拡張 GFT による DiGraph フィルターの実現

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参考文献:

- H. Kitamura, H. Yasuda, Y. Tanaka and S. Muramatsu, "Realization of DiGraph Filters Via Augmented GFT," 2023 IEEE International Conference on Image Processing (ICIP), Kuala Lumpur, Malaysia, 2023, pp. 2885-2889, doi: 10.1109/ICIP49359.2023.10222618.
- Abstract: This study proposes a filtering method for directed graph (digraph) signals. In order to realize digraph filtering, a novel graph Fourier transform (GFT), – Augmented GFT (AuGFT) –, is proposed by defining an Hermitian adjacency matrix. Although there has been the same method to give the adjacency matrix of digraphs, this study defines a novel digraph Laplacian. The existing digraph Laplacian does not give the graph signal variation considering the edge directions, while the novel one does. This paper introduces three important ideas. The first is the definition of a novel degree matrix to give the novel digraph Laplacian. The second is to decompose the symmetric and skew-symmetric components of the novel digraph Laplacian independently into their spectral components. The third is, based on the decomposition, to augment the conventional GFT for digraphs as an invertible real-valued dictionary. The new GFT is shown to provide a practical form of real-valued digraph filtering. The significance of the proposed method is verified through simulations of signal filtering on digraphs.
- URL: https://ieeexplore.ieee.org/stamp/stamp.jsp?tp=&arnumber=10222618&isnumber=10221892

謝辞：

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各種設定

```
isDiGraph = true; % 有向グラフとして解析
isRiver = true; % 河川データを解析

if ~exist('./gspbox','dir')
    setup
else
    addpath('gspbox')
end
gsp_start
```

```
GSPBox version 0.7.5. Copyright 2013-2015 LTS2-EPFL,
by Nathanael Perraudin, Johan Paratte, David Shuman and Vassilis Kalofolias
```

```
RESULTS_DIR = "../results/";
FIGEXT = ".png";
```

有向グラフの隣接行列 $\mathbf{A}$ の生成

有向グラフ $\mathcal{G}$ の定義

```
if ~isRiver
    s = [1 1 2 2 3 3 4 4 4 5];
    t = [2 3 4 5 6 7 8 9 10 4];
    G = digraph(s,t)
```

有向非巡回性の確認

```
    isdag(G)
```

有向グラフ $\mathcal{G}$ のエッジ $\mathcal{E}$

```
    G.Edges
```

有向グラフ $\mathcal{G}$ のノード $\mathcal{V}$

```
    G.Nodes
```

有向グラフ $\mathcal{G}$ の描画

```
    A = full(adjacency(G,'weighted'))
else
    Ariver = [
        0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...  %尾崎 1
        1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...  %荒町 2
        0 1 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...  %保明新田 3
        0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...  %臼井橋 4
        0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...  %新酒屋 5
        0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...  %帝石橋 6
        0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...  %西港 7
        1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ... %見附 8
        0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...%板井 9
        0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...%荒沢 10
        0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...%黒水 11
        0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...%渡部 12
```

```
        0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...%大河津 13
        0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...%長岡 14
        0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...%小千谷 15
        0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...%岩沢 16
        0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...%十日町 17
        0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...%宮野原 18
        0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...%堀之内 19
        0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...%小出 20
        0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...%浦佐 21
        0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...%六日町 22
        0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...%大倉 23
        0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...%清水 24
        0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...%大河原 25
        0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...%清水川原 26
        0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...%土樽 27
        0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
0 0; ...%塩名田 28
        0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
0 0; ...%生田 29
        0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
0 0; ...%杭瀬下 30
        0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
0 0; ...%立ヶ花 31
        0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...%柏尾橋 32
        0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
0 0; ...%下島橋 33
        0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
0 0; ...%熊倉 34
        0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
0 0; ...%陸郷 35
        0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
0 0; ...%小市 36
        0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
0 0; ...%島橋 37
```

```
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
1 0; ...%内膳落合 38
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0; ...%当ノ坂 39
    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
0 0; ...%大出橋 40
    ];
  Triver = {'尾崎','荒町','保明新田','臼井橋','新酒屋',...
    '帝石橋','西港','見附','板井','荒沢',...
    '黒水','渡部','大河津','長岡','小千谷',...
    '岩沢','十日町','宮野原','堀之内','小出',...
    '浦佐','六日町','大倉','清水','大河原',...
    '清水川原','土樽','塩名田','生田','杭瀬下',...
    '立ヶ花','柏尾橋','下島橋','熊倉','陸郷',...
    '小市','島橋','内膳落合','当ノ坂','大出橋'};

  TriverEng = {'Osaki','Aramachi','Homyosinden','Usuibashi','Shinsakaya',...
    'Teisekibashi','Nishiko','Mitsuke','Itai','Arasawa',...
    'Kuromizu','Watabe','Ookodu','Nagaoka','Ojiya',...
    'Iwasawa','Tookamachi','Miyanohara','Horinouchi','Koide',...
    'Urasa','Muikamachi','Ookura','Shimizu','Oogawara',...
    'Shimizugawara','Tsuchitaru','Shionada','Ikuta','Kuiseke',...
    'Tategahana','Kashiobashi','Shimojimabashi','Kumakura','Rikugo',...
    'Koichi','Shimabashi','Naizenochiai','Tounosaka','Ooidebashi'};

  locList
={'304031284401020','304031284401040','304031284401060','304031284401070','304031284
401080',...

'304031284401090','304031284401120','304031284401130','304031284401170','30403128440
1180',...

'304031284401200','304031284403020','304031284403030','304031284403060','30403128440
3070',...

'304031284403080','304031284403090','304031284403100','304031284403110','30403128440
3130',...

'304031284403140','304031284403150','304031284404010','304031284404030','30403128440
4040',...

'304031284404050','304031284404060','304031284416010','304031284416030','30403128441
6080',...

'304031284416100','304031284416135','304031284416160','304031284416170','30403128441
6180',...

'304031284416190','304031284416210','304031284419011','304031284419030','30403128442
0020'};
```

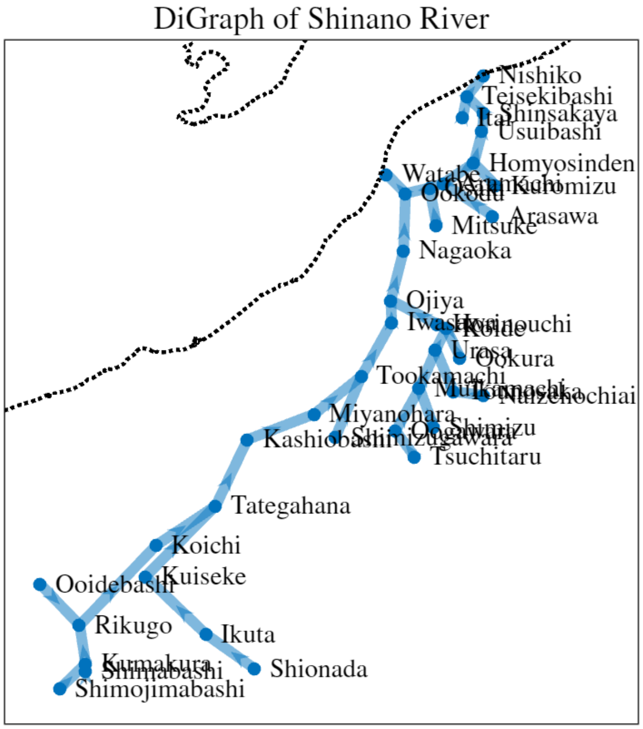
```
    Griver = digraph(Ariver,Triver,'omitselfloops');
    A = Ariver;
    G = Griver;

    lat = [37.6247,37.6375,37.6944,37.7839,37.8353,...
        37.8819,37.9406,37.5186,37.8219,37.5453,...
        37.6306,37.6636,37.6092,37.4481,37.3086,...
        37.2472,37.0958,36.9892,37.2419,37.2300,...
        37.1697,37.0617,37.1461,36.9528,36.9414,...
        36.9247,36.8694,36.2728,36.3697,36.5328,...
        36.7319,36.9172,36.2175,36.2869,36.3956,...
        36.6203,36.2672,37.0414,37.0528,36.5111,...
        ];

    long = [138.9097,138.9486,139.0342,139.0575,139.0619,...
        139.0144,139.0611,138.9292,139.0019,139.0881,...
        139.0958,138.7886,138.8419,138.8364,138.8008,...
        138.8031,138.7183,138.5858,138.9311,138.9567,...
        138.9272,138.8814,138.9958,138.9211,138.8142,...
        138.6456,138.8678,138.4186,138.2819,138.1119,...
        138.3094,138.3978,137.8708,137.9433,137.9267,...
        138.1417,137.9439,139.0611,138.9761,137.8158,...
        ];
end
```

```
figure
if isRiver
    h = plot(G,'LineWidth',4,'NodeLabel',TriverEng,'Layout','layered');

    h.XData = long;
    h.YData = lat;
    h.Interpreter = 'latex';
    h.NodeFontSize = 10;
```

ダウンロードサイト

- https://www1.gsi.go.jp/geowww/globalmap-gsi/download/data/gm-japan/gm-jpn-all_u_2_2.zip

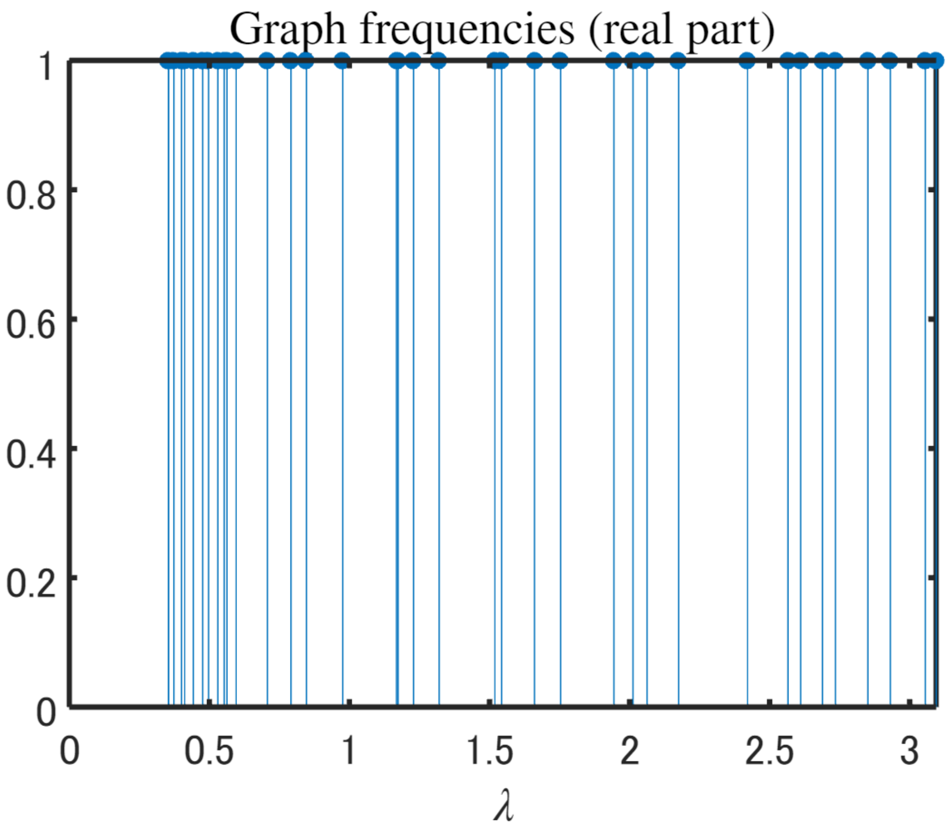
```
    coastl_jpn_file = "../data/gm-jpn-all_u_2_2/coastl_jpn.shp";

    if ~exist(coastl_jpn_file,"file")
        coastl_jpn_url = "https://www1.gsi.go.jp/geowww/globalmap-gsi/download/data/
gm-japan/gm-jpn-all_u_2_2.zip";
        %cdir = pwd;
        cdir = cd('../data');
        unzip(coastl_jpn_url)
        disp("Unzipped " + coastl_jpn_url)
        cd cdir;
    end
    CoastLine = shaperead('../data/gm-jpn-all_u_2_2/coastl_jpn.shp');
    mapshow(CoastLine,'LineWidth',1.5,'LineStyle',":",'Color',"black")
    title('DiGraph of Shinano River','Interpreter','latex')
    x_min = min(long);
    x_max = max(long);
    y_min = min(lat);
    y_max = max(lat);
    xlim([x_min-0.1 x_max+0.4])
    ylim([y_min-0.1 y_max+0.1])
    ax = gca;
    ax.FontSize = 11;
    hold off
    exportgraphics(ax,RESULTS_DIR+"DigraphShinano"+FIGEXT)
else
    h = plot(G,'LineWidth',4,'Layout','layered');
end
```



