

**Directions:** You have 35 minutes to complete these 9 problems. All answers must be written in accordance with the conventions on the Conventions page on the MMT website. Write all of your answers on the answer sheet. You may only use scratch paper provided by the MMT. No calculators allowed.

1. Find the remainder when  $2^0 + 2^1 + 2^2 + \cdots + 2^{99}$  is divided by 9.
2. A rectangle  $ABCD$  has side lengths  $AB = 2$  and  $BC = 4$ . Rotate  $ABCD$  about point  $A$  clockwise 315 degrees. Determine the total area swept out by rectangle  $ABCD$ .
3. A weird calculator has three buttons; one button adds 1 to whatever number you have, one button multiplies the number by 2, and one button multiplies the number by 3. The calculator starts with the number one. How many ways are there to get to the number 17 just by using these three buttons?
4. Given that  $a, b$  are the roots of the function  $f(x) = x^2 - x - 1$ , find  $\frac{1}{(a+1)^8} + \frac{1}{(b+1)^8}$ .
5. A sphere is internally tangent to a equilateral-triangle-based pyramid (apex of pyramid is directly above the center of the equilateral triangle) where the side length of the base is 12, and the height is 11. Find its radius.
6. For what real values (ranges) of  $b$  does the polynomial  $x^4 + 3x^3 + bx^2 + 3x + 1$  have real solution(s)?
7. Given triangle  $ABC$  with  $AB = 13, BC = 15, CA = 14$ , let  $I$  denote the incenter and  $O$  denote the circumcenter. Find the area of  $BIO$ .
8. Given a circle with 6 randomly selected points on the circle, find the probability that all 6 points lie on the same semicircle.
9. Find all  $x$  such that  $\sqrt[3]{x} + \sqrt[3]{x + 3\sqrt[3]{x} - 2} = \frac{4}{3}$ .