Sample 12-2

画像復元

フィルタ補正逆投影法 (FBP)

画像処理特論

村松 正吾

動作確認: MATLAB R2020a

Image restoration

Filtered backprojection (FBP)

Advanced Topics in Image Processing

Shogo MURAMATSU

Verified: MATLAB R2020a

準備

(Preparation)

```
clear
close all
```

パラメータ設定

(Parameter settings)

```
nSlices = 180;
lambda = 10^-5
```

```
lambda = 1.0000e-05
```

断層スライス画像の生成

(Creation of tomographic slice images)

- $u(\mathbf{p}), \mathbf{p} \in \mathbb{R}^2$: 原画像 (Original image)

PHANTOM関数を利用して改良型シェップ ローガン頭部ファントム画像 $\mathbf{u}$ を作成して表示 (Create the modified Shepp-Logan head phantom image by using function PHATOM and display it.)

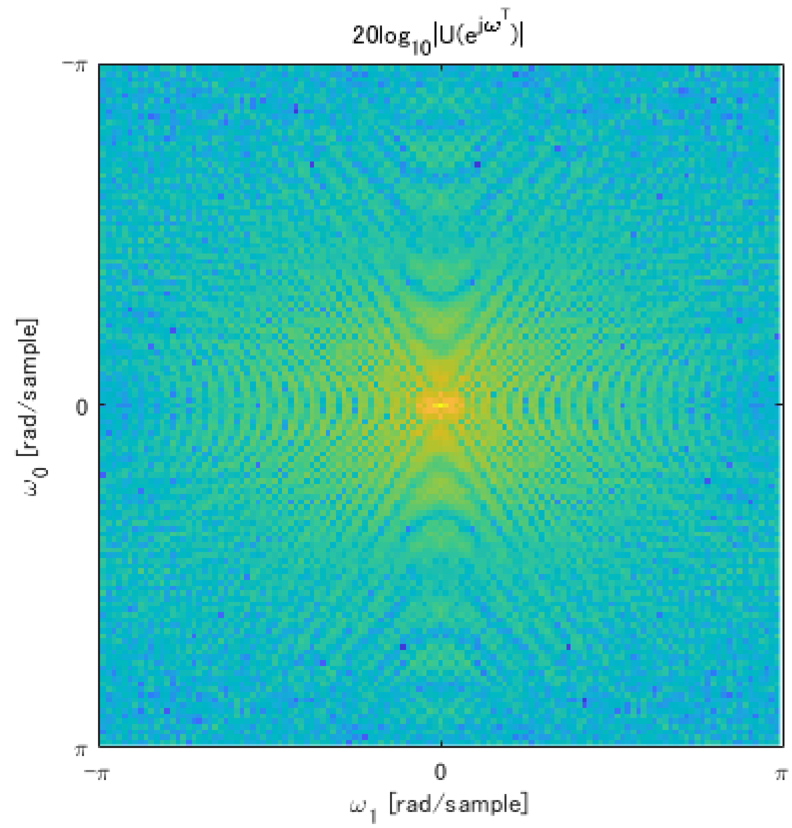
```
u = phantom(128);
figure(1)
imshow(u)
title('Original image u')
```

2変量DFT

(Bivariate DFT)

$$U[\mathbf{k}] = U(e^{j\boldsymbol{\omega}^T})|_{\boldsymbol{\omega}=2\pi\mathbf{Q}^{-T}\mathbf{k}}$$

```
nPointsQ = 2.^nextpow2(size(u));
U = fftn(u,nPointsQ);
figure(2)
w0 = linspace(-pi,pi-2*pi/nPointsQ(2),nPointsQ(2));
w1 = linspace(-pi,pi-2*pi/nPointsQ(1),nPointsQ(1));
imagesc(w1,w0,fftshift(20*log10(abs(U))));
axis square
ylabel('\omega_0 [rad/sample]')
xlabel('\omega_1 [rad/sample]')
title('20log_{10}|U(e^{j{\bf\omega}^T})|')
ax = gca;
ax.YLim = [-pi pi];
ax.XLim = [-pi pi];
ax.YTick = [-pi 0 pi];
ax.YTickLabel = { '-\pi', '0', '\pi'};
ax.XTick = [-pi 0 pi];
ax.XTickLabel = { '-\pi', '0', '\pi'};
```