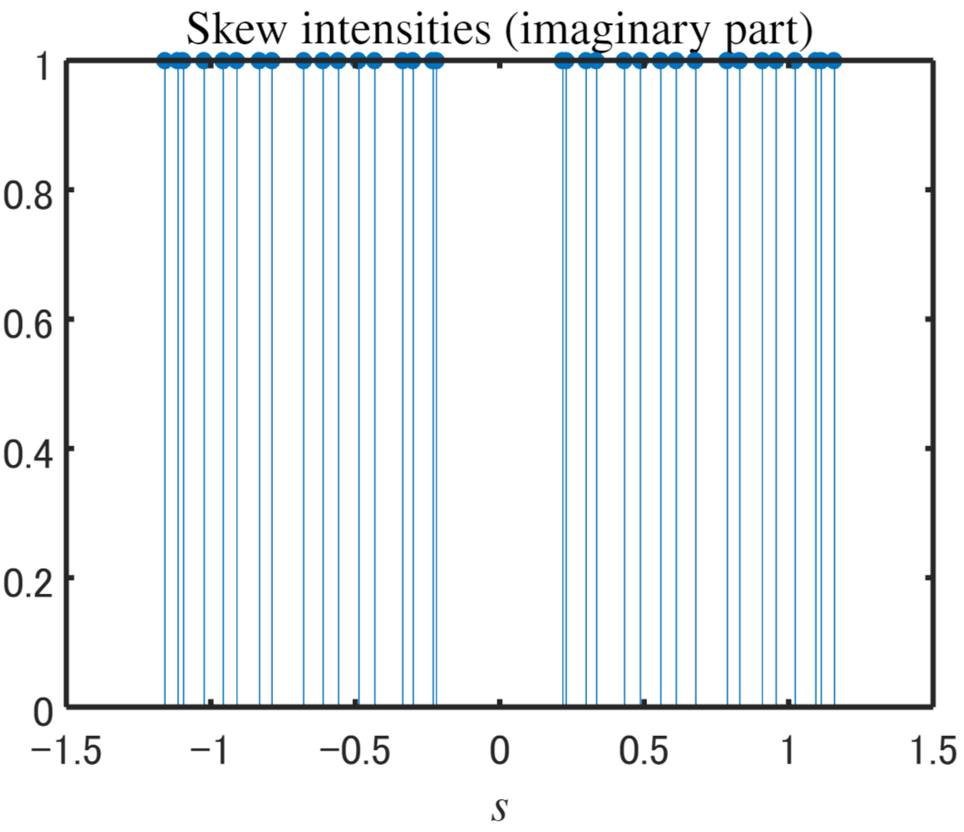
```
if ~isDiGraph
    if isa(A,'digraph') || isa(A,'graph')
        A = adjacency(A);
    end
    A = (A+A.')/2;
end
[U,Q,C,D,L,Lmd,Sgm] = fcn_digraphops(A)
```

```
U = 40×40
    0.0488    0.0441   -0.0093   -0.0053   -0.2644    0.0573   -0.0282    0.1100 ···
    0.0662    0.0661   -0.0218   -0.0035   -0.2443    0.0625   -0.0847   -0.1114
    0.0917    0.0960   -0.0368   -0.0017   -0.1776    0.0505   -0.0960   -0.2357
    0.1285    0.1375   -0.0561    0.0001   -0.0771    0.0247   -0.0561   -0.1546
    0.1808    0.1956   -0.0820    0.0020    0.0214   -0.0009   -0.0131   -0.0544
    0.2552    0.2774   -0.1177    0.0040    0.1204   -0.0265    0.0308    0.0526
    0.3607    0.3924   -0.1670    0.0060    0.1965   -0.0449    0.0582    0.1137
    0.0690    0.0625   -0.0131   -0.0079   -0.4315    0.0971   -0.0534    0.2379
    0.3607    0.3924   -0.1670    0.0060    0.1965   -0.0449    0.0582    0.1137
    0.0936    0.0936   -0.0309   -0.0052   -0.3987    0.1057   -0.1602   -0.2409
      ⋮

Q = 40×34
    0.4472   -0.0008    0.0000   -0.4372   -0.0293   -0.0005    0.0209   -0.0004 ···
    0.0007    0.4014    0.4905    0.0000    0.0007   -0.0374    0.0046    0.2172
   -0.3093    0.0006   -0.0000    0.4341    0.0355    0.0007   -0.3174    0.0067
    0.0003    0.1817    0.2807    0.0000    0.0004   -0.0240    0.0059    0.2771
    0.1116   -0.0002    0.0000   -0.1907   -0.0171   -0.0003    0.2496   -0.0053
   -0.0001   -0.0768   -0.1437   -0.0000   -0.0002    0.0134   -0.0049   -0.2336
   -0.0331    0.0001   -0.0000    0.0646    0.0061    0.0001   -0.1142    0.0024
    0.0004    0.1931    0.1964    0.0000    0.0002   -0.0134    0.0002    0.0102
    0.0331   -0.0001    0.0000   -0.0646   -0.0061   -0.0001    0.1142   -0.0024
   -0.1733    0.0003   -0.0000    0.2204    0.0171    0.0003   -0.1062    0.0022
      ⋮

C = 40×40 complex
   0.0000 + 0.0000i   0.5000 - 0.5000i   0.0000 + 0.0000i   0.0000 + 0.0000i ···
   0.5000 + 0.5000i   0.0000 + 0.0000i   0.5000 - 0.5000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.5000 + 0.5000i   0.0000 + 0.0000i   0.5000 + 0.5000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.5000 - 0.5000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.5000 - 0.5000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.5000 + 0.5000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.5000 + 0.5000i   0.0000 + 0.0000i   0.0000 + 0.0000i
      ⋮

D = 40×40
    2.1213         0         0         0         0         0         0         0 ···
         0    2.1213         0         0         0         0         0         0
         0         0    2.1213         0         0         0         0         0
         0         0         0    1.4142         0         0         0         0
         0         0         0         0    1.4142         0         0         0
         0         0         0         0         0    2.1213         0         0
         0         0         0         0         0         0    0.7071         0
         0         0         0         0         0         0         0    0.7071
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
      ⋮

L = 40×40 complex
   2.1213 + 0.0000i  -0.5000 + 0.5000i   0.0000 + 0.0000i   0.0000 + 0.0000i ···
  -0.5000 - 0.5000i   2.1213 + 0.0000i  -0.5000 + 0.5000i   0.0000 + 0.0000i
   0.0000 + 0.0000i  -0.5000 - 0.5000i   2.1213 + 0.0000i  -0.5000 - 0.5000i
   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.5000 + 0.5000i   1.4142 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i  -0.5000 + 0.5000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
  -0.5000 - 0.5000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i   0.0000 + 0.0000i
   0.0000 + 0.0000i  -0.5000 - 0.5000i   0.0000 + 0.0000i   0.0000 + 0.0000i
      ⋮

Lmd = 40×40
    0.3534         0         0         0         0         0         0         0⋯
         0    0.3537         0         0         0         0         0         0
         0         0    0.3546         0         0         0         0         0
         0         0         0    0.3735         0         0         0         0
         0         0         0         0    0.4007         0         0         0
         0         0         0         0         0    0.4117         0         0
         0         0         0         0         0         0    0.4426         0
         0         0         0         0         0         0         0    0.4759
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
      ⋮

Sgm = 34×34
         0    1.1583         0         0         0         0         0         0⋯
   -1.1583         0         0         0         0         0         0         0
         0         0         0    1.1129         0         0         0         0
         0         0   -1.1129         0         0         0         0         0
         0         0         0         0         0    1.0943         0         0
         0         0         0         0   -1.0943         0         0         0
         0         0         0         0         0         0         0    1.0230
         0         0         0         0         0         0   -1.0230         0
         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0
      ⋮
```

```
y = ones(1,size(Lmd,2));
stem(diag(Lmd),y,'filled')
title('Graph frequencies (real part)','Interpreter','latex')
xlabel('$\lambda$','Interpreter','latex')

ax = gca;
ax.LineWidth = 2;
ax.FontSize = 16;
exportgraphics(ax,RESULTS_DIR+"EigenValues_sym"+FIGEXT)
```

