Sample 13-1

辞書学習

カルーネン-レーベ変換（主成分分析）

画像処理特論

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動作確認: MATLAB R2023a

Dictionary learning

Karhunen–Loève transform (principle component analysis)

Advanced Topics in Image Processing

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Verified: MATLAB R2023a

準備

(Preparation)

```
clear
close all
import msip.download_img
msip.download_img
```

```
kodim01.png already exists in ./data/
kodim02.png already exists in ./data/
kodim03.png already exists in ./data/
kodim04.png already exists in ./data/
kodim05.png already exists in ./data/
kodim06.png already exists in ./data/
kodim07.png already exists in ./data/
kodim08.png already exists in ./data/
kodim09.png already exists in ./data/
kodim10.png already exists in ./data/
kodim11.png already exists in ./data/
kodim12.png already exists in ./data/
kodim13.png already exists in ./data/
kodim14.png already exists in ./data/
kodim15.png already exists in ./data/
kodim16.png already exists in ./data/
kodim17.png already exists in ./data/
kodim18.png already exists in ./data/
kodim19.png already exists in ./data/
kodim20.png already exists in ./data/
kodim21.png already exists in ./data/
kodim22.png already exists in ./data/
kodim23.png already exists in ./data/
kodim24.png already exists in ./data/
See Kodak Lossless True Color Image Suite
```

パラメータ設定

(Parameter settings)

- ブロックサイズ (Block size)

```
szBlk = [ 8 8 ];
```

画像の読込

(Read image)

- $\mathbf{u} \in \mathbb{R}^N$

```
file_uorg = './data/kodim23.png';
u = im2double(imread(file_uorg));
if size(u,3) == 3
    u = rgb2gray(u);
end
szOrg = size(u);
figure
imshow(u);
title('Original image u')
```

画像 yからのデータ行列 $\mathbf{Y}$ の生成

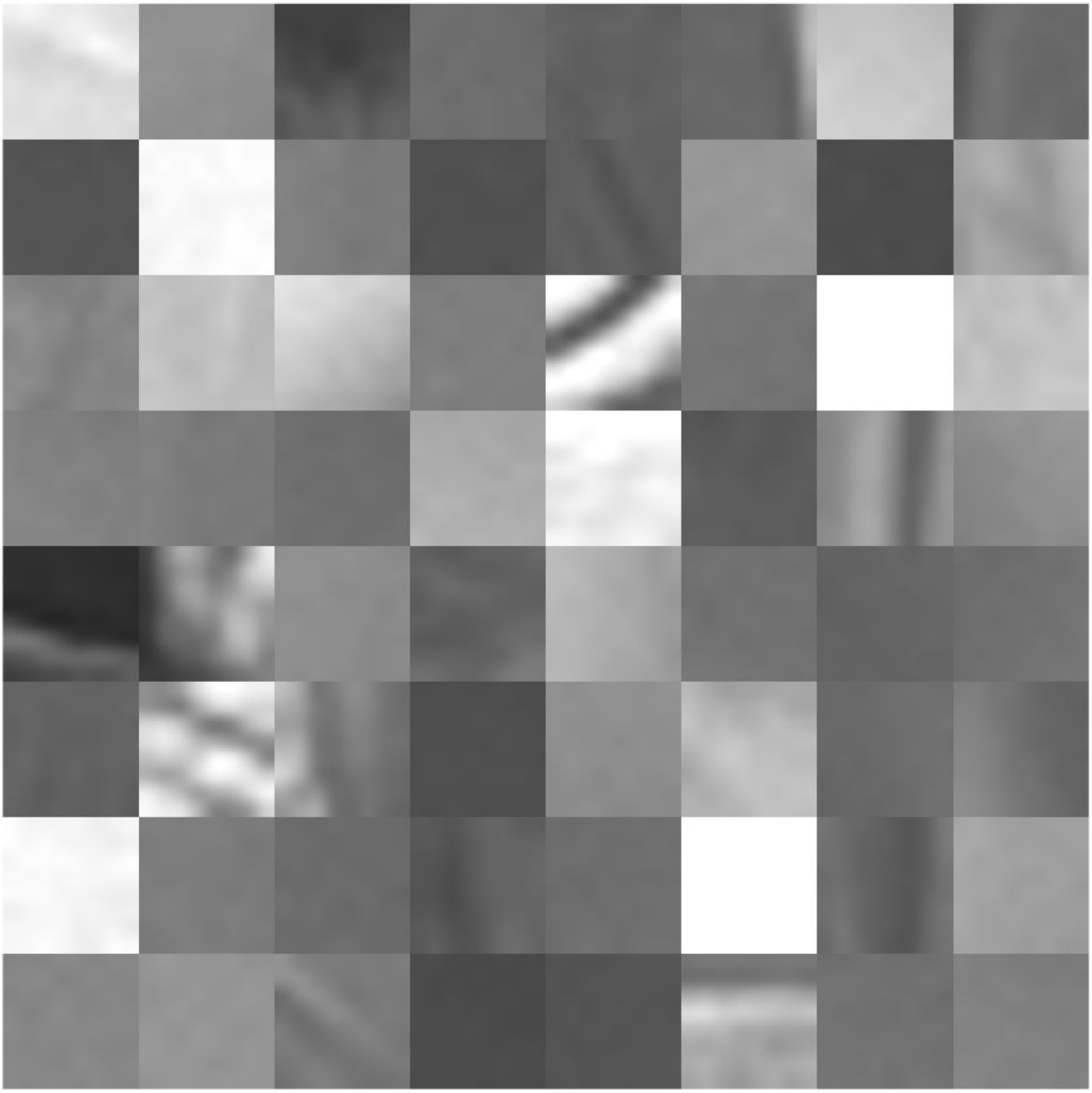
(Generate data matrices from images)

