Directions: You have 30 minutes to complete these 7 problems. All answers must be written in accordance with the conventions on the Conventions page on the MMM website. You may work with your team. Write all of your answers on the answer sheet. One answer sheet per team. You may only use scratch paper provided by the MMM. No calculators allowed.

Tier 1 (2 points each)

(1) Ralph is a reaper and Sally is a sower on the same farm. In order to have crop rotation, they grow corn at 2014 locations evenly spaced around a circle. On the first day, Sally starts at the Location 1 and sows a magical seed in all 2014 locations. (Magical seeds grow into crops immediately.) On the second day, Sally and Ralph return to Location 1, and skipping every other location, they "toggle" the crop; if there is already a crop at the location, Ralph reaps it, and otherwise, Sally sows a seed there. They continue this for 2014 days, such that on the n^{th} day, they start at Location 1, toggle every n^{th} location, and visit a total of 2014 locations. How many locations will have corn growing at the end of the 2014 days?

(2) On a 5×5 chessboard, how many rectangles with edges along the gridlines do not contain the center square?

(3) Because Jarry is very rich, he has 3 routers (Proloq, SyncFlo, and ATT Router) stationed at the three vertices (named P, S, and A respectively) of his triangular house. Because Jarry is also very popular, he is always at home checking his Facebook. At any given moment, he will automatically connect to the WiFi network closest to where he is. If $PS = 5$ meters and $AS = AP = \sqrt{13}$ meters, then the probability that Jarry is connected to the ATT Router can be expressed as a/b for relatively prime positive integers a and b. Compute $a + b$.

(4) Let $a_1, a_2, a_3, \cdots, a_{20}$ be an arithmetic sequence such that the sum of the first 10 terms is 400 and the sum of the last 10 terms is 6400. Compute the common difference.

Tier 2 (3 points each)

(5) Let your answer to (1) be a, and your answer to (2) be b. A lattice frog teleports according to the following rule: on a given day, if the frog is at point (x, y), then she may teleport to $(x + 2, y + 1)$, $(x + 1, y + 2)$, $(x - 1, y)$, or $(x, y - 1)$. If the lattice frog starts at $(0, 0)$, how many ways can it reach the point (a, b) in exactly 13 days?

(6) Let your answer to (3) be c, and your answer to (4) be d. Triangle ABC has side lengths $BC = c$, $AC = d$, and $AB = 24$. Consider a point P, and draw line l_A passing through P and parallel to BC. It meets line AB at point A_B and line AC at point A_C. Define lines l_B, l_C and points B_A, B_C, C_A, and C_B similarly. If $A_BA_C = B_AB_C = C_AC_B = x$, find x.

Tier 3 (4 points each)

(7) Let your answer to (5) be e, and your answer to (6) be f. Find at least one complex solution to the equation $x^4 + 20x^2 + 13 = ex^3 + fx$.