Team ID: ______________________________ Blitz Set # 1

1. Compute the value of $186 + 124 \times 31$.

1. ____________

2. What is the smallest positive integer n for which the last nonzero digit of the number $(5 \times 10 \times 15 \times \cdots \times (5n))^2 \times (1 \times 2 \times 3 \times \cdots \times n)$ is even?

2. ____________

3. Four cubes of side lengths 1, 2, 3, and 4 are stacked on top of one another in that order. The resulting figure is glued together. What is the surface area of the figure?

3. ____________

4. A $6 \times 10 \times 15$ box is cut into unit cubes. The main diagonal of the box cuts through the interior of n faces, not including edges of the cubes. Find n.

4. ____________

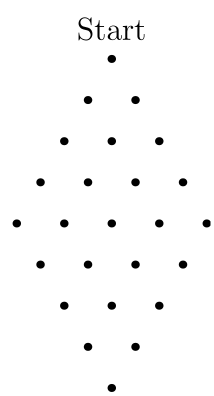
Team ID: ______________________________ Blitz Set # 2

5. If I pick a random number between 1 and 2013 inclusive, what is the probability that it is not a multiple of 3?

5. ____________

6. Bob is standing on the top dot in the diagram below and is trying to move to the bottom dot. Each turn, he randomly moves to an adjacent dot below where he is currently standing. If there is only one dot below and adjacent to him, he must move to that dot on his next turn. What is the probability that Bob passes through the middle dot on his path down?

6. ____________



7. A sequence of consecutive integers has a sum equal to 2013. What is the maximal number of terms in this sequence?

7. ____________

8. One triangle has altitudes of length $48/13$, $16/5$, and 12 and the other has altitudes $168/13$, $56/5$, and 12. Find the positive difference in the areas between the two triangles.

8. ____________

Team ID: ______________________________ Blitz Set # 3

9. If a and b are numbers such that $a + 2b = 9$ and $ab = 10$, find $a^2 + 4b^2$.

9. ____________

10. The probability that Lucky the Leprechaun will find a pot of gold in any given year is 1/3. In a four-year period, what is the probability that Lucky will find at least one pot of gold?

10. ____________

11. In acute triangle ABC, let E be an arbitrary point on side BC, and let N and M be the midpoints of AB and AC respectively. Lines NM and AE intersect at point P. Find AE/AP.

11. ____________

12. May has one small bottle, two medium bottles, and three large bottles. How many ways are there for her to arrange bottles in a row such that the bottles are in increasing order of size? She does not have to use all of the bottles, but she must use at least one bottle out of all six. Assume that all of the bottles are differently labeled.

12. ____________

Team ID: ______________________________ Blitz Set # 4

13. A student took the Individual Round of MagMaR, answering 12 questions and leaving the other 8 problems blank. If the student made careless mistakes on 75% of the problems that he answered, what percent of all of the problems did he answer correctly?

13. ____________

14. There are a total 1000 students at Video Game High School. Thirty percent of the boys and sixty percent of the girls are part of the school's math club. If there are 426 members in math club, how many girls are there at the school?

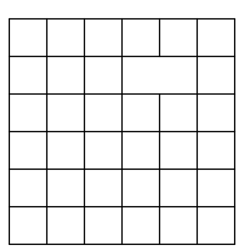
14. ____________

15. Isosceles triangle ABC has side lengths of $AB = BC = 13$ and $CA = 10$. Let M be the midpoint of CA, F be the foot of the altitude from A to BC, and X be the intersection of BM and AF. Find the length of AX.

15. ____________

16. How many ways are there to navigate the following grid from the bottom-left corner to the top-right corner if your only possible moves are one unit up or one unit right? Notice that there is a missing grid line.

16. ____________



Team ID: ______________________________ Blitz Set # 5

17. Five less than three times of a number is equal to twice the number added to seven. What is the number?

17. ____________

18. The writer of this problem, Aaron, has thought of a prime number to be the answer to this problem. However, when Jeffrey comes along and sees the number, he accidentally reverses its digits, and gets a larger prime number. It turns out that the sum of the two numbers is 1372. What is the smaller of the two numbers?

18. ____________

19. Find the sum of all distinct *prime* factors of 999999.

19. ____________

20. Segment PQ is inside of rectangle $ABCD$ such that $PQ \parallel AD$ and Q is closer to D than P is. Also, $PQ = 2$, $AD = 5$, and $AB = 12$. Lines BP and CQ intersect at point R, and AP and DQ at S. Find the maximum possible area of $PRQS$.

20. ____________

Team ID: ______________________________ Blitz Set # 6

21. A three digit integer is subtracted from the number formed by reversing its digits. What is the greatest common factor of all possible positives differences?

21. ____________

22. On the first day of Christmas, Steven studies for 60 minutes a day and Wallace studies for 40 minutes a day. On each successive day, Steven studies 3 minutes less than he did on the previous day and Wallace studies 5 more minutes than he did the day before. By the end of twelfth day of Christmas, Steven will have studied for a total of a minutes, and Wallace will have studied for a total of b minutes. Find $|a - b|$.

