

# MagMaR 2014

## Wild Round

Name: \_\_\_\_\_

School: \_\_\_\_\_

Team ID: \_\_\_\_\_

Grade: \_\_\_\_\_

Date: January 26, 2014

Problems: 40

Time: 20 minutes

Maximum Score:  $1 \times 40 = 40$

Type: Individual

Score: \_\_\_\_\_

### Do not start until instructed to do so!

Calculators, slide rules, books, computers, other electronic devices, are all prohibited. Similarly, graph paper, protractors, rulers, and compasses are not allowed at the competition. You may not collaborate with any other contestants during this round.

Please record your answers only in the blanks below; the ones provided on the test are only for convenience. Only answers recorded on this cover page will be graded.

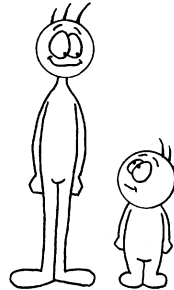
1.	2.	3.	4.	5.
6.	7.	8.	9.	10.
11.	12.	13.	14.	15.
16.	17.	18.	19.	20.
21.	22.	23.	24.	25.
26.	27.	28.	29.	30.
31.	32.	33.	34.	35.
36.	37.	38.	39.	40.

1. Compute  $(20 + 15)14 - (14 + 15)20$ .  
1. \_\_\_\_\_
2. In a future world, regular years (365 days) are to be split up into 5 regular-months and leap years (366 days) are to be split into 6 leap-months, so that each regular-month and leap-month has a fixed number of days in it. How many more days will there be in a regular-month than a leap-month?  
2. \_\_\_\_\_
3. Triangle  $ABC$  has side lengths of 3, 4, 5. What is its area?  
3. \_\_\_\_\_
4. Steven spends  $\frac{2}{5}$  of his monthly salary on rent while Amy spends  $\frac{1}{3}$  of her monthly salary on rent. If both spend the same amount of money on rent every month, what is the ratio of Steven's salary to Amy's salary?  
4. \_\_\_\_\_
5. How many of the first 2014 positive integers are divisible by 8?  
5. \_\_\_\_\_
6. What is the last digit of the sum  $1 + 22 + 333 + 4444 + \cdots + 999999999$ ?  
6. \_\_\_\_\_
7. While computing  $2^{25}$  for a certain math contest, Aaron received an answer of 33554433. If his answer is within 10 of the real value, compute actual value of  $2^{25}$ .  
7. \_\_\_\_\_
8. For every day that you are alive, starting today, you have a  $\frac{1}{2}$  chance of dying. What is the probability that you are still alive after the fourth day?  
8. \_\_\_\_\_
9. Five sheep are standing at the points  $(0, 0)$ ,  $(1, 3)$ ,  $(3, 0)$ ,  $(3, 4)$ , and  $(4, 0)$  on the plane. When the scientist presses "Start," each sheep immediately start walking towards its closest neighbor. How many loops (instances of sheep A following sheep B following other sheep following sheep A) are there?  
9. \_\_\_\_\_



10. At Asdfghjkl Middle School, 20% of students are short, 50% are happy, and 40% are neither short nor happy. If you select a short person from the school at random, what is the probability that he or she is happy?

10. \_\_\_\_\_



11. Your teacher is thinking of an integer between 1 and 100 inclusive, and whoever selects the closer number among you and your best friend wins. Selected integers must be distinct. Your best friend chooses the number 14. Assuming that you choose the number that maximizes your winning chances, what is the probability that you will win?

11. \_\_\_\_\_

12. You have four pencils in your backpack. Two are yellow and two are orange. You stick your hand into your backpack and grab two at once. What is the probability that one is orange and one is yellow?

12. \_\_\_\_\_

13. Nine coins are flipped. Find the probability that there are at least five heads.

13. \_\_\_\_\_

14. Compute  $201 \cdot 4 + 20 \cdot 14 + 2 \cdot 014$ .

14. \_\_\_\_\_

15. Brenda tries to compute the value of  $2014 - 14 \times 20$ , but does order of operations in the wrong order. What answer did she obtain?

15. \_\_\_\_\_

16. Jacob, Paul, and Andrew each start with \$80 in their pockets. Andrew steals half of Paul's money, then Jacob steals half of Andrew's money, and finally Paul steals half of Jacob's money. How much money does Paul have in the end?

