Sample 5-4

周波数解析

画像スペクトル

画像処理特論

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動作確認: MATLAB R2020a

Fourier analysis

Spectrum of images

Advanced Topics in Image Processing

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Verified: MATLAB R2020a

準備

(Preparation)

```
close all
```

サンプル画像の準備

(Preparation of sample image)

```
% Reading original image
u = im2double(imread('cameraman.tif'));
figure(1)
imshow(u)
title('Original')
```

画像（2変量信号） $\{u[\mathbf{n}] \in \mathbb{R}\}_{\mathbf{n}\in\mathbb{Z}^2}$ のスペクトル

(Spectrum of an image (bivariate signal) $\{u[\mathbf{n}]\in\mathbb{R}\}_{\mathbf{n}\in\mathbb{Z}^2}$)

$$U\left(e^{\jmath\boldsymbol{\omega}^T}\right)=\sum_{\mathbf{n}\in\Omega\subset\mathbb{Z}^2}u[\mathbf{n}]e^{-\jmath\boldsymbol{\omega}^T\mathbf{n}},\ \boldsymbol{\omega}\in\mathbb{R}^2$$

ただし，Ωは画像のサポート領域を意味する． (where Ω denotes the support region of the image.)

DFT(FFT)によるDSFTの周波数サンプル計算 (Frequency sampling of DSFT by DFT (FFT))

$$U[\mathbf{k}]=U\left(e^{j\boldsymbol{\omega}^T}\right)|_{\boldsymbol{\omega}=2\pi\mathbf{Q}^{-T}\mathbf{k}},\ \mathbf{k}\in\mathcal{N}(\mathbf{Q}^T)$$

以下では周期行列 $\mathbf{Q}$ を対角行列 (In the following, the periodic matrix $\mathbf{Q}$ is set to a diagonal matrix)

$$\mathbf{Q}=\begin{pmatrix}N_1 & 0\\ 0 & N_2\end{pmatrix}$$

に設定する．すなわち，(That is,)

$$\mathcal{N}(\mathbf{Q})=\mathcal{N}(\mathbf{Q}^T)=\{0,1,2,\cdots,N_1-1\}\times\{0,1,2,\cdots,N_2-1\}$$

$$N=|\mathcal{N}(\mathbf{Q})|=|\det(\mathbf{Q})|=N_1N_2$$

ただし，$\mathcal{N}(\cdot)$は基本周期内の整数ベクトル集合 (where $\mathcal{N}(\cdot)$ denotes a set of interger vectors in the fundamental pallalelpiped as)

$$\mathcal{N}(\mathbf{P}):=\{\mathbf{Px}\in\mathbb{Z}^D|\ \mathbf{x}\in[0,1)^D\}$$

である．ここでは，$\Omega\subseteq\mathcal{N}(\mathbf{Q})$ を仮定する． (Here, let us assume $\Omega\subseteq\mathcal{N}(\mathbf{Q})$.)

```
% Setting the number of frequency sample points in [0,2π)
nPoints1 = 256; % N_1
nPoints2 = 256; % N_2

% Spectrum of u[n]
U = fft2(u,nPoints1,nPoints2);
```

表示のための係数シフト

(Coefficient shift for display)

直流(DC)成分を配列の中心にシフト (Shift the direct current (DC) component to the center of the array)

```
% Shift the DC Coef. to the center
Usft = fftshift(U);

% Frequency sampling points
[w2,w1] = meshgrid(-pi:2*pi/nPoints2:pi-2*pi/nPoints2,-pi:2*pi/nPoints1:pi-2*pi/nPoints1);
```

