Week 3: AMC Geometry I

MSJ Math Club

October 4, 2012

1 Tips and Tricks

- Look for numbers that look like part of Pythagorean triples!

- When you have right angles or parallel lines, you usually want to make more right angles. If you have a diagram with few arcs (only lines), often times you will need to find the right angle (hahaha) to make.

- Most AIME geometry problems can usually be solved with making right angles and applying the Pythagorean Theorem. However, this is usually not true for other math contests.

- Right angles can sometimes make area calculation a useful tool, even if area is not part of the problem.

2 Examples

Note: $[x]$ denotes the area of figure x.

- *(2012 AIME II)* In the accompanying figure (which we will draw on the board), the outer square has side length 40. A second square S' of side length 15 is constructed inside S with the same center as S and with sides parallel to those of S. From each midpoint of a side of S, segments are drawn to the two closest vertices of S'. The result is a four-pointed starlike figure inscribed in S. The star figure is cut out and then folded to form a pyramid with base S'. Find the volume of this pyramid.

- Square $ABCD$, P inside, $AP = 7, BP = 5, CP = 1$. Find $[ABCD]$. Bonus: What if P is outside?

- *(2000 AIME II)* In trapezoid $ABCD$, leg $\overline{BC}$ is perpendicular to bases $\overline{AB}$ and $\overline{CD}$, and diagonals $\overline{AC}$ and $\overline{BD}$ are perpendicular. Given that $AB = \sqrt{11}$ and $AD = \sqrt{1001}$, find BC^2.

3 Practice Problems

1. *(Mathcounts)* A circular table is put in the corner of a room. A certain point on the table is 54 in away from one wall it is touching and 3 in from the other. What are all possible radii of the table?

2. *(2009 AMC 10B)* Rectangle $ABCD$ has $AB = 8$ and $BC = 6$. Point M is the midpoint of diagonal $\overline{AC}$, and E is on $\overline{AB}$ with $\overline{ME} \perp \overline{AC}$. What is the area of $\triangle AME$?

3. *(AMC 10, twisted)* Trapezoid has $AB||CD, AB = 25, BC = 7, CD = 50$, and $DA = 24$. What is the area of the trapezoid?

4. $\angle AOB = 90°$. For any A and B, what is the locus of C such that $\angle ACB = 90°$? (A locus is the set of all points which satisfy a given property.)

5. *(2009 SMT)* Equilateral triangle ABC has side length of 24. Points D, E, F lie on sides BC, CA, AB such that $AD \perp BC, DE \perp AC$, and $EF \perp AB$. G is the intersection of AD and EF. Find the area of the quadrilateral $BFGD$.

6. *(2009 HMMT)* In triangle ABC, D is the midpoint of BC, E is the foot of the perpendicular from A to BC, and F is the foot of the perpendicular from D to AC. Given that $BE = 5, EC = 9$, and the area of triangle ABC is 84, compute $|EF|$.

7. *(2007 HMMT)* ABCD is a convex quadrilateral such that $AB = 2, BC = 3, CD = 7$, and $AD = 6$. It also has an incircle. Given that $\angle ABC$ is right, determine the radius of this incircle.

8. *(HMMT)* Right triangle XYZ has right angle at Y and $XY = 228, YZ = 2004$. Angle Y is trisected, and the angle trisectors intersect XZ at P and Q so that X, P, Q, Z lie on XZ in that order. Find the value of $(PY + YZ)(QY + XY)$.