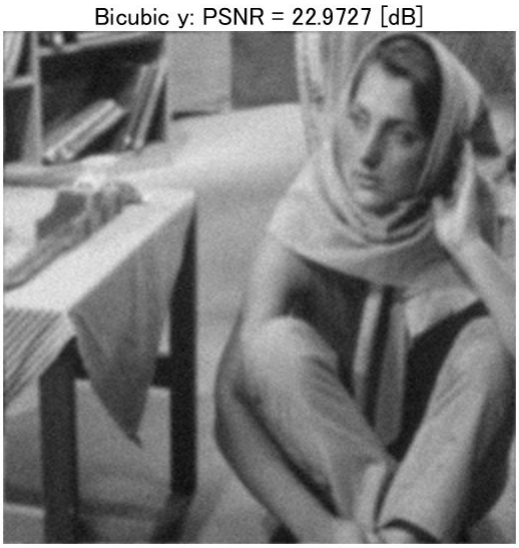
双三次補間

(Bicubic interpolation)

参考までに，双三次補間の結果を示す.

(For the sake of reference, the result with the bicubic interpolation is shown.)

```
% Bicubic
y = imresize(v,2,'bicubic');
figure(3)
imshow(y)
title(['Bicubic y: PSNR = ' num2str(psnr(u,y)) ' [dB]']);
```



単一画像超解像

(Single image super resolution)

上記の超改造問題は，冗長辞書 $\mathbf{D}$ を用いて ℓ_1-ノルム正則化崔譲二乗問題として

(The above super-resolution problem can be formulated by the ℓ_1-norm regularized least square problem with a redundant dictionary $\mathbf{D}$ as)

$$\hat{\mathbf{s}} = \arg\min_{\mathbf{s}} \frac{1}{2}\|\mathbf{v} - \mathbf{PDs}\|_2^2 + \lambda\|\mathbf{s}\|_1 .$$

のように定式化でき，その解は近接勾配アルゴリズムにより得られる.

(It is possible to obtain the solution by the proximal gradient algorithm)

$$\mathbf{s}^{(t+1)} \leftarrow \mathrm{prox}_{\gamma\lambda\|\cdot\|_1}(\mathbf{s}^{(t)} - \gamma\nabla f(\mathbf{s}^{(t)}))$$

$$\nabla f(\mathbf{s}) = \mathbf{D}^T\mathbf{P}^T(\mathbf{PDs} - \mathbf{v})$$

$$[\mathrm{prox}_{\gamma\lambda\|\cdot\|_1}(\mathbf{s})]_i = \mathcal{T}_{\gamma\lambda}([\mathbf{s}]_i) = \mathrm{sign}([\mathbf{s}]_i)\cdot\max(|[\mathbf{s}]_i| - \gamma\lambda, 0),$$

ただし，$0 < \gamma < 2/\beta$ であり，β は ∇f のリプシッツ定数である.

(where $0 < \gamma < 2/\beta$ and β is the Lipschitz constant of ∇f.)

以下では，サンプル 13-4 で訓練した NSOLT とともに近接勾配法を利用して単一画像超解像を行う.

(In the following, let us perform single image super-resolution (SISR) with a proximal gradient method using the nonseparate oversampled laped transform (NSOLT) trained in Sample 13-4 as a dictionary.)

$\mathbf{DD}^T = \mathbf{I}$ なので，(Since $\mathbf{DD}^T = \mathbf{I}$ holds，)

$$\beta = \lambda_{\max}(\mathbf{D}^T\mathbf{P}^T\mathbf{PD}) = (\mathbf{PDD}^T\mathbf{P}^T) = (\mathbf{PP}^T) = \lambda_{\max}(\mathbf{PP}^T) = \sigma_{\max}^2(\mathbf{P}).$$

