Sample 11-5

画像ノイズ除去

事前分布

画像処理特論

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動作確認: MATLAB R2020a

Image denoising

Prior distribution

Advanced Topics in Image Processing

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Verified: MATLAB R2020a

準備

(Preparation)

```
clear
close all
import msip.download_img
msip.download_img
```

```
lena.png already exists in ./data/
baboon.png already exists in ./data/
goldhill.png already exists in ./data/
barbara.png already exists in ./data/
```

パラメータ設定

(Parameter settings)

- sgm: ノイズ標準偏差 σ_w (Standard deviation of noise)
- nlevels: ウェーブレット段数 (Wavelet levels)

```
% Parameter settings
nlevels = 3;
```

画像の読込

(Read image)

```
img = "lena";
u = im2double(imread(['./data/' char(img) '.png']));
if size(u,3) == 3
    u = rgb2gray(u);
```

```
end
```

分析処理
(Analysis process)

直交ウェーブレット変換Symlet を利用．(Uses Symlet, which is an orthogonal wavelet transform.)

```
% Preperation of filters for wavelets
iswtb = license('checkout','wavelet_toolbox');
if iswtb % Functions in Wavelet Toolbox are used
    dwtmode('per')
    wname = "sym4";
    [h0,h1,f0,f1] = wfilters(wname);
    %save(['./data/' char(wname) '.mat'],'h0','h1','f0','f1')
else
    import msip.ezwavedec2
    import msip.ezwaverec2
    S = load('./data/sym4.mat');
    h0 = S.h0;
    h1 = S.h1;
    f0 = S.f0;
    f1 = S.f1;
    clear H  F
    % Analysis bivariate filters
    H.h00 = h0(:)*h0(:).';
    H.h01 = h0(:)*h1(:).';
    H.h10 = h1(:)*h0(:).';
    H.h11 = h1(:)*h1(:).';
    % Synthesis bivariate filters
    F.f00 = f0(:)*f0(:).';
    F.f01 = f0(:)*f1(:).';
    F.f10 = f1(:)*f0(:).';
    F.f11 = f1(:)*f1(:).';
end
```

```
**********************
**  DWT 拡張モード: 周期化  **
**********************
```

分析処理 (Analysis process)

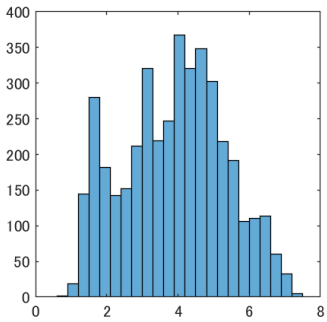
```
if iswtb
    [coefs,scales] = wavedec2(u,nlevels,h0,h1);
    % Reconstruction to check PR
    r = waverec2(coefs,scales,f0,f1);
else
    [coefs,scales] = ezwavedec2(u,nlevels,H);
    % Reconstruction to check PR%
    r = ezwaverec2(coefs,scales,F);
end
assert(norm(u-r,"fro")^2/numel(u)<1e-18,'Perfect reconstruction is violated.')
```

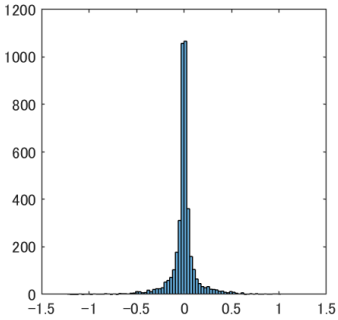
変換係数の抽出 (Extraction of coefficients)

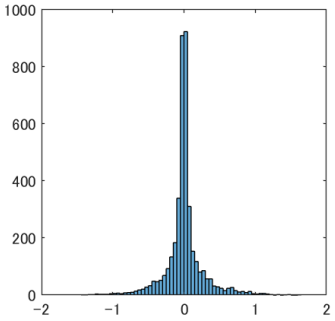
```
s = extractcoefs(coefs,scales);
```

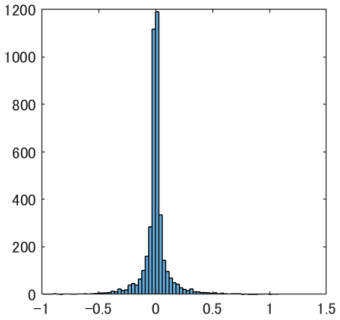
変換係数の分布
(Distribution of Coefs.)

