MagMaR 2013
Team Round

Name 1: _____

Name 2: _____

Name 3: _____

Name 4: _____

Name 5: _____

School: _____

Team ID: _____

Date: January 19, 2013

Problems: 15

Time: 30 minutes

Maximum Score: $10 \times 15 = 150$

Type: Team

Score: _____

Do not start until instructed to do so!

Calculators, slide rules, books, computers, other electronic devices, are all prohibited. Similarly, graph paper, protractors, rulers, and compasses are not allowed at the competition. This round is team-based; you may collaborate with your team members during this round.

Please record your answers only in the blanks below; the ones provided on the test are only for convenience. Only answers recorded on this cover page will be graded. Please turn in only one cover sheet per team.

1.	2.	3.	4.	5.
6.	7.	8.	9.	10.
11.	12.	13.	14.	15.

1. Carl Ray Jensen is naming his newborn daughter either "Yes", "No", or "Maybe."
   The probability that Carl chooses a certain name is directly proportional to the
   number of letters in the name. What is the probability that he calls her "Maybe"?

   1. _____

2. What are all possible months that the 30th Sunday can be in? Assume that it is
   not a leap year.

   2. _____

3. Kurt can type at an average of 50 words per minute on his QWERTY keyboard.
   Eleanor can type at an average speed of 80 words per minute on her Dvorak
   keyboard. How much more time, in seconds, does Kurt spend typing a 1000 word
   essay than Eleanor?

   3. _____

4. An isosceles trapezoid has side lengths of $5, 5, 5$, and 11. Marty draws in the two
   diagonals of the trapezoid and receives four small triangles. What is the area of
   the largest of these four triangles?

   4. _____

5. Find the sum of the positive odd factors of 2,700.

   5. _____

6. It costs \$4.15 to buy 3 erasers, 4 notebooks, and 5 pencils, \$4.40 to buy 4 erasers,
   5 notebooks, and 3 pencils, and \$3.45 to buy 5 erasers, 3 notebooks, and 4 pencils.
   How much does it cost to buy 1 eraser, 2 notebooks, and 3 pencils?

   6. _____

7. A duck walked up to a lemonade stand, and it said to the man running the stand,
   "Hey, if you pour half of the lemonade in your pitcher into my jug of grape juice,
   the mixture will be 40% grape juice." The lemonade man did so. The duck then
   poured 1/3 of his new mixture back into the lemonade pitcher. What fraction of
   the mixture in the pitcher is lemonade?

   7. _____

8. Three parallel cuts are made through a sphere of radius 2 such that each of the
   four pieces has a width of 1 unit. What is the total surface area of the four pieces?

   8. _____

9. The fourth house on a street has address number of 101 and the ninth house has
   an address number of 201. Given that the house numbers on the street form an
   arithmetic progression, what is the address of the first house on the street?

   9. _____

10. Lewis was asked to cut a deck of cards for a magician's trick, so he took a pair
    of scissors and cut each of his 20 cards in half, leaving 20 red cards and 20 black
    cards. What is the probability that two cards drawn (without replacement) from
    Lewis's newly made deck are both red?

    10. _____

11. Let n be a positive integer. The positive integers $a_0, a_1, \cdots, a_n$ satisfy

$$11a_0 - 1 = a_1^{10} + a_2^{10} + \cdots + a_n^{10}.$$

What is the smallest possible value of n?

11. _____

12. Winston is in the bottom left hand corner of a 4×4 grid, and Julia is in the top right hand corner of the same grid. Each minute, Winston randomly moves 1 step up or to the right along the edges, and Julia simultaneously randomly moves 1 step down or to the left along the edges. After four minutes, find the probability that they end up on the same point.

12. _____

13. All possible rearrangements of the letters 'AAGMMR' are placed in alphabetical order. On this list of 'words,' in what place is the word 'MAGMAR'?

13. _____

14. You are doing a quest in the game of Run Escape. You are standing at point A, in between two parallel rivers (River Lum and River Salve) that are 10 kilometers apart. You are 5 kilometers away from both rivers. To complete the quest, you must get a bucket of water from River Lum, get a bucket of water from River Salve, and talk to a person who is standing at point B, in that order. If the line AB is parallel to both rivers, and the distance between points A and B is 21 kilometers, what is length of the shortest path that you can take from point A to one river, then the other, and finally to point B, in kilometers?

14. _____

15. A polynomial function $P(x)$ with integer coefficients satisfies that $P(20) = P(13) = 2013$. What is the maximum number of integer roots that $P(x)$ can have?

15. _____