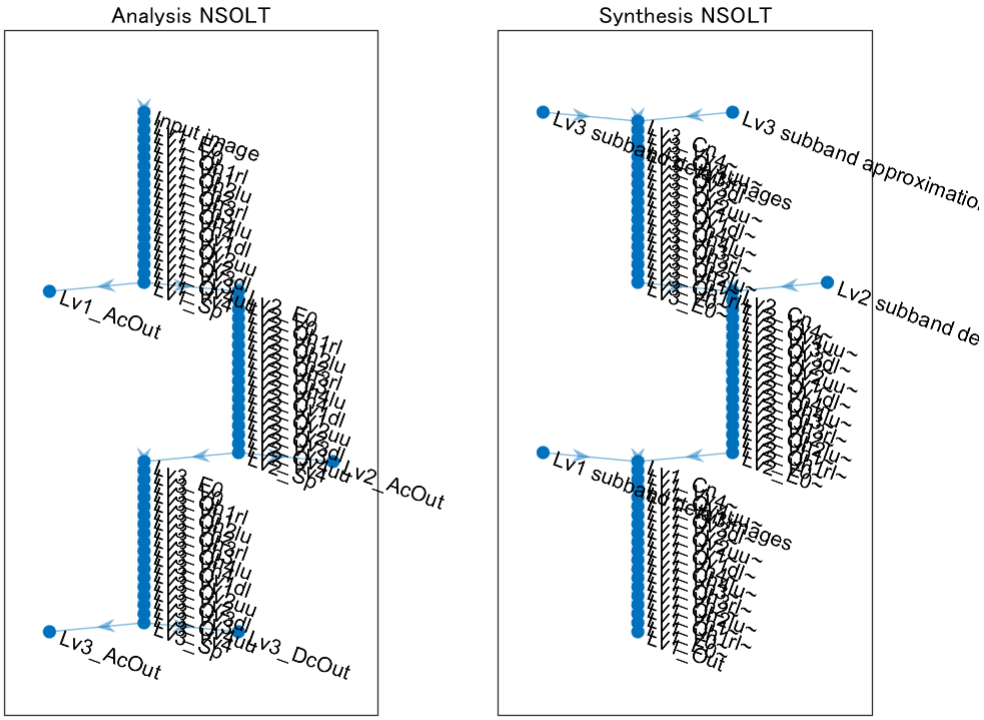
である．NSOLT の設計データは "data" フォルダの下の"nsoltdictionary.mat"にある.

(The design data of NSOLT is saved in "nsoltdictionary.mat" under the folder "data".)

NSOLT 辞書の読込み

(Loading NSOLT dictionary)

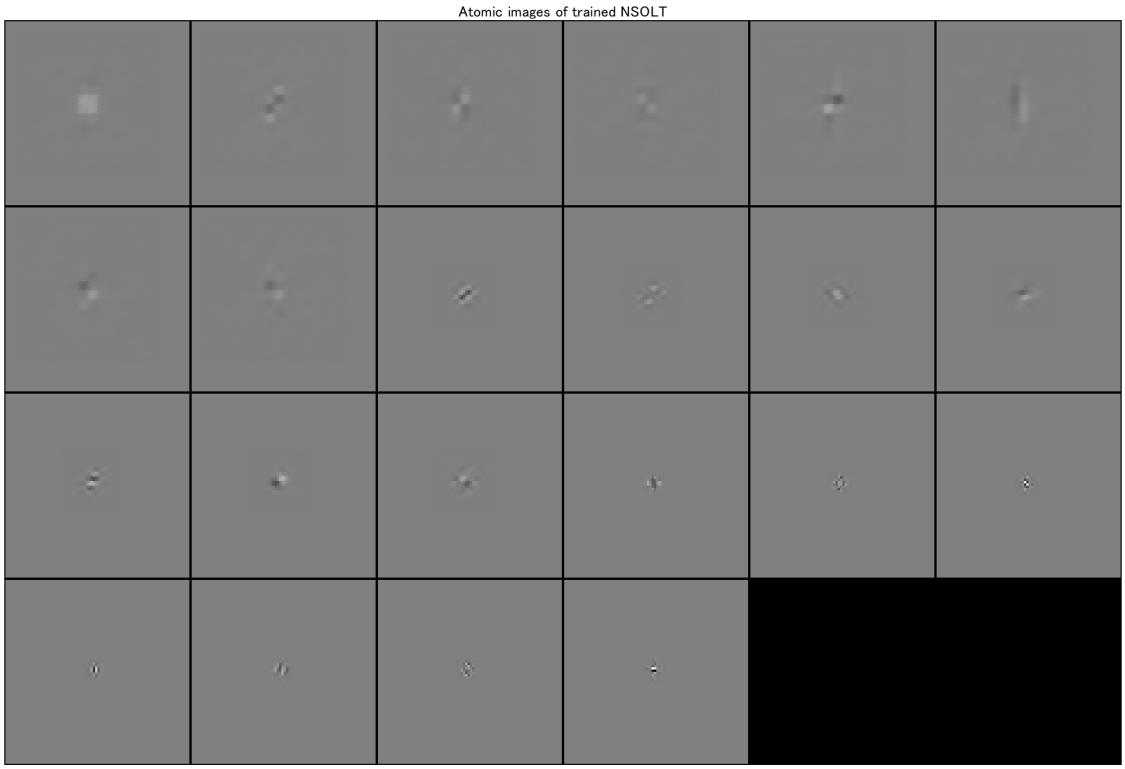
```
import msip.*
load ./data/nsoltdictionary.mat
analysislgraph = layerGraph(analysisnet);
synthesislgraph = layerGraph(synthesisnet);

figure(4)
subplot(1,2,1)
plot(analysislgraph)
title('Analysis NSOLT')
subplot(1,2,2)
plot(synthesislgraph)
title('Synthesis NSOLT')
```



