MagMaR 2016

Individual Round

Name:	______________
School:	______________
Team ID:	______________
Grade:	______________
Date:	
Problems:	20
Time:	60 minutes
Maximum Score:	$3 \times 20 = 60$
Type:	Individual
Score:	______________

Do not start until instructed to do so!

Calculators, slide rules, books, computers, other electronic devices, are all prohibited. Similarly, graph paper, protractors, rulers, and compasses are not allowed at the competition. You may not collaborate with any other contestants during this round.

Please record your answers only in the blanks below; the ones provided on the test are only for convenience. Only answers recorded on this cover page will be graded.

1.	2.	3.	4.	5.
6.	7.	8.	9.	10.
11.	12.	13.	14.	15.
16.	17.	18.	19.	20.

1. If $\frac{1}{10}$ of a is $\frac{3}{8}$ of b, what is $\frac{b}{a}$?

1. ____________

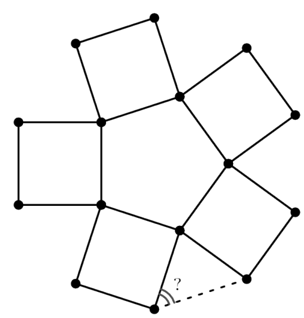
2. Julia can write one page of an essay in 48 minutes. How many hours will it take her to write her 10 page essay?

2. ____________

3. Girafarig is walking on a perfectly spherical planet with radius 50. If Girafarig's height is 5, how much more would his head have moved compared to his feet after traveling a circle along the equator of the planet?

3. ____________

4. Outside a regular pentagon, squares are constructed such that each square shares a side with the pentagon. Find the degree measure of the marked angle in the diagram.



4. ____________

5. How many 4-digit positive integers are divisible by 11?

5. ____________

6. In a regular 10-sided polygon, each pair of vertices is connected by a line segment. What is the probability that a randomly chosen line segment is a diagonal?

6. ____________

7. Two of the side lengths of a right triangle are 3 and 4. What is the smallest possible area of the triangle?

7. ____________

8. Aurick the Ant is on a corner of a unit cube. Let a be the shortest distance Aurick can travel by crawling on the surface of the cube to reach the opposite corner, and let b be the number of distinct paths of length a Aurick can take to reach the opposite corner. Compute the ordered pair (a, b).

8. ____________

9. Given that a, b, c, and $a^2 + b^2 + c^2$ are distinct positive primes, find the smallest of these primes.

9. ____________

10. Find $\sqrt{1} + \sqrt{121} + \sqrt{12321} + \sqrt{1234321} + \cdots + \sqrt{12345678987654321}$.

10. ____________

11. Let a be a positive integer. The greatest common divisor of a and 12 is 6, and the least common multiple of a and 30 is 90. Find the sum of all possible values of a.

11. ______________

12. Given that the area of a rectangle is 10 and the perimeter is $\sqrt{2016}$, find the length of the diagonal of the rectangle.

12. ______________

13. Jennifer and 4 other friends are having a nerf gun shootout. At a given time, they simultaneously shoot at a random person other than themselves. What is the probability that Jennifer is shot at least twice?

13. ______________

14. A farmhouse is in the shape of an equilateral triangle with a side length of 6. Bessie the cow is tied to one corner of the farmhouse with a rope of length 9. What is the total area that Bessie can roam around in, given that she cannot get inside the farmhouse?

14. ______________

15. Blake is eating cookies. When he eats them 3 at a time, he has 1 left over at the end. When he eats them 5 at a time, there are 3 left over. When he eats them 7 at a time, he has 5 left over. Given that Blake started with more than 2016 cookies, what is the least number of cookies Blake could have had initially?

15. ______________

16. A staircase has 16 steps. Maggie the MagMaR can hop 2 or 3 steps at a time. In how many ways can she reach the top of the staircase? **If she hops such that there is 1 step remaining, she cannot reach the top of the staircase.**

16. ______________

17. Real numbers x and y are independently and uniformly chosen at random in the interval $[0, 3]$. What is the probability that $x^2 + y^2 \leq 12$?

17. ______________

18. 6 points on the line $x = 0$ and 6 points on the line $x = 1$ are chosen. Each point on $x = 0$ is connected to each point on $x = 1$ by a line segment. Given that no 3 line segments concur at a single point, what is the total number of intersection points?

18. ______________

19. Given the equation $x^3 - kx - k = 0$, find k such that the equation has exactly two real solutions.

19. ______________

20. In $\triangle ABC$, W is on AB, X is on AC, and Y and Z are on BC such that $WXYZ$ is a rectangle. Given that $AB = 13$, $BC = 14$, and $CA = 15$, what is the maximum possible area of this rectangle?

20. ______________