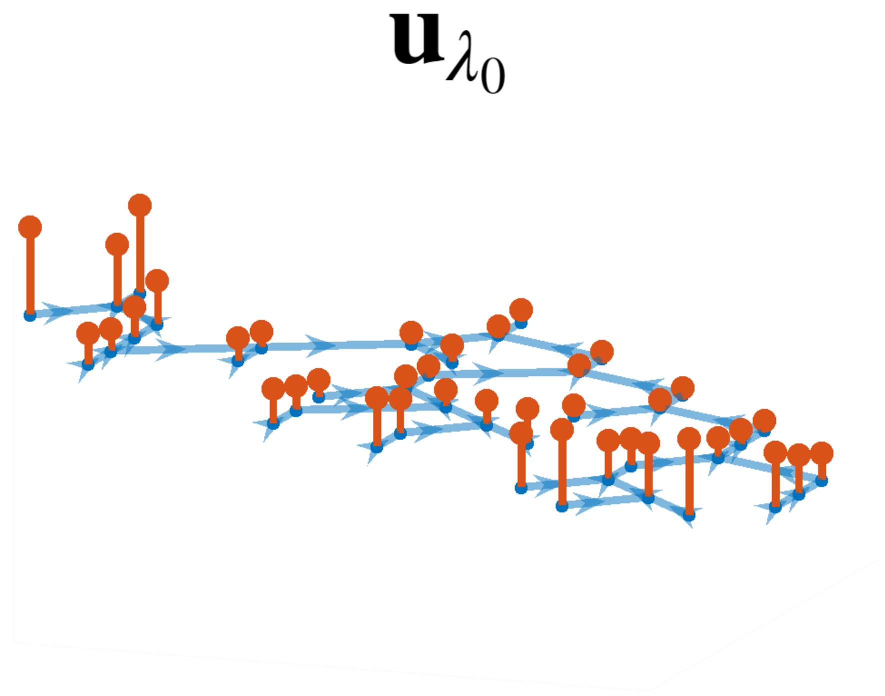


```
sgm = sort([diag(Sgm,1);diag(Sgm,-1)],'ascend');
idxg = find(sgm);
gma = sgm(idxg);
if ~isempty(gma)
    y = ones(1,length(gma));
    stem(gma,y,'filled')
    title('Skew intensities (imaginary part)','Interpreter','latex')
    xlabel('$s$','Interpreter','latex')

    ax = gca;
    ax.LineWidth = 2;
    ax.FontSize = 16;
    exportgraphics(ax,RESULTS_DIR+"EigenValues_skw"+FIGEXT)
end
```

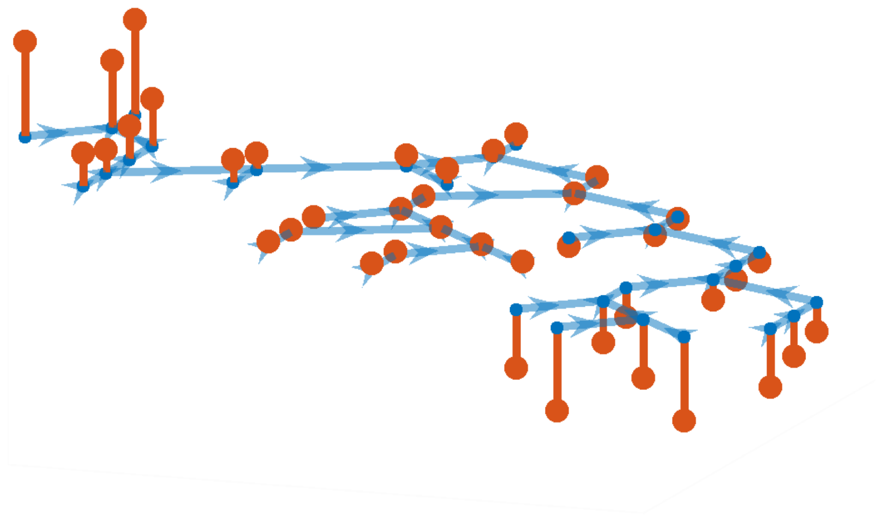


$\mathbf{u}_{\lambda_k}$ 拡張 GFT（対称成分）の基底ベクトル

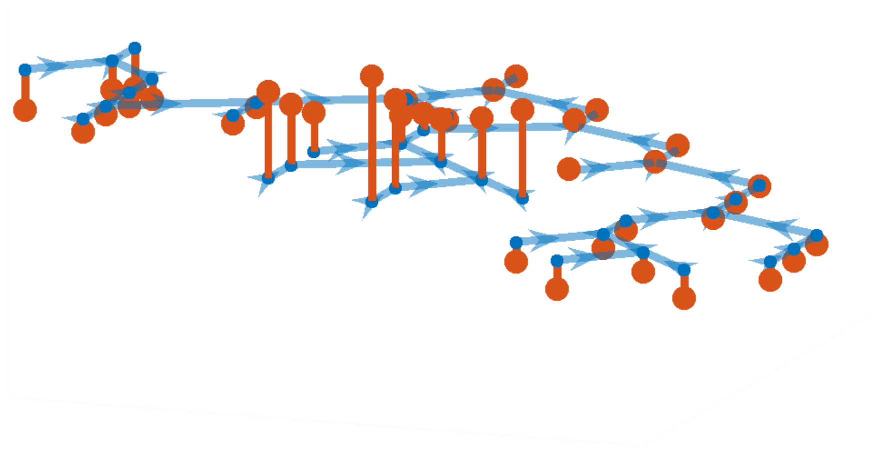
```
zscale = .8;
fontSize = 36;
lineWidth = 3;
arrowSize = 12;
az = 200;
el = 20;
for k = 0:5
    figure
    %uk = U(:,k+1)*Lmd(k+1,k+1);
    uk = U(:,k+1);
    %
    p1 = plot(G,'LineWidth',lineWidth);
    p1.ArrowSize = arrowSize;
    p1.NodeLabel = {};
    hold on
    stem3(p1.XData,p1.YData,uk,'fill','LineWidth',lineWidth);
    %zlim(zscale*[-1 1])
    %axis off
    title("$\mathbf{u}_{\lambda_"+num2str(k)+"}
$",'Interpreter','latex','FontSize',20)
    view(az,el)
    hold off
    ax = gca;
    ax.FontSize = fontSize;
    ax.ZLim = zscale*[-1 1];
    ax.Box = 'off';
    ax.XColor = .99*[1 1 1];
    ax.YColor = .99*[1 1 1];
    ax.ZColor = .99*[1 1 1];
    ax.XLim = [2 7];
    ax.YLim = [2 12];
    ax.Clipping = 'off';
    exportgraphics(ax,RESULTS_DIR+"u"+num2str(k)+FIGEXT)
end
```