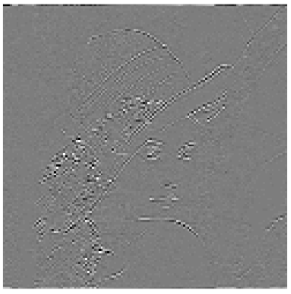
```
nchs = length(s);
for ich = 1:nchs
    figure(ich)
    subplot(1,2,1)
    if ich == 1
        imshow(s{ich}*pow2(-nlevels))
    else
        imshow(s{ich}+.5)
    end
    subplot(1,2,2)
    histogram(s{ich}(:))
    axis square
end
```

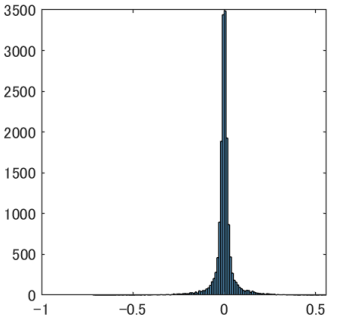


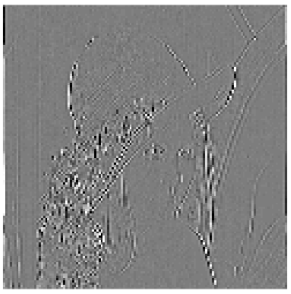

1200
1000
800
600
400
200
0
−1.5
−1
−0.5
0
0.5
1
1.5

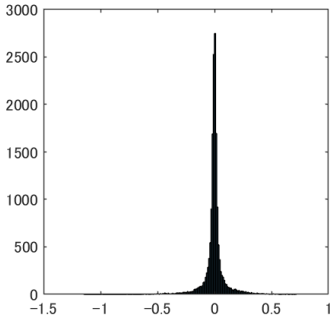

1000
800
600
400
200
0
−2
−1
0
1
2

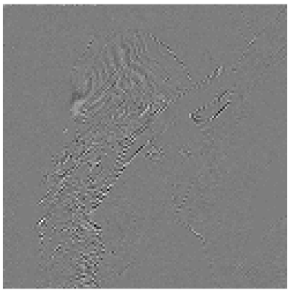

1200
1000
800
600
400
200
0
−1
−0.5
0
0.5
1
1.5

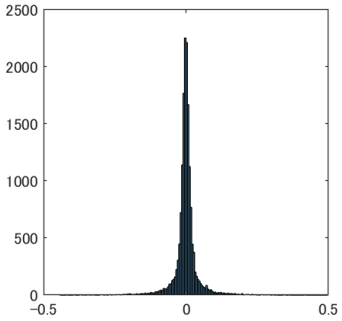


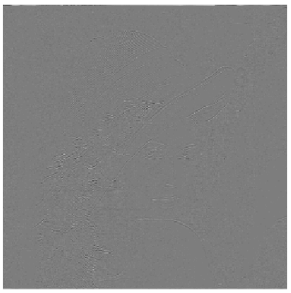

3500
3000
2500
2000
1500
1000
500
0
−1
−0.5
0
0.5

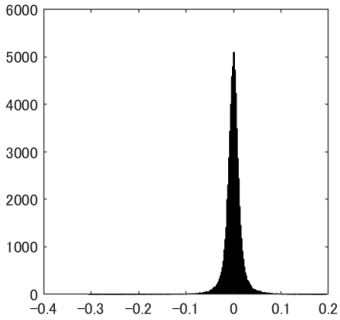


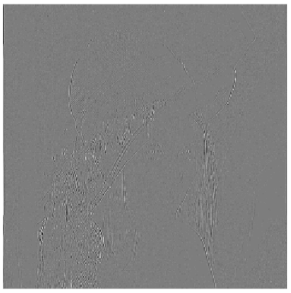

3000
2500
2000
1500
1000
500
0
−1.5
−1
−0.5
0
0.5
1

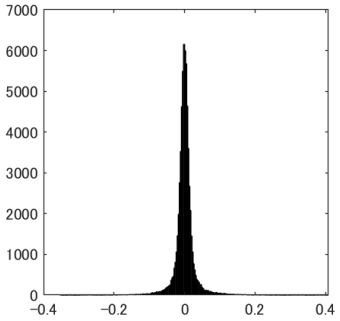


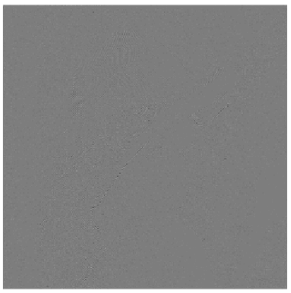

2500
2000
1500
1000
500
0
−0.5
0
0.5

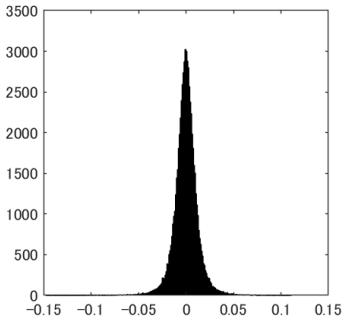



6000
5000
4000
3000
2000
1000
0
−0.4
−0.3
−0.2
−0.1
0
0.1
0.2




7000
6000
5000
4000
3000
2000
1000
0
−0.4
−0.2
0
0.2
0.4



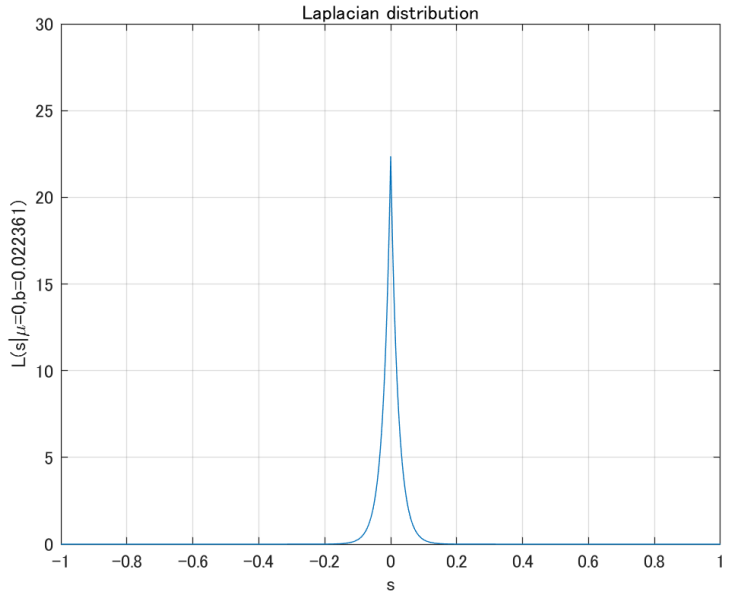


ラプラス分布
(Laplace distribution)

変換係数はラプラス分布に従う乱数と仮定．(Coefs. are assumed to be random numbers drawn from a Laplace distribution.)