要素画像の表示

(Show the atomic images)

```
figure(5)
atomicimshow(synthesisnet)
title('Atomic images of trained NSOLT')
```




```
import msip.*
analysislgraph = analysislgraph.replaceLayer('Input image',...
        imageInputLayer(size(y),'Name','Input image','Normalization','none'));
for iLayer = 1:length(analysislgraph.Layers)
    layer = analysislgraph.Layers(iLayer);
    %{
    if strcmp(layer.Name,'Sb_Srz')
        newsrzlayer = layer.setOriginalDimension(size(y));
        analysislgraph = analysislgraph.replaceLayer(...
            'Sb_Srz',newsrzlayer);
    end
    %}
    if strcmp(layer.Name,'Lv1_E0')
        decFactor = layer.DecimationFactor;
    end
    if strcmp(layer.Name,'Lv1_V0')
        nChannels = layer.NumberOfChannels;
    end
end

%isSerialize = false;
for iLayer = 1:length(synthesislgraph.Layers)
    layer = synthesislgraph.Layers(iLayer);
    %{
    if strcmp(layer.Name,'Sb_Dsz')
        newdszlayer = layer.setOriginalDimension(size(y));
        synthesislgraph = synthesislgraph.replaceLayer(...
```

```
            'Sb_Dsz',newdszlayer);
        isSerialize = true;
    end
    %}
    if contains(layer.Name,'subband detail images')
        nLevels = str2double(layer.Name(3));
        sbSize = size(y).*(decFactor.^(-nLevels));
        newsblayer = ...
            imageInputLayer([sbSize (sum(nChannels)-1)],'Name',layer.Name,'Normalization','none
        synthesislgraph = synthesislgraph.replaceLayer(...
            layer.Name,newsblayer);
    end
end
if contains(layer.Name,'subband approximation image')
    nLevels = str2double(layer.Name(3));
    sbSize = size(y).*(decFactor.^(-nLevels));
    newsblayer = ...
        imageInputLayer([sbSize 1],'Name',layer.Name,'Normalization','none');
    synthesislgraph = synthesislgraph.replaceLayer(...
        layer.Name,newsblayer);
end

%{
if isSerialize
    synthesislgraph = synthesislgraph.replaceLayer('Subband images',...
        imageInputLayer(newdszlayer.InputSize,'Name','Subband images','Normalization','none'));
end
%}

analysisnet = dlnetwork(analysislgraph);
synthesisnet = dlnetwork(synthesislgraph);
```

随伴関係（完全再構成）の確認

(Confirmation of the adjoint relation (perfect reconstruction))

NSOLT はパーセバルタイト性を満たすことに注意．(Note that NSOLT satisfy the Parseval tight property.)

```
x = rand(size(y),'single');
dlx = dlarray(x,'SSC'); % Deep learning array (SSC: Spatial,Spatial,Channel)
[dls{1:nLevels+1}] = analysisnet.predict(dlx);
dly = synthesisnet.predict(dls{1:nLevels+1});
display("MSE: " + num2str(mse(dlx,dly)))
```

```
"MSE: 2.8232e-09"
```