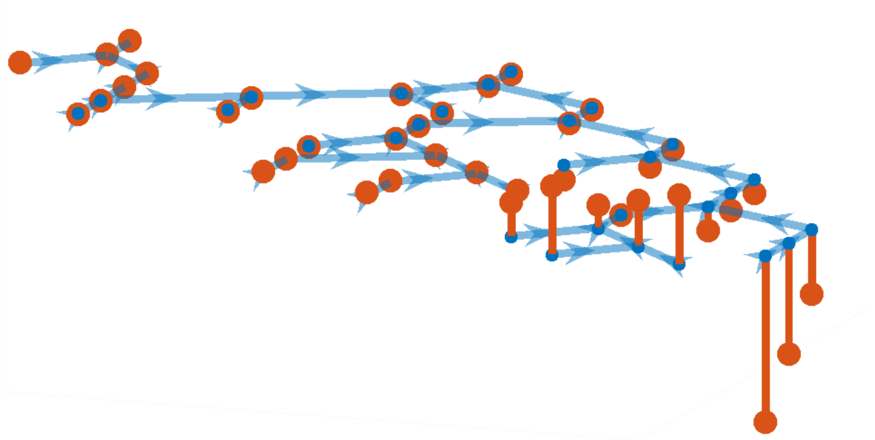
$\mathbf{u}_{\lambda_1}$



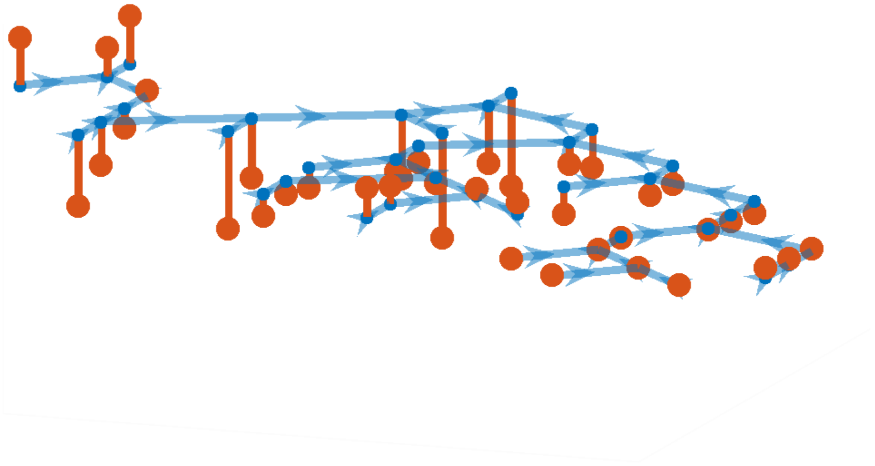
$\mathbf{u}_{\lambda_2}$

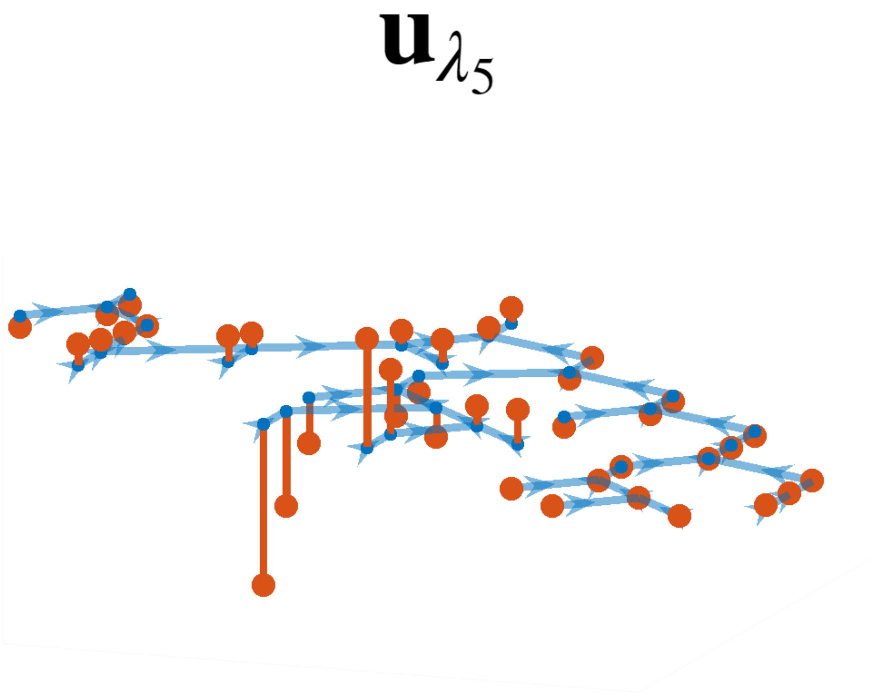


$$\mathbf{u}_{\lambda_3}$$



$$\mathbf{u}_{\lambda_4}$$

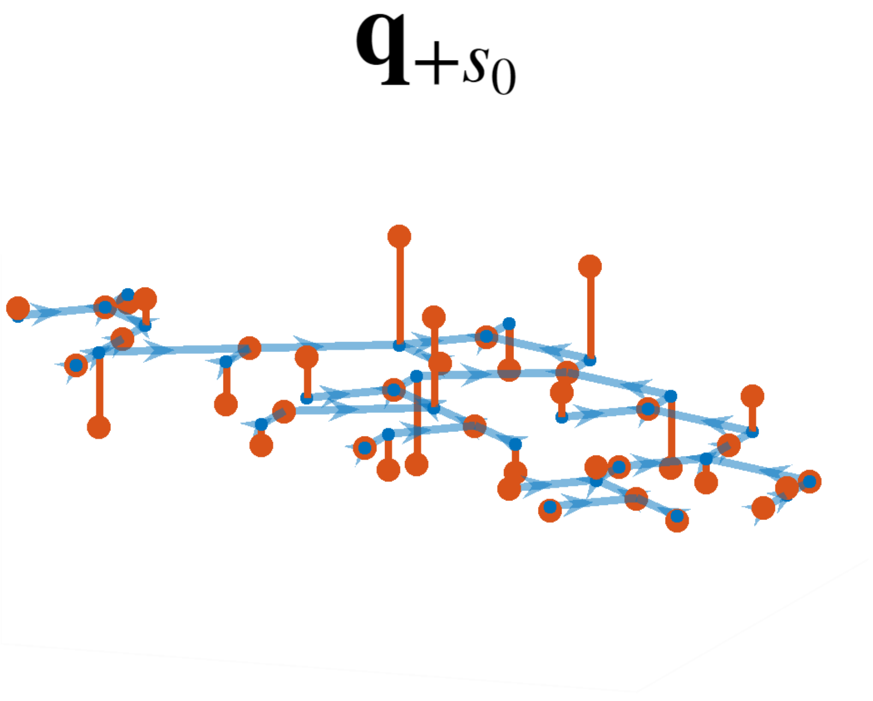




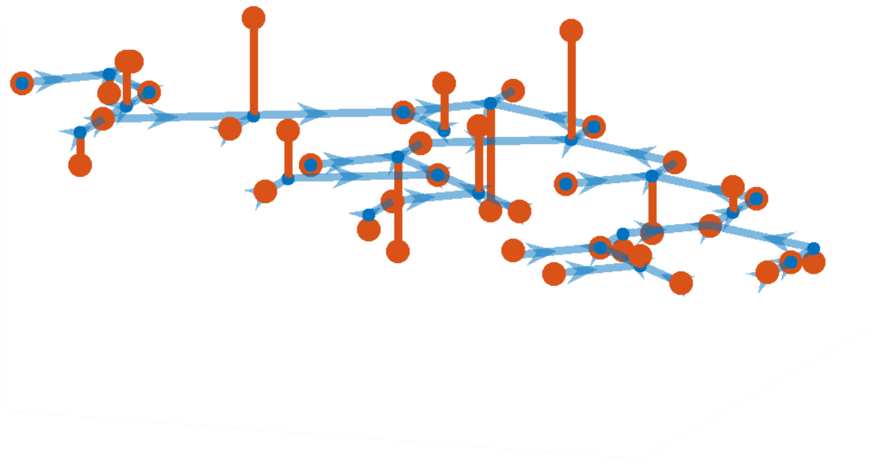
$\mathbf{u}_{\lambda_5}$

q_{s_k} 拡張 GFT（交代成分）の基底ベクトル

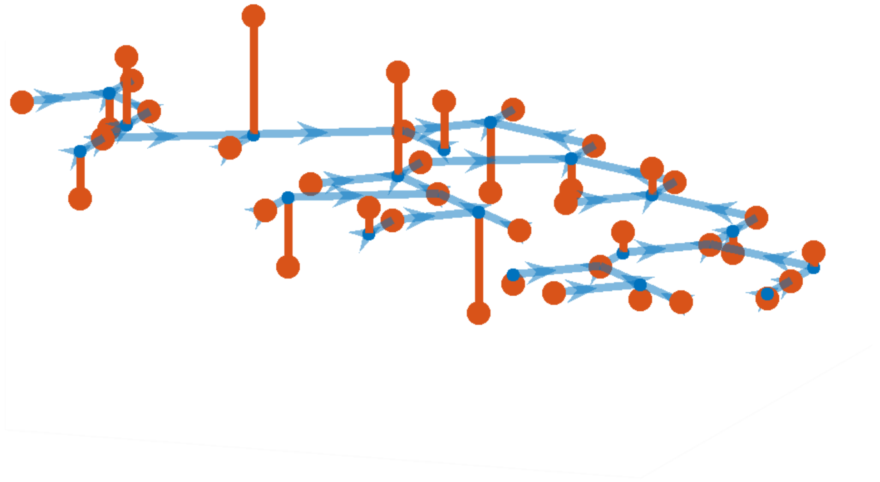
```
for k = 0:min(5,length(gma)/2-1)
    figure
    qkp = Q(:,2*k+1);
    % +sk
    p1 = plot(G,'LineWidth',lineWidth);
    p1.ArrowSize = arrowSize;
    p1.NodeLabel = {};
    hold on
    stem3(p1.XData,p1.YData,qkp,'fill','LineWidth',lineWidth);
    title("$\mathbf{q}_{+s_"+num2str(k)+"}$",'Interpreter','latex','FontSize',20)
    view(az,el)
    hold off
    ax = gca;
    ax.FontSize = fontSize;
    ax.ZLim = zscale*[-1 1];
    ax.Box = 'off';
    ax.XColor = .99*[1 1 1];
    ax.YColor = .99*[1 1 1];
    ax.ZColor = .99*[1 1 1];
    ax.XLim = [2 7];
    ax.YLim = [2 12];
    ax.Clipping = 'off';
    exportgraphics(ax,RESULTS_DIR+"qp"+num2str(k)+FIGEXT)
    %
    figure
    qkm = Q(:,2*k+2);
    % -sk
    p1 = plot(G,'LineWidth',lineWidth);
    p1.ArrowSize = arrowSize;
    p1.NodeLabel = {};
    hold on
    stem3(p1.XData,p1.YData,qkm,'fill','LineWidth',lineWidth);
    title("$\mathbf{q}_{-s_"+num2str(k)+"}$",'Interpreter','latex','FontSize',20)
    view(az,el)
    hold off
    ax = gca;
    ax.FontSize = fontSize;
    ax.ZLim = zscale*[-1 1];
    ax.Box = 'off';
    ax.XColor = .99*[1 1 1];
    ax.YColor = .99*[1 1 1];
    ax.ZColor = .99*[1 1 1];
    ax.XLim = [2 7];
    ax.YLim = [2 12];
    ax.Clipping = 'off';
    exportgraphics(ax,RESULTS_DIR+"qm"+num2str(k)+FIGEXT)
    %
end
```



