

# MagMaR 2014

## Team Round

Name 1: \_\_\_\_\_  
Name 2: \_\_\_\_\_  
Name 3: \_\_\_\_\_  
Name 4: \_\_\_\_\_  
Name 5: \_\_\_\_\_  
School: \_\_\_\_\_  
Team ID: \_\_\_\_\_  
Date: January 26, 2014  
Problems: 15  
Time: 30 minutes  
Maximum Score:  $10 \times 15 = 150$   
Type: Team  
  
Score: \_\_\_\_\_

### Do not start until instructed to do so!

Calculators, slide rules, books, computers, other electronic devices, are all prohibited. Similarly, graph paper, protractors, rulers, and compasses are not allowed at the competition. This round is team-based; you may collaborate with your team members during this round.

Please record your answers only in the blanks below; the ones provided on the test are only for convenience. Only answers recorded on this cover page will be graded. Please turn in only one cover sheet per team.

1.	2.	3.	4.	5.
6.	7.	8.	9.	10.
11.	12.	13.	14.	15.

1. Aaron says, "I have been alive for all or part of three separate decades". To the nearest year, what is the minimum possible value of his age?

1. \_\_\_\_\_

2. In the table below, the average wealth for the richest 1%, middle 19%, and bottom 80% of Americans are each shown in the table below. What is the average wealth of all Americans, in dollars?

2. \_\_\_\_\_

Class of population	Average wealth
Top 1%	\$10,500,000
Middle 19%	\$500,000
Bottom 80%	\$62,500

3. A rectangle's length and width are both prime numbers and has a perimeter of 120 units. How many different possible areas are there?

3. \_\_\_\_\_

4. Three vertices of a parallelogram are  $(1, 2)$ ,  $(5, 3)$ , and  $(2, 7)$ . If the fourth vertex lies in the first quadrant, what is the sum of its  $x$  and  $y$  coordinates?

4. \_\_\_\_\_

5. Maggie and Carl are playing chess. For every game that Maggie wins, she takes \$12 from Carl. For every game that Carl wins or draws, Carl takes \$15 from Maggie. After a positive number of games, each of them has the same amount that they each started with. What is the minimum number of games they could have played?

5. \_\_\_\_\_

6. Compute  $-1^2 + 2^2 - 3^2 + 4^2 - \dots - 99^2 + 100^2$ .

6. \_\_\_\_\_

7. A very small fish tank requires 60 square inches of wrapping paper to fully wrap and can hold  $\frac{1}{2}$  liter of water. If another fish tank with dimensions in a similar ratio requires 240 square inches of wrapping paper, how many liters of water can it hold inside?

7. \_\_\_\_\_

8. The side of a new roll of toilet paper is a circle with a radius of 3 inches, with a hollow circular center with a radius of 1 inch. Calvin completely unrolls a new roll of toilet paper, uses half of the toilet paper, and flushes it down the toilet. If he rolls the remaining half of the toilet paper back into a cylinder, what is the radius of the used roll?

8. \_\_\_\_\_

9. In a five-pointed star, four of the angles are marked as shown. What is the angle of the fifth point?

9. \_\_\_\_\_

10. A factorial, or  $n!$ , is equal to  $n \times (n-1) \times (n-2) \times \cdots \times 2 \times 1$ . Define a “reverse factorial”, which is denoted as  $n_{\downarrow}$ , to be  $n \div ((n-1) \div ((n-2) \div \cdots 3 \div (2 \div 1)))$ . For example,  $5_{\downarrow} = 5 / (4 / (3 / (2 / 1))) = \frac{5 \cdot 3 \cdot 1}{4 \cdot 2} = 15/8$ .

What is the largest positive integer  $m$  such that  $m!$  is a factor of  $100! \cdot 100_{\downarrow}$ ?

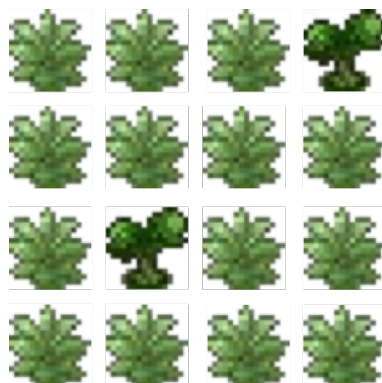
10. \_\_\_\_\_

11. Carolyn and her younger brother are splitting a  $4 \times 4$  square cake, as shown below. She tricks him into thinking that it is fair if and only if the perimeters of their pieces are equal. If she must cut along the gridlines, what is the maximum area of cake that she can take?

11. \_\_\_\_\_

12. A Youngster, Lass, Bug Catcher, and School Kid each want to stand in the  $4 \times 4$  grid of grass patches shown below, but no two want to stand in the same row or same column as another person. If no one may stand on the two squares occupied by trees, how many different arrangements are possible?

12. \_\_\_\_\_



13. Peter is buying a car that costs \$100,000, and uses two 20% off coupons and two 25% off coupons (because they expire today). If Peter uses all four coupons in an order that minimizes the final cost, how much does he pay?

13. \_\_\_\_\_

14. If  $a + b + c = 7$ ,  $ab + bc + ca = 14$ ,  $abc = 8$ , what is  $(1+a)(1+b)(1+c)$ ?

14. \_\_\_\_\_

15. Every second, you will have a fixed probability  $p$  of falling asleep. In six minutes, you have a  $37/64$  chance of being asleep. What is the probability that you will be asleep in eight minutes?

15. \_\_\_\_\_