A1. [**2**] A factorial, or $n!$, is equal to $n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$. Find the sum of the last eight digits of $(((4!)!)!)!$.

A1. ______________

A2. [**3**] A date is written on the board in the form $MM - DD - YY$, where MM represents the month, DD represents the date, and YY represents the last two digits of the year. However, Alvin misreads this as the expression $(MM - D) \cdot (D - YY)$. List all dates in 2014 for which this expression is a positive perfect cube in $MM - DD - YY$ format.

A2. ______________

A3. [**3**] In a high school curriculum, one must learn Pre-Algebra before learning either Algebra or Probability. Additionally, Geometry and Calculus each require an Algebra background, while Statistics and Discrete Math each require Probability as a prerequisite. If Arthur must learn all seven subjects (Pre-Algebra, Algebra, Probability, Geometry, Calculus, Statistics, and Discrete Math), in how many orders can he learn the subjects?

A3. ______________

A4. [**4**] A convex n-gon $P_1P_2\ldots P_n$ is given. The sum of the interior angles at P_1, P_2, $\ldots$, P_{n-1} and the supplement of the interior angle at P_n is $2000°$. Compute the largest possible value of n.

A4. ______________

B1. [**2**] Jared is shooting arrows at a target. So far, he has hit the target 13 times out of a total of 37 shots. How many more consecutive hits must he make in order to have a 75% success rate?

B1. ______________

B2. [**3**] Four cubes, each with 5 green sides and 1 red side, are arranged together on the table to form a larger square prism. What is the probability that no red sides are visible from the sides or top?

B2. ______________

B3. [**3**] Let m be the result when $\sqrt{41}$ is rounded to the nearest tenth. Find $10m$.

B3. ______________

B4. [**4**] You walk into a random store and buy a random item priced uniformly at random between \$0.01 and \$99.99. (The cost is an integer number of cents.) Since you do not know how much the item costs, you hand the cashier a \$100 bill. The cashier tries to give you change in as few coins as possible. If he is out of quarters and does not have half-dollars, what is the expected number of coins that you will receive? (Remember that for change that exceeds \$0.99, the cashier will pay you the whole dollars with paper bills.)

B4. ______________

C1. [**2**] In a weird variation of Rock-Paper-Scissors (members' edition), each of two players can choose from 101 different signs to play on his or her turn, such that each symbol trumps 50 symbols and loses to 50 other symbols. What is the probability that in a single round of Rock-Paper-Scissors (members' edition), the two players tie?

C1. ________

C2. [**3**] The positive difference between the largest two angles of triangle ABC is equal to the positive difference between the smallest two angles. Suppose that $BC < CA < AB = 10$, and let D be the foot of the altitude from A to $\overline{BC}$. Find BD.

C2. ________

C3. [**3**] Compute the number of ways to tile a 3×12 grid with L-trominoes (**not** tetrominos). (An L-tromino is a shape composed of three squares in an L-shape, as depicted below.)

C3. ________

C4. [**4**] Square $ABCD$ and $AEFG$ have the same side length and are oriented in the same direction. (In other words, $ABCD$ and $AEFG$ are either both clockwise or counterclockwise.) If the length of $ED = 20\sqrt{13}$ and $BG = 20\sqrt{14}$. Compute the side length of each square.

C4. ________

D1. [**2**] A circular piece of cardboard weighs 20 mg and has circumference 1 meter. Mr. Recycle Bin comes and eats part of the paper. The remaining piece of paper weighs 5mg and is also circular. How many centimeters are in its circumference now?

D1. ________

D2. [**3**] What is the smallest positive integer that can be written as the sum of two positive squares in two distinct ways?

D2. ________

D3. [**3**] You start on the center square of a 3×3 grid. Every minute, you move to an adjacent square (up, down, left, or right). What is the probability that you are on the center square after 7 minutes?