ラドン変換
(Radon transform)

$$\mathrm{r}_u(x,\theta) = \int_{\mathbb{R}} u(x\boldsymbol{\theta} + y\boldsymbol{\theta}^{\perp})dy$$

ただし，(where)

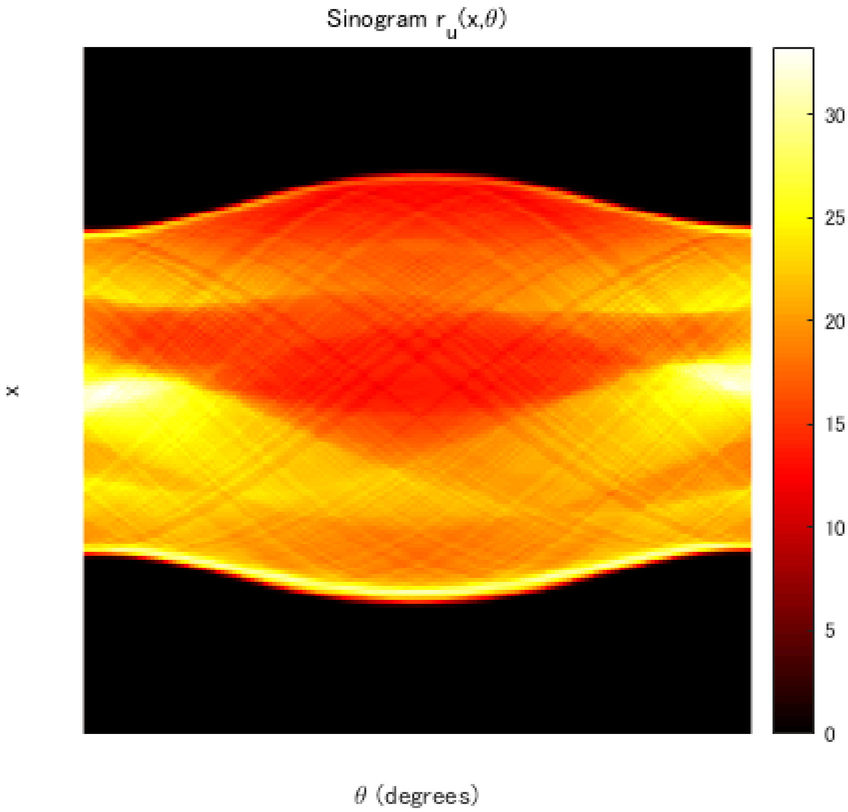
- $\boldsymbol{\theta} = (\cos\theta, \sin\theta)^T$
- $\boldsymbol{\theta}^{\perp} = (-\sin\theta, \cos\theta)^T$

RADON関数を利用 (Use function RADON)

```
thetaset = linspace(0,180-180/nSlices,nSlices);
[r,p] = radon(u,thetaset);
```

サイノグラムの表示 (Show sinogram)

```
figure(3)
imshow(r,[],'Xdata',thetaset,'Ydata',p,'InitialMagnification','fit')
xlabel('\theta (degrees)')
ylabel('x')
title('Sinogram r_u(x,\theta)')
colormap(gca,hot), colorbar
```




```
iptsetpref('ImshowAxesVisible','off')
```

スライスの単変量DFT

(Univariate DFT of each slice)

中央スライス定理(Central slice theorem)

$$R_u(j\Omega_x, \theta) = U(j(\sin\theta \;\; \cos\theta)^T\Omega_x)$$

※座標を垂直，水平の順序で表示していることに注意 (NOTE that the coordinates are shown in vertical and horizontal order.)

