

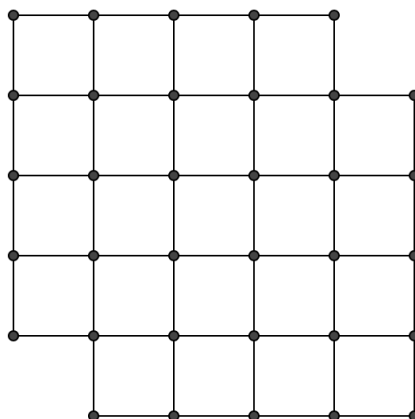
**Directions:** This is Day 2 of the Individual Round portion of the 2015 Mission Math Tournament, and is used as a factor in determining our MSJ teams in future on-site competitions like the Stanford and Berkeley Math Tournaments. There are 6 short-answer problems to be solved in 30 minutes, and problems are weighted 2x of Day 1 problems. *Only answers written on your answer sheet will be scored.* Good luck!

**Problem 1:** Let  $O$  be the midpoint of  $\overline{AC}$  in rectangle  $ABCD$ . Find the probability that a randomly chosen point inside rectangle  $ABCD$  is closer to point  $O$  than any vertex of the rectangle.

**Problem 2:** Find the value of  $\sum_{n=2}^{\infty} \frac{1}{n^2 - 1} = \frac{1}{2^2 - 1} + \frac{1}{3^2 - 1} + \frac{1}{4^2 - 1} + \cdots$ .

**Problem 3:** The function  $f(x)$  outputs the truncation of the result when  $x$  is divided by 5. For example,  $f(4) = 0$  and  $f(12) = 2$  and  $f(20) = 4$ . The function  $g(x)$  is defined to be  $f$  applied enough times until the result is less than 5. For example,  $g(2) = 0$  and  $g(33) = g(6) = g(1) = 1$  and  $g(64) = g(12) = g(2) = 2$ . For how many positive integers  $x$  such that  $1 \leq x \leq 300$  is it true that  $g(x) = 2$ ?

**Problem 4:** How many rectangles are in the figure below? (Remember that squares are rectangles.)



**Problem 5:** Given that  $x$  is a real number that satisfies  $x^3 + 4x = 8$ , find the value of  $x^7 + 64x^2$ .

**Problem 6:** Circles  $\omega_1$ ,  $\omega_2$ , and  $\omega_3$  are mutually externally tangent, and their radii are 3, 4, and 5, respectively. Let their points of tangency be  $A$ ,  $B$ , and  $C$ . Find the area of  $\triangle ABC$ .