- $\mathbf{s} \sim \mathrm{Lap}(\mathbf{s}|\boldsymbol{\mu} = \mathbf{0}, b)$
- $\mathrm{Lap}(\mathbf{s}|\boldsymbol{\mu} = \mathbf{0}, b) = \frac{1}{2b}\exp\left(-\frac{\|\mathbf{s} - \boldsymbol{\mu}\|_1}{b}\right)$

```
% Laplacian parameters
mu = 0;
sgm2 = 0.001;
b = sqrt(sgm2/2);
ifig = nchs + 1;
% Laplacian distribution
spdf = @(x) 1/(2*b)*exp(-abs(x-mu)/b);
figure(ifig)
h = fplot(spdf);
xlabel('s')
ylabel(['L(s|\mu=0,b=' num2str(b) ')'])
title('Laplacian distribution')
grid on
axis([-1 1 0 30])
```



観測画像
(Observation image)

パラメータ設定 (Parameter settings)

```
sgmuint8 = 30;
sgm = sgmuint8/255;
```

ノイズ付加 (Add noise)

```
v = imnoise(u,'gaussian',0,sgm^2);
```

分析処理 (Analysis process)

```
if iswtb
    [coefs,scales] = wavedec2(v,nlevels,h0,h1);
else
    [coefs,scales] = ezwavedec2(v,nlevels,H);
end
```

ウェーブレット縮退処理
(Wavelet shrinkage)

問題設定 (Problem settings)

- $\hat{\mathbf{s}} = \arg\min_{\mathbf{s}} \frac{1}{2}\|\mathbf{v} - \mathbf{D}\mathbf{s}\|_2^2 + \lambda\|\mathbf{s}\|_1$
- $\mathbf{D}\mathbf{D}^T = \mathbf{D}^T\mathbf{D} = \mathbf{I}$ (Orthonormal)

パラメータ設定 (Parameter settings)

```
isbayesshrink = false;
if ~isbayesshrink
    isaprxleft = true;
    lambda = 10^-0.8
end
```

```
lambda = 0.1585
```

ソフト閾値処理 (Soft-thresholding)

