MagMaR 2013

Individual Round

Name: _____

School: _____

Team ID: _____

Grade: _____

Date: January 19, 2013

Problems: 20

Time: 40 minutes

Maximum Score: $3 \times 20 = 60$

Type: Individual

Score: _____

Do not start until instructed to do so!

Calculators, slide rules, books, computers, other electronic devices, are all prohibited. Similarly, graph paper, protractors, rulers, and compasses are not allowed at the competition. You may not collaborate with any other contestants during this round.

Please record your answers only in the blanks below; the ones provided on the test are only for convenience. Only answers recorded on this cover page will be graded.

1.	2.	3.	4.	5.
6.	7.	8.	9.	10.
11.	12.	13.	14.	15.
16.	17.	18.	19.	20.

1. SpongeRobert is a rectangular prism 4 inches thick, 18 inches tall, and 12 inches wide. What is the volume of SpongeRobert in cubic feet?

1. _____

2. Ash chooses one of his six Pokemon at random for a Pokemon battle. What is the probability that in his next battle, he will choose the same Pokemon?

2. _____

3. This test was printed out 2 days ago on January 17, at 3:00 P.M. Right now, it is January 19, and the time is 10:40 A.M. (No really, it is.) How many minutes ago was this test printed out?

3. _____

4. The number $3 \cdot \sqrt{3} \cdot \sqrt[3]{3} \cdot \sqrt[4]{3}$ can be expressed in the form 3^a for some value of a. Find a.

4. _____

5. Evaluate the sum $1 + 2 + 4 + 5 + 7 + 8 + \cdots + 1000 + 1001$, in which multiples of 3 are excluded from the first 1001 positive integers.

5. _____

6. Professor Oak's 3rd grade class is trying to split into groups of equal size for a class project. However, whether they have groups of 2, 3, or 4 people, there is always exactly one person who does not have a group. If there are between 15 and 30 students in the class, how many students are in Professor Oak's class?

6. _____

7. How many positive integers between 1 and 2013, inclusive, share at least one prime factor with 2013?

7. _____

8. A row of houses is labeled from 1 to 100. Woody rings the doorbell of every third house, starting with houses $1, 4, 7, \cdots$. Meanwhile, Buzz Lightyear rings the doorbell of every fourth house starting from the back, starting with houses $100, 96, 92, \cdots$. How many houses are left undisturbed?

8. _____

9. For positive numbers a, b, c, and d, we have that $|a - b| = 1$, $|b - c| = 3$, and $|c - d| = 5$. If $a = 10$, what is the sum of all possible distinct values of d?

9. _____

10. Evan gets a large paycheck at the beginning of each month. Of his monthly salary, half of it is spent on honey, one-sixth of it on toothpaste, and one-fourth of it on berries. He also spends \$9001 each month on books. If Evan breaks even each month (i.e. he does not gain or lose money in the long run), how large is his paycheck in dollars?

10. _____

11. One vertical line and one horizontal line is drawn in a rectangle with perimeter 1, forming four small rectangles. Find sum of the perimeters of all of the rectangles of all sizes, including the original rectangle.

11. _____

12. Square $ABCD$, points P and Q are on segments BC and CD respectively. If $[APC] = 20$, $[AQC] = 13$, and $[AQD] = 2[APB]$, find area of the square. (Here, $[SHAPE]$ denotes the area of the shape.)

12. _____

13. What is the tens digit of the number 2^{2013}?

13. _____

14. A book with $4n$ pages is made by stacking multiple sheets of paper on top of each other, and folding the stack in half. (For example, a book with 12 pages requires three sheets of paper: one with the pages 1, 2, 11, 12, another with the pages 3, 4, 9, 10, and the last one with pages 5, 6, 7, 8.) Victoria notices that in her book, the sheet with pages 36 and 93 are missing. What is the page number of the last page of the book?

14. _____

15. One morning, Romeo leaves the city of Verona at 9:00 in the morning, driving at a constant speed of 40 miles per hour. An hour later at 10:00, Juliet notices that Romeo is missing, and chases after Romeo at a constant speed of 50 miles per hour. If their parents realize that both of them are missing at 11:00 in the morning, how fast must they drive, in miles per hour, if they want to catch up to Romeo and Juliet at the same time?

15. _____

16. What is the volume of a regular tetrahedron inscribed in a sphere of radius 1?

16. _____

17. The points on the spiral are numbered in increasing order as shown below. What is the number of the point that is 10 units to the left of point 0?

17. _____

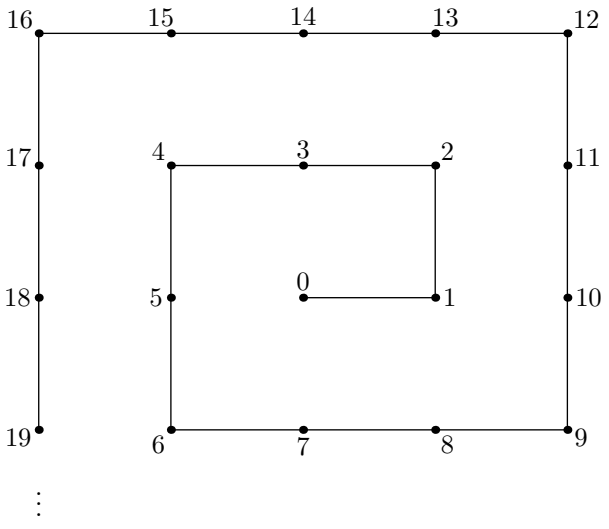

Figure 1: Problem 17

18. The diagram shown below consists of four isosceles triangles with congruent side lengths of 20 around a square of side length 24. When the four triangles are folded up, the resulting object is a triangular pyramid. Find the volume of the pyramid.

18. _____

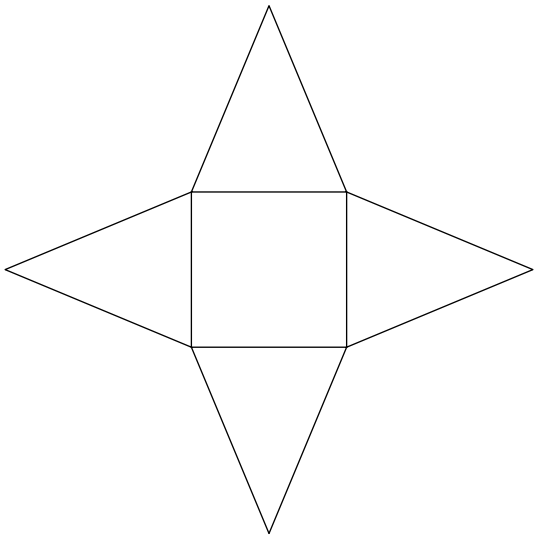

Figure 2: Problem 18

19. Compute the sum $0.\overline{01} + 0.\overline{03} + 0.\overline{05} + \cdots + 0.\overline{99}$.

19. _____

20. *Problem removed.*

20. _____