MSJ Math Club

Week 8: Incircles

November 28, 2012

1 Tips and Tricks

- Given two intersecting lines, the locus of points P such that the distance from P to both lines is the same are the internal and external angle bisectors of the two lines. This can be used to prove the concurrency of the angle bisectors.
- Problem: Prove that the area K of a triangle ABC is given by $K = rs$, where r is the inradius, and s is the semiperimeter (half of the perimeter). This is a nice formula that comes up often. However, the ideas behind this proof are seen practically *everywhere* on math contests.
- No seriously, know the proof for the point above.

2 Practice Problems

1. (SMT 2007-08) What is the area of the incircle of a triangle with side lengths 10040, 6024, and 8032?
2. In triangle ABC with $\angle A = 90°$, point I is the incenter of the triangle. If $Area(BIC) = 14$ and $Area(ABC) = 32$, what is the length of BC?
3. (NIMO Summer 2011) Triangle ABC with $\angle A = 90°$ has incenter I. A circle passing through A with center I is drawn, intersecting BC at E and F such that $BE < BF$. If $BE/EF = 2/3$, then find CF/FE.
4. (HMMT 2011-12) Let ABC be a triangle with incenter I. Let the circle centered at B and passing through I intersect side AB at D and let the circle centered at C passing through I intersect side AC at E. Suppose DE is the perpendicular bisector of AI. What are all possible measures of angle BAC in degrees?
5. (BMT 2011-12) Let $ABCD$ be a cyclic quadrilateral, with $AB = 7$, $BC = 11$, $CD = 13$, and $DA = 17$. Let the incircle of ABD hit BD at R and the incircle of CBD hit BD at S. What is RS?
6. (AoPS) In triangle ABC, let D be an arbitrary point on side BC. Let ω_1 and ω_2 denote the incircles of triangles ABD and ACD respectively. The common external tangent to ω_1 and ω_2 aside from BC intersects segment AD again at point K. Show that the length of AK is independent of your choice of point D.
7. (AIME 1993) Let $\overline{CH}$ be an altitude of $\triangle ABC$. Let R and S be the points where the circles inscribed in the triangles ACH and BCH are tangent to $\overline{CH}$. If $AB = 1995$, $AC = 1994$, and $BC = 1993$, then find RS.
8. (HMMT 2007-08) Let ABC be a triangle, and I its incenter. Let the incircle of ABC touch side BC at D, and let lines BI and CI meet the circle with diameter AI at points P and Q, respectively. Given $BI = 6$, $CI = 5$, and $DI = 3$, determine the value of $(DP/DQ)^2$.