D3. ________

D4. [**4**] Gary lives in a triangular house BRN with $BR = BN = 3\sqrt{10}$ meters and $RN = 6$ meters, with one exit at each corner of the house. One day, while burning CDs, his computer overheats and starts a fire in his house. Immediately, he runs to the closest exit. What is the probability that he exits through corner B?

D4. ________

E1. [**2**] Let ABC be a triangle with area 800 and let P be a point inside it. Let D, E, F be the midpoints of $\overline{AP}$, $\overline{BP}$, $\overline{CP}$. What is the area of DEF?

E1. ______________

E2. [**3**] Each of the four basic operators $+, -, \times$, and $\div$ is filled into one of the blanks in the expression $1 \_\_ 2 \_\_ 3 \_\_ 4 \_\_ 5$. What is the largest value attainable from evaluating the expression?

E2. ______________

E3. [**3**] Given a regular hexagon, what is the ratio of the area of the largest equilateral triangle that can fit inside the hexagon to the area of the smallest equilateral triangle that the hexagon can fit into?

E3. ______________

E4. [**4**] Mrs. Phair is generating a sequence of numbers $a_1, a_2, a_3, \cdots$ on her whiteboard. In the first minute, she writes $a_1 = 1$. During the n^{th} minute for $n \geq 2$, Mr. Unfair randomly orders the numbers $\{1, 2, 3, \cdots, n-1\}$ and lets them equal to $b_1, b_2, \cdots, b_{n-1}$ respectively. Mrs. Phair then lets $a_n = a_1b_1 + a_2b_2 + a_3b_3 + \cdots + a_{n-1}b_{n-1}$. Let k be the smallest positive integer such that the maximal and minimal possible values for a_k differ by more than 2013. Compute this difference.

E4. ______________

F1. [**2**] What is the sum of the distinct positive factors of 72?

F1. ______________

F2. [**3**] A cone has a height of 12 units and a circular base of radius 4. A cut is made through the middle of the cone's height, parallel to its base, forming a mini cone and another solid. What is the volume of this other solid?

F2. ______________

F3. [**3**] One card is missing from a pack of 52 standard playing cards. (Standard playing cards have four suits – spades, hearts, diamonds, clubs – each with one of 13 possible face values.) From the remaining cards in the pack, one card is drawn and is found to be spades. Find the probability the missing card is also a spade.

F3. ______________

F4. [**4**] Let $\Pi(n)$ denote the product of the digits of a positive integer n. If n only has one digit, then $\Pi(n) = n$. How many prime numbers $p \leq 100$ have the property that $\Pi(p)$ is also a prime number?

F4. ______________

G1. [**2**] Compute $1 - 3 + 5 - 7 + \cdots + 101$.

G1. ______________

G2. [**3**] A five-digit positive integer $n = 123AB$ with A and B as digits is chosen. For what value of A is n never divisible by 11?

G2. ______________

G3. [**3**] How many positive integers n less than 9000 can be written in the form 2014_b, for some base $b \geq 5$? (The subscript denotes a number base. When we write a numeral, such as 421, the value of it is $4 \cdot 10^2 + 2 \cdot 10^1 + 1 \cdot 10^0$ because we use base 10.)

G3. ______________

G4. [**4**] We say $(A, B; X, Y)$ is a *harmonic bundle* if A, B, X, Y are four points satisfying the following conditions: (i) the points are distinct and collinear, (ii) $\frac{XA}{XB} = \frac{YA}{YB}$ and (iii) the segments $\overline{AB}$ and $\overline{XY}$ overlap. For example, $A = (-5, 0)$, $B = (5, 0)$, $X = (1, 0)$ and $Y = (25, 0)$ form a harmonic bundle.

Suppose X is selected on segment $\overline{AB}$ such that it is impossible to find Y for which $(A, B; X, Y)$ is a harmonic bundle. If $AB = 100$, what is AX?

