

Directions: This is the Team Round portion of the 2015 Mission Math Tournament. There are 10 short-answer problems to be solved in 30 minutes, and you are permitted to work together with your teams. *Only answers written on your team answer sheet will be scored.* Good luck!

Problem 1: A square with side length 6 is rotated 90° about its center. What is the area swept out by the rotation of the square?

Problem 2: Given that $a + b = 6$ and $ab = 2$, find the value of $a^4 + b^4$.

Problem 3: Let p_1, p_2, p_3 be primes such that $p_1 < p_2 < p_3$. It is known that $p_1^2 + p_2^2 + p_3^2$ is also a prime. Find p_1 .

Problem 4: Let ABC be a 5-12-13 right triangle. Let X be a point located inside triangle ABC . Find the minimum possible value of $AX^2 + BX^2 + CX^2$.

Problem 5: Find the number of triples of **integers** (x, y, z) such that $xyz = 1260$.

Problem 6: Find the maximum value of the function $f(\theta) = \sin^3 \theta - 9 \sin \theta$.

Problem 7: Professor Snape randomly chooses 10 out of the 30 people in his class to receive an A, 10 to receive a B, 5 to receive a C, and 5 to receive a D. What is the probability that each of four friends in the class receives a different letter grade?

Problem 8: Given that $(x^3 - 5x)^3 - 5(x^3 - 5x) = x$, find all possible values of x .

Problem 9: The numbers 1 through n are written in order on a board. Two players take turns playing a game such that on any turn, a player may switch a and b if $b > a$ and b is after a . The first player who is unable to make a move loses. Assuming both players play perfectly, for how many values of $1 \leq n \leq 2015$ will the first player win?

Problem 10: A circle with radius 8 is centered at the origin on a coordinate plane. The line $y = 3$ intersects the circle at points A and C , with A left of C . The line $x = 3$ intersects the circle at points B and D , with D below B . Call the intersection of these two lines I . Find the sum of the areas of regions $[AID]$ and $[BIC]$.

Note that the region $[AID]$ denotes the area bounded by segments AI and ID , and the arc \widehat{AD} , and that the region $[BIC]$ denotes the area bounded by segments BI and IC , and the arc \widehat{BC} .