

MagMaR 2015

Team Round

Name 1: _____
Name 2: _____
Name 3: _____
Name 4: _____
Name 5: _____
School: _____
Team ID: _____
Date: March 29, 2015
Problems: 15
Time: 40 minutes
Maximum Score: $10 \times 15 = 150$
Type: Team

Score: _____

Do not start until instructed to do so!

Calculators, slide rules, books, computers, other electronic devices, are all prohibited. Similarly, graph paper, protractors, rulers, and compasses are not allowed at the competition. This round is team-based; you may collaborate with your team members during this round.

Please record your answers only in the blanks below; the ones provided on the test are only for convenience. Only answers recorded on this cover page will be graded. Please turn in only one cover sheet per team.

1.	2.	3.	4.	5.
6.	7.	8.	9.	10.
11.	12.	13.	14.	15.

1. A unit cube has its corners cut off such that the initial faces are now regular octagons. How many vertices does the resulting object have?
1. _____
2. Bob has N cookies. He makes enough cookies so he can evenly distribute them among his 9 friends. Sadly, one of his friends is allergic to cookies, so Bob now only has 8 friends to share his cookies with. Fortunately, he could still distribute his cookies evenly among his 8 friends. Suddenly, another one of his friends fainted, so Bob now only has 7 friends to distribute the cookies to. But no worries! Bob can still evenly distribute his N cookies to his 7 friends. What is the smallest possible value of N ?
2. _____
3. Compute the number of distinct permutations of the letters of MSJMATH. A permutation is a way to rearrange the letters in the word.
3. _____
4. How many ordered pairs (x, y) of integers satisfy the equation $\frac{20}{x} = \frac{y}{15}$?
4. _____
5. Calculate $(\frac{1}{2015} + \frac{2}{2015}) + (\frac{2}{2015} + \frac{3}{2015}) + (\frac{3}{2015} + \frac{4}{2015}) + \cdots + (\frac{2013}{2015} + \frac{2014}{2015})$.
5. _____
6. Circle O has radius 2. Circles A and B each have radius 1, are externally tangent to each other, and are internally tangent to circle O . Circle X is externally tangent to circles A and B , and is internally tangent to O . Find the radius of circle X .
6. _____
7. The kingdom of Magmarland currently has 4 isolated towns. In order to unify Magmarland, King Kao wishes to build 3 roads each connecting 2 towns so that it is possible to reach any town from any other town. How many ways can this be done?
7. _____
8. How many three digit numbers are there such that both the number and the sum of its digits are divisible by 5?
8. _____
9. Bessie accidentally broke her analog clock, making it go 2015 times faster. How many times during the day is her clock correct?
9. _____
10. Define three functions: $f_1(x) = 3x - 1$, $f_2(x) = 3x$, and $f_3(x) = 3x + 1$. Chianteng is playing a game in which he starts off with the number 1. On each turn, he applies one of the functions $\{f_1, f_2, f_3\}$ to his number. If he wishes to end up with the number 2015 after 7 turns, please help him come up with the order of functions he should use. Write your answer as a sequence of subscripts of the functions.
10. _____

11. Consider a square $ABCD$. Let E, F, G, H be the midpoints of AB, BC, CD , and DA respectively. The line segments DE, AF, BG , and CH intersect at 4 points to form a smaller square. Find the ratio of the area of the small square to the area of the big square.
11. _____
12. Three mathematicians and five physicists are sitting at a round table. Because mathematicians strongly dislike each other, they refuse to sit next to each other. How many distinct ways can they be seated?
12. _____
13. Triangle ABC has sides $AB = 13, BC = 14, CA = 15$. Construct squares $ADEB, BFGC, CHIA$ outside of triangle ABC . Determine the area of hexagon $DEFGHI$.
13. _____
14. Find the last 2 digits of 2014^{2015} .
14. _____
15. Find all real x such that $\frac{x^2+8x+15}{2x^2+4x+4} = \frac{x^2+2x}{2x^2+16x+34}$.
15. _____