G4. ______________

H1. [**2**] Choose a point O on a plane. Let $\mathcal{A}$ be the set of all points of distance at most 2 units from O. Let $\mathcal{B}$ be the set of all points of distance at most 3 from some point in $\mathcal{A}$. Let $\mathcal{C}$ be the set of all points of distance at most 5 from some point in $\mathcal{B}$. To the nearest integer, what is the area of $\mathcal{C}$?

H1. ______________

H2. [**3**] Let M be the midpoint of side BC of triangle ABC with $AB = AC = 13$ and $BC = 10$. A semicircle centered at point M is tangent to sides AB and AC. What is the radius of the semicircle?

H2. ______________

H3. [**3**] The answer to this question is a two-digit number which can be written in the form $10a + b$, where a and b are positive integers such that $0 < a, b < 10$. Find $2ab$.

H3. ______________

H4. [**4**] A man is choosing among three different entrees for his dinner. He first considers plate A, eating it with a 1/3 chance and moving on to plate B with 2/3 chance. In general, from whatever plate he is considering consuming, he has a 1/3 chance of eating it and 2/3 of considering the next plate alphabetically (C after B after A after C). What is the probability that he ends up eating plate A?

H4. ______________

I1. [**2**] A random day in January 2014 is chosen. What is the probability that it is a Saturday or Sunday? (Today, January 26, 2014, is a Sunday.)

I1. ____________

I2. [**3**] In triangle ABC, side $AB = 5$, $AC = 13$, and $\angle B = 96^\circ$. How many points P on segment BC (distinct from points B and C) are there such that the length of AP is an integer?

I2. ____________

I3. [**3**] Two congruent equilateral triangles overlap as shown below. If the side length of each triangle is 12 and the height h marked is $7\sqrt{3}$, compute the area of the region of overlap.

I3. ____________

I4. [**4**] How many integers $m < 100$ of the form pq for distinct prime numbers p, q cannot be written in the form $\binom{n}{r}$, for positive integers $n \geq r \geq 0$?

I4. ____________

J1. [**2**] John and his brother are running around a 400 meter track. John runs at 8 m/s while his brother runs at 3 m/s. If they start running from the same spot at the same time in the same direction, how many seconds will have passed when John laps his brother the second time?

J1. ______________

J2. [**3**] What is the slope of the line connecting the two points of intersection of $20x + 14y = 24$ and $x^2 + y^2 = 42$?

J2. ______________

J3. [**3**] If $\dfrac{x-2}{3}$, $\dfrac{x-4}{5}$, $\dfrac{x-6}{7}$ are all positive integers, what is the smallest possible value of x?

J3. ______________

J4. [**4**] What is the area of a triangle with the coordinates $(0, 0, 0)$, $(3, 4, 4)$, and $(-6, -8, 12)$?

J4. ______________

K1. [**2**] On a 20-question True-False quiz, Brenda guesses on all of the questions and gets 16 of the questions correct. If she flips the answers to each of the first 10 questions, she will have 12 questions correct. How many questions will she answer right if she changes the answer to each of the last 10 problems as well?

K1. ______________

K2. [**3**] Forty 2×3 rectangles are arranged together into a figure with no gaps inside. What is the minimum possible perimeter of this figure?

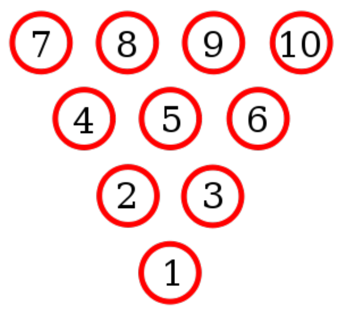
K2. ______________

K3. [**3**] The median and mean of the five numbers in $\{5, 9, x, 14, 6\}$ are both equal. What is the sum of all possible values of x?