$\hat{\mathbf{s}} = \mathcal{T}_{\lambda}(\mathbf{s}) = \mathrm{sign}(\mathbf{s}) \odot \max(\mathrm{abs}(\mathbf{s}) - \lambda\mathbf{1}, \mathbf{0})$

```
% サブバンド適応ソフト縮退処理
if isbayesshrink
    import msip.bayesshrink
    coefs = bayesshrink(coefs,scales);
else
    if isaprxleft
        mask = ones(size(coefs));
        mask(1:prod(scales(1,:))) = 0;
        lambda = lambda * mask;
    end
    softshrink = @(x) sign(x).*max(abs(x)-lambda,0);
    coefs = softshrink(coefs);
end
```

合成処理 (Synthesis process)

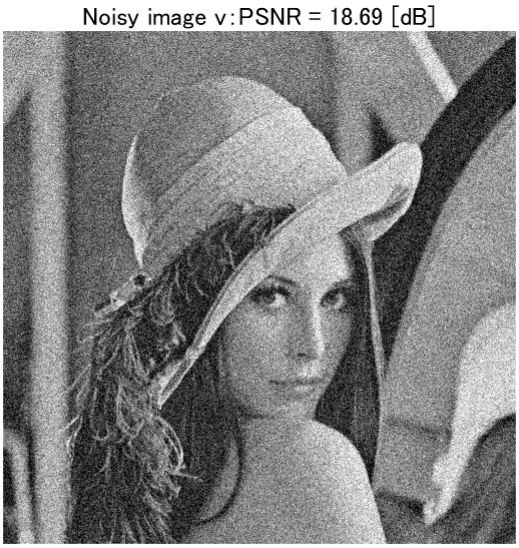
```
if iswtb
    r = waverec2(coefs,scales,f0,f1);
else
    r = ezwaverec2(coefs,scales,F);
end
```

画像表示

(Image show)

```
ifig = ifig + 1;
figure(ifig)
imshow(u);
title('Original image u')
```


```
ifig = ifig + 1;
figure(ifig)
imshow(v)
title(sprintf('Noisy image v : PSNR = %5.2f [dB]',psnr(u,v)))
```




```
ifig = ifig + 1;
figure(ifig)
imshow(r)
title(sprintf('Denoised image r : PSNR = %5.2f [dB]',psnr(u,r)))
```

Denoised image r : PSNR = 26.78 [dB]

ウェーブレット画像ノイズ除去関数
(Wavelet image denoising function)

参考資料 (Reference)

```
if iswtb
    help wthcoef2
    help wdenoise2
end
```

```
wthcoef2 - Wavelet coefficient thresholding 2-D

    This MATLAB function returns the horizontal (vertical or diagonal, respectively)
    coefficients obtained from the wavedec2 decomposition structure [C,S] (see
    wavedec2 for more information), by soft (if SORH ='s') or hard (if SORH ='h')
    thresholding defined in vectors N and T.

    NC = wthcoef2('type',C,S,N,T,SORH)
    NC = wthcoef2('type',C,S,N)
    NC = wthcoef2('a',C,S)
    NC = wthcoef2('t',C,S,N,T,SORH)

    参考 wavedec2, wthresh

    wthcoef2 のドキュメンテーション
wdenoise2 - Wavelet image denoising

    This MATLAB function denoises the grayscale or RGB image IM using an empirical
    Bayesian method.

    IMDEN = wdenoise2(IM)
    IMDEN = wdenoise2(IM,LEVEL)
    [IMDEN,DENOISEDCFS] = wdenoise2(___)
    [IMDEN,DENOISEDCFS,ORIGCFS] = wdenoise2(___)
    [IMDEN,DENOISEDCFS,ORIGCFS,S] = wdenoise2(___)
    [IMDEN,DENOISEDCFS,ORIGCFS,S,SHIFTS] = wdenoise2(___)
    [___] = wdenoise2(___,Name,Value)
```

wdenoise2(___)

参考 wavedec2, wdenoise

wdenoise2 のドキュメンテーション

変換係数の抽出
(Extraction of Coefs.)

```
function s = extractcoefs(coefs,scales)
nscales = size(scales,1)-1;
s = cell(3*(nscales-1)+1,1);
sidx = 1;
ndims = scales(1,:);
eidx = sidx + prod(ndims) - 1;
s{1} = reshape(coefs(sidx:eidx),ndims);
sidx = eidx + 1;
ich = 2;
for iscale = 2:nscales
    ndims = scales(iscale,:);
    for iband = 1:3
        eidx = sidx + prod(ndims) - 1;
        s{ich} = reshape(coefs(sidx:eidx),ndims);
        sidx = eidx + 1;
        ich = ich + 1;
    end
end
end
```