

**Directions:** Welcome to the Fun Round! This round does **not** factor into MSJ team placement, but it does factor into cash prize awards and MMT results. (The MMT 2015 Awards Ceremony is scheduled for next Friday). There are 14 short-answer problems to be solved in 30 minutes. You may only use a pen, and any scratch work will be penalized! The first 12 problems are worth 8 points each, and the last 2 are worth 12 points each for a maximum of 120 points. *Only answers written on your answer sheet will be scored.* Good luck!

**Problem 1:** Find the value of  $2015^2$ .

**Problem 2:** Let  $p$  be your answer to this problem, and let  $a$  be the answer to Problem #3. Write down a prime number less than  $a$ . If no one else writes the prime number you write, you will be awarded  $8 \times \frac{p}{a}$ . Otherwise, you will be awarded 0 points.

**Problem 3:** How many ways are there to walk from  $(0, 0)$  to  $(5, 3)$  if you can only move up and right to lattice points?

**Problem 4:** Let  $a$  be the answer to Problem #5. Submit an integer between 0 and 2,147,483,647, inclusive, for this problem. What is the  $a$ th largest answer submitted for this problem?

**Problem 5:** Let  $\triangle ABC$  have  $AB = 5$ ,  $BC = 12$ , and  $CA = 15$ . Let the incircle of  $\triangle ABC$  touch side  $BC$ ,  $CA$ , and  $AB$  at points  $D$ ,  $E$ , and  $F$ . Find  $AF$ .

**Problem 6:** Estimate  $\log_{10}(a!)$ , where  $a$  is the answer to Problem #7. Based on how close you are to the actual question, the most accurate answers will receive 8, 7, 6, 6, 5, 5, 4, 4, 4, 3, 3, 3, 2, 2, 2, 2, 1, 1, 1, 1 points, respectively.

**Problem 7:** Ally, Betty, Caddy, Daisy, Elly, Freddy, and Gracie line up in a line. It is known that Ally is ahead of both Betty and Caddy. It is also known that Daisy is ahead of Freddy, who is ahead of Gracie. In how many ways could they be lining up?

**Problem 8:** Let  $a$  be the answer to Problem #9. Let  $b = \frac{a}{100}$ . A 2-dimensional  $b$  is being cut by straight lines (there is no typo here). What is the most number of pieces of  $b$  that can result after 10 straight-line cuts?

**Problem 9:** Vincent Crabbe is eating cupcakes. If he puts them in his mouth 5 at a time, there will be 4 left over. If he puts them in his mouth 7 at a time, there will be 6 left over. If he puts them in his mouth 9 at a time, there will be 8 left over. What is the least number of cupcakes that Vincent can have?

**Problem 10:** Let your answer from Problem #12 be  $n$ . Let  $a = \max\{9 - n, 0\}$ . Write the  $a$ th prime larger than 100. (If  $a = 0$ , write the largest prime less than 100.)

**Problem 11:** Qingyue decides to write down all positive integers which are neither divisible by 7 nor contain the digit 7, in increasing order. What is the 70th number that Qingyue writes?

**Problem 12:** Let  $a$  be the answer to Problem #10,  $b$  be the answer to Problem #13, and  $c$  be the answer to Problem #14 (Confusing, huh?). When you see this question, walk up to a proctor, point to this question, and ask "What does this word say?" If you do so, the proctor will write your rank  $n$  on the board (which you should copy down as your answer for this problem), and you will receive  $\max\{9 - n, 0\}$  points. Please do not come up to a proctor more than once for this problem.

**Problem 13:** Let  $a$  be the answer to Problem #3, and let  $b$  be the answer to Problem #5. Let  $n = a + b$ . Now let  $p$  be the product of all the positive integral factors of  $n$ . Determine  $\frac{p}{6}$ .

**Problem 14:** Find the sum of the first  $a$  positive integer squares, where  $a$  is the answer to Problem #11. If you forgot the formula for the sum of the first  $n$  squares, you may ask a proctor, for a 4-point deduction.