CP Algorithms Handbook

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1 Sorting and Searching

1.1 Quick Sort

```
void qwik_sort(int p, int q, int tab[]){
       int a = p, b = q;
int m = tab[(p+q)/2];
3
       while(a <= b){</pre>
5
           while(m > tab[a])
6
                a++;
           while(m < tab[b])</pre>
8
9
               b--;
           if(a <= b){
10
                swap(tab[a],tab[b]);
11
12
                    a++;
13
           }
14
15
     if(b > p)
16
17
           qwik_sort(p, b, tab);
       if(a < q)
18
           qwik_sort(a, q, tab);
19
20 }
```

1.2 Merge Sort

```
void merge(int left, int mid, int right, int t[]){
     int a = left, b = mid;
      int i = left;
      while(a < mid && b < right)</pre>
          tab[i++] = (t[a] < t[b] ? t[a++] : t[b++]);
      while(a < mid)</pre>
          tab[i++] = t[a++];
    while(b < right)</pre>
9
          tab[i++] = t[b++];
      for(int k = left; k < right; k++)</pre>
10
          t[k] = tab[k];
11
12 }
13
void mergesort(int left, int right, int t[]){
   if(left+1 != right){
       int m = (left+right)/2;
16
17
          mergesort(left, m, t);
         mergesort(m, right, t);
18
          merge(left, m, right, t);
19
      }
20
```

2 Dynamic Programming

2.1 State representaion

2.1.1 All subsets

This way you can generate all subsets in order ready for DP.

```
int sub = whole_set;
while(sub > 0){

sub = (sub-1)&whole_set;
}
```

3 Graph Algorithms

3.1 Shortest Path

3.1.1 Djikstra

```
priority_queue < pair < long long int ,int >> q;
2 \text{ dist}[1] = 0;
3 q.push({0, 1});
4 while(!q.empty()){
      int a = q.top().second;
       q.pop();
       if(vis[a]) continue;
       vis[a] = true;
       for(auto v : adj[a]){
   int b = v.first;
9
10
            int w = v.second;
11
            if (dist[b] > dist[a] + w) {
    dist[b] = dist[a] + w;
12
13
                  q.push({-dist[b], b});
14
15
            }
16
       }
17 }
```

3.1.2 Belman-Ford

```
1 dist[1] = 0;
2 for(int i = 1; i <= n-1; i++){</pre>
        for(auto e : edges){
   int a, b, w;
             tie(a, b ,w) = e;
if(dist[b] > dist[a]+w){
    par[b] = a;
6
                   dist[b] = dist[a]+w;
             }
9
10
11 }
12 for(auto e : edges){
     int a, b, w;
tie(a, b ,w) = e;
13
14
      if(dist[b] > dist[a]+w){
15
             found = true;
16
             node = b;
17
             par[b] = a;
dist[b] = dist[a]+w;
19
20 }
```

3.1.3 Floyd-Warshal

```
1 long long int dist[SIZE][SIZE] {};
2 for(int i = 0; i <= n; i++){</pre>
      for(int k = 0; k <= n; k++){
            dist[i][k] = __INT64_MAX__/2;
4
5
6 }
7 for(int i = 1; i <= n; i++){</pre>
      dist[i][i] = 0;
       for(auto v : adj[i]){
9
           int b = v.first;
10
           long long int w = v.second;
11
           dist[i][b] = min(w, dist[i][b]);
dist[b][i] = min(w, dist[i][b]);
12
13
14
15 }
16
for(int k = 1; k <= n; k++){</pre>
     for(int i = 1; i <= n; i++){
18
           for(int j = 1; j <= n; j++){</pre>
19
                dist[i][j] = min(dist[i][j], dist[i][k] + dist[k][j]);
20
21
22
23 }
```

3.2 Union Find

```
int s_size[MX];
2 int link[MX];
4 int max_size = 0;
5 int num;
7 int find(int x){
    while(x != link[x])
         x = link[x];
9
     return x;
10
11 }
12
bool same(int a, int b){
14
   return find(a) == find(b);
15 }
16
void unite(int a, int b){
     a = find(a);
18
     b = find(b);
     if(s_size[a] < s_size[b])</pre>
20
         swap(a, b);
21
    s_size[a] += s_size[b];
22
    max_size = max(max_size, s_size[a]);
23
      link[b] = a;
24
25
      num --;
26 }
27
28 int main(){
for(int i = 0; i < MX; i++){
          s_size[i] = 1;
30
          link[i] = i;
31
32
33
     int n, m;
34
35
    cin >> n >> m;
     num = n;
for(int i = 0; i < m; i++){</pre>
36
37
       int a, b;
cin >> a >> b;
38
39
          if(!same(a, b))
40
          unite(a, b);
cout << num << " " << max_size << endl;
41
42
43
     return 0;
44
45 }
```

