**Directions:** This is Day 1 of the Individual Round portion of the 2015 Mission Math Tournament, and is used as a factor in determining our MSJ teams in future on-site competitions like the Stanford and Berkeley Math Tournaments. There are 12 short-answer problems to be solved in 35 minutes. *Only answers written on your answer sheet will be scored.* Good luck!

**Problem 1:** What is the greatest 9-digit number that is divisible by 9, all of whose digits are distinct?

**Problem 2:** What is the probability that the sum of 2 independent dice rolls is greater than 7?

**Problem 3:** TC can buy packs of playing cards in the following deals:

- 2 packs for \$10
- 3 packs for \$14
- 7 packs for \$29

What is the least amount of money TC needs to buy at least 22 packs of cards?

**Problem 4:** In right triangle $\triangle ABC$ with $\angle C = 90°$, we have $D, E, F$ on $AB$ such that, in order, we have $A, D, E, F, B$ and $AD : DE : EF : FB = 1 : 2 : 3 : 4$. Given that $BC = 5$ and $AC = 12$, find [DEC] + [FBC], where brackets indicate area.

**Problem 5:** Points $A$, $B$, and $C$ lie on the parabola $y = x^2$. If the slope of $AB$ is 4, the slope of $BC$ is 8, and the slope of $AC$ is 16, what are the possible **x-coordinates** of $A$?

**Problem 6:** A rectangle $ABCD$ has $AB = 2$ and $BC = 3$. A circle with radius 1 is inscribed inside, externally tangent to $DA$, $AB$, and $BC$. Another circle $\omega$ is also inscribed inside, externally tangent to the first circle, $BC$, and $CD$. Find the radius $r$ of $\omega$.

**Problem 7:** A $5 \times 5$ grid of squares is randomly filled in with 0's and 1's. A square is *happy* if all of its edge-neighbors have the same value as itself. What is the expected number of *happy* squares in the $5 \times 5$ grid?

**Problem 8:** Triangle $\triangle ABC$ has $D, E, F$ the midpoints of $BC$, $CA$, and $AB$, respectively. Also, let G be the intersection of $AD$ and $FE$, let H be the intersection of $BE$ and $DF$, and let I be the intersection of $CF$ and $DE$. Given that [ABC] = 112, find [GHI], where brackets indicate area.

**Problem 9:** A sequence is given by $a_1 = 5$ and $a_n = 3a_{n-1} + 8$ for $n > 1$. Find a closed form for $a_n$ (one without summations or references to previous terms in the sequence).

**Problem 10:** Find all real solutions $x$ that satisfy the equation $x^4 - 8 = 2x(2x^2 - 3x + 2)$.

**Problem 11:** A rectangular piece of paper has length 2 and width 1. A dotted line is drawn from two opposite vertices. The paper is then folded flat along the dotted line to create a new shape. What is the area of this new shape?

**Problem 12:** A permutation $\sigma$ is a function that maps a finite set to itself. How many permutations $\sigma$ acting on the set $\{1, 2, 3, 4, 5, 6, 7\}$ are there such that $\sigma(\sigma(\{1, 2, 3, 4, 5, 6, 7\})) = \{1, 2, 3, 4, 5, 6, 7\}$? In other words, how many self-inverse permutations that act on a set of 7 distinct elements are there? For example, if our permutation were $\delta = (3, 2, 1)$, then $\delta(\delta(\{1, 2, 3\})) = \delta(\{3, 2, 1\}) = \{1, 2, 3\}$. Therefore $\delta = (3, 2, 1)$ is a self-inverse permutation that acts on a set of 3 distinct elements.