Directions: You have 30 minutes to complete these 9 problems. All answers must be written in accordance with the conventions on the Conventions page on the MMT website. You may work with your team. Write all of your answers on the answer sheet. You may only use scratch paper provided by the MMT. No calculators allowed.

1. Given $x(x^2+3) = 3x^2+1$, find all possible values of x.

2. Find all positive integers n such that n^2+1 is divisible by $n+1$.

3. A fractal is described as follows: Let O be the center of a unit circle and let A, B, C, D be four equally spaced points on the circle in that order. The sector OAB is shaded. Circle O_1 is inside sector OBC such that it is tangent to the curved triangle. The diameter of circle O_2 is OD. The interior of circle O is schematically equivalent to the interior of the circles O_1 and O_2, except that it is scaled. Find the shaded area of the fractal.

4. If $f(x) = x^2 + ux + v$ has integer roots a, b, with $a < b$, and $v - u = 2013$, find all possible pairs (a, b).

5. Two rays, AB and AC, extend out from point A such that $\angle BAC < 90^\circ$. Circle ω_1 is externally tangent to rays AB and AC and within $\angle BAC$. Let ω_n be the circle externally tangent to rays AB and AC, within $\angle BAC$, and tangent to ω_{n-1} on the side opposite of A. Given that ω_1 has area π and ω_7 has area 729π, find the sum of the areas of $\omega_1, \omega_2, \omega_3, \cdots, \omega_7$.

6. A classroom has a 3 by 5 grid of desks, and the students have a set seating arrangement. One day, the teacher wants them to change seats, so he says they must each move to a seat that's a Manhattan distance of exactly 3 seats away. In how many ways can they do this? (The Manhattan distance between two points (x_1, y_1) and (x_2, y_2) is defined as $D = |x_1 - x_2| + |y_1 - y_2|$.)

7. Given that x, y, z are positive reals satisfying $2x^2y^2 + 4yz^4 + xz^2 = 6xyz^2$, find y.

8. Find the smallest positive integer x for which $x, x+1$, and $x+2$ all have exactly six factors.

9. A hypersphere is inscribed in a four-dimensional simplex of edge-length 12 (5 vertices, all edges, faces and volumes are equal and regular). Find its radius.