Name: _____ **Team:** _____

Directions: You have 30 minutes to complete these 3 problems. All answers must be written in accordance with the conventions on the Conventions page on the MMM website. Write all of your answers on the this sheet. You may only use scratch paper provided by the MMM. No calculators allowed. Please remember to check your work!

1. Mrs. Phair is listing a sequence of numbers $a_1, a_2, a_3, \cdots$ on her whiteboard. In the first minute, she writes $a_1 = 1$. In the n^{th} minute for $n \geq 2$, Mrs. Unfair randomly generates a permutation $b_1, b_2, \cdots, b_{n-1}$ of the numbers $\{1, 2, 3, \cdots, n-1\}$, and Mrs. Phair calculates $a_n = a_1 b_1 + a_2 b_2 + a_3 b_3 + \cdots + a_{n-1} b_{n-1}$. Let k be the smallest positive integer such that the maximal and minimal possible values for a_k differ by more than 2013. Compute this difference.

1. _____

2. What is the greatest real root of $x^4 + 8x^3 - 40x^2 + 32x + 16 = 0$?

2. _____

3. Generate two random numbers x, y in the range $(0, 1]$. What is the probability that $\lfloor 1/x \rfloor = \lfloor 1/y \rfloor$? It may be useful to note that $\displaystyle\sum_{i=1}^{\infty} \frac{1}{i^2} = \frac{\pi^2}{6}$.

3. _____