22. ____________

23. Find all solutions x to $|x - 1| + |x + 3| = 8$.

23. ____________

24. On a 5×5 grid of squares, each of the 25 small squares has an area of 1 square inch. What is the sum of the areas of all rectangles determined by the lines of the grid?

24. ____________

Team ID: ____________________ Blitz Set # 7

25. Barack is standing 4 meters away from John and 3 meters away from Mitt. What is the maximum possible distance between John and Mitt?

25. ____________

26. The Magikarp Salesman is a merchant selling fishing rods to his customers. He first raises the prices of all of his items by 30 percent. After that, he holds a sale, and marks off 40 percent of the modified price of each item. During the sale, how much does a customer pay for a Super Rod that was originally \$100 before the price changes?

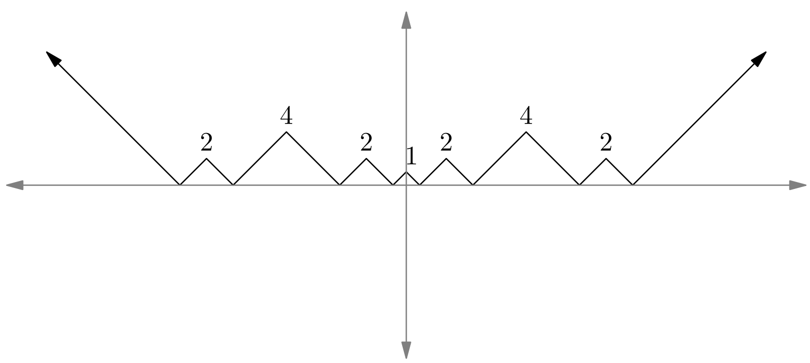
26. ____________

27. A single loaf of sliced bread has 16 slices of bread. Mark eats 5 slices every day in the morning. At the end of the day, if the number of slices left is between 1 and 4 inclusive, his mom buys another loaf. If there are no slices left, then she doesn't do anything. Mark will cry when there is no bread left in the morning. If he finds 8 slices left this morning (a Saturday), on which day will Mark first cry?

27. ____________

28. The following graph is in the form $||||x| - a| - b| - c|$. The numbers on top of the peaks indicate the y-value of the peak. All line segments have slopes of either 1 or -1. Find $a + b + c$.

28. ____________



Team ID: ____________________ Blitz Set # 8

29. Find the greatest common factor of 14^3 and 15^2.

29. ____________

30. The radius of the Earth is approximately 4000 miles. Disregarding the Earth's revolution around the sun, how fast does a rock sitting at the Equator travel on average, in miles per hour?

30. ____________

31. Two congruent 4×4 grids of squares, one with red line segments and the other with blue line segments, are laid directly on top of each other. The red grid is then rotated $45°$ about the center of the grid, so that some red grid lines intersect with some blue grid lines. Find the number of such intersection points. (If more than three lines pass through a single point, that counts as 1 intersection point.)

31. ____________

32. Find the remainder when 10^{100} is divided by 111.

32. ____________

Team ID: ______________________ Blitz Set # 9

33. How many different arrangements of the word 'MEWTWO' are there?

33. ________

34. The ScienceCounts Competition awards right-triangle-shaped trophies. At the Chapter level, trophies are $4\frac{1}{4}$ inches wide and 6 inches tall. If the National level trophies are similar in shape to the Chapter trophies, and are 16 inches tall, how wide are National level trophies?

34. ________

35. Blaziken is jumping up a building with 10 stories, numbered in order (starting from the bottom) as $1, 2, \cdots, 10$. From any floor, the he can jump to any higher-numbered floor as long as the two floors' numbers are relatively prime. (For example, it can jump from 6 to 7 or 1 to 8, but not from 6 to 10 or 6 to 5.) If the Blaziken starts from floor 1, how many ways are there for it to get to floor 10?

35. ________

36. Simplify $\dfrac{(2+\sqrt{3})^{2-\sqrt{3}}}{(2-\sqrt{3})^{2+\sqrt{3}}}$.

36. ________

Team ID: ______________________ Blitz Set # 10

37. A 4×4 grid of squares is initially blank, except for a 1 in the bottom left square. Becky fills in the grid so that the number in any given cell is equal to the sum of all of the numbers below in the same column and all of the numbers to the left in the same row. What is the number in the top right cell?

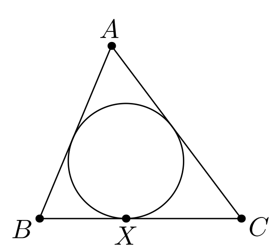
37. ________

38. In chess, a knight moves in an 'L' shape. In other words, in one turn, the knight travels two squares either horizontally or vertically, and then one more square in a direction perpendicular to the original direction of travel. For example, in the middle of a board, a knight can move to 8 different squares. A knight is randomly placed on one of the 64 squares of an empty chessboard. On average, to how many squares can it move in one turn?