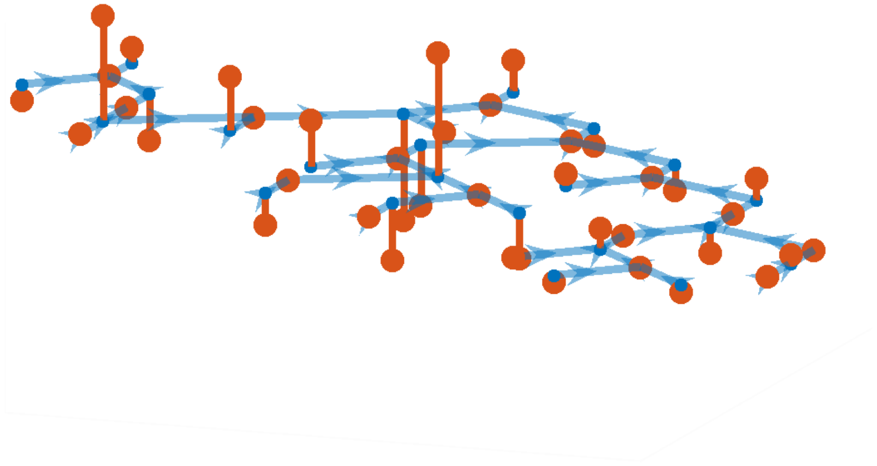
$\mathbf{q}_{-s_0}$



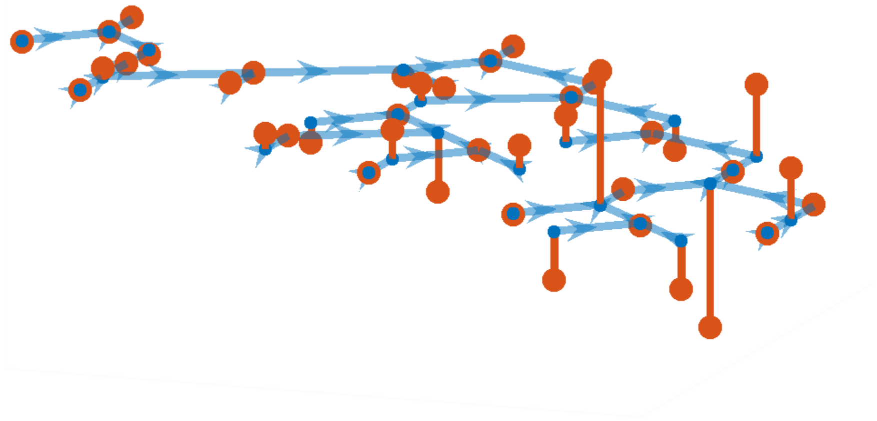
$\mathbf{q}_{+s_1}$



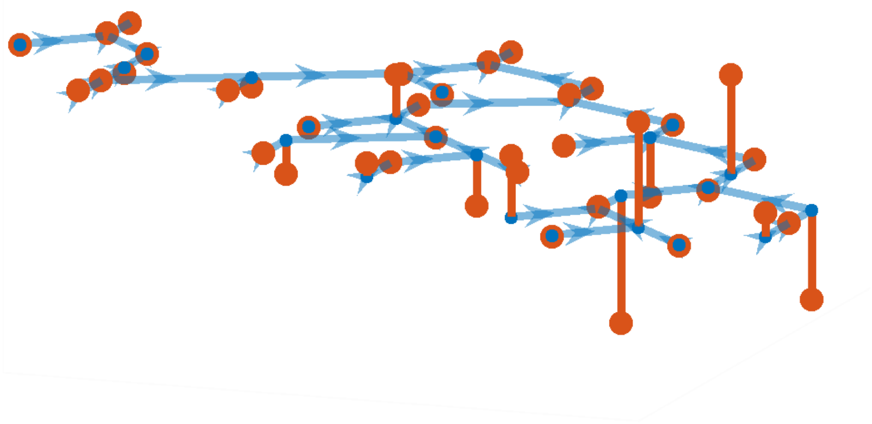
$$\mathbf{q}_{-s_1}$$



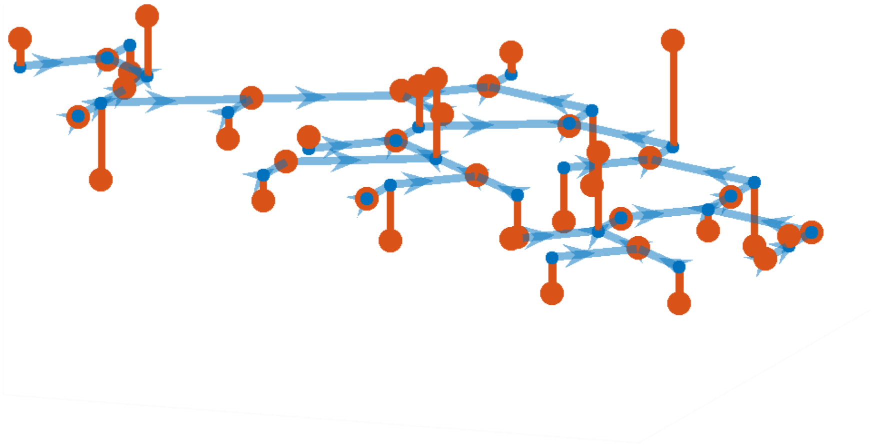
$$\mathbf{q}_{+s_2}$$



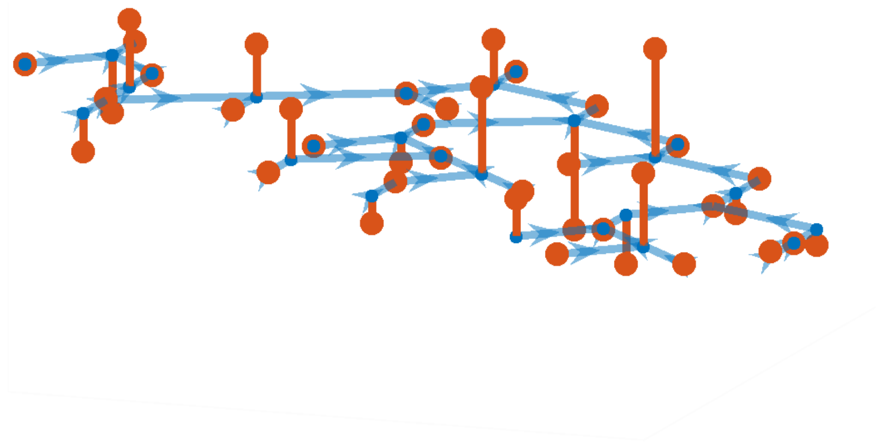
$$\mathbf{q}_{-s_2}$$



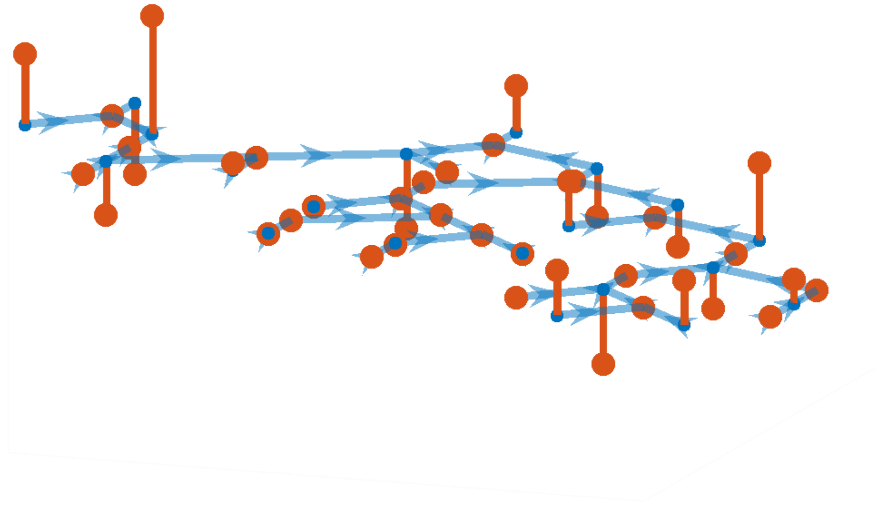
$$\mathbf{q}_{+s_3}$$



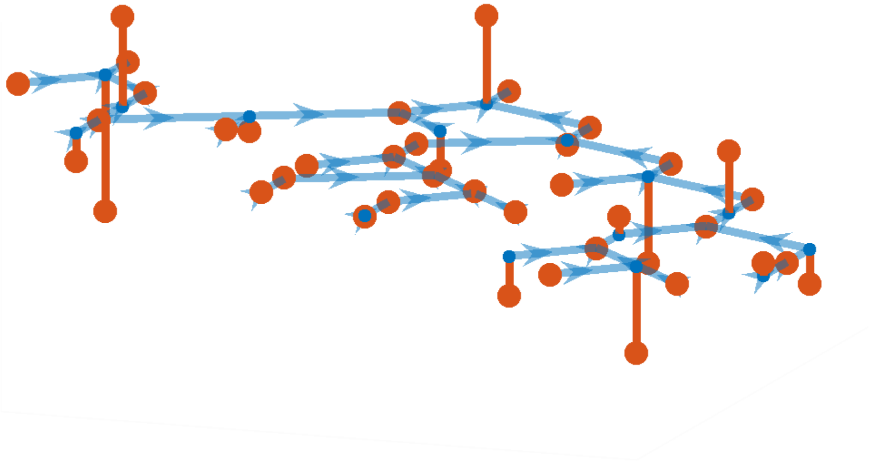
$$\mathbf{q}_{-s_3}$$



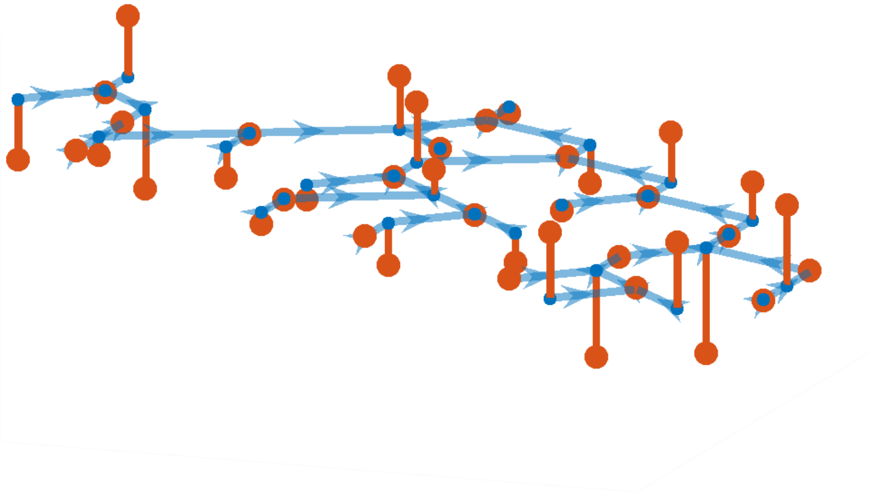
$$\mathbf{q}_{+s_4}$$



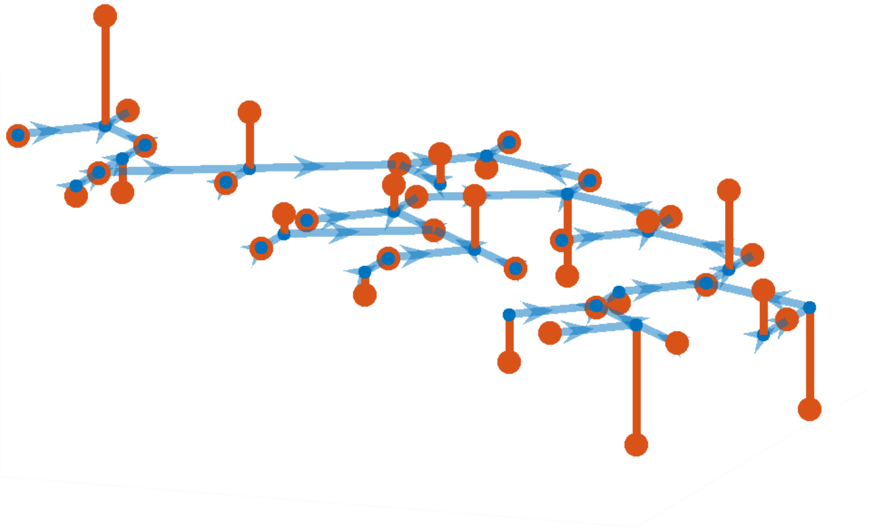
$$\mathbf{q}_{-s_4}$$



$$\mathbf{q}_{+s_5}$$



$$\mathbf{q}_{-s_5}$$



変動の確認

$$\mathbf{L} = \begin{pmatrix} d_{11} & -b_{12} & -b_{13} & -b_{14} \\ -\bar{b}_{12} & d_{22} & -b_{23} & -b_{24} \\ -\bar{b}_{13} & -\bar{b}_{23} & d_{33} & -b_{34} \\ -\bar{b}_{14} & -\bar{b}_{24} & -\bar{b}_{34} & d_{44} \end{pmatrix}$$

$$\underbrace{\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix}}_{\mathbf{y}} = \underbrace{\begin{pmatrix} d_{11} & -b_{12} & -b_{13} & -b_{14} \\ -\bar{b}_{12} & d_{22} & -b_{23} & -b_{24} \\ -\bar{b}_{13} & -\bar{b}_{23} & d_{33} & -b_{34} \\ -\bar{b}_{14} & -\bar{b}_{24} & -\bar{b}_{34} & d_{44} \end{pmatrix}}_{\mathbf{L}} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}}_{\mathbf{x}}$$

$$d_{k,k} := \sum_{\ell \in \mathcal{N} \setminus \{k\}} |c_{k,\ell}| = \sum_{\ell \in \mathcal{N} \setminus \{k\}} |c_{\ell,k}| = \sum_{\ell \in \mathcal{N} \setminus \{k\}} \sqrt{c_{k,\ell} b_{\ell,k}} = \sum_{\ell \in \mathcal{N} \setminus \{k\}} \sqrt{c_{k,\ell} \bar{c}_{k,\ell}}$$

ただし，　$\mathcal{N} := \{1, 2, \cdots, N\}$。

$$y_k = d_{k,k}x_k - \sum_{\ell=k+1}^{N} b_{k,\ell}x_\ell - \sum_{\ell=1}^{k-1} \bar{b}_{k,\ell}x_\ell$$

$$\Delta_{\mathbf{L}}(\mathbf{x}) = \mathbf{x}^T\mathbf{L}\mathbf{x} = \sum_{k\in\mathcal{N}} \sum_{\ell=k+1}^{N} |b_{k,\ell}| \cdot |x_k - e^{-j\angle b_{k,\ell}}x_\ell|^2$$

行列計算

```
nDims = size(L,1);
x = randn(nDims,1);
xLx = x.'*L*x
```

```
xLx = 67.9390 - 0.0000i
```

累積加算

```
tv = 0;
for iRow = 1:nDims
    xk = x(iRow);
    for iCol = iRow+1:nDims
        ckl = C(iRow,iCol);
        xl = x(iCol);
        tv = tv + abs(ckl)*abs(xk-exp(1j*angle(ckl))*xl)^2;
    end
end
tv
```

```
tv = 67.9390
```