観測過程の随伴作用素

(Adjoint operator of the measurement process)

```
% Adjoint process P.'
upsample2 = @(x,m) ipermute(upsample(permute(upsample(x,m),[2 1 3]),m),[2 1 3]);
adjproc = @(x) imfilter(upsample2(x,2),h,'corr','circ');
```

近接勾配法(ISTA)の実行

(Execution of proximal-gradient method (ISTA))

- "synthesisnet" and "analysisnet" are objects of "dlnetwork."
- "dlnetwork" takes "dlarray."
- Function "extractdata" is used for extracting ordinary array data from "dlarray."

```
% Parameter settings
lambda = 0.0007;
gamma = 2./(beta+1e-2);
nIters = 20;
isDcSkip = true;
```

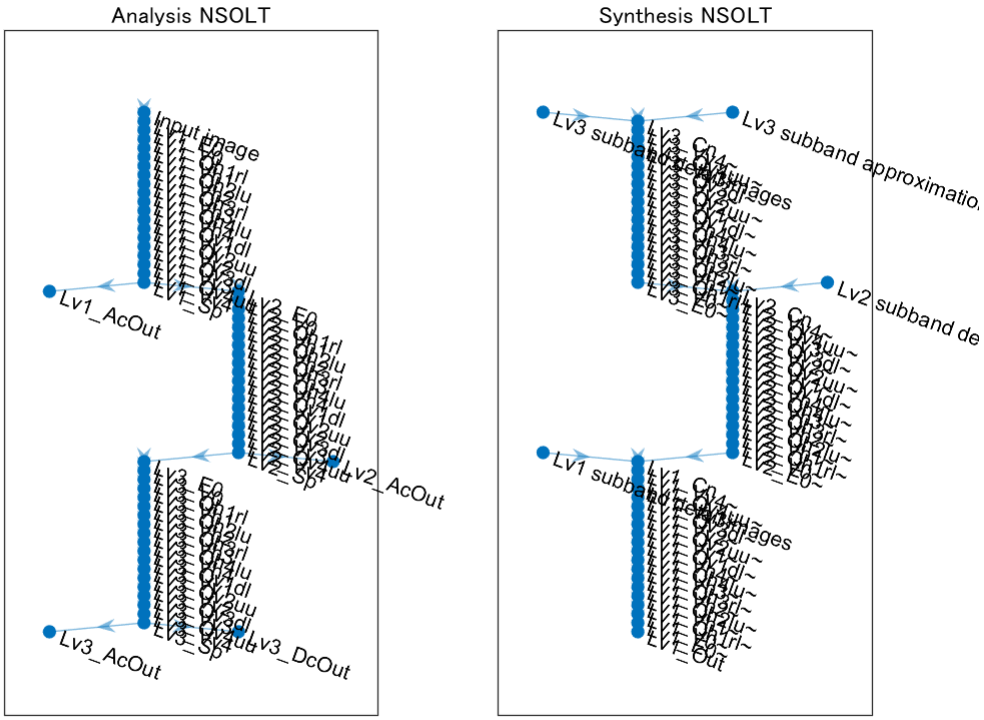
```
% Initalization
z = dlarray(zeros(size(y),'like',y),'SSC'); % Deep learning array (SSC: Spatial,Spatial,Channel
[coefs{1:nLevels+1}] = analysisnet.predict(z);

% profile on
for itr = 1:nIters
    x = linproc(extractdata(synthesisnet.predict(coefs{1:nLevels+1})));
    [g{1:nLevels+1}] = analysisnet.predict(dlarray(adjproc(x-v),'SSC'));
    % Gradient descnet
    for iOutput = 1:nLevels+1
        coefs{iOutput} = coefs{iOutput}-gamma*g{iOutput};
        if ~isDcSkip || iOutput < nLevels+1
            % ISTA for detail (w/o soft-thresholding for approximation components)
            coefs{iOutput} = wthresh(coefs{iOutput},'s',gamma*lambda);
        end
    end
end
% profile off
% profile viewer
```

```
% Reconstruction
r = extractdata(synthesisnet.predict(coefs{1:nLevels+1}));
```

結果の表示

(Display the result)

```
figure(6)
imshow(r)
```




```
title(['Restoration r: PSNR = ' num2str(psnr(u,r)) ' [dB]'])
```