DFTでの解釈 (Interpretation as DFT)

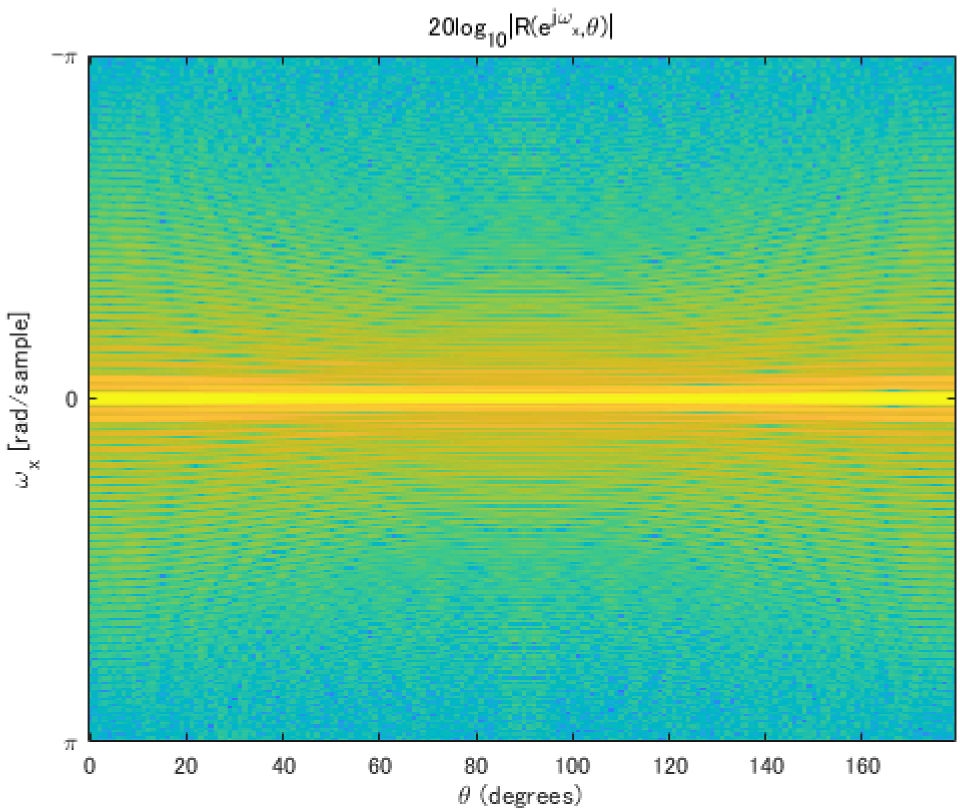
$$R_{u,\theta}[k] = R_u(e^{j\omega}, \theta)|_{\omega=2\pi k/N} = U(e^{j\boldsymbol{\omega}^T})|_{\boldsymbol{\omega}=\begin{pmatrix}\sin\theta\\ \cos\theta\end{pmatrix}2\pi k/N}$$

```
nPoints = 2^nextpow2(max([size(U,1) length(p)]));
rc = circshift(padarray(r,nPoints-size(r,1),'post'),p(1));
R = fft(rc,nPoints,1);
```

スライス毎のスペクトルの表示 (Display of spectra for each slice)

```
figure(4)
wx = linspace(-pi,pi-2*pi/nPoints,nPoints);
imagesc(thetaset,wx,fftshift(20*log10(abs(R)),1))
ylabel('\omega_x [rad/sample]')
```

```
xlabel('\theta (degrees)')
title('20log_{10}|R(e^{j\omega_x},\theta)|')
ax = gca;
ax.YLim = [-pi pi];
ax.YTick = [-pi 0 pi];
ax.YTickLabel = { '-\pi', '0', '\pi'};
```




```
iptsetpref('ImshowAxesVisible','off')
```

逆ラドン変換

(Inverse radon transform)