標本平均ブロックを引く代わりに，予め零平均化したデータで学習(Instead of subtracting the sample average block, training with pre-zero averaged data)

```
meansubtract = @(x) x-mean(x,"all");
y = meansubtract(u);

% # of patches
nPatches = prod(szOrg./szBlk);

npos = randsample(prod(szOrg-szBlk),nPatches);
ybs = zeros(szBlk(1),szBlk(2),nPatches,'like',y);
szSrchy = szOrg(1)-szBlk(1);
for iPatch = 1:nPatches
    ny_ = mod(npos(iPatch)-1,szSrchy)+1;
    nx_ = floor((npos(iPatch)-1)/szSrchy)+1;
    ybs(:,:,iPatch) = y(ny_:ny_+szBlk(1)-1,nx_:nx_+szBlk(2)-1);
end
figure
montage(ybs+0.5,'Size',[8 8]);
```



```
drawnow

Y = reshape(ybs,prod(szBlk),[]);
```

カルーネン-レーベ変換（主成分分析）

(Karhunen–Loève transform (principle component analysis))

問題設定 (Problem setting):

$$\widehat{\mathbf{\Phi}} = \arg \max_{\mathbf{\Phi} \in \mathbb{R}^{M \times M}} \mathrm{tr}\left(\mathbf{\Phi}_{:,1:p}^T \widehat{\mathbf{\Sigma}}_y \mathbf{\Phi}_{:,1:p}\right), \text{ s.t. } \mathbf{\Phi}^T \mathbf{\Phi} = \mathbf{I}_M, \forall p \in \{1, 2, \cdots, M\}$$

ただし， $\widehat{\mathbf{\Sigma}}_y$は 観測ベクトル $\{\mathbf{y}_n\}_n$ （零平均を仮定）の標本分散共分散行列 (where, $\widehat{\mathbf{\Sigma}}_y$ is the sample covariance matrix of the observation vectors $\{\mathbf{y}_n\}_n$ (assumed to have zero mean:)

$$\widehat{\mathbf{\Sigma}}_y = \frac{1}{S-1} \sum_{n=1}^{S} \mathbf{y}_n \mathbf{y}_n^T$$

解 (Solution):

固有値分解 (Eigendecomposition)

$$\widehat{\mathbf{\Phi}}^T \widehat{\mathbf{\Sigma}}_y \widehat{\mathbf{\Phi}} = \mathbf{\Lambda}$$

ただし， $\mathbf{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_M)$． $\lambda_1 \geq \lambda_2 \geq \cdots \lambda_M$ は $\widehat{\mathbf{\Sigma}}_y$の固有値． (where, $\mathbf{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_M)$． $\lambda_1 \geq \lambda_2 \geq \cdots \lambda_M$ are the eigenvalues of $\widehat{\mathbf{\Sigma}}_y$.)

標本分散共分散行列 $\widehat{\mathbf{\Sigma}}_y$の計算 (Calculation of sample covariance matrix $\widehat{\mathbf{\Sigma}}_y$)

```
SigmaY = cov(Y.');
```

標本分散共分散行列 $\widehat{\mathbf{\Sigma}}_y$の固有値分解 (Eigendecomposition of sample covariance matrix $\widehat{\mathbf{\Sigma}}_y$)

```
[Phi_pca,Lambda] = eig(SigmaY);
```

固有値 λ の大きさの降順に列ベクトルをソート (Sorting column vectors in the descending order of the eigenvalues λ)

```
[~,idx] = sort(diag(Lambda),'descend');
Phi_pca = Phi_pca(:,idx);
```

固有ベクトルを基底画像に変換 (Reshape the eigenvectures into basis images)

```
nBases = prod(szBlk);
basisImagesPca = zeros(szBlk(1),szBlk(2),nBases);
for iBasis = 1:nBases
    basisImagesPca(:,:,iBasis) = reshape(Phi_pca(:,iBasis),szBlk(1),szBlk(2));
end
```

基底画像の表示

(Show basis images)

```
figure
montage(imresize(basisImagesPca,8,'nearest')+.5,'BorderSize',[2 2])
title('Basis images of KLT(PCA)')
```