

Team ID#: _____ Team Name: _____

1. In how many ways can I split 5 indistinguishable cookies among Grace and Carolyn? It is okay if someone receives zero cookies.

Answer 1: _____ Answer 2: _____ Answer 3: _____

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2. How many positive factors does 4096 have?

Answer 1: _____ Answer 2: _____ Answer 3: _____

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3. Anna currently has a 92% in her Algebra class. Given that the final exam is worth 20% of her grade, what is the lowest grade, as a percent, she can get on her final such that Anna ends the class with at least a 90%?

Answer 1: _____ Answer 2: _____ Answer 3: _____

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4. A woman in Fortree City believes you can deduce whether or not you have hidden powers of your own. If you can guess which hand she is hiding a coin in three times in a row, she will gift you the TM10 (Hidden Power). What is the probability you receive TM10 on your first try at her challenge?

Answer 1: _____ Answer 2: _____ Answer 3: _____

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5. Rhombus $ABCD$ has $AB = BC = CD = DA = 4$, and $AC = 6$. Find BD^2 .

Answer 1: _____ Answer 2: _____ Answer 3: _____

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6. Given that a has a remainder of 2 when divided by 3, and that b has a remainder of 4 when divided by 5, find the maximum possible remainder when ab is divided by 15.

Answer 1: _____ Answer 2: _____ Answer 3: _____

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7. On the coordinate plane, a triangle is placed such that its three vertices lie within or on the boundary of the region defined by $0 \leq x \leq 10$ and $0 \leq y \leq 10$. What is the maximum possible area of the triangle?

Answer 1:_____ Answer 2:_____ Answer 3:_____

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8. In $\triangle ABC$, $\angle A = 50^\circ$ and $\angle B = 70^\circ$. Let O be the center of the circumcircle (the circle that passes through the 3 vertices of the triangle). Find $\angle OAB$ in degrees.

Answer 1:_____ Answer 2:_____ Answer 3:_____

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9. From January 1st, 2017, 0:01 to January 2nd, 2017, 0:01, how many times does the minute hand cross the hour hand?

Answer 1:_____ Answer 2:_____ Answer 3:_____

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10. Tiancheng has two pizza tokens, each of which he can use to buy a slice of cheese, pepperoni, sausage, hawaiian, veggie, or combo pizza. How many distinct ways can he order two slices of pizza?

Answer 1:_____ Answer 2:_____ Answer 3:_____

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11. In rectangle $ABCD$ (A, B, C, D are in clockwise order), let M be the midpoint of CD . Let the intersection of BM and AC be I . Find the ratio $AI : IC$.

Answer 1:_____ Answer 2:_____ Answer 3:_____

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12. Lauren writes the numbers $1, 2, \dots, n$ on a sheet of paper. In total, she writes 2016 digits. What is n ?

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13. How many positive integers $1 \leq x \leq 26$ are there such that $x^2 - x$ is divisible by 26?

Answer 1:_____ Answer 2:_____ Answer 3:_____

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14. At a party, each pair of people shakes hands with each other at most once. Given that there are 228 handshakes in total, what is the least possible number of people at the party?

Answer 1:_____ Answer 2:_____ Answer 3:_____

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15. Three concentric circles ω_1 , ω_2 , and ω_3 have radii 1, 2, and 3 respectively. Let A , B be on the ω_3 such that AB is tangent to ω_1 . AB intersects ω_2 at C and D , with C closer to A than B . Find $AC + BD$.

Answer 1:_____ Answer 2:_____ Answer 3:_____

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16. Let S be a set of n consecutive positive integers starting with a_1 . Pick two numbers, remove them from the set, and replace the two numbers with their sum. Continue this process until one number is left. If 162 is the number that is left, find the least possible value for a_1 .

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17. Point E is outside square $ABCD$ such that BCE is an equilateral triangle with $BC = 10$. Let O be the point on the plane such that $OE = OA = OD = k$. Find k .

Answer 1:_____ Answer 2:_____ Answer 3:_____

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18. Let S_n be the set of three-digit numbers in base n . For how many values of n is there an element in S_n which has the same value as the decimal number 2016?

Answer 1:_____ Answer 2:_____ Answer 3:_____

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19. In the Pseudoh region, Poke Balls cost \$200 and Ultra Balls cost \$1200. Suppose that against a full health level 50 Zapdos, Ultra Balls have a 4% capture rate while Poke Balls have a 0.5% capture rate. What is the expected number of dollars saved by capturing Zapdos with Ultra Balls instead of Poke Balls?

Answer 1:_____ Answer 2:_____ Answer 3:_____

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20. Aurich the Ant sees an opening in a cylindrical can of soup with height 6 and radius $\frac{5}{\pi}$. Climbing along the sides or bases of the cylindrical can, what is the minimum distance Aurich has to travel to get from a point at the bottom of the lateral face to a point diametrically opposite at the top of the lateral face?

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21. Find the 2016th smallest positive integer that is not divisible by 2, 3, or 5.

Answer 1:_____ Answer 2:_____ Answer 3:_____

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22. A level 100 Groudon with Fissure and a level 100 Kyogre with Sheer Cold are battling. Groudon moves first, then Kyogre and Groudon take turns making moves. Assuming both moves can be used infinitely many times, and that Groudon and Kyogre always use Fissure and Sheer Cold, respectively, what is the probability that Kyogre wins the battle? (Note: against opposing Pokemon of the same level, Fissure and Sheer Cold both have a 30% chance of a 1-hit-KO.)

Answer 1:_____ Answer 2:_____ Answer 3:_____

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23. Find $1 \cdot 99 + 2 \cdot 98 + 3 \cdot 97 + 4 \cdot 96 + \cdots + 98 \cdot 2 + 99 \cdot 1$.

Answer 1:_____ Answer 2:_____ Answer 3:_____

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24. Let $ABCDEF$ be a regular hexagon. Connect the midpoints of AB , CD , and EF to form a triangle, and connect the midpoints of BC , DE , and FA to form a second triangle. What is the ratio of the area of the intersection of the two triangles to hexagon $ABCDEF$?

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25. Let $\lfloor x \rfloor$ denote the greatest integer less than or equal to x . Find x if $x \lfloor x \rfloor = 2016$.

Answer 1:_____ Answer 2:_____ Answer 3:_____

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26. How many three-digit positive integers contain digits that are, reading from left to right, in a geometric sequence?

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27. Let α , β , and γ be the solutions to the equation $x^3 - x^2 - x + 2 = 0$. Find $\alpha^3 + \beta^3 + \gamma^3$.

Answer 1:_____ Answer 2:_____ Answer 3:_____

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28. For how many integers x such that $1 \leq x \leq 100$ is $\frac{3x+5}{20x+34}$ a reduced fraction?

Answer 1:_____ Answer 2:_____ Answer 3:_____

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29. Let A be the number of ways to choose 2016 people out of a group of 2020 people. Given that 2011 is prime number, find the remainder when A is divided by 2011.

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30. In $\triangle ABC$, $\angle A = 60^\circ$, $AB = 3$, and $AC = 5$. Let D be on BC , E be on CA , and F be on AB . Let D , E , and F be positioned such that the perimeter of $\triangle DEF$ is minimized. In this scenario, find the length EF .

Answer 1:_____ Answer 2:_____ Answer 3:_____