Magnetism

MSJ Physics Club

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It should be noted that these handouts are concise and do not take the place of a full physics class. A bit of familiarity with calculus is assumed.

6 Magnetic Fields

Magnetism is related to electricity. You probably have seen it when magnets interact and on refrigerators and stuff. The magnetic field at a point is denoted $\vec{B}$. and points away from the north pole of a magnet towards the south pole. You can think of the north and south poles of a magnet as magnetic charges (like + and − for electricity), but magnetic monopoles have never been found in nature. (Doesn't mean they necessarily don't exist!).

7 Dot Product

Before we get to magnetism, we should cover some math. The cross product of two vectors $\vec{a} \cdot \vec{b}$ is defined as the scalar $ab \cos\theta$ where θ is the angle between the two vectors.

7.1 Cross product

The cross product of two vectors $\vec{a}$ and $\vec{b}$ is defined as the vector perpendicular to both vectors with magnitude $ab \sin\theta$ where θ is the angle between the two vectors. Naturally, there are two possible directions for this vector. If you point you forefinger in the direction of the first vector and you middle finger in the direction of the second, your thumb is the direction of the product. (1st Right-Hand rule)

7.2 Magnetic force on a Moving Charge

The force $\vec{F}$ exerted on a charge q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by:

$$\vec{F} = q\vec{v} \times \vec{B}$$

The above equation defines the magnetic field $\vec{B}$ in terms of the force exerted on a moving charge. The SI unit for magnetic field is tesla (T).

When a wire carries a current into a magnetic field, there is a force on the wire that is equal to the sum of the magnetic forces on the charged particles within. If a wire has cross-sectional area A and length L carrying current I, the total force on the wire segment is:

$$\vec{F} = (q\vec{v} \times \vec{B})nAL = I\vec{L} \times \vec{B}$$

7.3 Magnetic Field Generated By a Wire; Ampere's Law

The magnetic field generated by a current carrying wire is in rings around the wire. You can find the direction using the second right-hand rule: Imagine your hand wrapped around a wire with your thumb pointed in the direction of the current. Your fingers are pointed in the direction of the magnetic field generated.

The strength of the magnetic field is given by the following formula:

$$\vec{B} = \frac{\mu_0 I}{2\pi r}$$

There's a generalization to this called Ampere's law, which states:

$$\oint \vec{B} \cdot dl = \mu_0 I$$

Basically, we have a imaginary closed path around a current carrying wire. The sum of the magnetic field multiplied (dot product) by the infinite infinitesimal lengths is equal to the right side of the equation.

7.4 Maxwell's Correction to Ampere's Law

James Clerk Maxwell realized that a current might not always pass through a ring to create a magnetic field, in the case of a capacitor (sth that is able to store charge). This type of storage always creates an increasing electric field, so Maxwell's correction added a term to Ampere's equation:

$$\oint \vec{B} \cdot dl = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi}{dt}$$

Where Φ is the electric flux through the ring.

8 Practice

1. An infinitely long charged insulating cylinder has a uniform charge density of q_m (charge per unit length) and radius r. This cylinder is then rotated about its axis at an angular velocity ω. Find the magnitude of the magnetic field at the center of the cylinder.

2. A charged metal sphere moves at a constant velocity v along the axis of a ring. At time $t = 0$ the sphere is at the exact center. Find the magnetic field at the ring.

3. A charged particle with mass m and charge q is moving with velocity v in a plane perpendicular to a uniform magnetic field B. Find the period of the particle's circular motion.

4. A circular wire loop of radius R, mass m, and current I lies on a horizontal surface. There is a horizontal field $\vec{B}$. How large can the current I be before one edge of the loop will lift off the surface?

5. A current carrying wire is bent into a semicircular loop of radius R that lies in the xy plane. There is a uniform magnetic field $\vec{B} = B\hat{k}$ perpendicular to the plane of the loop. Verify that the force acting on the loop is 0.