問題設定 **(Problem setting)**

$$\hat{\mathbf{u}} = \arg\min_{\mathbf{u}} \frac{1}{2} \left\| \begin{pmatrix} \mathbf{r}_{\theta_0} \\ \mathbf{r}_{\theta_1} \\ \vdots \\ \mathbf{r}_{\theta_{I-1}} \end{pmatrix} - \begin{pmatrix} \mathbf{W}_1^{-1}\mathbf{S}_{\downarrow\theta_0} \\ \mathbf{W}_1^{-1}\mathbf{S}_{\downarrow\theta_1} \\ \vdots \\ \mathbf{W}_1^{-1}\mathbf{S}_{\downarrow\theta_{I-1}} \end{pmatrix} \mathbf{W}_2\mathbf{u} \right\|_2^2 + \frac{\lambda}{2}\|\nabla\mathbf{u}\|_2^2$$

ただし，(where)

- $\mathbf{u} \in \mathbb{R}^N$: Vectorized expression of discretized array of $u(\mathbf{p})$
- $\mathbf{r}_{\theta_i} \in \mathbb{R}^m$: Vectorized expression of discretized slice line of $r_u(x,\theta)$
- $\mathbf{W}_d$: d-dimensional DFT matrix

- $\mathbf{S}_{\downarrow\theta_i}$: Slice operator for direction θ_i
- ∇: Gradient operator

解 **(Solution)**

$$\hat{\mathbf{u}} = \mathbf{W}_2^{-1}\left(\left(\sum_{i=0}^{I-1}\mathbf{S}_{\downarrow\theta_i}^T\mathbf{S}_{\downarrow\theta_i}\right) + \frac{m}{N}\lambda\mathbf{W}_2\nabla^T\nabla\mathbf{W}_2^{-1}\right)^{-1}\left(\sum_{i=0}^{I-1}\mathbf{S}_{\downarrow\theta_i}^T\mathbf{W}_1\mathbf{r}_{\theta_i}\right)$$

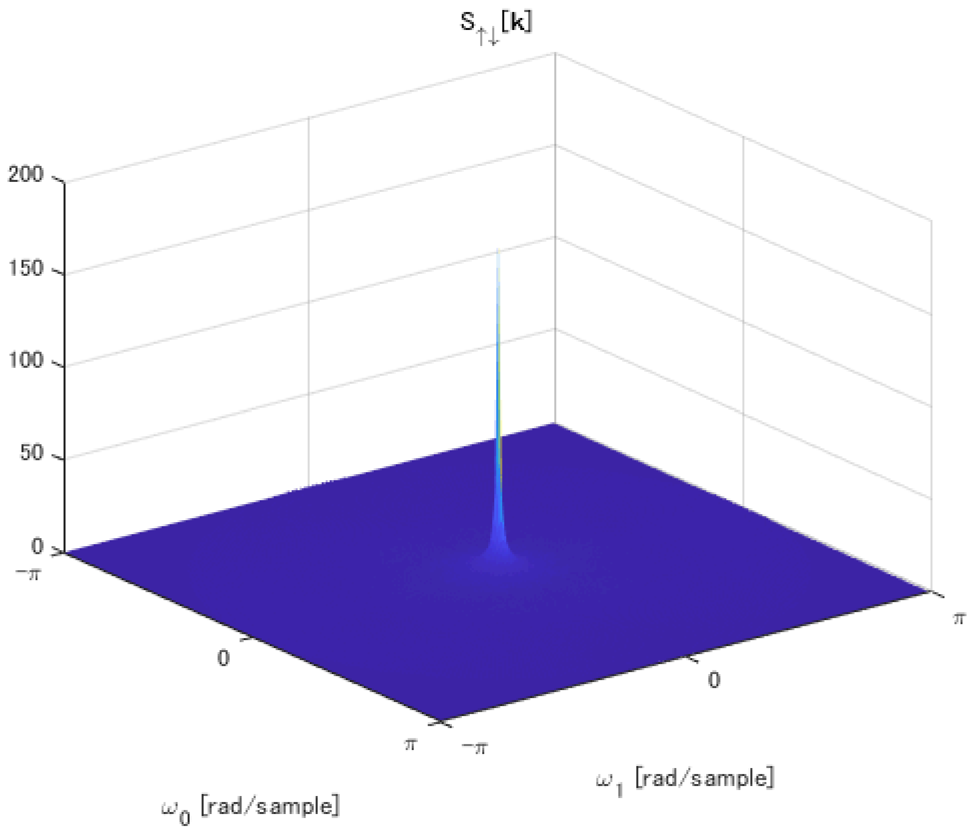
$$\overset{\text{DFT}}{\longleftrightarrow} \mathrm{U}[\mathbf{k}] = \left(S_{\uparrow\downarrow}[\mathbf{k}] + \frac{m}{N}\lambda\left(|\mathrm{G}_0[\mathbf{k}]|^2 + |\mathrm{G}_1[\mathbf{k}]|^2\right)\right)^{-1}\left(\sum_{i=0}^{I-1}\mathrm{R}_{\theta_i}[\mathbf{k}]\right)$$