3.3 Path through all edges

```
set <int> adj[MX];
3 bool vis[MX];
 5 void zero(){
for(int i = 0; i < MX; i++)
vis[i] = false;</pre>
8 }
9
int dfs1(int a){
    int ret = 0;
11
     vis[a] = true;
for(int b : adj[a]){
12
13
          ret += 1;
14
           if(!vis[b])
15
               ret += dfs1(b);
16
17
18
      return ret;
19 }
20
21 int main(){
int n, m;
cin >> n >> m;
```

```
for(int i = 0; i < m; i++){
   int a, b;
   cin >> a >> b;
24
25
26
             adj[a].insert(b);
27
28
             adj[b].insert(a);
29
30
31
        zero();
       if (dfs1(1)/2 != m) {
    cout << "IMPOSSIBLE";</pre>
32
33
             return 0;
34
35
36
       for(int i = 1; i <= n; i++){
   if(adj[i].size() & 1){</pre>
37
38
                  cout << "IMPOSSIBLE";</pre>
39
                  return 0;
40
             }
41
42
43
        vector < int > ans;
45
46
        stack<int> s;
47
        s.push(1);
        while(!s.empty()){
48
49
            int a = s.top();
50
             if(adj[a].size() == 0){
                 ans.push_back(a);
51
                  s.pop();
53
             }
             else{
54
                  int b = *adj[a].begin();
                  adj[a].erase(adj[a].begin());
56
                  adj[b].erase(a);
57
                  s.push(b);
58
             }
59
60
61
       for(int a : ans)
62
            cout << a << " ";
63
64
65
        return 0;
66
67 }
```

3.4 Topo Sort

```
int n, m;
stack <int> res;
3 vector < int > g[100007];
4 int vis[100007];
5 bool imp = false;
7 void dfs(int n){
     vis[n] = 1;
for(int v : g[n]){
9
10
           if(vis[v] == 0)
               dfs(v);
11
           else if(vis[v] == 1)
12
13
               imp = true;
14
      vis[n] = 2;
15
16
      res.push(n);
17 }
18
19 int main(){
     cin >> n >> m;
20
       for(int i = 0 ; i < m; i++){</pre>
21
           int a, b;
22
           cin >> a >> b;
23
           g[a].push_back(b);
24
25
26
```

```
for(int i = 1; i <= n; i++){
   if(vis[i] == 0){</pre>
27
28
                  dfs(i);
29
30
31
32
        if(!imp){
33
34
             while(!res.empty()){
                   cout << res.top() << " ";
35
36
                   res.pop();
37
        }
38
        else{
             cout << "IMPOSSIBLE";</pre>
40
41
42
        return 0;
43
44 }
```

3.5 Flows and Cuts

All problems in this section can be solved using the same basic algorithm defined in 3.5.1.