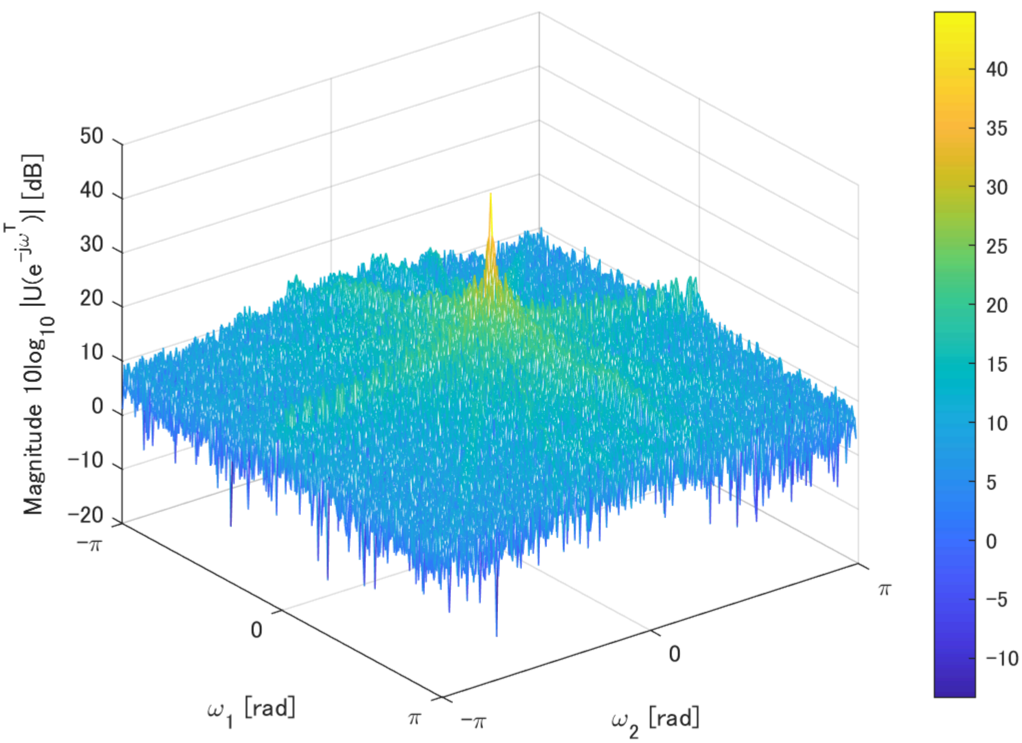
振幅スペクトル $\left|U\left(e^{\jmath\boldsymbol{\omega}^T}\right)\right|$ の表示

Display of magnitude spectrum $|U(e^{j\omega^T})|$

$$|U(e^{j\omega^T})| = \sqrt{\Re(U(e^{j\omega^T}))^2 + \Im(U(e^{j\omega^T}))^2}$$

```
% Calculation of the magnitude spectrum
Umag = abs(Usft);

% Display the magnitude spectrum
figure(2)
mesh(w1,w2,10*log10(Umag))
ax = gca;
xlabel('\omega_2 [rad]')
ylabel('\omega_1 [rad]')
zlabel('Magnitude 10log_{10} |U(e^{-j\omega^T})| [dB]')
axis ij
ax.XLim = [-pi pi];
ax.XTick = [ -pi 0 pi ];
ax.XTickLabel = { '-\pi', '0', '\pi'};
ax.YLim = [-pi pi];
ax.YTick = [ -pi 0 pi ];
ax.YTickLabel = { '-\pi', '0', '\pi'};
colorbar(ax)
```