- $G_0[\mathbf{k}]$: DFT of 1st-order difference filter in vertical direction
- $G_1[\mathbf{k}]$: DFT of 1st-order difference filter in horizontal direction

```
% Accumuration of backprojected spectrum Rθi[k]
SR = zeros(nPoints+1);
% Accumuration of slice lines in frequency domain
SS = zeros(nPoints+1);
% Zero array
S0 = zeros(nPoints+1);
S1 = S0;
S1(1,:) = 1;
for iSlice = 1:length(thetaset)
    theta = thetaset(iSlice);
    Si = S0;
    Si(1,:) = [R(:,iSlice).' 0];
    SR = SR + imrotate(fftshift(Si),theta,'crop');
    SS = SS + imrotate(fftshift(S1,1),theta,'crop');
end
SR = ifftshift(SR(1:end-1,1:end-1));
SS = ifftshift(SS(1:end-1,1:end-1));
```

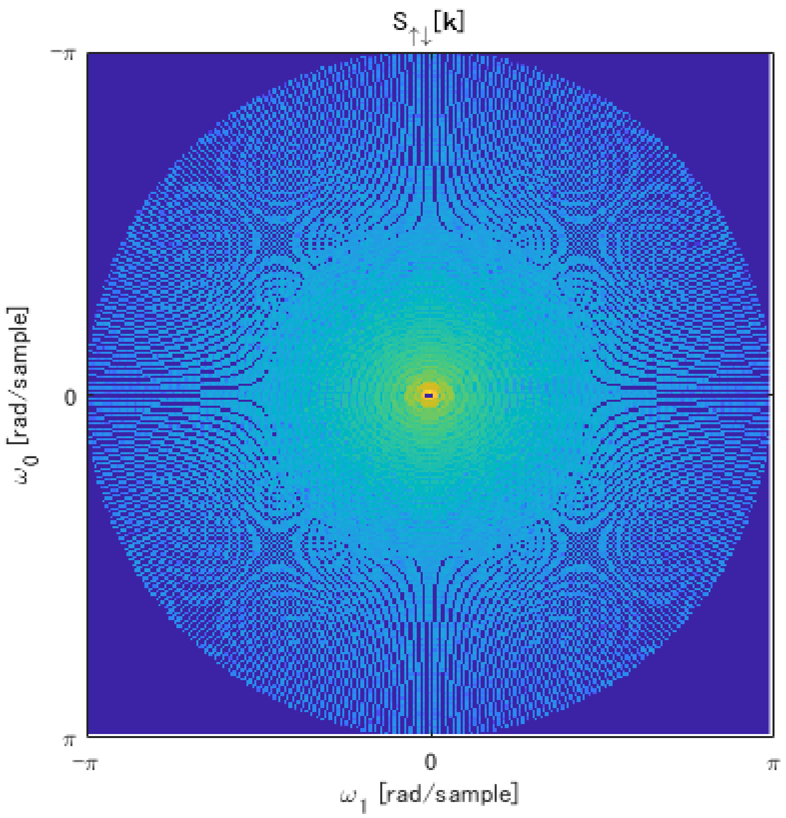
累積スライスラインの表示 (Display accumulated slice lines)

```
figure(5)
[k0,k1] = ndgrid(linspace(-pi,pi-2*pi/nPoints,nPoints));
surf(k0,k1,fftshift(SS),'EdgeColor','none')
ylabel('\omega_0 [rad/sample]')
xlabel('\omega_1 [rad/sample]')
title('S_{\uparrow\downarrow}[{\bfk}]')
ax = gca;
ax.YDir = 'reverse';
ax.YLim = [-pi pi];
ax.XLim = [-pi pi];
ax.YTick = [-pi 0 pi];
ax.YTickLabel = { '-\pi', '0', '\pi'};
ax.XTick = [-pi 0 pi];
ax.XTickLabel = { '-\pi', '0', '\pi'};
```



