MSJ Math Club

AMC 10/12 Practice

29 January 2015

1 Introduction

You are probably familiar with the fact that the AMC 10/12 are 25 question, 75 minute contests. This year, the A-contest will be held 2/3/2015 and the B-contest 2/25/2015. Qualification for AIME through AMC 10 is 120 points or top 2.5%, whichever is more inclusive. Qualification for AIME through AMC 12 is 100 points or top 5%, whichever is more inclusive. This handout will simply consist of a hodgepodge of AMC problems to help you prepare for these upcoming contests.

2 Tips and Tricks

- The only way to become a lot better at math is to simply do tons of problems, especially previous years' problems of similar contest difficulty. For the AMCs, this would be previous AMCs, SMTs, BMTs, and AIMEs.
- Part of getting good at math is learning to make connections between different types of problems. In particular, changing a problem you have never seen before into a more familiar problem is an extremely useful technique.

3 Examples

1. (2011 AMC 10A) Two counterfeit coins of equal weigh are mixed with 8 identical genuine coins. The weight of each of the counterfeit coins is different from the weight of each of the genuine coins. A pair of coins is selected at random without replacement from the 10 coins. A second pair is selected at random without replacement from the remaining 8 coins. The combined weight of the first pair is equal to the combined weight of the second pair. What is the probability that all 4 selected coins are genuine?
2. (2011 AMC 10A) Let R be a square region and $n \geq 4$ an integer. A point X in the interior of R is called n-ray partitional if there are n rays emanating from X that divide R into n triangles of equal area. How many points are 100-ray partitional but not 60-ray partitional?
3. (2014 AMC 10A) The number 5^{867} is between 2^{2013} and 2^{2014}. How many pairs of integers (m, n) are there such that $1 \leq m \leq 2012$ and
$$5^n < 2^m < 2^{m+2} < 5^{n+1}?$$
4. (2014 AMC 12A) Let $f_0(x) = x + |x - 100| - |x + 100|$, and for $n \geq 1$, let $f_n(x) = |f_{n-1}(x)| - 1$. For how many values of x is $f_{100}(x) = 0$?
5. (2014 AMC 12A) In $\triangle BAC$, $\angle BAC = 40°$, $AB = 10$, and $AC = 6$. Points D and E lie on $\overline{AB}$ and $\overline{AC}$, respectively. What is the minimum possible value of $BE + DE + CD$?
6. (Corollary) Given $\triangle ABC$, find the point F inside the triangle such that the sum $AF + BF + CF$ is minimized. This point is known as the Fermat Point of $\triangle ABC$.

4 Practice Problems

1. (2011 AMC 10B) A pyramid has a square base with sides of length 1 and has lateral faces that are equilateral triangles. A cube is placed within the pyramid so that one face is on the base of the pyramid and its opposite face has all its edges on the lateral faces of the pyramid. What is the volume of this cube?

2. (2009 AMC 10A) Three distinct vertices of a cube are chosen at random. What is the probability that the place determined by these three vertices contains points inside the cube?

3. (2010 AMC 10B) Let $a > 0$, and let $P(x)$ be a polynomial with integer coefficients such that

$$P(1) = P(3) = P(5) = P(7) = a$$

$$P(2) = P(4) = P(6) = P(8) = -a.$$

   What is the smallest possible value of a?

4. (2010 AMC 10B) A high school basketball game between the Raiders and Wildcats was tied at the end of the first quarter. The number of points scored by the Raiders in each of the four quarters formed an increasing geometric sequence, and the number of points scored by the Wildcats in each of the four quarters formed an increasing arithmetic sequence. At the end of the fourth quarter, the Raiders had won by one point. Neither team scored more than 100 points. What was the total number of points scored by the two teams in the first half?

5. (2011 AMC 10B) What is the hundreds digit of 2011^{2011}?

6. (2011 AMC 12A) Consider all quadrilaterals $ABCD$ such that $AB = 14$, $BC = 9$, $CD = 7$, and $DA = 12$. What is the radius of the largest possible circle that fits inside or on the boundary of such a quadrilateral?

7. (2014 AMC 12A) The fraction

$$\frac{1}{99^2} = 0.\overline{b_{n-1}b_{n-2}\cdots b_2b_1b_0},$$

   where n is the length of the period of the repeating decimal expansion. What is the sum $b_0 + b_1 + \cdots + b_{n-1}$?

8. (2010 AMC 12B) The set of real numbers x for which

$$\frac{1}{x-2009} + \frac{1}{x-2010} + \frac{1}{x-2011} \geq 1$$

   is the union of intervals of the form $a < x \leq b$. What is the sum of the lengths of these intervals?