Directions: For each question, teams submit a closed range $[n, m]$, representing a lower bound $n \neq 0$ and an upper bound m for the answer. A correct answer means that the actual answer is within the range given. Your score is determined by the following formula, (where n_i is the lower bound given for correctly-answered question i, m_i is the upper bound given for question i, x is the number of incorrect answers, k is the number of correctly-answered questions, and $[x]$ is the greatest integer function):

$$\textbf{Score} = 2^x \left(10 + \sum_{i=1}^{k} \left[\frac{m_i}{n_i}\right]\right)$$

Problem 1: Let N be the number of people on MSJ Math Club's email list. Estimate N^2.

Problem 2: Given that $\tan(60°) = 1.732...$ and that $\tan(75°) = 3.732...$, estimate $\tan(89°)$.

Problem 3: Coined by Edward Casner's 9-year old nephew, a googol is defined as 10^{100}, and a googol plex is defined as 10^{googol}. Estimate $\dfrac{\ln(\text{googol plex})}{e^{e^{\pi}}}$.

Problem 4: A *smoot* is a humorous unit of measurement named after Oliver R. Smoot, who repeatedly laid down on Harvard Bridge so that his MIT fraternity brothers could use his height to measure the length of the bridge. One *smoot* is equal to Oliver Smoot's height at the time of the prank. Let N be the number *smoots* long the Great Wall of China is. Estimate $\ln(N)^2$.

Problem 5: TC and Brian are playing a game of seven-card stud, in which each player is randomly dealt seven cards from a standard 52 card deck, as the name suggests. Estimate the probability (as a decimal) that TC can form a full house (3 of one rank, 2 of some other rank) using some 5-card combination of his seven cards?

Problem 6: TC, Brian, Tomas, and Austin are now playing a game of bridge, which is played with a standard 52 card deck (13 cards per player). Estimate the probability (as a decimal) that Brian has a void. (i.e. no cards in one or more of the four suits)

Problem 7: Perfect numbers are numbers whose sum of their proper divisors is equal to the number itself. It is known that all even perfect numbers are of the form $2^{p-1} \times (2^p - 1)$, where $2^p - 1$ is a Mersenne prime. The first few perfect numbers are 6, 28, 496, and the 10th known perfect number has 54 digits. Estimate the number of digits in the 30th known perfect number.

Problem 8: Is it possible for numbers to be not just perfect but amicable? Two numbers are amicable if the sum of the proper factors of each is equal to the other number. For example, the smallest pair of amicable numbers is (220, 284); for the proper divisors of 220 are 1, 2, 4, 5, 10, 11, 20, 22, 44, 55 and 110, of which the sum is 284; and the proper divisors of 284 are 1, 2, 4, 71 and 142, of which the sum is 220. The 5th pair of amicable numbers is (6232, 6368) and the 10th pair of amicable numbers is (66928, 66992). Estimate the product of the two numbers in the 20th pair of amicable numbers.

Problem 9: The Catalan numbers are defined recurrence $C_n = C_0C_{n-1} + C_1C_{n-2} + \cdots + C_{n-1}C_0 = \sum_{k=0}^{n-1} C_kC_{n-1-k}$, where $C_0 = 1$, and have many useful applications in combinatorics. In particular, the first few Catalan numbers are 1, 2, 5, 14. Estimate the 15th Catalan number.

Problem 10: Let N be the area of a regular heptagon with side length 1. Estimate 10^N.

Problem 11: In $\triangle ABC$, $AB = 70$, $BC = 119$, and $AC = 184$. Estimate the radius of the A-excircle (the circle tangent to the line AB, the line AC, and the segment BC on the opposite side of A).

Problem 12: Congratulations, you've won 15.5 trillion Zimbabwe Dollars! Estimate the number of times more you would have to win the same jackpot in order to afford a Tesla Model S. Use conversion rates from Zimbabwe Dollars to USD as of June 15, 2015 and the current price for a Tesla Model S for estimations. For this problem only, if you guess a correct range, your ratio will be $\log([\frac{\text{upper}}{\text{lower}}])$ instead of just $[\frac{\text{upper}}{\text{lower}}]$.