

Sample 5-4

周波数解析

画像スペクトル

画像処理特論

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動作確認: MATLAB R2020a

Fourier analysis

Spectrum of images

Advanced Topics in Image Processing

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Verified: MATLAB R2020a

準備

(Preparation)

```
close all
```

サンプル画像の準備

(Preparation of sample image)

```
% Reading original image  
u = im2double(imread('cameraman.tif'));  
figure(1)  
imshow(u)  
title('Original')
```

Original



画像（2 変量信号） $\{u[\mathbf{n}] \in \mathbb{R}\}_{\mathbf{n} \in \mathbb{Z}^2}$ のスペクトル

(Spectrum of an image (bivariate signal) $\{u[\mathbf{n}] \in \mathbb{R}\}_{\mathbf{n} \in \mathbb{Z}^2}$)

$$U(e^{j\omega^T}) = \sum_{\mathbf{n} \in \Omega \subset \mathbb{Z}^2} u[\mathbf{n}] e^{-j\omega^T \mathbf{n}}, \quad \omega \in \mathbb{R}^2$$

ただし、 Ω は画像のサポート領域を意味する。(where Ω denotes the support region of the image.)

DFT(FFT)による DSFT の周波数サンプル計算 (Frequency sampling of DSFT by DFT (FFT))

$$U[\mathbf{k}] = U(e^{j\omega^T}) \Big|_{\omega=2\pi\mathbf{Q}^{-T}\mathbf{k}}, \quad \mathbf{k} \in \mathcal{N}(\mathbf{Q}^T)$$

以下では周期行列 \mathbf{Q} を対角行列 (In the following, the periodic matrix \mathbf{Q} is set to a diagonal matrix)

$$\mathbf{Q} = \begin{pmatrix} N_1 & 0 \\ 0 & N_2 \end{pmatrix}$$

に設定する。すなわち、(That is,)

$$\mathcal{N}(\mathbf{Q}) = \mathcal{N}(\mathbf{Q}^T) = \{0, 1, 2, \dots, N_1 - 1\} \times \{0, 1, 2, \dots, N_2 - 1\}$$

$$N = |\mathcal{N}(\mathbf{Q})| = |\det(\mathbf{Q})| = N_1 N_2$$

ただし、 $\mathcal{N}(\cdot)$ は基本周期内の整数ベクトル集合 (where $\mathcal{N}(\cdot)$ denotes a set of interger vectors in the fundamental pallalelpiped as)

$$\mathcal{N}(\mathbf{P}) := \{\mathbf{P}\mathbf{x} \in \mathbb{Z}^D \mid \mathbf{x} \in [0, 1)^D\}$$

である。ここでは、 $\Omega \subseteq \mathcal{N}(\mathbf{Q})$ を仮定する。(Here, let us assume $\Omega \subseteq \mathcal{N}(\mathbf{Q})$.)

```
% Setting the number of frequency sample points in [0,2π)
nPoints1 = 256; % N_1
nPoints2 = 256; % N_2

% Spectrum of u[n]
U = fft2(u,nPoints1,nPoints2);
```

表示のための係数シフト

(Coefficient shift for display)

直流(DC)成分を配列の中心にシフト (Shift the direct current (DC) component to the center of the array)

```
% Shift the DC Coef. to the center
Usft = fftshift(U);

% Frequency sampling points
[w2,w1] = meshgrid(-pi:2*pi/nPoints2:pi-2*pi/nPoints2,-pi:2*pi/nPoints1:pi-2*pi/nPoints1);
```