16. \_\_\_\_\_



17. At the local supermarket, Brandon purchases an Apple Macbook on sale for \$10. If the original price is \$800, what percent *discount* did Brandon receive?  
17. \_\_\_\_\_
18. Suppose that  $2014L + 1000 = 2014 + 1000L$ . What is  $L$ ?  
18. \_\_\_\_\_
19. If the largest element of the set  $\left\{\frac{1}{2}, \frac{3}{4}, \frac{5}{6}, \dots, \frac{99}{100}\right\}$  is  $\frac{m}{n}$  in lowest terms, find  $m + n$ .  
19. \_\_\_\_\_
20. Alex dyes his hair a different color every day. If he can choose from cyan, magenta, yellow, or black every morning, and cannot have the same color hair on two consecutive days, how many different hair color schemes can he have in a 4-day block?  
20. \_\_\_\_\_
21. A square  $ABCD$  has integer side lengths and a diagonal of length at most 30. What is the maximal possible area of the square?  
21. \_\_\_\_\_
22. Alvin the Cow eats 10 kilograms of grass every day. If Alvin burns off 30 grams of excess weight in one minute of running, how long must Alvin run each day (in seconds) in order to not gain weight?  
22. \_\_\_\_\_



23. The mean of four distinct positive integers is 5. What is the largest possible value for the second-largest number?  
23. \_\_\_\_\_
24. A random permutation (or rearrangement) of the letters 'AAGMMRR' is chosen. What is the probability that either 'GRAMMAR' or 'MAGMARR' is selected?  
24. \_\_\_\_\_
25. The greatest common divisor (GCD) of two positive integers is 2, and the least common multiple (LCM) of the same two numbers is 140. How many different unordered pairs of integers satisfy this?  
25. \_\_\_\_\_

26. A triangle has side lengths of 20, 14, and  $m$ , where  $m$  is an integer. How many different values for  $m$  are there?  
26. \_\_\_\_\_
27. Given a triangle with two side lengths of 4 and 6 and an area of 12, find the length of the third side of the triangle.  
27. \_\_\_\_\_
28. The slope of the line connecting the points  $(1, 3)$  and  $(4, 7)$  is equal to the slope of the line connecting the points  $(-2, 5)$  and  $(x, 13)$ . Compute  $x$ .  
28. \_\_\_\_\_
29. An isosceles trapezoid has bases of length 20 and 14. If the area of the trapezoid is 102, what is the length of each of the legs (the sides of the trapezoid that are not parallel)?  
29. \_\_\_\_\_
30. How many ways can \$15 be paid off with some combination of quarters and dimes if at least one of each coin must be used?  
30. \_\_\_\_\_
31. Two mangoes and three pencils cost \$1714. Two mangoes and five pencils cost \$2014. How much would two mangoes and seven pencils cost?  
31. \_\_\_\_\_
32. During detention, Anna lists all ten-digit positive integers that use each of the ten digits exactly once, such as 1234567890 and 9876543210. What is the greatest common divisor (GCD) of all of the numbers that Anna writes?  
32. \_\_\_\_\_
33. On the  $n^{\text{th}}$  day of January 2014, your mean friend steals  $\$n$  dollars from you and your average friend donates  $\$(16 - n)$  to you. If you had \$100 at the start of the year, how much money will you have at the end of the fifteenth day of January?  
33. \_\_\_\_\_
34. Ryan sets his clock to emit a *ring* every 25 minutes and a *ding* every 45 minutes, starting at midnight. Later that day, he looks over to the clock just as it goes *ring* and *ding* at the same time, for the first time on that day. What does the clock say? (Express your answer in  $HH : MM$  notation, where  $HH$  represents the hour and  $MM$  represents the minutes.)  
34. \_\_\_\_\_



35. A 1-watt lightbulb is connected to several transformers, each of which doubles the power output. What is the fewest number of transformers needed in order for the final power level to be over 9000 watts?

35. \_\_\_\_\_

36. An Online Problemsolving Site (AOPS) and Acronym of Problemsolving Society (AoPS) both started in 2003, each with 1000 community members. The AOPS community grows by 10% every year, while AoPS attracts 100 new members every year. At the end of the third year, how many more members will AOPS have than AoPS?

36. \_\_\_\_\_

37. How many of the fractions in the set  $\left\{\frac{1}{7}, \frac{2}{8}, \frac{3}{9}, \dots, \frac{93}{99}, \frac{94}{100}\right\}$  are fully reduced?

37. \_\_\_\_\_

38. Compute  $\sqrt{2^{16} + 2^9 + 1}$ .

38. \_\_\_\_\_

39. The problem writers for a self-centered math contest count the total number of problems that mention the contest year, and divide that number by 135, the total number of problems. What is the probability that the computed decimal has an infinite number of decimal digits? (For example,  $1/135 = 0.00740740740\dots$ , which has an infinite number of decimal digits.)

39. \_\_\_\_\_

40. Your English teacher is very slow at grading papers. On her own, she can finish grading the essays in 30 days, but if both of you grade the papers together, you two can finish in 20 days. How many days would you take you to grade the essays on your own, assuming that each of you work at a constant rate?

40. \_\_\_\_\_

