Week 2: AMC Combinatorics I

MSJ Math Club

September 27, 2012

1 Examples

- *(2010 AIME II)* Dave arrives at an airport which has twelve gates arranged in a straight line with exactly 100 feet between adjacent gates. His departure gate is assigned at random. After waiting at that gate, Dave is told the departure gate has been changed to a different gate, again at random. Find the probability that Dave walks 400 feet or less to the new gate.
- *(2005 AIME II)* A hotel packed breakfast for each of three guests. Each breakfast should have consisted of three types of rolls, one each of nut, cheese, and fruit rolls. The preparer wrapped each of the nine rolls and once wrapped, the rolls were indistinguishable from one another. She then randomly put three rolls in a bag for each of the guests. Find the probability that each guest got one roll of each type.
- *(2008 AMC 12A)* A permutation $(a_1, a_2, a_3, a_4, a_5)$ of $(1, 2, 3, 4, 5)$ is *heavy-tailed* if $a_1 + a_2 < a_4 + a_5$. What is the number of heavy-tailed permutations?

2 Practice Problems

Difficulty ranging from Mathcounts to hard AIME. You should try them all for maximum fun.

1. *(2002 AIME I)* Many states use a sequence of three letters followed by a sequence of three digits as their standard license-plate pattern. Given that each three-letter three-digit arrangement is equally likely, find the probability that such a license plate will contain at least one palindrome (a three-letter arrangement or a three-digit arrangement that reads the same left-to-right as it does right-to-left).
2. How many diagonals can you draw in a regular polygon of n sides?
3. *(2008 AMC 10A)* Jacob uses the following procedure to write down a sequence of numbers. First he chooses the first term to be 6. To generate each succeeding term, he flips a fair coin. If it comes up heads, he doubles the previous term and subtracts 1. If it comes up tails, he takes half of the previous term and subtracts 1. What is the probability that the fourth term in Jacob's sequence is an integer?
4. *(AoPS Intro to C/P)* There are 11 students in Mr. Palmer's History class, including the Baker triplets Annika, Billy, and Catherine. The teacher calls all 11 students in random order one at a time to his desk to tell each student his or her final grade. However, the first time he calls a student to his desk, Billy is in the bathroom and therefore can't be called up, but hes back in time for the rest of the students to be called. What is the probability that Billy is the first of the triplets to be called to the desk?

5. *(AoPS Intro to C/P)* What is the probability that if three points are chosen at random on the circumference of a circle, then the triangle formed by connecting the three points does not have a side with length greater than the radius of the circle?

6. *(2001 AIME II)* Club Truncator is in a soccer league with six other teams, each of which it plays once. In any of its 6 matches, the probabilities that Club Truncator will win, lose, or tie are each $\frac{1}{3}$. Find the probability that Club Truncator will finish the season with more wins than losses.

7. *(AoPS Intro to C/P)* Three points are selected randomly on the circumference of a circle. What is the probability that the triangle formed by these three points contains the center of the circle?

8. *(1988 AIME)* A convex polyhedron has for its faces 12 squares, 8 regular hexagons, and 6 regular octagons. At each vertex of the polyhedron one square, one hexagon, and one octagon meet. How many segments joining vertices of the polyhedron lie in the interior of the polyhedron rather than along an edge or a face?

9. *(2005 AIME I)* Robert has 4 indistinguishable gold coins and 4 indistinguishable silver coins. Each coin has an engraving of one face on one side, but not on the other. He wants to stack the eight coins on a table into a single stack so that no two adjacent coins are face to face. Find the number of possible distinguishable arrangements of the 8 coins.