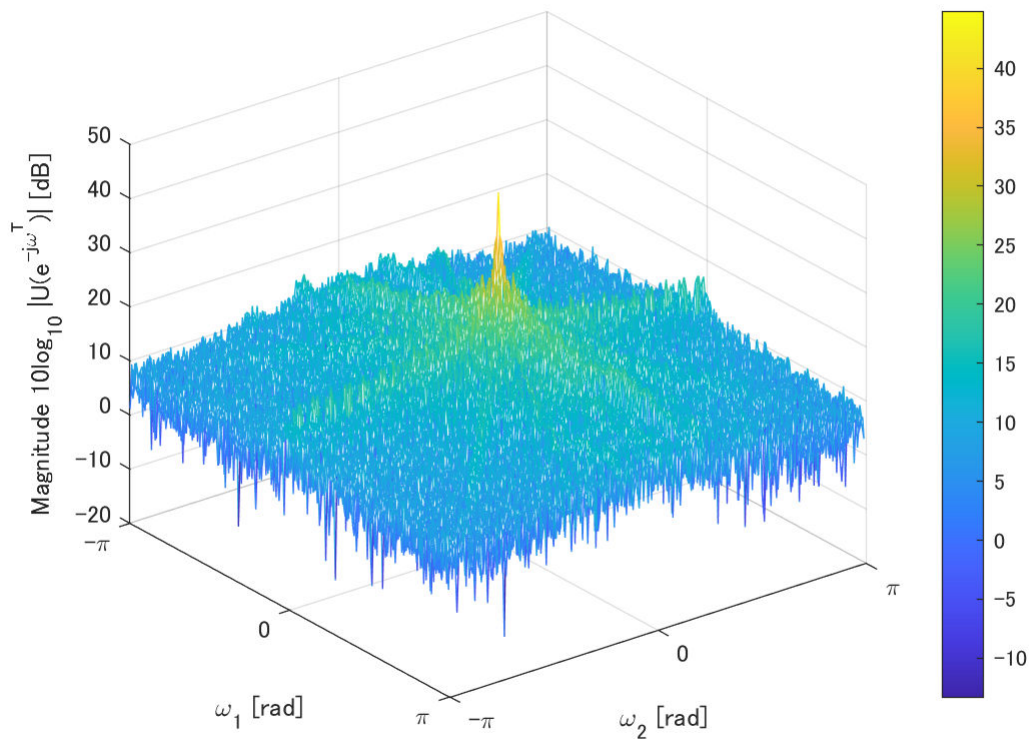
振幅スペクトル $|U(e^{j\omega^T})|$ の表示

Display of magnitude spectrum $|U(e^{j\omega^T})|$

$$|U(e^{j\omega^T})| = \sqrt{\Re(U(e^{j\omega^T}))^2 + \Im(U(e^{j\omega^T}))^2}$$

```
% Calculation of the magnitude spectrum
Umag = abs(Usft);

% Display the magnitude spectrum
figure(2)
mesh(w1,w2,10*log10(Umag))
ax = gca;
xlabel('\omega_2 [rad]')
ylabel('\omega_1 [rad]')
zlabel('Magnitude 10log_{10} |U(e^{-j\omega^T})| [dB]')
axis ij
ax.XLim = [-pi pi];
ax.XTick = [ -pi 0 pi ];
ax.XTickLabel = { '-\pi', '0', '\pi'};
ax.YLim = [-pi pi];
ax.YTick = [ -pi 0 pi ];
ax.YTickLabel = { '-\pi', '0', '\pi'};
colorbar(ax)
```



位相スペクトル $\angle U(e^{j\omega^T})$ の表示

(Display of phase spectrum $\angle U(e^{j\omega^T})$)

$$\angle U(e^{j\omega^T}) = \tan^{-1} \frac{\Im(U(e^{j\omega^T}))}{\Re(U(e^{j\omega^T}))}$$

```
% Calculation of the magnitude spectrum
```

```
Uphs = angle(Usft);
```

```
% Display the magnitude spectrum
```

```
figure(3)
```

```
mesh(w1,w2,Uphs)
```

```
ax = gca;
```

```
xlabel('\omega_2 [rad]')
```

```
ylabel('\omega_1 [rad]')
```

```
zlabel('Phase \angle U(e^{-j\omega^T}) [rad]')
```

```
axis ij
```

```
ax.XLim = [-pi pi];
```

```
ax.XTick = [ -pi 0 pi ];
```

```
ax.XTickLabel = { '-\pi', '0', '\pi'};
```

```
ax.YLim = [-pi pi];
```

```
ax.YTick = [ -pi 0 pi ];
```

