MSJ Math Club

Week 1: Areas

September 20, 2012

Note: In this handout, $[P]$ will denote the area of a figure P, and not the empirical formula of a compound.

1 Examples

- Aaron, Jeffrey, and Minmin are standing at the points $(0,0),(1,0)$, and $(0,1)$ respectively. Because your math club officers have really weird walking habits, when one of them moves, he must move on the line parallel to the line formed by where the other two officers stand, and through his starting point. Is it ever possible for the three people to stand on the points $(0,0),(1,0)$, and $(0,2)$ (not necessarily in order)?
- In triangle ABC, point D is on side BC such that AD is the angle bisector of $\angle BAC$. If $AB = 9$, $AD = 12$, and $AC = 15$, compute $\cos\frac{\angle BAC}{2}$.

2 Practice Problems

1. Square A has area of 4. Square B has one of its vertices as the center of square A and has area of 9. What is the area of the region inside at least one of the squares?
2. *(CAML 2011-12)* A regular dodecagon has a side length of 2. What is the difference in areas of the circumscribed circle (the circle drawn around it) and the inscribed circle (the circle drawn in it)?
3. *(Math Olympiad Treasures)* A square $ABCD$ has points M and N on sides AB and BC respectively. Let $DM \cap AN = P, MC \cap AN = Q$, and $DN \cap MC = R$. Show that $[PQRD] = [APM] + [MQNB] + [RNC]$. (The $\cap$ symbol means "intersect".)
4. Find locus of points P inside ΔABC such that $[ABP] = [ACP]$. What if P can be outside of ΔABC?
5. *(MSJHSSMTTST 2011-12)* Rectangles $ABCD$ and $ACEF$ are constructed so that points E, B, and F are in a line and the area of $ADCEF$ is 105. If $BD = 10$, then what is CE?
6. In quadrilateral $ABCD$, points K and M are the midpoints of sides AB and CD respectively. Let L and N be points on sides BC and AD respectively, such that $BL/LC = DN/NA$. Show that $[KLMN] = \frac{1}{2}[ABCD]$.
7. In triangle ABC, let I be the incenter of the triangle. Show that $AI \cdot BC + BI \cdot CA + CI \cdot AB \geq 4[ABC]$.