38. ________

39. Triangle ABC with has side lengths of $AB = 5$, $BC = 6$, and $CA = 7$. A circle inside of ABC is tangent to each of the three sides. Let X be the point where the circle touches segment BC. Find the ratio of the lengths: BX/XC.

39. ________



40. Brock is thinking of an 8-digit number. He tells his girlfriend that it leaves a remainder of 2013 upon division by 10000 and a remainder of 9001 upon division by 10001. What is the number?

40. ________

Team ID: ______________________________ Blitz Set # 11

41. Evaluate $5 \times 15 \times 25 \times 2 \times 12 \times 22$.

41. ____________

42. There are initially only red and blue marbles in a bag, and at least one of each color. The number of red marbles is doubled, and the number of blue marbles is tripled. If there are a total of 42 marbles in the bag after that, what is the minimum number of marbles that the bag could have started with?

42. ____________

43. Triangles ABC and DEF satisfy that $\angle BAC = \angle EDF = 30°$, $AB = 6$, $DE = 12$, and $EF = 2 \cdot BC$. If $AC = 3\sqrt{3} + 3$, what is the sum of all possible side lengths of DF?

43. ____________

44. Find the value of x that satisfies:

$$\frac{x-1}{y-1} = \frac{x-7}{z+2} = \frac{5x-13}{2y+z}$$

44. ____________

Team ID: ______________________________ Blitz Set # 12

45. How many prime numbers are there between 1 and 50?

45. ____________

46. Find the sum of the squares of the factors of 6^4.

46. ____________

47. How many permutations of the letters A,B,C,D are there such that none of the strings "AB", "BC", "CD", and "DA" appear? (In other words, the letter B can't follow the letter A and so on.)

47. ____________

48. A square $ABCD$ with side length of 2 is rotated about point A to obtain a new square $AXYZ$ so that sides BC and YZ intersect at point P. If the area of $ABPZ$ is 2, find the ratio $BP : PC$.

48. ____________

Team ID: ______________________________ Blitz Set # 13

49. What is the average of the distinct prime factors of 1365?

49. __________

50. In the first 6 days of a week, Clippy helped an average of 37 people each day. How many people must Clippy help on the last day in order to meet his quota of helping 40 people per day on average in a week?

50. __________

51. The numbers 1 through 80 are written uniformly around a paper circle in increasing order. The circle is folded in half so that the numbers 20 and 13 overlap and each number is folded on top of another number. How many pairs of overlapping numbers have a sum of 113?

51. __________

52. In quadrilateral $ABCD$ with points A and D on the same side of line BC, $\angle BAC = \angle BDC = 90°$, $\angle DBC = 45°$, and $\angle ACB = 30°$. If $BC = \sqrt{2}$, what is the length of AD?

52. __________

Team ID: ______________________________ Blitz Set # 14

53. Henry borrows \$50,000 to pay for his college tuition. Henry pays off his loan by paying \$1250 back each month for 4 years. After his fourth year of college, Henry will have paid d dollars in *interest*. What percent of his original loan is d?

53. __________

54. Misty has a total of \$3.60 in quarters and nickels only. If she has three times as many nickels as quarters, how many coins does Misty have in total?

54. __________

55. A rectangle has an area of 198 and a diagonal length of 30. What is the perimeter of the rectangle?

55. __________

56. Let $f(x)$ denote the greatest integer less than or equal to x. For example, $f(\pi) = 3$ and $f(2) = 2$. Evaluate the sum $f(\sqrt{1}) + f(\sqrt{3}) + f(\sqrt{5}) + \cdots + f(\sqrt{625})$.

56. __________

57. A customer buying refrigerator priced at \$600 has a 30% off, 50% off, and a 20% off coupon. If he uses all three coupons, how much does he pay for the refrigerator? (He does not get the refrigerator for free.)

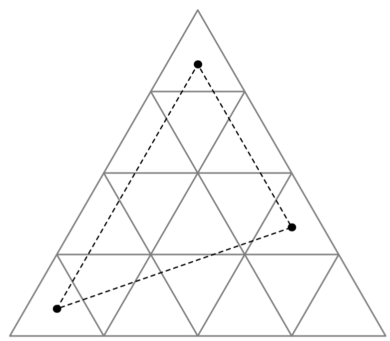
57. ____________

58. Draco draws two squares, one with a side length 4 times of the smaller square's side length. What is the ratio of the area of the smaller square to the area of the larger square?

58. ____________

59. In the adjoining figure, the area of each of the smallest equilateral triangles is 1. What is the area of the dotted triangle?

59. ____________



60. Let a, b, and c be distinct positive integers from the set of numbers $\{1, 3, 5, 7\}$. Of all 24 ways to select the values of a, b, and c, find the average of all the values of $(a + b)^2 + (b - c)^2 + (c + 2a)^2$.

60. ____________