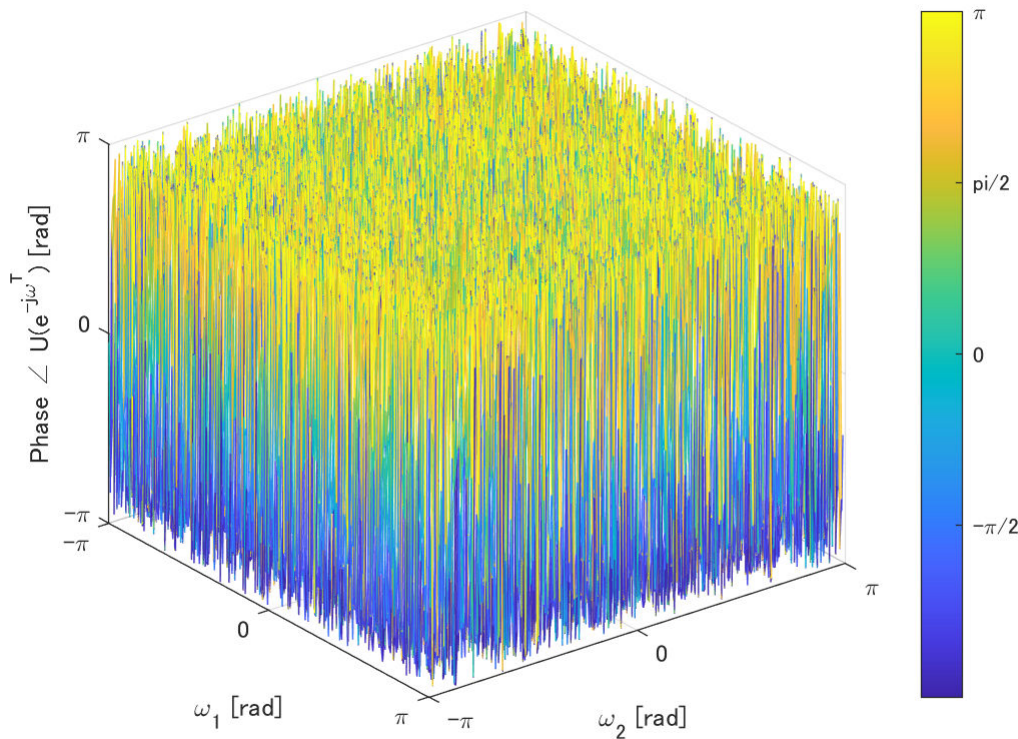
```
ax.YTickLabel = { '-\pi', '0', '\pi'};
```

```
ax.ZLim = [-pi pi];
```

```
ax.ZTick = [ -pi 0 pi ];
```

```
ax.ZTickLabel = { '-\pi', '0', '\pi'};
```

```
colorbar(ax,'Ticks',[ -pi -pi/2 0 pi/2 pi], 'TickLabels', { '-\pi', '-\pi/2', '0', 'pi/2', '\pi'}
```



スペクトル $U(e^{j\omega^T})$ からの画像再構成

(Reconstruction from the spectrum $U(e^{j\omega^T})$)

$$u[\mathbf{n}] = \frac{1}{(2\pi)^2} \int_{\omega \in [0, 2\pi)^2} U(e^{j\omega^T}) e^{j\omega^T \mathbf{n}} d\omega, \quad \mathbf{n} \in \Omega \subset \mathbb{Z}^2$$

