Wild Round

Name: _____

School: _____

Team ID: _____

Grade: _____

Date: February 28, 2016

Problems: 40

Time: 20 minutes

Maximum Score: $1 \times 40 = 40$

Type: Individual

Score: _____

Do not start until instructed to do so!

Calculators, slide rules, books, computers, other electronic devices, are all prohibited. Similarly, graph paper, protractors, rulers, and compasses are not allowed at the competition. You may not collaborate with any other contestants during this round.

Please record your answers only in the blanks below; the ones provided on the test are only for convenience. Only answers recorded on this cover page will be graded.

1.	2.	3.	4.	5.
6.	7.	8.	9.	10.
11.	12.	13.	14.	15.
16.	17.	18.	19.	20.
21.	22.	23.	24.	25.
26.	27.	28.	29.	30.
31.	32.	33.	34.	35.
36.	37.	38.	39.	40.

1. What is the units digit of $2016^{2016^{2016}}$?

   1. _____

2. Let a be the answer to this question. What is $5a - 8$?

   2. _____

3. A triangle has side lengths 5, 12, and 13. What is the area of the triangle?

   3. _____

4. Brian is adding the numbers from 1 to 10, inclusive. However, he accidentally left out a number, so that his final sum was 51. What number did Brian leave out?

   4. _____

5. A box has side lengths 1, 2, and 2. What is the length of the longest rigid rod that can be fit inside this box?

   5. _____

6. A rectangle with side lengths 9 and 16 has the same area as a rectangle of with side lengths 12 and r. Find r.

   6. _____

7. If 3 *prtk*s are worth 5 *ghrm*s, and 7 *ghrm*s are worth 6 *fpy*s, how many *prtk*s are worth the same as 12 *fpy*s?

   7. _____

8. Tiancheng decides to write a sequence, starting with 77, such that each successive term is the product of the digits of the previous term. What is the first one-digit number Tiancheng writes?

   8. _____

9. A tree has one trunk, which has 11 branches, each of which has 11 twigs. What is the total number of twigs, branches, and trunks this tree has?

   9. _____

10. Jim has a $\frac{1}{2}$ chance of getting into Berkeley and a $\frac{1}{5}$ chance of getting into MIT. What is the probability that Jim gets into at least one of these two colleges?

    10. _____

11. What is the tens digit of 101^{2016}?

    11. _____

12. Ash is trying to catch them all! Given that there exist 720 pokemon, and she can catch one every 5 minutes, how many hours will it take her to catch them all?

    12. _____

13. The mean, median, and unique mode of $\{14, 15, 12, 19, x\}$ are all the same number. What is this number?

    13. _____

14. A one-inch long rod is cut into 7 sections that have lengths that are in the ratio $1 : 2 : 3 : 4 : 5 : 6 : 7$. How long, in inches, is the largest section?

    14. _____

15. An isosceles trapezoid has bases with lengths 10 and 16, and the distance between these two bases is 4. Find the perimeter of the trapezoid.

15. _____

16. Alex has 3 pairs of socks, 4 sweaters, and 2 pairs of jeans, all of different sizes. How many outfits can Alex make?

16. _____

17. Let $f(x)$ be a function such that $f(x) \geq x$ and $f(f(x)) = x$ for all x. Find $f(20) - f(16)$.

17. _____

18. Anne, Bill, Carl, Dylan, Eshaan, Fred, and Greg are lining up in a completely random order. What is the probability that Anne is ahead of Bill, who is ahead of Carl?

18. _____

19. In a competition, each judge randomly gives either a pass or a fail mark for each competitor. Given that there are 3 competitors and 3 judges, what is the probability that the judges agree for each of the 3 competitors?

19. _____

20. If a and b are the roots of $x^2 + 11x + 5$, what is the value of $\frac{1}{a} + \frac{1}{b}$?

20. _____

21. What is the sum of the distinct prime factors of 8099?

21. _____

22. Triangle ABC has side lengths 6, 8, and 10, and is inscribed inside a circle. What is the area of that circle?

22. _____

23. Oddish is capable of learning exactly 8 moves: Absorb, Sweet Scent, Poisonpowder, Stun Spore, Sleep Powder, Acid, Moonlight, and Petal Dance. How many possible combinations of 4 moves can Oddish learn?

23. _____

24. In quadrilateral $ABCD$, $AB = 15$, $BC = 20$, $CD = 24$, $DA = 7$, and $AC = 25$. Find the area of quadrilateral $ABCD$.

24. _____

25. There are several bicycles and tricycles in a room. If there are 11 vehicles and 26 wheels, how many bicycles are there?

25. _____

26. The square of the sum of two real numbers is twice their product. What is the difference of these two numbers?

26. _____

27. Tiancheng spins two spinners. The first spinner has an equal chance of landing on 1, 3, or 6. The second spinner has an equal chance of landing on 2, 4, or 5. What is the probability that the sum of the two spins is even?

27. _____

28. A coin is flipped 7 times. What is the probability of getting more heads than tails?

28. _____

29. A rectangle with integers as side lengths has an area of 999. What is the smallest possible perimeter of the rectangle?

29. _____

30. A circle and an equilateral triangle have the same center. Given that both shapes have the same area, find $A - B$, where A is the area inside the circle but outside the triangle and B is the area inside the triangle but outside the circle.

30. _____

31. If a Pokemon is frozen, it has a 20% chance of thawing out during each turn. What is the probability that it thaws by the end of the second turn?

31. _____

32. Tiancheng decides to write a sequence starting with 12, such that each successive term is the sum of the previous term's proper factors (all positive factors except itself). How many distinct terms are there in the sequence?

32. _____

33. A unit cube has its "corners" (the triangular pyramids formed by each vertex and the plane passing through the three adjacent midpoints of edges) cut off. What is the volume of the resulting solid?

33. _____

34. A random two-digit number is constructed by placing each of the digits from 4 to 9 (inclusive) in a bag and picking two of them, without replacement, so the first number picked is the tens digit and the second number picked is the units digit. What is the probability that this number is divisible by 3?

34. _____

35. A circle with radius 1 rolls around the exterior of a square with side length 3. What is the length of the path that the center of the circle traces out?

35. _____

36. Ten lines are drawn in a plane such that no three lines intersect at one point and no pair of lines are parallel. How many regions do the lines divide the plane into?

36. _____

37. A square is inscribed in an isosceles triangle with a base of length 8 and sides of length 5. If the square shares a side with the base of the triangle, what is the side length of the square?

37. _____

38. How many distinct paths are there to go from (0,0) to (6,2), moving only right or up one unit at a time, without passing through the point (3,1)?

38. _____

39. A dartboard is formed by three concentric circles, with radii 1, 2, and 3. Hitting the innermost region is worth 7 points, the middle region worth 4 points, and the outer region worth 2 points. Given that a dart is thrown randomly onto the dartboard, what is the expected number of points you will get?

39. _____

40. Find the value of

$$1 + \frac{1}{2} + \frac{1}{2 \cdot 3} + \frac{1}{2^2 \cdot 3} + \frac{1}{2^2 \cdot 3^2} + \frac{1}{2^3 \cdot 3^2} + \frac{1}{2^3 \cdot 3^3} + \frac{1}{2^4 \cdot 3^3} + \frac{1}{2^4 \cdot 3^4} + \cdots$$

40. _____