K3. ______________

K4. [**4**] Bob is bowling, only knocking down the first pin (marked as 1 in the diagram below). When a pin is toppled, each of the one or two pins has a $\frac{1}{2}$ chance of being knocked over as well (independent of the other pin's results). What is the probability that Bob bowls a strike (knocks all 10 pins down)?

K4. ______________



L1. [**2**] Farmer Don wants to build a closed fence around his (rectangular) house farm to keep neighbors out and buys a bunch of posts. If he puts them each 1 foot apart, he will still need 150 more posts to fully surround his plot of land, but if he puts them 1 yard apart, he will have 90 poles too many. How many fenceposts does he have?

L1. ______________

L2. [**3**] What is the sum of the decimal digits of $11 \times 101 \times 10,001 \times 100,000,001$?

L2. ______________

L3. [**3**] For how many values of n does a regular n-gon have integer interior angle measures?

L3. ______________

L4. [**4**] Find the sum of the decimal digits of $999{,}999{,}999^2 - 899{,}999{,}999^2$.

L4. ______________

M1. [**2**] The sequence $a_0, a_1, \ldots, a_9$ satisfies $a_0 = 0$ and $a_n = n + a_{n-1}$ for each integer n. Find a_9.

M1. ______________

M2. [**3**] A total of 73 students registered online for MagMaR 2014. Henry, Evan, and Aaron handled the registrations, and the average number of students they each checked in was 20. Only one student checked in who had not registered online. How many students who registered online did not show up?

M2. ______________


figure 1

M3. [**3**] In the diagram below, three circles with radii of 4, 8, and 12 have the same center. Compute the area of the shaded region.

M3. ______________

M4. [**4**] Compute the area of the region bounded by the graph of

$$y = ||x + 20| + |x + 13|| - 14$$

and the x-axis.

M4. ______________

N1. [**2**] Ernesto is gluing 27 unit cubes together to form a $3 \times 3 \times 3$ cube. What is the fewest pairs of faces he must glue together in order for the large cube to not fall apart?

N1. ______________

N2. [**3**] Without using calculus, one can intuitively prove that the area of a circle is πr^2 by splitting a circle into many equally sized sectors and arranging them into a parallelogram, as shown in the left diagram in the figure below. Since the two bases have a total length equal to the circumference of the circle, $2\pi r$, and the height is the radius of the circle, then the area of the parallelogram and the area of the circle is $r \cdot (2\pi r)/2$.

A circle of radius 2 is split into 6 sectors and arranged similarly into a parallelogram. What is the area of the analogous parallelogram, as marked in red in the right diagram in the figure below?

N2. ______________

N3. [**3**] A line segment has endpoints at $(3, 7)$ and $(38, 72)$. Through the interiors of how many unit squares does this line segment cut?

N3. ______________

N4. [**4**] Necklaces can have beads of two colors: red and gold. Call a necklace *Au-rich* if the majority of its beads are gold. How many distinct necklaces with 11 beads are *Au-rich*? (Two necklaces are the same if one can be reached by a rotation of the other. However, flipping a necklace counts as a different configuration.)

N4. ______________

O1. [**2**] A spinner is split into eight congruent sections, labelled 1 through 8 clockwise, as shown below. Sean flicks the spinner. Using the number that he lands on, he moves the arrow that many spaces counter-clockwise. At the end of this process, what is the probability that the arrow is pointing to the number '8'?

O1. ____________

O2. [**3**] How many positive integers less than 100 have exactly 4 factors?

O2. ____________

O3. [**3**] Aurick chooses a point P in space such that a 90° rotation about point P sends $(0, 0, 0)$ to $(20, 14, 2)$. What is the minimum possible distance from the origin to point P?

O3. ____________

O4. [**4**] Your mean friend stole your items and is charging you money in return for the items. If you must pay $14 to get your laptop, 1 folder, and 1 pencil back and $20 to receive your laptop, 3 folders, and 5 pencils, how much must you pay your friend to get your laptop, all 4 folders, and all 7 pencils back?

O4. ____________