IDFT(IFFT)による再構成 (Reconstruction by IDFT (IFFT))

$$u[\mathbf{n}] = \frac{1}{|\det(\mathbf{Q})|} \sum_{\mathbf{k} \in \mathcal{N}(\mathbf{Q})} U[\mathbf{k}] e^{j2\pi \mathbf{k} \mathbf{Q}^{-1} \mathbf{n}}, \quad \mathbf{n} \in \Omega \subseteq \mathcal{N}(\mathbf{Q})$$

```
% Reconstruction from the spectrum
```

```
r = ifft2(U,nPoints1,nPoints2);
```

```
% Clipping to the support region \Omega
```

```
urec = r(1:size(u,1),1:size(u,2));
```

```
figure(4)
```

```
imshow(urec)
```

```
% MSE
```

```
mymse = @(x,y) mean((double(x)-double(y)).^2,'all');
```

```
title(['Reconstruction MSE: ' num2str(mymse(u,urec))])
```

Reconstruction MSE: 2.3016e-32



振幅スペクトル $|U(e^{j\omega^T})|$ からの画像再構成

(Reconstruction from the spectrum $|U(e^{j\omega^T})|$)

$$u_{\text{mag}}[\mathbf{n}] = \frac{1}{(2\pi)^2} \int_{\omega \in [0, 2\pi)^2} |U(e^{j\omega^T})| e^{j\omega^T \mathbf{n}} d\omega, \quad \mathbf{n} \in \Omega \subset \mathbb{Z}^2$$

IDFT(IFFT)による計算 (Calculation by IDFT (IFFT))

$$u_{\text{mag}}[\mathbf{n}] = \frac{1}{|\det(\mathbf{Q})|} \sum_{\mathbf{k} \in \mathcal{N}(\mathbf{Q}^T)} |U[\mathbf{k}]| e^{j2\pi \mathbf{k}^T \mathbf{Q}^{-1} \mathbf{n}}, \quad \mathbf{n} \in \Omega \subseteq \mathcal{N}(\mathbf{Q})$$

```
% Reconstruction from the spectrum
rmag = ifft2(fftshift(Umag),nPoints1,nPoints2);

% Clipping to the support region  $\Omega$ 
umag = rmag(1:size(u,1),1:size(u,2));
figure(5)
imshow(umag+.5)
title('Magnitude only')
```

Magnitude only



位相スペクトル $\angle U(e^{j\omega^T})$ からの画像再構成

(Reconstruction from the spectrum $\angle U(e^{j\omega^T})$)

$$u_{\text{phs}}[\mathbf{n}] = \frac{1}{(2\pi)^2} \int_{\boldsymbol{\omega} \in [0, 2\pi)^2} e^{j\angle U(e^{j\boldsymbol{\omega}^T})} e^{j\boldsymbol{\omega}^T \mathbf{n}} d\boldsymbol{\omega}, \quad \mathbf{n} \in \Omega \subset \mathbb{Z}^2$$

IDFT(IFFT)による計算 (Calculation by IDFT (IFFT))

$$u_{\text{phs}}[\mathbf{n}] = \frac{1}{|\det(\mathbf{Q})|} \sum_{\mathbf{k} \in \mathcal{N}(\mathbf{Q}^T)} e^{j\angle U[\mathbf{k}]} e^{j2\pi \mathbf{k}^T \mathbf{Q}^{-1} \mathbf{n}}, \quad \mathbf{n} \in \Omega \subseteq \mathcal{N}(\mathbf{Q})$$

```
% Reconstruction from the spectrum
```

```
rphs = ifft2(exp(1j*ifftshift(Uphs)),nPoints1,nPoints2);
```

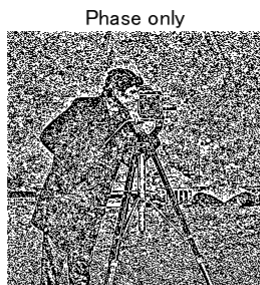
```
% Clipping to the support region  $\Omega$ 
```

```
uphs = rphs(1:size(u,1),1:size(u,2));
```

```
figure(6)
```

```
imshow(nPoints1*nPoints2*real(uphs)+.5)
```

```
title('Phase only')
```



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