

Directions: You have 35 minutes to complete these 10 problems. All answers must be written in accordance with the conventions on the Conventions page on the MSJHSSBMTTPSTMT website. You may work with your team. Write all of your answers on the answer sheet. One answer sheet per team. You may only use scratch paper provided by the MSJHSSBMTTPSTMT. No calculators allowed.

1. If $-x = 3$, what is the value of x ?
2. Eliezer Yudkowsky, of LessWrong fame, is a staunch supporter of cryonics. He decides to preserve his own body to be able to witness the singularity sometime in the future. In order to perform the preservation of his body, a doctor has to deal with myriad cellular configurations, one of which is modeled by a virtually infinite 2D triangle of cells arranged like Pascal's triangle. It is also similar to Pascal's triangle in this respect: if the number which would have appeared in Pascal's triangle at a particular cell's position is even, the cell is frozen. Otherwise, the cell is warm. What is the probability that a randomly-selected cell from the 18th row of this cellular triangle is warm?

Note: Pascal's triangle can be generated by writing 1 at the top, and adding the two top adjacent numbers to get the next number at the bottom. Thus, the second row is 1 1, the third row is 1 2 1, and the fourth row is 1 3 3 1.

3. While walking home from school, you see a mangled sheet of paper. On it contains a problem from the 2102 MMT. However, due to the sheer stress of time travel, all the numbers fell out and you cannot reassemble exactly where the givens are. The givens are 1, 2, 3, 4, and 5. After thinking about it for a while, you decide to spend the rest of the day computing the answer to all possible arrangements of givens. Determine the arithmetic mean of all the answers that you get.
4. Once upon a time, there was a pony. This pony emitted brain waves, just like all other ponies. One day, one such brain wave found its way, alone, to the corridor of the Great Equine Despot, who for some reason got a kick out of slicing poor brain waves that happened to come into the corridor of the Great Equine Despot. The brain wave can be modeled by $f(x) = 1000 \sin(2\pi x) - \cos(2\pi x)$ and the Great Equine Despot's spear can be modeled by

$$g(x) = \begin{cases} 1000x^{10}, & \text{if } 0 \leq x \leq 10 \\ -x + 2000, & \text{if } x > 10. \end{cases}$$

The shine factor of the spearing of the brain wave is given by $10n + 3$, where n is the number of intersection points of the brain wave and the Great Equine Despot's spear. Find the shine factor.

5. On December 16, 1997, Porygon, in all its rapidly-flashing, iridescent glory, was transmitted to the television sets of millions of Japanese children. Quite a few of them were inflicted with seizures, and so they had to be rushed to the hospital. The paramedic truck that transported some of these children has wheels of radius 5 feet. The chassis of the car is a rectangle $ABCD$ where \overline{AB} is over \overline{CD} . There is a point X on \overline{CD} such that protective flap \overline{AX} , behind which lie the children, exists and $AD = 20$ feet. A huge barrel from the truck's former use, whose cross-section is a circle with center O , rests right up against the flap so that it is tangent to the flap and the floor of the truck. It is also tangent to the ceiling. $AO = 30$. Given all possibilities for the height of the truck, which gives the lowest margin between the ceiling of the truck and an overhead bridge that lies 60 feet above the road?
6. A man is screaming as he attempts to run away from a velociraptor. He was the tower guard of the Panopticon, so in a fit of irrationality, he decides that he has to keep close sight of the tower in case it spontaneously burns down. Thus he runs such that in the n th minute, he runs n units in the direction 45° clockwise from how he last ran. How far is he from the Panopticon tower after 2012 minutes?
7. Farmer John has a plot of land that lies between two perfectly straight and parallel rivers. He sends two of his cows to mark off plots in the following manner. The cows start at separate rivers, and mark their starting points with a cowshoe. They begin walking in the same direction, each with a

constant pace for two minutes, each dropping another cowshoe at the one minute and two minute marks. Finally, Farmer John connects up the zero minute cowshoes, the one minute cowshoes, and two minute cowshoes together. However, it turns out that the zero minute cowshoes are 40 meters apart, the one minute cowshoes are 30 meters apart, and the two minute cowshoes are 50 meters apart, as the cows did not travel at the same pace. Compute the total area enclosed within the rivers and the zero and two minute cowshoes.

8. In the game of Volcano Falldown, your goal is to navigate down an (infinite) mountainside with the volcanic eruption of Eyjafjallajökull right behind your tracks. However, there are a lot of trees blocking your way. Suppose that every 50 meters, there is a 350-meter wide band of trees. A lumberjack decided to capitalize on this clearing of trees before you and fortunately randomly cleared some of the blocking trees. However, due to strict Icelandic zoning laws, exactly one swath of land 50 meters long is cut out each clearing. Furthermore, the slope is so steep that if you attempt to walk downward, you necessarily must keep going downward until you hit some trees. You walk at a pace of 5 meters a second, and the lava flow runs at a pace of 3 meters a second. If you start off with a 300 meters head start with respect to the lava flow, and the next tree clearing is 50 meters away, what is the expected vertical distance that you will be able to cover? The thicket of trees is so dense that you can only see the band of trees right in front of you.
9. Speeding on the Autobahn is a beloved pastime of frequent tourists to Germany who are avid motorists. One day, Susan decides to up the ante and not only speed on the Autobahn (legally I may add), but solve a math problem while she's at it. She feels that a difficult problem, one you could find on #9 of a team test, would be appropriate for her level of experience. She would like to find all solutions to $(x^2 - 2x + 5)(5x^2 + 4x + 1) = ((x^2 - x) + (4x + 2))^2$. Without the burden of speeding on the Autobahn, solve this problem.
10. Since I'm too lazy to write a theme for this problem, I'll just give this vanilla problem to you. Triangle ABC inscribed in circumcircle Ω . A circle ω is tangent to sides \overline{AB} and \overline{AC} , and internally tangent to arc BC of Ω at points M, N , and P respectively. If $PM = 5$, $PN = 8$, and $\angle BAC = 60^\circ$, find BC .