Individual Round

Name: _____

School: _____

Team ID: _____

Grade: _____

Date:

Problems: 20

Time: 60 minutes

Maximum Score: $3 \times 20 = 60$

Type: Individual

Score: _____

Do not start until instructed to do so!

Calculators, slide rules, books, computers, other electronic devices, are all prohibited. Similarly, graph paper, protractors, rulers, and compasses are not allowed at the competition. You may not collaborate with any other contestants during this round.

Please record your answers only in the blanks below; the ones provided on the test are only for convenience. Only answers recorded on this cover page will be graded.

1.	2.	3.	4.	5.
6.	7.	8.	9.	10.
11.	12.	13.	14.	15.
16.	17.	18.	19.	20.

1. If $a - b = 5$ and $b - c = 9$, what is $a - c$?

    1. _____

2. What is the length of the longest diagonal in a regular hexagon with a perimeter of 6?

    2. _____

3. Alex has 1-cent, 6-cent, and 7-cent coins. What is the minimum number of coins he needs to make 100 cents?

    3. _____

4. Magby draws a circle with a center at $(1, 2)$ and goes through the point $(7, 10)$. What is the radius of the circle?

    4. _____

5. A regular hexagon and a square have the same perimeter. What is the ratio of the area of the hexagon to the area of the square?

    5. _____

6. Alvin, Alex, and Jerome split a pizza into slices of equal size. After finishing the pizza and falling into pizza-induced comas, they forget how many slices of pizza they ate. All they remember is that everyone ate a whole number of slices, Alex ate 3 times as much pizza as Alvin did and Jerome ate $\frac{5}{11}$ of the pizza. What is the minimum number of slices Jerome could have eaten?

    6. _____

7. Two distinct integers are randomly chosen from between 1 and 10 inclusive. What is the probability that their product is prime?

    7. _____

8. What is the last digit of $2^{2015} + 0^{2015} + 1^{2015} + 5^{2015}$?

    8. _____

9. Caroline, Carolyn, Coraline, Keroline each flip two coins. What is the probability Carolyn is the only person who flips any heads?

    9. _____

10. Aaron the Aron wants to evolve to a Lairon! However, in order to do so, he needs to battle some other Pokemon and gain EXP. At level 22, Aaron requires 1 EXP to advance to the next level, and each level after that requires twice as much EXP as the previous level. If each Magmar gives Aaron 33 EXP, how many Magmars will Aaron have to defeat to reach level 32, the level at which he evolves into Lairon?

    10. _____

11. Unit square $ABCD$ is inscribed in a circle. The circle is then inscribed in a square, and the resulting square is again inscribed in a circle. What is the area of the largest circle?

    11. _____

12. Find the sum of the digits of all integers between 1 and 100 (inclusive).

   12. _____

13. A point P is chosen on the edge of a paper circle with radius 1; then, the circle is folded so that P coincides with the center of the circle. Compute the area of the resulting shape.

   13. _____

14. Given that a, b are the two values that satisfy $x^2 - x - 1 = 0$, Find $\frac{1}{(a+1)^2} + \frac{1}{(b+1)^2}$.

   14. _____

15. Ten distinct cities are arranged in a circle. Every pair of cities is connected by a straight road. Gennady the tourist wishes to visit all the cities once. He can start from any city, but the path he takes cannot cross itself. How many different ways can Gennady visit all the cities?

   15. _____

16. Find the coefficient of the x^{10} term when $(x^2 + x + 1)^5 (x+1)^2$ is multiplied out.

   16. _____

17. Consider 100 evenly-spaced points drawn around a circle. How many rectangles can be formed using these points as vertices of the rectangle?

   17. _____

18. For how many integers $2 \le n \le 2015$ is $\frac{5n}{n^2-1}$ a reduced fraction? A reduced fraction is one in which no number, except 1, can be divided evenly into the numerator and denominator.

   18. _____

19. Players Alpha, Beta, Gamma, and Delta are playing in a four person round robin tournament, in which each person plays one match against everyone else. Each match results in either a win or a loss for each player, and each player has an equal probability of winning each match. What is the probability that player Gamma has the most wins (includes tying for most wins)?

   19. _____

20. Triangle AB_1C_1 has $AB_1 = 5$, $B_1C_1 = 3$, $C_1A = 4$. Let B_2 be on AB_1 so that CB_2 is perpendicular to AB_1. Let C_2 be on AC_1 so that B_2C_2 is perpendicular to AC_1. Let B_3 be on AB_2 so that C_2B_3 is perpendicular to AB_2, and let this process continue infinitely. For $i \ge 1$, shade in all triangles $B_iC_iB_{i+1}$. What is the area of the shaded regions?

   20. _____