

Sample 10-6

冗長変換

ℓ_1 -ノルム最小化

画像処理特論

村松 正吾

動作確認: MATLAB R2020a

Redundant transforms

ℓ_1 -norm minimization

Advanced Topics in Image Processing

Shogo MURAMATSU

Verified: MATLAB R2020a

準備

(Preparation)

```
close all
```

非線形近似の設定

(Settings of non-linear approximation)

```
% # of Coefs.  
K = 32;
```

入力信号の生成

(Generation of input sequence)

```
% # of input samples  
nSamples = 128;  
  
% Random process in AR(1) model  
rng('default');  
w = 0.1*randn(nSamples,1);  
w(floor(end/2)) = 1;  
u = filter(1,[1 -0.95],w);
```

合成辞書

(Synthesis dictionary)

```
% Synthesis filters
```


3

[illegible]

28 列から 54 列

[illegible]

[illegible]

ℓ_1 -ノルム最小化による非線形近似
(Non-linear approximation with ℓ_1 -norm minimization)

$$\hat{\mathbf{s}} = \arg \min_{\mathbf{s} \in \mathbb{R}^L} \|\mathbf{s}\|_1 \text{ s.t. } \mathbf{v} = \mathbf{D}\mathbf{s}$$

凸緩和法による分析処理と係数選択 (Analysis process and coefficient selection by a convex-relaxation)

- 基底追跡法 (Basis Pursuit; BP)

線形計画問題に帰着させる. (Reduced to a linear programming problem)

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x} \in \mathbb{R}^{2L}} \mathbf{1}^T \mathbf{x} \text{ s.t. } \mathbf{v} = \mathbf{D}(\mathbf{I} \quad -\mathbf{I})\mathbf{x} \cap \mathbf{x} \in [0, \infty)^{2L}$$

ただし, (where)

$$\mathbf{s} = \mathbf{s}_+ - \mathbf{s}_- \in \mathbb{R}^L$$

$$\mathbf{x} = \begin{pmatrix} \mathbf{s}_+ \\ \mathbf{s}_- \end{pmatrix} \in \mathbb{R}^{2L}$$

```
% Initialization
M = size(D,2);
f = ones(2*M,1);
lu = zeros(2*M,1);
% Linear programming
z = linprog(f,[],[],[D -D],u,lx,[]);
```

最適解が見つかりました。

```
s = z(1:M) - z(M+1:end);
```

係数選択 (Coefficient selection)

```
s = s(:);
[~,ix] = sort(abs(s), 'descend');
s(ix(K+1:end)) = 0;
```

近似結果 (Approximation result)

```
v = D*s;
```

近似誤差 (Residual)

```
r = u - v;
```

グラフ描画

(Graph plot)

```
figure(1)

% Input
subplot(3,1,1)
stem(0:nSamples-1,u,'filled')
```

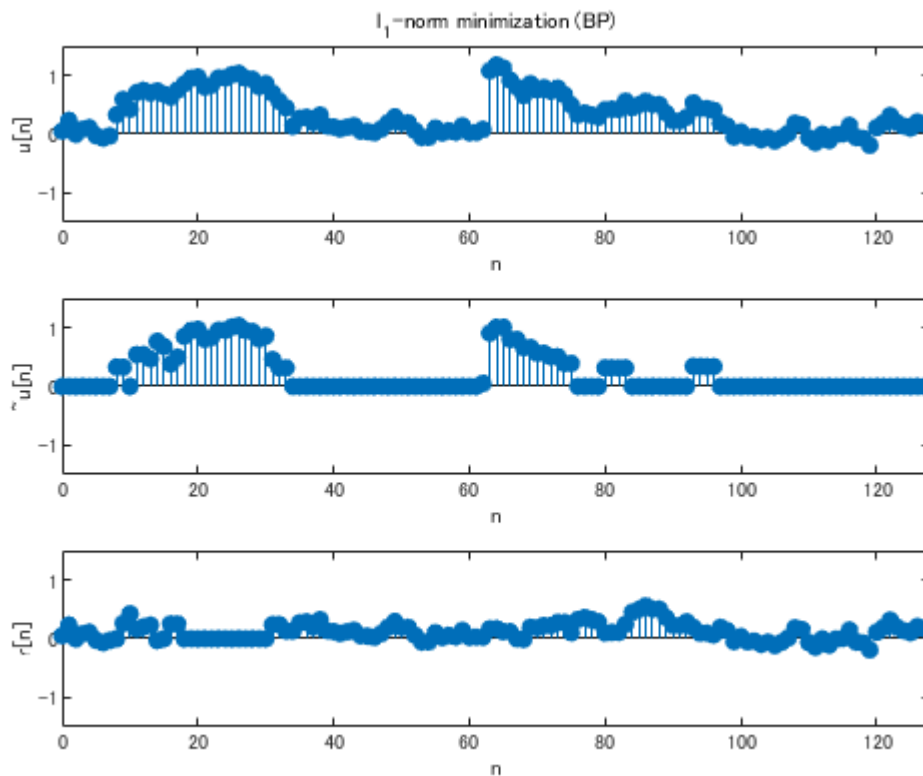
```
axis([0 nSamples -1.5 1.5])
xlabel('n')
ylabel('u[n]')
title('l1-norm minimization (BP)')
```

```
% NLA
```

```
subplot(3,1,2)
stem(0:nSamples-1,v,'filled')
axis([0 nSamples -1.5 1.5])
xlabel('n')
ylabel('~u[n]')
```

```
% Residual
```

```
subplot(3,1,3)
stem(0:nSamples-1,r,'filled')
axis([0 nSamples -1.5 1.5])
xlabel('n')
ylabel('r[n]')
```



MSE評価 (MSE evaluation)

```
mymse = @(x,y) mean((x(:)-y(:)).^2);
fprintf('mse = %f\n',mymse(u,v));
```

```
mse = 0.035184
```

© Copyright, Shogo MURAMATSU, All rights reserved.