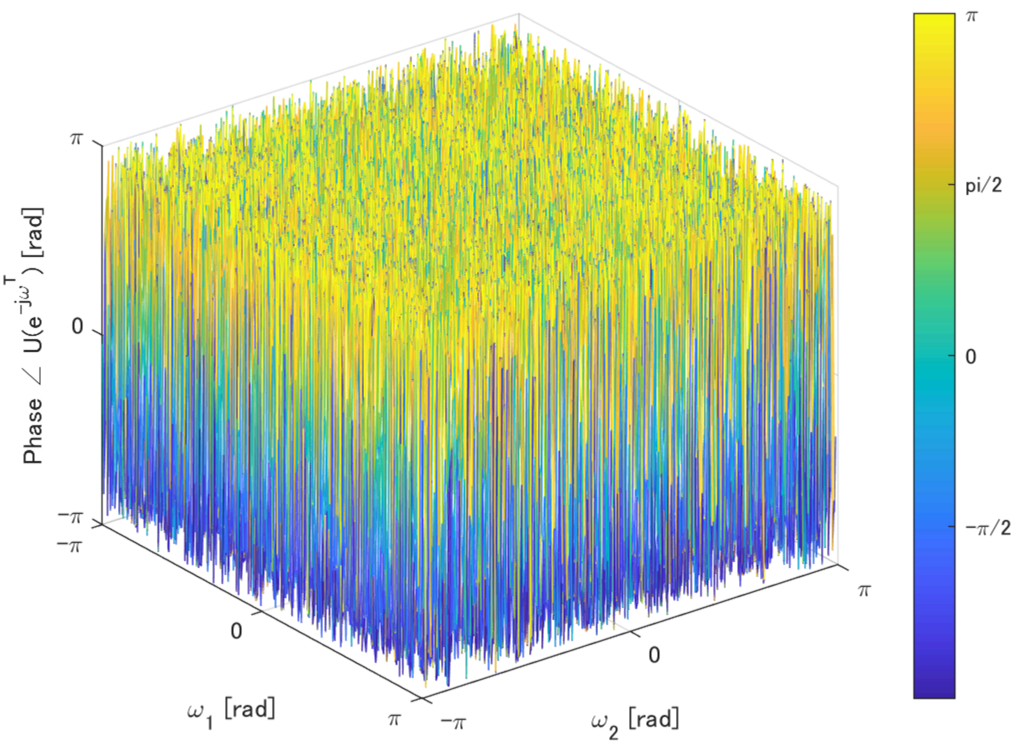
位相スペクトル $\angle U(e^{j\omega^T})$ の表示

(Display of phase spectrum $\angle U\left(e^{j\omega^T}\right)$)

$$\angle U\left(e^{j\omega^T}\right) = \tan^{-1}\frac{\mathfrak{I}\left(U\left(e^{j\omega^T}\right)\right)}{\mathfrak{R}\left(U\left(e^{j\omega^T}\right)\right)}$$

```
% Calculation of the magnitude spectrum
Uphs = angle(Usft);

% Display the magnitude spectrum
figure(3)
mesh(w1,w2,Uphs)
ax = gca;
xlabel('\omega_2 [rad]')
ylabel('\omega_1 [rad]')
zlabel('Phase \angle U(e^{-j\omega^T}) [rad]')
axis ij
ax.XLim = [-pi pi];
ax.XTick = [ -pi 0 pi ];
ax.XTickLabel = { '-\pi', '0', '\pi'};
ax.YLim = [-pi pi];
ax.YTick = [ -pi 0 pi ];
ax.YTickLabel = { '-\pi', '0', '\pi'};
ax.ZLim = [-pi pi];
ax.ZTick = [ -pi 0 pi ];
ax.ZTickLabel = { '-\pi', '0', '\pi'};
colorbar(ax,'Ticks',[ -pi -pi/2 0 pi/2 pi],'TickLabels', { '-\pi', '-\pi/2', '0', 'pi/2', '\pi'
```



スペクトル $U\left(e^{\jmath\boldsymbol{\omega}^T}\right)$ からの画像再構成

(Reconstruction from the spectrum $U\left(e^{\jmath\boldsymbol{\omega}^T}\right)$)

$$u[\mathbf{n}] = \frac{1}{(2\pi)^2}\int_{\boldsymbol{\omega}\in[0,2\pi)^2} U\left(e^{\jmath\boldsymbol{\omega}^T}\right)e^{\jmath\boldsymbol{\omega}^T\mathbf{n}}d\boldsymbol{\omega},\ \mathbf{n}\in\Omega\subset\mathbb{Z}^2$$

