Directions: You have 35 minutes to complete these 12 problems. All answers must be written in accordance with the conventions on the Conventions page on the MSJHSSBMTTPSTMT website. Write all of your answers on the answer sheet. You may only use scratch paper provided by the MSJHSSBMTTPSTMT. No calculators allowed.

1. What is the maximum possible area of a rectangle of perimeter 16?

2. Find all y that satisfy the equation $\int_0^y (x^2 - 4x + 4)\, dx = \int_y^4 (x^2 - 4x + 4)\, dx$.

3. Evaluate $\int_0^{3\pi/2} \sin x \cos x\, dx$.

4. A man is absentmindedly trying to draw a circle with a compass. The compass can be represented by an isosceles triangle with a vertex angle of θ and two legs of length 9 cm. The length of the base of the triangle is the radius r of the arc that is drawn. When the man starts drawing, $\theta = \pi/4$ radians, and the compass turns at 1 radian per second (so that he would draw a full circle in 2π seconds). Due to the man's absentmindedness and the poor construction of the compass, θ steadily increases by $1/24$ radian per second, and the man finally notices and stops drawing 2π seconds later. The length in cm of the spiral that he drew can be expressed in the form $a(\sqrt{b} + \sqrt{c}) - d\sqrt{e + \sqrt{f}}$, where a, b, c, d, e, and f are positive integers, $b \geq c$, and b, c, and f are square-free. Find the ordered hextuple (a, b, c, d, e, f).

5. Find $\int_{-1}^{-3} \frac{1}{x^2 + 4x + 5}\, dx$.

6. Find $\int_1^e \frac{e^x(1 + x \ln x)}{x}\, dx$.

7. Find the equation of the tangent line to the graph of the 19th derivative of $y = \prod_{n=1}^{57} (x - n)^n$ at $x = 38$.

8. Find an elementary real function $f(x)$ containing no trigonometric functions (e.g. sin, cos, sec, sinh) such that the sum of $f'(x)$ and one of the antiderivatives of $f(x)$ is $\frac{x^2 \ln x + 1}{x} - x$.

9. Evaluate $\int_{-2012}^{2012} x^{2011} \sec x\, dx$.

10. Given that $17^{2013} - 12^{2013} \approx 7.8285 \times 10^{2476}$, find $\left\lfloor \log_{10} \sum_{n=1}^{2012} \frac{5}{2012}\left(12 + \frac{5n}{2012}\right)^{2012} \right\rfloor$.

11. Evaluate $\sum_{n=0}^{\infty} \frac{1}{2n^2 + 7n + 6}$.

12. Evaluate $\int_1^{\infty} \frac{\sqrt{x^2 - 1}}{x^3}\, dx$.