

1. Find  $-20 - 19 - 18 - \dots + 22 + 23$ .

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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2. What is  $\frac{2+4+6}{1+3+5} - \frac{1+3+5}{2+4+6}$ ?

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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3. Of the 120 students at a particular school, 62 students are in the swimming club, 74 students are in the basketball club, and 28 are in neither club. How many students are in both the swimming and the basketball club?

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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4. How many paths are there from  $(-4,-4)$  to  $(3,3)$ , if one can only move up or right in the path?

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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5. Find  $99999^2$ .

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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6. Aurich the ant is standing on the center of a face of a unit cube. He spots a piece of chocolate at a vertex not on the face he is currently standing on. What is the minimum distance he must travel to arrive at the chocolate, given he must travel on the faces of the cube?

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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7. What is the smallest positive integer  $x$  such that  $x^2 + 8x + 12$  and  $x^2 + 14x + 49$  are not relatively prime?

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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8. Annie plays at Badminton tournaments every weekend. In any given round during the tournament, the probability of her doing well in the round is  $\frac{2}{3}$ . The probability of her partner doing well in the round is  $\frac{3}{5}$ . The probability of them winning the round is 0 if neither are doing well,  $\frac{1}{2}$  if one of the pair is doing well, and 1 if both are doing well. What is the probability that they win both of their two knockout rounds?

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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9. A square with side length 6 is rotated 360 degrees about its center. Determine the area swept out by the full rotation.

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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10. Let  $y$  be the answer to #16. Find  $2y - 1$ .

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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11. Find the remainder when  $(2^0 + 2^1 + 2^2 + \cdots + 2^{99})$  is divided by 9.

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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12. Circle  $\omega$  has radius 6. Rays  $AB$  and  $AC$  are tangent to  $\omega$ , with  $\angle BAC = 60^\circ$ . Find the distance from  $C$  to  $AB$ .

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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13. Find

$$\sqrt{\frac{12}{1 + \sqrt{\frac{12}{1 + \sqrt{\frac{12}{1 + \cdots}}}}}}$$

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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14. There are 9 people in a math class. The teacher wants to choose 4 students to form a problem-writing committee, but Lili and Mili cannot be on the committee at the same time because they laugh too much. How many different possibilities for a committee are there?

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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15. The MagMaR individual rounds are held in three separate distinct rooms. Given that there are nine indistinguishable proctors, and each room must have at least 2 proctors, compute the number of ways the proctors can be distributed between the three rooms.

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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16. Let  $x$  be the answer to #10. Find  $x^3$ .

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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17. Jsun and Jmoon decided to play a game with a pile of  $N$  coins. Both players take turns removing between 1 and 3 coins (inclusive) from the pile, and the person to remove the last coin wins. If Jsun moves first, for how many positive integers  $N$  less than or equal to 100 does Jsun have a winning strategy?

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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18. If  $a$ ,  $b$ ,  $c$ , and  $d$  are distinct integers, find the minimum possible value of  $a^2b^2 + b^2c^2 + c^2d^2 + d^2a^2$ .

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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19. In regular hexagon  $ABCDEF$  with side length 1, equilateral triangles  $ACE$  and  $BDF$  are drawn. Compute the area of the intersection of these two triangles.

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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20. The roots of  $f(x) = x^3 - 12x + 7$  are  $r_1$ ,  $r_2$ , and  $r_3$ . What is  $r_1^2 + r_2^2 + r_3^2$ ?

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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21. Triangle  $ABC$  is in the plane such that  $A = (-1, 14)$ ,  $B = (11, 2)$ ,  $C = (11, -4)$ . Points  $A, B$ , and  $C$  are reflected across line  $x = 9$  to points  $A', B'$ , and  $C'$  respectively. Find the area of the intersection of triangles  $ABC$  and  $A'B'C'$ .

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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22. Find the remainder when  $(1^2 + 1 * 2 + 2^2) + (2^2 + 2 * 3 + 3^2) + (3^2 + 3 * 4 + 4^2) + \dots + (2014^2 + 2014 * 2015 + 2015^2)$  is divided by 2015.

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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23. Trapezoid  $ABCD$  has right angles at  $A$  and  $D$ , and diagonals  $AC$  and  $BD$  intersect at  $E$ . The area of triangle  $ABE$  is 25 and the area of triangle  $DEC$  is 81, and  $AD = 7$ . What is the area of trapezoid  $ABCD$ ?

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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24. If we place a knight on every square of a 2 by 11 chessboard, then how many ways are there to rearrange the knights such that each knight either stays in place or makes one move? (A knight is a piece that can move two squares horizontally and one square vertically, or two squares vertically and one square horizontally.)

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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25. Bob the ant is standing on one vertex of a 4 by 5 by 6 rectangular prism. He wants to get to the vertex that is directly opposite (as in the vertex that does not share a face with the vertex he is on). What is the square of the length of the shortest path he can take if he only moves along the surface of the prism?

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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26. Congratulations on reaching the last 5 problems! What was the answer to #1?

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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27. Find all integer solutions  $(x, y)$  that satisfy the following equation:  $(x - y - 2)^2 + (2x + y)^2 = 1$ .

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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28. A fair, six-sided die is rolled twice. What is the expected value of the greater of the two rolls?

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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29. Define  $f(n) = n + (\text{largest prime factor of } n)^2$ . Find all values of  $n$  such that  $f(f(n)) = 2015$ .

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_

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30. Farmer John has red, blue, and green paint. How many ways can he paint his six-sided barn such that adjacent sides have different colors?

Answer 1: \_\_\_\_\_ Answer 2: \_\_\_\_\_ Answer 3: \_\_\_\_\_