

Directions: You have 30 minutes to complete these 9 problems. All answers must be written in accordance with the conventions on the Conventions page on the MMM website. Write all of your answers on the answer sheet. You may only use scratch paper provided by the MMM. No calculators allowed.

1. Chianteng is racing himself on the escalators. The amount of time he takes to run up and down the “up” escalator is the same as the amount of time for him to ride the “up” escalator to the next floor. If the escalator steps move 1 meter in 2 seconds, what is Chianteng’s running speed, assuming it is constant?
2. Napoleon, a prime minister, and a secretary sit around a round table with 4 other people for dinner. Unfortunately, the cooks did not add salt. The only salt shaker is to be passed around the table clockwise. The prime minister gets the salt shaker first, but Napoleon must get the salt shaker before the secretary does. For how many seating arrangements is this possible?
3. For a positive prime $p < 100$, the GCD of $5p - 8$ and $8p - 5$ is 1. Similarly, the GCD of $11p + 14$ and $14p + 11$ is also 1. Find the sum of all possible values of p .
4. Let p be the probability that an eight-digit number in the form of $AB2014CD$ is divisible by 9. Let q be the probability that an eight-digit number in the form of $2014ABCD$ is divisible by 9. Compute $|p - q|$.
5. How many integers $m < 100$ of the form pq for distinct prime numbers p, q cannot be written in the form $\binom{n}{r}$, for positive integers $n \geq r \geq 0$?
6. Points A, B and C are on a circle with center of O . The two tangents to the circle at points A and C intersect a point P . Line PB intersects the circle again at point D . If $AB = 20$, $BC = 14$, and $CD = 42$, then what is the length of DA ?
7. Let H be the orthocenter of triangle ABC . If $AH = 20$ and $BC = 13$, find the length of the circumradius R .
8. On an airplane ride from Fremont to Alpine, Marianna, Bowei, Ray, Jeffrey, and Aaron are sitting together in that order. Unfortunately, each of them was unsatisfied with their respective neighbor(s) on the plane ride to Alpine (they each took up too much elbow room). Thus, on the airplane ride back, none of them wanted to sit next to someone they had previously neighbored. How many ways can they seat themselves to satisfy this restriction?
9. CAML the Camel’s back is modeled by the equation $-x^4 - 4x^3 + 4x^2 + 20x + 13 = 0$. What is the equation of the line tangent to the two humps?

