Directions: You have 35 minutes to complete these 9 problems. All answers must be written in accordance with the conventions on the Conventions page on the MMT website. Write all of your answers on the answer sheet. You may only use scratch paper provided by the MMT. No calculators allowed.

1. Rectangle $ABCD$ has $AB = 2$ and $BC = 4$. Equilateral triangles ABW, BCX, CDY, DAZ are constructed outward of the rectangle. Find the area of $AWBXCYDZ$.

2. The digits of a 5-digit positive integer are 1's, 2's, and 3's with at least one of each. What is the smallest such integer that is divisible by 8 and 9?

3. Find all **positive integer** solutions (x, y) to this equation:

$$\sqrt{6+\sqrt{y}}+\sqrt{6-\sqrt{y}}=\sqrt{2x}$$

4. Aurich the ant finds an ice cream cone with height $2\sqrt{2}$ and base of radius 1. He enters the cone at point P on the edge of the base. Let Q be diametrically opposite P, and let O be the vertex of the cone. There is a bit of chocolate at point R on the cone, where R is a point on QO such that $RO = 2$. What is the shortest distance he must travel, while crawling on the cone, to get to the chocolate?

5. If I randomly select 6 of the first 20 positive integers, what is the expected number of selected integers x such that $x + 1$ is also in the selection?

6. Find the remainder when $100x^{99} + 99x^{98} + 98x^{97} + \cdots + 3x^2 + 2x + 1$ is divided by $x^3 - x$.

7. Let $S = 1, 2, 3, 4, 5, 6, 7$. Find the number of functions $f : S \to S$ such that $f(f(f(f(x)))) = x$ for all $x = 1, 2, 3, \cdots, 7$.

8. Given that a, b, c are positive reals such that $a + b + c = 9$, find the minimum possible value of $a^2 + 2b^2 + 3c^2$.

9. Define point $P = (\frac{21}{2}, 9)$. A line through P intersects the lines $y = \frac{4}{3}x$ and the x-axis at points A, B respectively. Find the minimum value of $PA \cdot PB$.