逆投影スペクトルの表示 (Display of backprojected spectrum)

```
figure(6)
imagesc(w0,w1,fftshift(20*log10(abs(SR))))
axis square
ylabel('\omega_0 [rad/sample]')
xlabel('\omega_1 [rad/sample]')
title('S_{\uparrow\downarrow}[{\bfk}]')
ax = gca;
ax.YDir = 'reverse';
ax.YLim = [-pi pi];
ax.XLim = [-pi pi];
ax.YTick = [-pi 0 pi];
ax.YTickLabel = { '-\pi', '0', '\pi'};
ax.XTick = [-pi 0 pi];
ax.XTickLabel = { '-\pi', '0', '\pi'};
```




```
iptsetpref('ImshowAxesVisible','off')
```

勾配フィルタのパワースペクトル

(Power spectrum of gradient filter)

$$\nabla^T\nabla \overset{\mathrm{DFT}}{\longleftrightarrow} |\mathrm{G}_0[\mathbf{k}]|^2 + |\mathrm{G}_1[\mathbf{k}]|^2$$

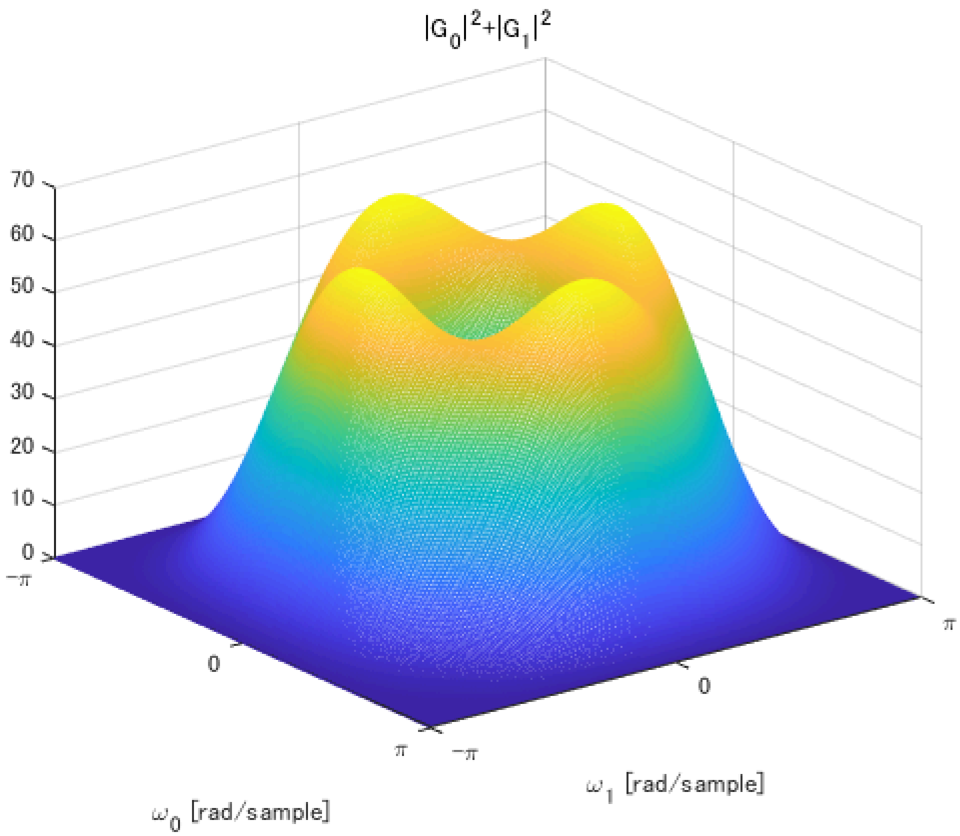
ソーベルフィルタを利用 (Adopt Sobel filter)

```
g0 = fspecial('sobel');
g1 = g0.';
GG = abs(fftn(g0,nPoints*[1 1])).^2+abs(fftn(g1,nPoints*[1 1])).^2;
LL = nPoints*GG;
```