誤差評価

```
assert(abs(xLx - tv)<1e-6,'Invalid TV')
```

```
if isRiver
    %解析期間設定
    year = "2022";
    month1 = "0801";
    num_day1 = 31; % month1 が何日間か指定(最新のデータで月の途中の場合なども可)

    bgn_day =1; % 解析開始日
    bgn_time =1; %解析開始時間

    end_day =5 ;% 解析終了日(月をまたぐ場合はまたぐ場合の日付)
    end_time =24;%解析終了時間
    %月をまたぐ場合の機能
    pathmonth = 0; %月をまたぐ場合は 1 に設定
    month2 = "0401"; %month1 の次の月を指定
    num_day2 = 31;%month2 が何日までなのか指定

    errorcatch1 = "終了時間の方が開始時間よりも前にあります！";
    errorcatch2 = "min が max よりも大きいです";


    % error message
    if pathmonth == 0
        if bgn_day*24+bgn_time > end_day*24+end_time
            errorcatch1
        end
    end

    er_month1 = erase(month1,'01');
    er_month1 = str2double(er_month1);
    er_month2 = erase(month2,'01');
    er_month2 = str2double(er_month2);

    BGN = (bgn_day-1)*24+(bgn_time-1);

    if pathmonth == 1
        END = (num_day1*24) + (end_day-1)*24 + end_time;
    else
        END = (end_day-1)*24 + end_time;
    end

    hours = END - BGN;

    if hours < 0
        errorcatch1
    end

    formatSpec = 'Start: %d/%d %d:00, Finish: %d/%d %d:00, Time: %d hours';
    if pathmonth == 0
        DATA_period = 
sprintf(formatSpec,er_month1,bgn_day,bgn_time,er_month1,end_day,end_time,hours);
    else
        DATA_period = 
sprintf(formatSpec,er_month1,bgn_day,bgn_time,er_month2,end_day,end_time,hours);
    end

    % ana_hours = 35 %round(hours*30/100)
    t_duration = hours + 1;
```

データ収集

```
    str1 = "http://www1.river.go.jp/cgi-bin/DspWaterData.exe?KIND=2&ID=";
    bg = "&BGNDATE=";
    en = "&ENDDATE=";
    str2 = "&KAWABOU=NO";
    enddate = "1231"; % ここは変更しない　絶対触るな
    check = "0000ff"; %暫定値の時でも抽出できる用（絶対消すな）
    % check2 = "。。"
    error1 = "#ff00ff"; %閉局・欠測も抽出できる用（絶対消すな）
    options = weboptions('CharacterEncoding','Shift_JIS');

    locs =40;
    waterlevel1 = zeros(num_day1*24,locs);
    waterlevel2 = zeros(num_day2*24,locs);

    for iloc = 1:locs
        ID = locList(iloc);
        url1 = str1 + ID + bg + year + month1 + en + year + enddate + str2;
        response1 = string(webread(url1,options));
        response1 = extractBetween(response1,'<TBODY>','</TBODY>');
        response1 = extractAfter(response1,'<TH bgcolor="#AAFFAA">');

        if contains(response1,check) == 1
            response1 = extractBetween(response1,'<FONT size="-1" color=','</
FONT>');
            response1 = extractAfter(response1,'>');

        else
            response1 = extractAfter(response1,'</TR>');
            response1 = extractBetween(response1,'<TD align="','</FONT></TD>');
            response1 = extractAfter(response1,'<FONT size="-1">');
        end

        waterlevel1(:,iloc) = response1;

    end
    waterlevel1 = transpose(waterlevel1);
    waterlevel1 = fillmissing(waterlevel1,'linear',2,'EndValues','nearest');
    %  月を跨いで解析するよう（エラーの原因がわからないので保留）最悪 if 文無くして配列結合
して処理する
    if pathmonth == 1
        for iloc = 1:locs
            ID = locList(iloc);
            url2 = str1 + ID + bg + year + month2 + en + year + enddate + str2;
            response2 = string(webread(url2,options));
            response2 = extractBetween(response2,'<TBODY>','</TBODY>');
            response2 = extractAfter(response2,'<TH bgcolor="#AAFFAA">');

            if contains(response2,check) == 1
                response2 = extractBetween(response2,'<FONT size="-1" color=','</
FONT>');
                response2 = extractAfter(response2,'>');
            else
                response2 = extractAfter(response2,'</TR>');
                response2 = extractBetween(response2,'<TD align="','</FONT></TD>');
                response2 = extractAfter(response2,'<FONT size="-1">');
            end

            % 欠損データ補間(NaN の場合に直前のデータと同じにする)
            response2 = fillmissing(response2,"previous");

            waterlevel2(:,iloc) = response2;
        end
        waterlevel2 = transpose(waterlevel2);
        waterlevel2 = fillmissing(waterlevel2,'linear',2,'EndValues','nearest')
    end

    if pathmonth == 12
        waterlevel = cat(2,waterlevel1,waterlevel2);
    else
        waterlevel = waterlevel1;
    end

    %ゼロ点高足し合わせ(水文水質データベースより)
    %%{
    waterlevel(8,:)=waterlevel(8,:)+14.800;
    waterlevel(10,:)=waterlevel(10,:)+58.5;
    waterlevel(11,:)=waterlevel(11,:)+39.9;
    waterlevel(23,:)=waterlevel(23,:)+232.0;
    waterlevel(24,:)=waterlevel(24,:)+575.577;
    waterlevel(25,:)=waterlevel(25,:)+357.495;
    waterlevel(26,:)=waterlevel(26,:)+468.000;
    waterlevel(27,:)=waterlevel(27,:)+604.213;
    waterlevel(28,:)=waterlevel(28,:)+610.870;
    waterlevel(29,:)=waterlevel(29,:)+463.600;
    waterlevel(30,:)=waterlevel(30,:)+355.950;
    waterlevel(31,:)=waterlevel(31,:)+324.255;
    waterlevel(32,:)=waterlevel(32,:)+300.480;
    waterlevel(33,:)=waterlevel(33,:)+640.130;
    waterlevel(34,:)=waterlevel(34,:)+550.010;
    waterlevel(35,:)=waterlevel(35,:)+498.570;
    waterlevel(36,:)=waterlevel(36,:)+360.120;
    waterlevel(37,:)=waterlevel(37,:)+560.880;
    waterlevel(38,:)=waterlevel(38,:)+601.850;
    waterlevel(39,:)=waterlevel(39,:)+269.420;
    waterlevel(40,:)=waterlevel(40,:)+765.000;
    %%}
    % 解析期間を設定してデータを成型
    waterlevel(:,END+1:end) = [];
    waterlevel(:,1:BGN) = []
else
    waterlevel = randn(nDims)
```

```
end
```

```
waterlevel = 40×120
    7.6500    7.6500    7.6500    7.6500    7.6400    7.6400    7.6400    7.6300 ⋯
    6.3000    6.3000    6.3000    6.2900    6.2900    6.2700    6.2700    6.2600
    1.9700    1.9800    1.9900    2.0000    2.0000    1.9900    1.9800    1.9700
    1.0300    1.0400    1.0600    1.0800    1.1100    1.1200    1.1300    1.1400
    0.8600    0.8700    0.8900    0.9400    0.9600    0.9800    0.9800    0.9900
    0.7400    0.7600    0.8000    0.8400    0.8500    0.8700    0.8800    0.8800
    0.7300    0.7800    0.8300    0.8700    0.8600    0.8800    0.8700    0.8500
   15.7900   15.7900   15.7900   15.7600   15.7500   15.7300   15.7200   15.7200
    1.2700    1.2800    1.3000    1.3200    1.3400    1.3700    1.4000    1.4100
   58.8200   58.8200   58.8200   58.8200   58.8200   58.8300   58.8300   58.8300
       ⋮
```

```
x = waterlevel;
freqXSet = fcn_fwdAuGFT(x,U,Q)
```

```
freqXSet = 74×120
10^3  ×
    1.1351    1.1352    1.1352    1.1352    1.1352    1.1352    1.1352    1.1352 ⋯
   -0.7974   -0.7974   -0.7974   -0.7973   -0.7973   -0.7973   -0.7972   -0.7972
    0.5410    0.5410    0.5410    0.5410    0.5410    0.5410    0.5410    0.5410
   -0.1347   -0.1349   -0.1349   -0.1349   -0.1349   -0.1349   -0.1348   -0.1348
   -0.0817   -0.0817   -0.0818   -0.0818   -0.0818   -0.0818   -0.0818   -0.0818
   -0.1316   -0.1316   -0.1316   -0.1316   -0.1316   -0.1316   -0.1316   -0.1317
    0.2364    0.2364    0.2364    0.2365    0.2365    0.2365    0.2366    0.2366
   -0.1275   -0.1275   -0.1274   -0.1274   -0.1274   -0.1274   -0.1274   -0.1275
   -0.1476   -0.1476   -0.1476   -0.1477   -0.1476   -0.1477   -0.1477   -0.1477
    0.2248    0.2248    0.2248    0.2248    0.2248    0.2248    0.2249    0.2249
       ⋮
```

```
y = fcn_invAuGFT(freqXSet,U,Q)
```

```
y = 40×120
    7.6500    7.6500    7.6500    7.6500    7.6400    7.6400    7.6400    7.6300 ⋯
    6.3000    6.3000    6.3000    6.2900    6.2900    6.2700    6.2700    6.2600
    1.9700    1.9800    1.9900    2.0000    2.0000    1.9900    1.9800    1.9700
    1.0300    1.0400    1.0600    1.0800    1.1100    1.1200    1.1300    1.1400
    0.8600    0.8700    0.8900    0.9400    0.9600    0.9800    0.9800    0.9900
    0.7400    0.7600    0.8000    0.8400    0.8500    0.8700    0.8800    0.8800
    0.7300    0.7800    0.8300    0.8700    0.8600    0.8800    0.8700    0.8500
   15.7900   15.7900   15.7900   15.7600   15.7500   15.7300   15.7200   15.7200
    1.2700    1.2800    1.3000    1.3200    1.3400    1.3700    1.4000    1.4100
   58.8200   58.8200   58.8200   58.8200   58.8200   58.8300   58.8300   58.8300
       ⋮
```

```
rmse = @(rx,ry) norm(rx(:)-ry(:),2)/sqrt(numel(rx));
rmse(waterlevel,y)
```

```
ans = 2.1326e-13
```

```
assert(norm(x(:)-y(:))<1e-6)
```

フィルタ処理実験

```
x_in = waterlevel
```

```
x_in = 40×120
    7.6500    7.6500    7.6500    7.6500    7.6400    7.6400    7.6400    7.6300 ···
    6.3000    6.3000    6.3000    6.2900    6.2900    6.2700    6.2700    6.2600
    1.9700    1.9800    1.9900    2.0000    2.0000    1.9900    1.9800    1.9700
    1.0300    1.0400    1.0600    1.0800    1.1100    1.1200    1.1300    1.1400
    0.8600    0.8700    0.8900    0.9400    0.9600    0.9800    0.9800    0.9900
    0.7400    0.7600    0.8000    0.8400    0.8500    0.8700    0.8800    0.8800
    0.7300    0.7800    0.8300    0.8700    0.8600    0.8800    0.8700    0.8500
   15.7900   15.7900   15.7900   15.7600   15.7500   15.7300   15.7200   15.7200
    1.2700    1.2800    1.3000    1.3200    1.3400    1.3700    1.4000    1.4100
   58.8200   58.8200   58.8200   58.8200   58.8200   58.8300   58.8300   58.8300
      ⋮
```

```
lmax = max(diag(Lmd))
```

```
lmax = 3.0943
```

```
g_dr = gsp_design_regular(lmax);
G_A = 1.0;
G_B = 1.0;
```

