Sample 8-5

離散コサイン変換

DFT との関係

画像処理特論

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動作確認: MATLAB R2020a

Discrete cosine transform

Relation to DFT

Advanced Topics in Image Processing

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Verified: MATLAB R2020a

準備

(Preparation)

```
close all
```

DCT 行列

(DCT matrix)

$$[\mathbf{C}_M]_{k,n} = \sqrt{\frac{2}{M}}\alpha_k \cos\frac{k(n+1/2)\pi}{M},\ \ k,n=0,1,\cdots,M-1$$

$$\alpha_k = \begin{cases} \frac{1}{\sqrt{2}} & k=0 \\ 1 & k=1,2,\cdots,M-1 \end{cases}$$

```
% DCT points
nPoints = 4;
C = dctmtx(nPoints)
```

```
C = 4×4
    0.5000    0.5000    0.5000    0.5000
    0.6533    0.2706   -0.2706   -0.6533
    0.5000   -0.5000   -0.5000    0.5000
    0.2706   -0.6533    0.6533   -0.2706
```

OTDFT 行列

(OTDFT matrix)

OTDFT は DCT と深い関係にある。(OTDFT is closely related to DCT.)

OTDFT: Odd-time discrete Fourier transform (GDFT w/ $a = 0, b = 1/2$)

- 一般化 DFT (GDFT: generalized DFT)

$$X_N^{(a,b)}[k] = \sum_{n=0}^{N-1} x[n] W_N^{(k+a)(n+b)},\ k = 0, 1, \cdots, N-1$$

- OTDFT 行列 (OTDFT matrix)

$$[\mathbf{W}_N^{\left(0,\frac{1}{2}\right)}]_{k,n} = e^{-\jmath\frac{\pi}{N}k} e^{-\jmath\frac{2\pi}{N}kn}$$

```
% DFT points
nPointsDft = 2 * nPoints;

% OTDFT matrix
k=0:nPointsDft-1;
Wdft = dftmtx(nPointsDft);
Lambda = diag(exp(-1j*pi/nPointsDft*k));
Wotdft = Lambda * Wdft
```

```
Wotdft = 8×8 complex
   1.0000 + 0.0000i   1.0000 + 0.0000i   1.0000 + 0.0000i   1.0000 + 0.0000i ···
   0.9239 - 0.3827i   0.3827 - 0.9239i  -0.3827 - 0.9239i  -0.9239 - 0.3827i
   0.7071 - 0.7071i  -0.7071 - 0.7071i  -0.7071 + 0.7071i   0.7071 + 0.7071i
   0.3827 - 0.9239i  -0.9239 + 0.3827i   0.9239 + 0.3827i  -0.3827 - 0.9239i
   0.0000 - 1.0000i  -0.0000 + 1.0000i   0.0000 - 1.0000i  -0.0000 + 1.0000i
  -0.3827 - 0.9239i   0.9239 + 0.3827i  -0.9239 + 0.3827i   0.3827 - 0.9239i
  -0.7071 - 0.7071i   0.7071 - 0.7071i   0.7071 + 0.7071i  -0.7071 + 0.7071i
  -0.9239 - 0.3827i  -0.3827 - 0.9239i   0.3827 - 0.9239i   0.9239 - 0.3827i
```

OTDFT による DCT

(DCT through OTDFT)

OTDFT と DCT の関係 (Relation between OTDFT and DCT)

$$[\mathbf{C}_M]_{k,n} = \frac{\alpha_k}{\sqrt{2M}} \left[\mathbf{W}_{2M}^{\left(0,\frac{1}{2}\right)} \mathbf{E}_M\right]_{k,n},\ k, n = 0, 1, \cdots, M-1$$

ただし、 $\mathbf{E}_M$ は対称拡張行列 (where $\mathbf{E}_M$ is the symmetric extension matrix defined by)

$$
\mathbf{E}_M = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 1 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \end{pmatrix}.
$$

である。

```
% Symmetric extension matrix
E = [ eye(nPoints) ; fliplr(eye(nPoints)) ]
```

```
E = 8×4
     1     0     0     0
     0     1     0     0
     0     0     1     0
     0     0     0     1
     0     0     0     1
     0     0     1     0
     0     1     0     0
     1     0     0     0
```

```

% DCT matrix through OTDFT
D = 1/sqrt(2*nPoints)*...
    diag([1/sqrt(2) ones(1,nPoints-1) ])*...
    Wotdft(1:nPoints,:)*E
```

```
D = 4×4 complex
   0.5000 + 0.0000i   0.5000 + 0.0000i   0.5000 + 0.0000i   0.5000 + 0.0000i
   0.6533 + 0.0000i   0.2706 + 0.0000i  -0.2706 + 0.0000i  -0.6533 + 0.0000i
   0.5000 + 0.0000i  -0.5000 - 0.0000i  -0.5000 - 0.0000i   0.5000 + 0.0000i
   0.2706 + 0.0000i  -0.6533 - 0.0000i   0.6533 + 0.0000i  -0.2706 - 0.0000i
```

誤差の評価 (Evaluation of error)

```
norm(D-C,'Fro')
```

```
ans = 4.8552e-16
```

Wdft による変換(DFT)には、高速フーリエ変換(FFT)を適用できる。(The Fast Fourier Transform (FFT) can be applied to the transform with Wdft (DFT).)