

Directions: You have 50 minutes to complete these 10 problems. All answers must be written in accordance with the conventions on the Conventions page on the MSJHSSBMTPSTMT website. Write all of your answers on the answer sheet. You may only use scratch paper provided by the MSJHSSBMTPSTMT. 2 calculators per team. All calculators are allowed (in particular, TI-89s and TI-Nspires are allowed), but computers are prohibited.

1. Calculate the sum of the first 2012 squares.
2. What is the 30th Fibonacci number? Use the convention that the first and second Fibonacci numbers are both 1.
3. Determine the number of lattice points (x, y) that satisfy the inequality $x^2 + y^2 \leq 1000000$.
4. Define the *permutation index* of a permutation to be its lexicographical order in the set of permutations. Thus, the permutation indices of $\{11, 22, 33\}$, $\{11, 33, 22\}$, $\{22, 33, 11\}$, and $\{33, 22, 11\}$ are 1, 2, 4, and 6 respectively. Compute the permutation index of $\{3, 14, 15, 9, 2, 6, 5, 35, 8, 979\}$.
5. Compute the number of digits in $1000!$.
6. Determine the minimum absolute value in the following set of numbers: $\sin 1, \sin 2, \sin 3, \dots, \sin 1337$. Express your answer as $\sin x$, where x is an integer. Note: this problem is in radians!
7. A twin prime pair is a pair of primes that differ by exactly 2. Thus, 101 and 103 is a pair, while 103 and 107 is not. How many twin prime pairs with both primes less than 1000 are there?
8. The Collatz sequence is defined as follows: for an even x , the next number is $x/2$, and for an odd x , the next number is $3x + 1$. It is hypothesized (and tested up to 10^{18} or so) that every positive integer eventually reaches the number 1. However, this process can take a long time. For instance, the number 27 takes 111 steps to reach 1, climbing as high as 9232 before reaching 1. Determine the sum of all of these numbers. Thus, you are looking for the sum $27 + 82 + 41 + 124 + 62 + 31 + \dots + 4 + 2 + 1$.
9. You have a series of 32 lamps, each initially on. Every second, each lamp that is currently on has a 50% probability of turning off. What is the expected number of seconds for all lamps to turn off? Round your answer to the nearest millionth.
10. An ant travels along the edges of a regular icosahedron. Every minute, it moves to a neighboring vertex. After 30 minutes, what is the probability that it ends up on the starting vertex? Round your answer to the nearest millionth.