HPF LPF 関数表示,抽出

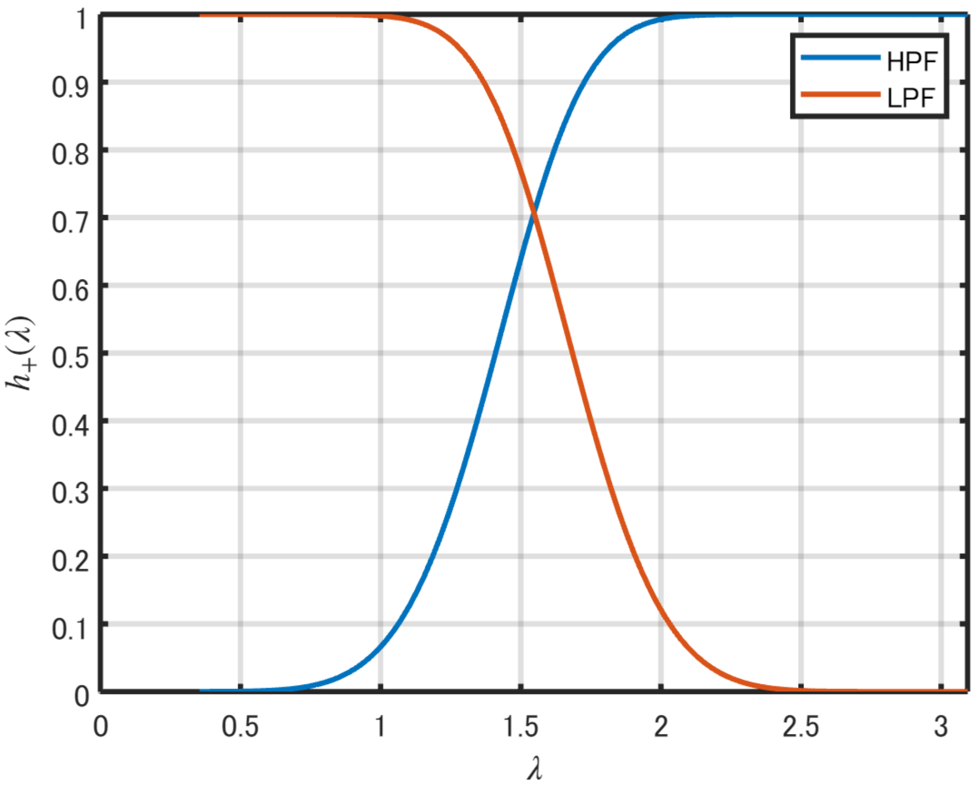
```
g_dr(1) % HPF
```

```
ans = 1×1 の cell 配列
    {@(x)regular(x*(2/lmax),d)}
```

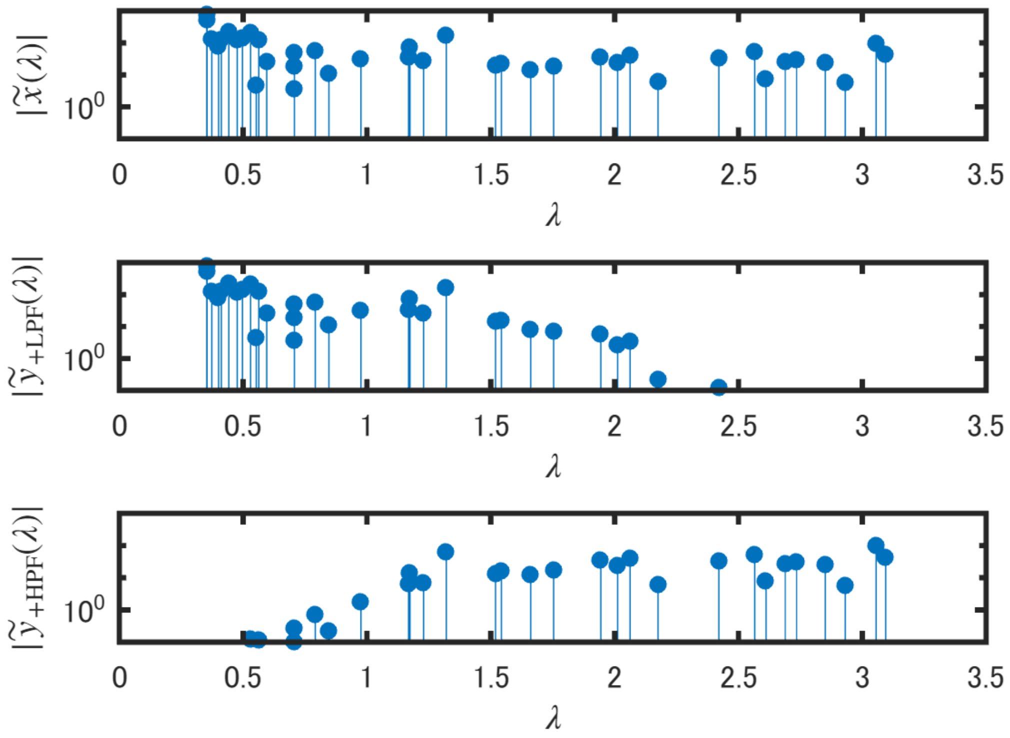
```
g_dr(2) % LPF
```

```
ans = 1×1 の cell 配列
    {@(x)real(sqrt(1-(regular(x*(2/lmax),d)).^2))}
```

```
lambdas = linspace(Lmd(1),lmax);
fd_H = gsp_filter_evaluate(g_dr(1),lambdas);
fd_L = gsp_filter_evaluate(g_dr(2),lambdas);
figure
plot(lambdas,fd_H,'LineWidth',2)
xlim(full([0 lmax]));
xlabel('$\lambda$','Interpreter','latex','FontSize',12)
ylabel('$h_{+}(\lambda)$','Interpreter','latex','FontSize',12)
%title('HPF')
hold on

%figure
plot(lambdas,fd_L,'LineWidth',2)
xlim(full([0 lmax]));
hold off
grid on
ax = gca;
legend({'HPF','LPF'})
ax.LineWidth = 2;
ax.FontSize = 12;
exportgraphics(ax,RESULTS_DIR+"filterbank"+FIGEXT)
```



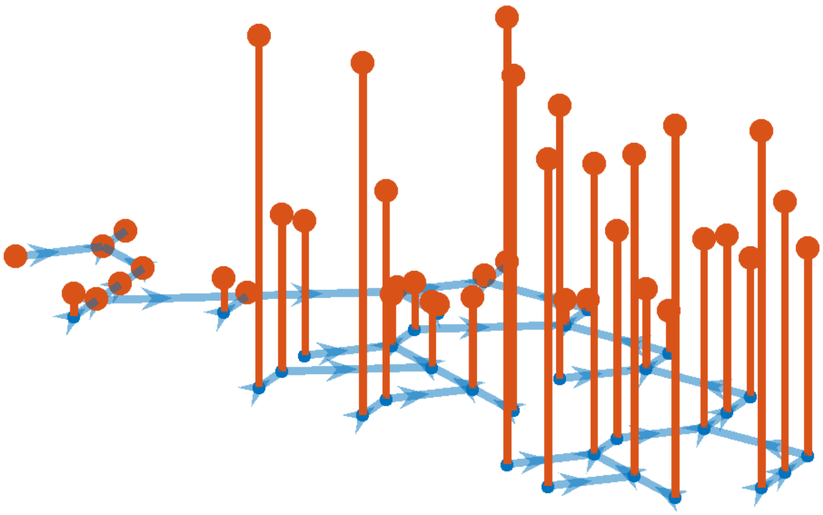
```
ylim_ = [0.1 1000];

lambdaList = diag(Lmd);
filterList_high = gsp_filter_evaluate(g_dr(1),lambdaList);
H_high = filterList_high; % Rename
filterList_low = gsp_filter_evaluate(g_dr(2),lambdaList);
H_low = filterList_low; % Rename

jikoku = min(90,size(freqXSet,2));
N = length(lambdaList);
freqX = freqXSet(:,jikoku);
freqY_high = diag(H_high)*freqX(1:N,:);
freqY_low  = diag(H_low)*freqX(1:N,:);

%スペクトル
figure
subplot(3,1,1)
stem(lambdaList,abs(freqX(1:N,:)),'Filled')
xlabel('$\lambda$','Interpreter','latex')
ylabel('$|\tilde{x}(\lambda)|$','Interpreter','latex')
ax = gca;
ax.FontSize = 12;
ax.LineWidth = 2;
ax.YScale = 'log';
ax.YLim = ylim_;
exportgraphics(ax,RESULTS_DIR+"X"+FIGEXT)

subplot(3,1,2)
stem(lambdaList,abs(freqY_low),'filled')
xlabel('$\lambda$','Interpreter','latex')
ylabel('$|\tilde{y}_\mathrm{+LPF}(\lambda)|$','Interpreter','latex')
ax = gca;
ax.FontSize = 12;
ax.LineWidth = 2;
ax.YScale = 'log';
ax.YLim = ylim_;
exportgraphics(ax,RESULTS_DIR+"Ylpf"+FIGEXT)

subplot(3,1,3)
stem(lambdaList,abs(freqY_high),'filled')
xlabel('$\lambda$','Interpreter','latex')
ylabel('$|\tilde{y}_\mathrm{+HPF}(\lambda)|$','Interpreter','latex')
ax = gca;
ax.FontSize = 12;
ax.LineWidth = 2;
ax.YScale = 'log';
ax.YLim = ylim_;
exportgraphics(ax,RESULTS_DIR+"Yhpf"+FIGEXT)
```



```
% x
x_in = waterlevel(:,jikoku);
xSet = waterlevel;
y = x_in;
real(y.'*L*y)
```

```
ans = 1.7761e+06
```

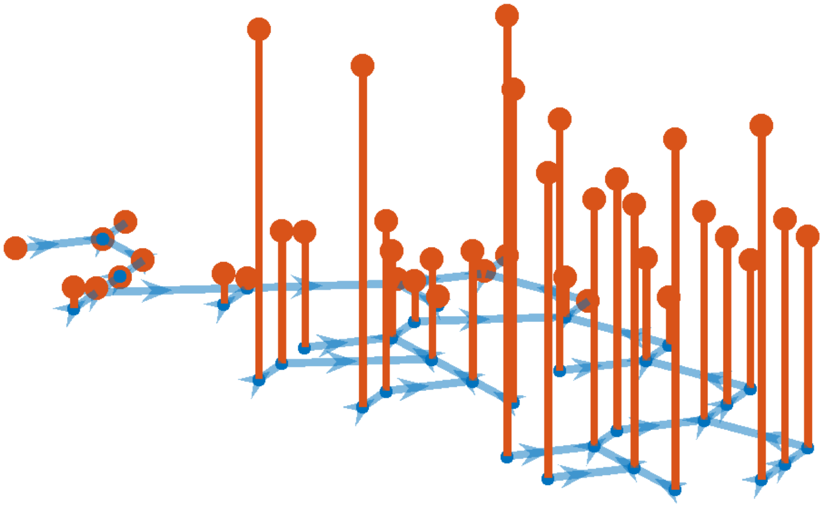
```
yfile = "orig";
mygstem3(G, x_in, RESULTS_DIR, yfile, FIGEXT);
```