IDFT(IFFT)による再構成 (Reconstruction by IDFT (IFFT))

$$u[\mathbf{n}] = \frac{1}{|\det(\mathbf{Q})|}\sum_{\mathbf{n}\in\mathcal{N}(\mathbf{Q})} U[\mathbf{k}]e^{\jmath 2\pi\mathbf{k}\mathbf{Q}^{-1}\mathbf{n}},\ \mathbf{n}\in\Omega\subseteq\mathcal{N}(\mathbf{Q})$$

```
% Reconstruction from the spectrum
r = ifft2(U,nPoints1,nPoints2);

% Clipping to the support region Ω
urec = r(1:size(u,1),1:size(u,2));
figure(4)
imshow(urec)
% MSE
mymse = @(x,y) mean((double(x)-double(y)).^2,'all');
title(['Reconstruction MSE: ' num2str(mymse(u,urec))])
```

Reconstruction MSE: 2.3016e-32

振幅スペクトル $|U\left(e^{\jmath\boldsymbol{\omega}^T}\right)|$ からの画像再構成

(Reconstruction from the spectrum $|U\left(e^{\jmath\boldsymbol{\omega}^T}\right)|$)

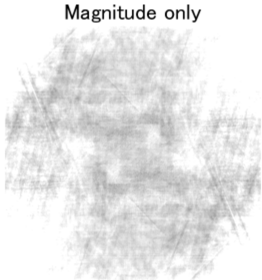
$$u_{\mathrm{mag}}[\mathbf{n}] = \frac{1}{(2\pi)^2}\int_{\boldsymbol{\omega}\in[0,2\pi)^2} |U\left(e^{\jmath\boldsymbol{\omega}^T}\right)|e^{\jmath\boldsymbol{\omega}^T\mathbf{n}}d\boldsymbol{\omega},\ \mathbf{n}\in\Omega\subset\mathbb{Z}^2$$

IDFT(IFFT)による計算 (Calculation by IDFT (IFFT))

$$u_{\mathrm{mag}}[\mathbf{n}] = \frac{1}{|\det(\mathbf{Q})|}\sum_{\mathbf{k}\in\mathcal{N}(\mathbf{Q}^T)} |U[\mathbf{k}]|e^{\jmath 2\pi\mathbf{k}^T\mathbf{Q}^{-1}\mathbf{n}},\ \mathbf{n}\in\Omega\subseteq\mathcal{N}(\mathbf{Q})$$

```
% Reconstruction from the spectrum
rmag = ifft2(ifftshift(Umag),nPoints1,nPoints2);
```

```
% Clipping to the support region Ω
umag = rmag(1:size(u,1),1:size(u,2));
figure(5)
imshow(umag+.5)
title('Magnitude only')
```



位相スペクトル $\angle U\left(e^{\jmath\boldsymbol{\omega}^T}\right)$ からの画像再構成

(Reconstruction from the spectrum $\angle U\left(e^{\jmath\boldsymbol{\omega}^T}\right)$)

$$u_{\mathrm{phs}}[\mathbf{n}] = \frac{1}{(2\pi)^2}\int_{\boldsymbol{\omega}\in[0,2\pi)^2} e^{\jmath\angle U\left(e^{\jmath\boldsymbol{\omega}^T}\right)} e^{\jmath\boldsymbol{\omega}^T\mathbf{n}} d\boldsymbol{\omega},\ \mathbf{n}\in\Omega\subset\mathbb{Z}^2$$

IDFT(IFFT)による計算 (Calculation by IDFT (IFFT))

$$u_{\mathrm{phs}}[\mathbf{n}] = \frac{1}{|\det(\mathbf{Q})|}\sum_{\mathbf{k}\in\mathcal{N}(\mathbf{Q}^T)} e^{\jmath\angle U[\mathbf{k}]} e^{\jmath 2\pi\mathbf{k}^T\mathbf{Q}^{-1}\mathbf{n}},\ \mathbf{n}\in\Omega\subseteq\mathcal{N}(\mathbf{Q})$$

```
% Reconstruction from the spectrum
rphs = ifft2(exp(1j*ifftshift(Uphs)),nPoints1,nPoints2);

% Clipping to the suppor region Ω
uphs = rphs(1:size(u,1),1:size(u,2));
figure(6)
imshow(nPoints1*nPoints2*real(uphs)+.5)
title('Phase only')
```

Phase only