MSJ Math Club

Week 17: Computational Angle Chasing

April 18, 2013

Warning: This is a *very* difficult handout. In many of these problems, extra constructions or other synthetic observations may be needed. Alternatively, you can view this as good trig bashing practice. I (Aaron) haven't actually solved everything in here yet, so I won't be able to provide hints. ;)

1 Problems

1. Triangle ABC is isosceles with $\angle A = 100°$ and $AB = AC$. A point D is constructed outside the triangle such that $BD = AC$ and $\angle DBC = 20°$. What is the measure of $\angle BCD$?

2. $ABCD$ is a convex quadrilateral such that $AB < AD$. The diagonal AC bisects $\angle BAD$, and $\angle ABD = 130°$. Let E be a point on segment AD. Given that $\angle BAD = 40°$ and that $BC = CD = DE$, determine $\angle ACE$ in degrees.

3. (AMC10 2008) Quadrilateral $ABCD$ has $AB = BC = CD$, $\angle ABC = 70°$, and $\angle BCD = 170°$. What is the degree measure of $\angle BAD$?

4. In isosceles triangle ABC with $AB = AC$ and $\angle BAC = 82°$, a point P is located inside the triangle such that $BP = BA$ and $\angle ABP = 38°$. Find the measure of $\angle PCA$.

5. In acute triangle ABC, O is the circumcenter, point M is the midpoint of side BC, and point N is the midpoint of segment OA. If $\angle ABC = 4\angle ONM$ and $\angle ACB = 6\angle ONM$, what are the angle measures of the triangle?

6. Point P is constructed in triangle ABC such that $\angle PAB = \angle PAC = 22°$, $\angle PBA = 8°$, and $\angle PBC = 30°$. What is the measure of $\angle PCA$?

7. In isosceles triangle ABC with $AB = BC$ and $\angle ABC = 20°$, points D and E are on sides AB and BC respectively such that $\angle BAE = 20°$ and $\angle BCD = 30°$. Find the measure of $\angle AED$.

8. In triangle ABC, point P is located inside so that $\angle PBA = 16°$, $\angle PBC = 34°$, $\angle PCB = 24°$, and $\angle PCA = 2°$. Find the measure of $\angle PAC$.

9. (AIME 2003) Triangle ABC is isosceles with $AC = BC$ and $\angle ACB = 106°$. Point M is in the interior of the triangle so that $\angle MAC = 7°$ and $\angle MCA = 23°$. Find the number of degrees in $\angle CMB$.

10. Square $ABCD$ has a point E inside such that $\angle EBC = \angle ECB = 15°$. What is $\angle EDA$?

11. Let ABC be a triangle such that $\angle A = 60°$ and $\angle B = 100°$. Point E is the midpoint of side BC, and D is on side AC such that $\angle DEC = 80°$. If the length of AC is 1, evaluate $[ABC] + 2[CDE]$.

12. (CGMO 2007) Point D lies inside triangle ABC such that $\angle DAC = \angle DCA = 30°$ and $\angle DBA = 60°$. Point E is the midpoint of segment BC. Point F lies on segment AC with $AF = 2FC$. Prove that $DE \perp EF$.

13. (JBMO 2007) Let $ABCD$ be a convex quadrilateral with $\angle DAC = \angle BDC = 36°$, $\angle CBD = 18°$ and $\angle BAC = 72°$. The diagonals and intersect at point P . Determine the measure of $\angle APD$.

14. (USAMO 1996) Let ABC be a triangle, and M an interior point such that $\angle MAB = 10°$, $\angle MBA = 20°$, $\angle MAC = 40°$ and $\angle MCA = 30°$. Prove that the triangle is isosceles.