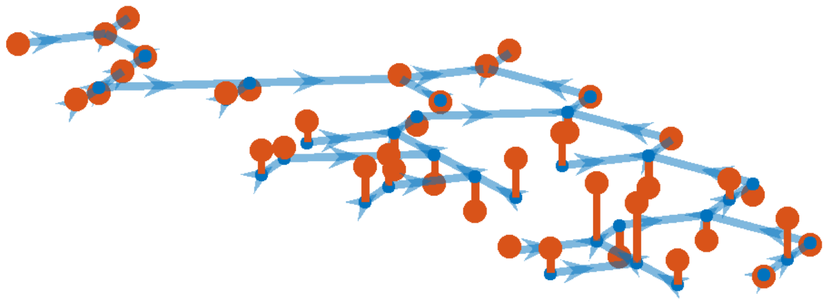
ユニタリ GFT(ParaGFT)による処理

```
[W,~,~,~,~,paraLmd] = fcn_paragraphops(A);
paraH_high = gsp_filter_evaluate(g_dr(1),diag(paraLmd));
paraH_low = gsp_filter_evaluate(g_dr(2),diag(paraLmd));
%
hModeSet = {  'lowpass', 'highpass'}; % 'direct',
for idxHMode = 1:length(hModeSet)
    hMode = hModeSet{idxHMode};
    if strcmp(hMode,'direct')
        hp = ones(N,1);
    elseif strcmp(hMode,'lowpass')
        hp = paraH_low;
    elseif strcmp(hMode,'highpass')
        hp = paraH_high;
    else
        error('Invalide HMode')
    end
    disp("==========")
    disp("HMode: " + hMode)
    % y
    y = real(W*(hp.*W')*x_in); % 実部を抽出
    real(y.'*L*y)
    yfile = "y_h_" + hMode + "_paragft";
    mygstem3(G, y, RESULTS_DIR, yfile, FIGEXT);
end
```

```
==========
HMode: lowpass
ans = 1.6142e+06
```

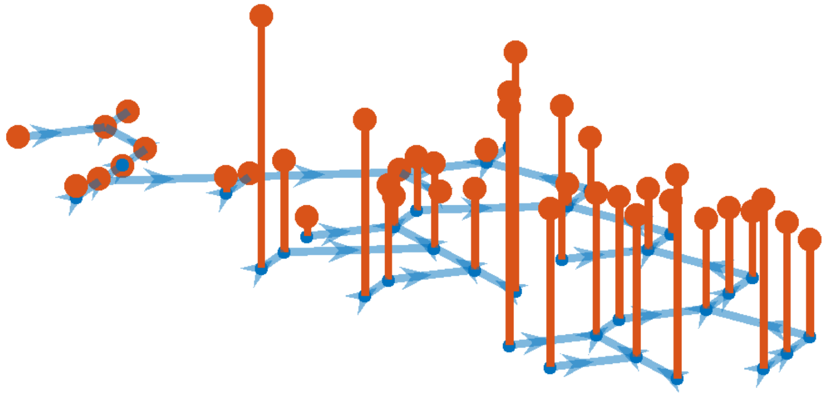


```
==========
HMode: highpass
ans = 9.5854e+04
```

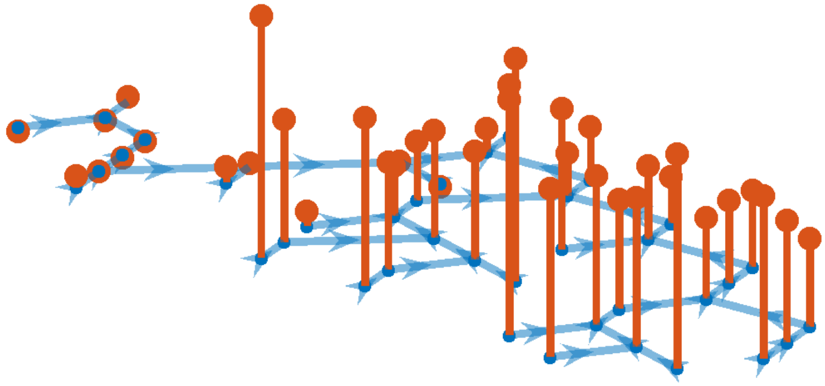


```
hpModeSet = {'lowpass', 'highpass'};
hmModeSet = { 'zeros', 'opts' };
r = size(Q,2);
sc = 1; %sqrt(2); % h+ フィルタ利得

for idxHpMode = 1:length(hpModeSet)
    hpMode = hpModeSet{idxHpMode};
    if strcmp(hpMode,'direct')
        hp = ones(N,1);
    elseif strcmp(hpMode,'lowpass')
        hp = sc*H_low;
    elseif strcmp(hpMode,'highpass')
        hp = sc*H_high;
    else
        error('Invalide HpMode')
    end
    disp("==========")
    disp("HpMode: " + hpMode)

    for idxHmMode = 1:length(hmModeSet)
        if r == 0
            warning('HmMode N/A')
        else
            hmMode = hmModeSet{idxHmMode};
            if strcmp(hmMode,'zeros')
                hm = zeros(r/2,1);
            elseif strcmp(hmMode,'ones')
                hm = ones(r/2,1);
            elseif strcmp(hmMode,'negones')
                hm = -ones(r/2,1);
            elseif strcmp(hmMode,'opts')
                hm0 = zeros(r/2,1);
                rng default % For reproducibility
                opts = optimoptions(@fminunc);
                mycost_(hp,hm0,U,Q,L,xSet)
                problem = createOptimProblem('fminunc',...
                    'objective', @(hm) mycost_(hp,hm,U,Q,L,xSet),... % 最適はデータセットを使うように変更
                    'x0',hm0,'options',opts);
                ms = MultiStart;
                hm = run(ms,problem,20);
            else
                error('Invalid HmMode')
            end
        end
        disp("- HmMode: "+hmMode)
        cost = mycost_(hp,hm,U,Q,L,x_in)
        %

        % y
        y = myfilt_(x_in,hp,hm,U,Q);
        real(y.'*L*y)
        yfile = "y_hp_" +hpMode+"_hm_" + hmMode;
        mygstem3(G, y, RESULTS_DIR, yfile, FIGEXT);

    end

end
```

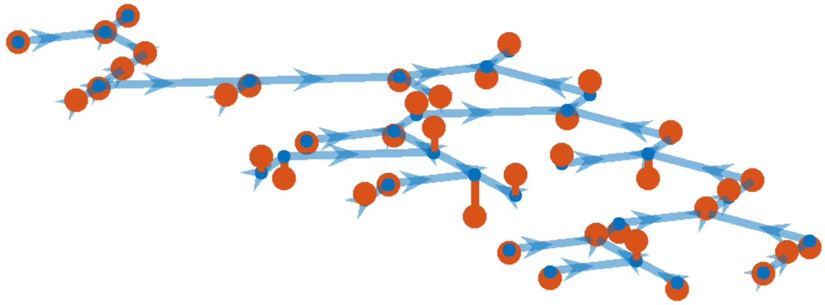
```
==========
HpMode: lowpass
- HmMode: zeros
cost = 5.5735e+05
ans = 5.5735e+05
```



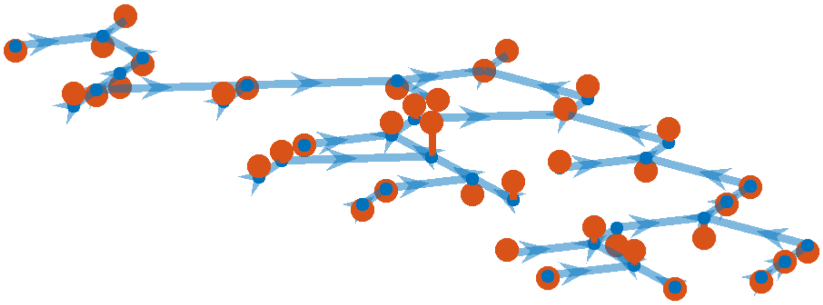
```
ans = 5.5691e+05
MultiStart は、すべての開始点からの実行を完了しました。

20 回すべてのローカル ソルバーの実行が、ローカル ソルバーの正の終了フラグで収束しました。
- HmMode: opts
cost = 5.3849e+05
ans = 5.3849e+05
```



```
==========
HpMode: highpass
- HmMode: zeros
cost = 4.1275e+04
ans = 4.1275e+04
```



```
ans = 4.1184e+04
MultiStart は、すべての開始点からの実行を完了しました。

20 回すべてのローカル ソルバーの実行が、ローカル ソルバーの正の終了フラグで収束しました。
- HmMode: opts
cost = 2.2190e+04
ans = 2.2190e+04
```



自作関数:グラフ信号の可視化

```
function mygstem3(G, y, RESULTS_DIR, yfile,FIGEXT)
zscale = 400;
fontSize = 36;
lineWidth = 3;
arrowSize = 12;
az = 200;
el = 20;
%
figure
p1 = plot(G,'LineWidth',lineWidth);
p1.ArrowSize = arrowSize;
p1.NodeLabel = {};
hold on
stem3(p1.XData,p1.YData,y,'fill','LineWidth',lineWidth);
axis off
view(az,el)
hold off
ax = gca;
```

```
%
ax.FontSize = fontSize;
ax.ZLim = zscale*[-1 1];
ax.Box = 'on';
ax.XColor = .99*[1 1 1];
ax.YColor = .99*[1 1 1];
ax.ZColor = .99*[1 1 1];
ax.XLim = [2 7];
ax.YLim = [2 12];
ax.Clipping = 'off';
%
exportgraphics(ax,RESULTS_DIR+yfile+FIGEXT)
end

function cost =  mycost_(hp,hm,U,Q,L,x)
y = myfilt_(x,hp,hm,U,Q);
v = L*y;
cost = 0;
for idx = 1:size(v,2)
    cost = cost + real(y(:,idx)'*v(:,idx));
end
cost = cost/size(v,2);
end

function y = myfilt_(x,hp,hm,U,Q)
N = size(x,1);
r = size(Q,2);
freqX = fcn_fwdAuGFT(x,U,Q);
freqXp = freqX(1:N,:);
freqYp = hp.*freqXp;
freqXm = freqX(N+1:end,:);
%
if r == 0
    freqYm = [];
else
    tmp = [1 0 ; -1 0]*ones(2,r/2)*diag(hm); % FIXME: h_(Σ)
    tmp = upsample(tmp.',2).';
    tmp = tmp(:,1:end-1);
    h_Sgm = diag(tmp(1,:),1)+diag(tmp(2,:),-1);
    freqYm = h_Sgm * freqXm; % TODO: h_Sgm
end
%
freqY = vertcat(freqYp,freqYm);
y = fcn_invAuGFT(freqY,U,Q);

end
```