パワースペクトルの表示 (Display the power spectrum)

```
figure(7)
mesh(k1,k0,fftshift(GG))
ylabel('\omega_0 [rad/sample]')
xlabel('\omega_1 [rad/sample]')
title('|G_0|^2+|G_1|^2')
ax = gca;
ax.YDir = 'reverse';
ax.YLim = [-pi pi];
ax.XLim = [-pi pi];
ax.YTick = [-pi 0 pi];
```

```
ax.YTickLabel = { '-\pi', '0', '\pi'};
ax.XTick = [-pi 0 pi];
ax.XTickLabel = { '-\pi', '0', '\pi'};
```



画像再構成
(Image reconstuction)

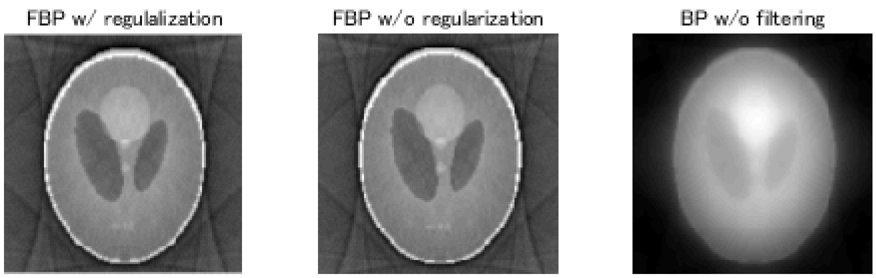
零割を避けるために分母に EPS を加算 (Add EPS to the denominator to avoid dividing by zero.)

```
figure(8)
% FBP w/ regulalization
Ur = SR./(SS+lambda*LL+eps);
ur = circshift(real(ifft2(Ur)),size(u)/2);
ur = ur(1:size(u,1),1:size(u,2));
subplot(1,3,1)
imshow(ur,[])
title('FBP w/ regulalization')

% FBP w/o regulalization
Uo = SR./(SS+eps);
uo = circshift(real(ifft2(Uo)),size(u)/2);
uo = uo(1:size(u,1),1:size(u,2));
subplot(1,3,2)
imshow(uo,[])
title('FBP w/o regularization')

% BP w/o filtering
```

```
ub = circshift(real(ifft2(SR)),size(u)/2);
ub = ub(1:size(u,1),1:size(u,2));
subplot(1,3,3)
imshow(ub,[])
title('BP w/o filtering')
```



IRADON関数

(Functon IRADON)

逆ラドン変換を行うMATLAB関数IRADONの利用例 (An example of using the MATLAB function IRADON to perform the inverse radon transformation)

Perform filtered backprojection.

```
I1 = iradon(r,thetaset);
```

Perform unfiltered backprojection.

```
I2 = iradon(r,thetaset,'linear','none');
```

Display the reconstructed images.

```
figure(9)
subplot(1,2,1)
imshow(I1,[])
title('Filtered Backprojection')
subplot(1,2,2)
```

```
imshow(I2,[])
title('Unfiltered Backprojection')
```