MagMaR 2014

Individual Round

Name:	______________________
School:	______________________
Team ID:	______________________
Grade:	______________________
Date:	
Problems:	20
Time:	40 minutes
Maximum Score:	$3 \times 20 = 60$
Type:	Individual
Score:	______________________

Do not start until instructed to do so!

Calculators, slide rules, books, computers, other electronic devices, are all prohibited. Similarly, graph paper, protractors, rulers, and compasses are not allowed at the competition. You may not collaborate with any other contestants during this round.

Please record your answers only in the blanks below; the ones provided on the test are only for convenience. Only answers recorded on this cover page will be graded.

1.	2.	3.	4.	5.
6.	7.	8.	9.	10.
11.	12.	13.	14.	15.
16.	17.	18.	19.	20.

1. A brand of hand sanitizer claims to kill 99.99% of germs. If a drop of sneeze contains 200 million germs, how many germs in a single drop of sneeze are not killed from using the hand sanitizer?

   1. ____________

2. If x is the answer to this question, what is $11x - 70$?

   2. ____________

3. Define $a \star b = b^2 - (a^2 + a)$. Compute $x = ((((1 \star 2) \star 3) \star 4) \star 5) \star 6$.

   3. ____________

4. The height and circumference of the base of a cylindrical soda can are both equal to 2π units. What is the volume of the can in units^3?

   4. ____________

5. The mass of five U.S. dimes is equal to the mass of two U.S. quarters. If Danny has 42 kilograms of dimes and 21 kilograms of quarters, what is the ratio of the number of dimes to the number of quarters that Danny has?

   5. ____________

6. Solve for x: $\sqrt{1x} + \sqrt{4x} + \sqrt{9x} + \sqrt{16x} = 1 + 4 + 9 + 16$.

   6. ____________

7. A perfect power is a positive integer that can be written in the form n^m for a positive integer n and $m \geq 2$. Catalan's Conjecture states that $2^3 = 8$ and $3^2 = 9$ are the only two consecutive positive perfect powers among all positive integers. Find the average of the smallest pair of perfect powers that differ by exactly 2.

   7. ____________

8. Quadrilateral $GRAM$ is inscribed in a circle, and $\angle GAM \cong \angle RMA$. If $RM = 8$, what is the ratio of the area of triangle MAG to that of triangle MAR?

   8. ____________

9. An arithmetic sequence is a list of terms such that the difference any between two consecutive terms is constant. For example, $\cdots, 1, 5, 9, 13, \cdots$ is an arithmetic sequence. How many decreasing arithmetic sequences with only integer terms have 20 and 14 as two of the terms?

   9. ____________

10. Alex and Bob are counting numbers. Alex starts at 2014 and counts down by 2's while Bob starts at 9 and counts up by 3's. If they count at the same rate, at what number do they meet?

    10. ____________

11. Al and Cy are standing together, 60 meters away from Bo. When the clock strikes midnight, Al and Cy will each begin moving towards Bo, at 2 m/s and 4 m/s respectively. Simultaneously, Bo will begin moving towards Al at 1 m/s.

When Cy meets Bo, he will turn around and begin moving towards Al at 4 m/s. How many seconds after midnight will Cy meet Al?

11. ____________

12. Penny flips a fair coin five times. Let p be the probability that she flips 5 heads in a row, and let q be the probability that she flips $HTTHH$ (here, H denotes a head and T denotes a tail flip). Compute q/p.

12. ____________

13. You have an unopened bag of chips and can of salsa. You realize that each bag of chips uses up 5/7 of a can of salsa. If you finish a bag of chips but have leftover salsa, you buy another bag of chips. If you finish a can of salsa but have leftover chips, you buy another can of salsa. By the time that you run out of chips and salsa at the same time, how many bags of chips will you have consumed?

13. ____________

14. Find the smallest positive integer n that satisfies the following:

$$\begin{aligned} n &\equiv 14 \pmod{20} \\ n &\equiv 20 \pmod{14} \end{aligned}$$

(Here, $a \equiv b \pmod{m}$ means that a and b leave the same remainder when each is divided by m. For example, $20 \equiv 14 \pmod{6}$ because both 20 and 14 leave a remainder of 2 upon division by 6.)

14. ____________

15. Six points on the plane form a regular hexagon. Let a be the number of distinct triangles with three vertices as three distinct points. Let b be the number of distinct triangles with three vertices on these points such that the center of the hexagon lies on or in the triangle. Compute $100a + b$.

15. ____________

16. In the card game War, two players simultaneously play a card from his or her deck. The person who plays the higher card takes both played cards and adds them to his or her deck. This process is repeated until one person, the winner, holds all of the cards. Maxie starts with the cards 3, 4, 5, and 10 and Minnie starts with 6, 7, 8, 9. If one of them wins after two days of playing War, what is the probability that Maxie was the winner?

16. ____________

17. Let $f(x)$ be a function such that $f(f(x)) = x$ for all x. If $f(a^2) = f(a+20)$ for some $a > 0$, what is a?

17. ______

18. Let r be the sum of the positive factors of 108 and let s be the sum of the squares of the positive factors of 108. Compute s/r.

18. ______

19. Let p be the probability that an eight-digit number in the form of $AB2014CD$ is divisible by 9 (the digits are not necessarily distinct). Let q be the probability that an eight-digit number in the form of $2014ABCD$ is divisible by 9. Compute $|p - q|$.

19. ______

20. In triangle ABC, points D and E are on sides AB and AC respectively so that DE is parallel to side BC. Let F be the intersection of segments BE and CD. If the ratio of the areas of triangles FBC and ABC is $13 : 37$, compute the ratio $DE : BC$.

20. ______