MagMaR 2016

Team Round

Name 1:	______________________
Name 2:	______________________
Name 3:	______________________
Name 4:	______________________
Name 5: (if needed)	______________________
School:	______________________
Team ID:	______________________
Date:	February 28, 2016
Problems:	15
Time:	40 minutes
Maximum Score:	$10 \times 15 = 150$
Type:	Team
Score:	______________________

Do not start until instructed to do so!

Calculators, slide rules, books, computers, other electronic devices, are all prohibited. Similarly, graph paper, protractors, rulers, and compasses are not allowed at the competition. This round is team-based; you may collaborate with your team members during this round.

Please record your answers only in the blanks below; the ones provided on the test are only for convenience. Only answers recorded on this cover page will be graded. Please turn in only one cover sheet per team.

1.	2.	3.	4.	5.
6.	7.	8.	9.	10.
11.	12.	13.	14.	15.

1. Given that $x^2 + y^2 - 6y + 9 = 0$, find the unique ordered pair (x, y).

1. ____________

2. Two of the side lengths of a non-degenerate triangle are 3 and 5. What is the sum of all possible integer values of the third side length?

2. ____________

3. Given that there are 8 distinct circles drawn on the plane, what is the maximum number of intersection points?

3. ____________

4. An equilateral triangle ABC is inscribed in a unit circle. What is the radius of the largest circle that can fit between the line AB and the minor arc AB?

4. ____________

5. In the Pseudoh region, Acro Bikes move at 12 mph and Mach Bikes move at 30 mph. Ash rides down Cycling Road with the Acro Bike but rides back up Cycling Road with the Mach Bike. What is the average speed of his trip, in mph?

5. ____________

6. A circle of radius 8 and a circle of radius 24 are externally tangent. A rubber band is slipped around the two circles. What is the length of the stretched rubber band?

6. ____________

7. The *factorial* is a function defined on positive integers n, such that $n!$ (read as "n factorial") is equal to $n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot 2 \cdot 1$. Compute the remainder when

$$1! + 2! + 3! + \cdots + 2016!$$

is divided by 2016.

7. ____________

8. A batch of cookies has 5 chocolate chip cookies, 5 red velvet cookies, and 5 snickerdoodles. If Tom chooses 3 cookies randomly, what is the probability that he is missing at least one flavor?

8. ____________

9. Recall from problem 7 the definition of the factorial. Find the number of zeros at the end of

$$100! \cdot 99! \cdot 98! \cdot 97! \cdot \dots \cdot 3! \cdot 2! \cdot 1!$$

9. ____________

10. For how many integers $1 \le x \le 1000$ is $x^3 + 19x + 4$ divisible by 8?

10. ____________

11. A wheel of radius 5 and a wheel of radius 4 are connected at their centers by an axle of length 3. The larger wheel is soaked in ink and the two wheels are placed on a piece of paper and given a small push so that the ink traces a circle on the

paper. How many revolutions will the wheels undergo before the larger wheel traces out a full circle of ink on the paper?

11. ____________

12. The three real roots of the equation $2x^3 - 19x^2 + 57x + k = 0$ form an increasing geometric sequence. Find the largest of the roots.

12. ____________

13. In a square with side length 4, Aurich the Ant starts at the bottom left corner. With each step, Aurich may move 1 unit up, 1 unit right, or $\sqrt{2}$ units $45°$ up-right. In how many ways can Aurich reach the top right corner of this square?

13. ____________

14. The floor function is defined on real numbers x such that $\lfloor x \rfloor$ (read as "floor of x") is the greatest integer less than or equal to x. Find all ordered pairs (a, b) of positive real numbers such that

$$a^{\lfloor b \rfloor} = 20$$

$$b^{\lfloor a \rfloor} = 16$$

14. ____________

15. In isosceles right triangle $\triangle ABC$, $\angle B = 90°$. There is a point P inside the triangle such that $AP = 13$, $BP = 6\sqrt{2}$, and $CP = 5$. Let X be the reflection of P across AB, Y be the reflection of P across BC, and Z be the reflection of P across CA. Find the area of $\triangle XYZ$.

15. ____________