Directions: You have 35 minutes to complete these 12 problems. All answers must be written in accordance with the conventions on the Conventions page on the MSJHSSBMTTPSTMT website. Write all of your answers on the answer sheet. You may only use scratch paper provided by the MSJHSSBMTTPSTMT. No calculators allowed.

A5. Find all complex solutions to the equation $|x-4| = |x^2+5x-5|$.

G5. Let $ABCD$ be a square. There is a circle such that the center is at A and two sides of $ABCD$ are radii of the circle. A ray from point C is drawn so that it intersects the circle at two points P and Q. What is the length of the locus of the midpoint of P and Q, as the ray varies? (The *locus* of a property is the set of all points which satisfy a condition. The locus of points in the plane which have distance 9001 from a particular point is a circle of radius 9001).

T5. If $a = 291600$ and $b = 9001$, compute
$$\frac{gcd(a,b)lcm(a,b)}{a(a-b)}$$
.

A6. Evaluate $\sin\frac{\pi}{3}\sin\frac{\pi}{12} + \sin\pi\sin\frac{\pi}{4} + \sin\frac{\pi}{6}\sin\frac{5\pi}{12}$.

G6. A circle with center O has a point P outside and a point T on the circle such that PT is tangent to it. Ray PAB with A and B on the circle is drawn such that $PT = 20$ and $PA = 12$. A point N is on segment AB with $ON = NB = 8$. What is the radius of the circle?

T6. In the game of SET, there are 81 cards, each of which is distinct. On each card, there are 4 different attributes, and each attribute can have 3 different values. (Note that $81 = 3^4$; i.e. there is exactly one card for each possible set of attribute values.) A "SET" is defined as a set of 3 cards such that for each attribute, all 3 cards either all have the same value, or they all have different values. Compute the number of possible SETs that can be made (with replacement of cards).

A7. Consider the polynomial $x^4 - 3x^3 + 12x - 1$, with roots r_1, r_2, r_3, and r_4. Find $r_1^2 + r_2^2 + r_3^2 + r_4^2$.

G7. Convex heptagon $ABCDEFG$ has the property that the midpoints of AB, CD, EF, and GA form a square of side length 16. Furthermore, $BC = 15, DE = 14$, and $FG = 13$. Compute the area of the heptagon.

T7. Suppose A is playing a game with a computer. The computer selects two real numbers in the range $[0,1]$ at random, and A gets to select one of them. With this knowledge, A has to guess which number is larger. With optimal play, let the probability that A wins be a/b where a, b are relatively prime positive integers. Compute $100a + b$.

A8. Find all solutions to
$$\left(1-\sqrt{1-\frac{2}{x+1}}\right)\left(1-\sqrt{1+\frac{2}{x-1}}\right) = -\frac{1}{2}.$$

G8. Let ABC be a triangle such that $AB = 15$, $BC = 8$, and $CA = 16$. There is a semicircle with diameter endpoints X and Y on AB and AC and tangent to BC. If XY is parallel to BC, find the radius of the semicircle.

T8. A *hyperknight* is a piece that moves 2 squares in two directions and 1 square in the other direction in a three-dimensional grid. The hyperknight starts at point $(1,1,1)$, and can only travel in and on a $2\times 2\times 2$ cubical region centered at the origin and oriented parallel to the three axes. What is the probability that, after 6 moves, the hyperknight is at point $(1,1,-1)$?