



**Contest Number R1**

Calculators are prohibited. Answers must be exact and satisfy conventions for simplification.

**December 12, 2012**

Name _____	Teacher _____	Grade Level _____	Score _____
<i>Time Limit: 35 minutes</i>			<i>Answer Column</i>
A1.	For what positive real number $a$ is the parabola $y = -x^2 + ax - 3$ tangent to the line $y = x + 4$ ?		A1.
G1.	Triangle $ABC$ has an angle bisector $AD$ such that $D$ is on side $BC$ . $E$ and $F$ are the feet of $D$ on $AB$ and $AC$ respectively ( $DE \perp AB$ and $DF \perp AC$ ). If $AE = 3$ , $EB = 2$ , $CF = 4$ , and $BC = 8$ , find the length of $AD$ .		G1.
T1.	There are 3 lights, each initially on. At the end of each second, one light is chosen at random and toggled. What is the expected number of lights that are on after 5 seconds?		T1.
A2.	A distribution consists of the integers from 1 to 100, inclusive, such that the frequency of each integer $n$ is $2^{n-1}$ . What is the sum of the terms in this sequence?		A2.
G2.	Let triangle $ABC$ have an area of $2012 \cdot AB^2$ . Extend the perpendicular of $AC$ passing through $A$ and the perpendicular of $AB$ passing through $B$ to meet at point $D$ . Compute the ratio $AC/AD$ .		G2.
T2.	A blind man on a train has a bag of 100 cookies, each of which is either a chocolate chip cookie or a raisin cookie. He chooses a random cookie and runs it through a cookie detector. If it is a chocolate chip cookie, the cookie detector has an 84% chance of telling the blind man it is a chocolate chip cookie and a 16% chance of telling him it is a raisin cookie. If the cookie is a raisin cookie, the cookie detector has a 96% chance of telling the blind man it is a raisin cookie and a 4% chance of telling the blind man it is a chocolate chip cookie. Given that the cookie detector tells the man it is a raisin cookie and there is a 30% chance that it is actually a chocolate chip cookie, how many chocolate chip cookies are in the bag?		T2.
A3.	Find the sum of the nonreal solutions to $3(x^2 - 2x + 2) = 3x - 1 + \sqrt{-x^4 + 9x^2 - 6x - 3}$ .		A3.
G3.	A diagonal is drawn on a $2012 \times 2015$ grid from the bottom-left corner to the top-right corner. Consider the triangles formed by two perpendicular grid lines and the diagonal, where the diagonal is above one of the legs if the grid is oriented with grid lines parallel to the axes, and the interiors of the triangles do not contain any portion of a grid line. Assign the leftmost triangle the number 1, the next leftmost 2, and so on. If $i$ is the assigned number of a triangle and $a_i$ is the area of the triangle whose number is $i$ , find $\sum_i ia_i$ .		G3.

		<i>Answer Column</i>
T3.	Define $\sigma(n)$ to be the sum of the positive divisors of an integer $n$ , and $\rho(n)$ to be the sum of the reciprocals of the positive divisors of an integer $n$ . How many positive integers $n$ satisfy $\sigma(n)/\rho(n) = 2012$ ?	T3.
A4.	Find $1^4 + 2^4 + \cdots 50^4$ .	A4.
G4.	Given triangle $ABC$ , let point $X$ be a point in the interior of triangle $ABC$ . Extend $AX$ to meet $BC$ at $D$ , $BX$ to $AC$ at $E$ , and $CX$ to $AB$ at $F$ . If $AF = FB = AX = FX = CE = 1$ , compute $AE$ .	G4.
T4.	<p>A woman once had a great fortune made of nickels and dimes. After she died, her will specified that a circle of coins be arranged such that the coins are equally spaced apart, 2012 nickels are on one half of the circle, and 2012 dimes are on the other half. She also declared that a dime be placed at the center of the circle. At the funeral, coins next to each other on the circle were bonded magically by light, and all the coins on the circle were bonded to the center dime. One fortunate grandson was chosen to place both of his feet on a nickel of his choice. He was told that a new nickel would suddenly appear in his hand. (This property holds true for the dimes as well.) How many paths can this grandson take if he can only move along the light bonds, he always steps on destination coins with both feet, and he wants to get three nickels and then get four dimes?</p> 	T4.