3.5.1 Maximum Flow

This implementation of Ford-Fulkerson algorithm (known as Edmonds-Karp algorithm) uses BFS to check if it's possible to expand the flow through the graph and assign depths from the source. Later it uses DFS to expand te flow.

```
1 ll flow[MX][MX];
2 11 used[MX][MX];
4 vector < int > adj[MX];
5 vector<int> radj[MX];
7 bool vis[MX];
9 bool bfs(int s, int lvl[], const int n){
       queue < int > q;
       q.push(s);
       lv1[s] = 1;
13
       while(!q.empty()){
           int p = q.front();
14
15
           q.pop();
           for(int v : adj[p]){
16
                if(lvl[v] == 0 and flow[p][v] - used[p][v] > 0){
    lvl[v] = lvl[p]+1;
18
                     q.push(v);
19
                }
20
21
           for(int v : radj[p]){
22
                if(lvl[v] == 0 and used[v][p] > 0){
                     lvl[v] = lvl[p]+1;
24
                     q.push(v);
25
26
27
           }
       }
28
       return lvl[n] > 0;
29
30 }
31
  int dfs(int x, ll val, const int lvl[], const int n){
32
       vis[x] = true;
if(x == n){
33
34
           return val;
35
36
       for(int v : adj[x]){
37
            if(lvl[v] == lvl[x]+1 \text{ and } flow[x][v] - used[x][v] > 0 \text{ and } !vis[v]){
38
                int r = dfs(v, min(val, flow[x][v]-used[x][v]), lvl, n);
39
                if(r > 0){
40
                     used[x][v] += r;
41
                     return r;
                }
43
           }
```

```
45
       for(int v : radj[x]){
46
           if(lvl[v] == lvl[x]+1 \text{ and } used[v][x] > 0 \text{ and } !vis[v])
47
                int r = dfs(v, min(val, used[v][x]), lvl, n);
48
                 if(r > 0){
49
                     used[v][x] -= r;
50
51
                     return r;
52
           }
53
54
       }
55
       return 0;
56 }
```

To find the flow:

```
long long int res = 0;

while(bfs(1, lvl, n)){
    res += dfs(1, __INT_MAX__, lvl, n);
    for(int i = 0 ; i < MX; i++){
        vis[i] = false;
        lvl[i] = 0;
}
</pre>
```

3.5.2 Minimum Cut

To find Minimum Cut in a graph we need to find the Maximum Flow and check which edges connect two created disjont sets of nodes. This modification to finds them:

```
long long int res = 0;
        while(bfs(1, lvl, n)){
            res += dfs(1, __INT_MAX__, lvl, n);
for(int i = 0 ; i < MX; i++){
                  vis[i] = false;
                  lvl[i] = 0;
7
             }
8
9
        cout << res << endl;</pre>
10
        for(int i = 0 ; i < MX; i++){
    vis[i] = false;</pre>
11
12
             lv1[i] = 0;
13
14
15
16
       bfs(1, lvl, n);
        for(int a = 1; a <= n; a++){</pre>
17
             if(lvl[a] > 0){
18
19
                  for(int b : adj[a]){
20
                       if(lvl[b] == 0){
                            cout << a << " " << b << endl;
21
22
                  }
23
             }
24
```

3.5.3 Maximum Matching

3.6 2SAT

Given logical formula in the conjunctive normal form:

$$(a_1 \vee b_1) \wedge (a_2 \vee b_2) \wedge ... \wedge (a_n \vee b_n)$$

we can eliminate disjunction by repacing each $(a_i \vee b_i)$ element with pair:

$$\neg a_i \to b_i \land \neg b_i \to a_i$$

```
vector < int > adj [MX];
vector < int > Tadj [MX];
3 stack<int> scc_stack;
5 int scc[MX];
7 bool vis[MX];
8 int scc_val[MX];
int not_node(int a){
    if(!(a&1))
11
           return a+1:
12
13
         return a-1;
14
15 }
16
void dfs1(int a){
      vis[a] = true;
18
       for(int b : adj[a]){
19
           if(!vis[b])
20
21
                dfs1(b);
22
23
       scc_stack.push(a);
24 }
25
void dfs2(int a, int scc_n){
    vis[a] = true;
for(int b : Tadj[a]){
27
28
           if(!vis[b])
                dfs2(b, scc_n);
30
31
32
       scc[a] = scc_n;
33 }
34
35 void zero(){
   for(int i = 0 ; i < MX; i++)
    vis[i] = false;</pre>
36
37
38 }
39
40 int main(){
      int n, m;
41
42
       cin >> n >> m;
       for(int i = 0 ; i < n; i++){</pre>
43
           char op;
44
45
           int a, b;
           cin >> op >> a;
if(op == '+'){
46
47
                a *= 2;
           }
49
50
            else{
               a *= 2;
51
                a++;
52
           }
53
           cin >> op >> b;
if(op == '+'){
54
55
                b *= 2;
56
57
            else{
58
                b *= 2;
59
                b++;
60
61
            Tadj[a].push_back(not_node(b));
62
            Tadj[b].push_back(not_node(a));
63
            adj[not_node(b)].push_back(a);
65
            adj[not_node(a)].push_back(b);
66
67
       for(int i = 1 ; i <= 2*m+1; i++){</pre>
68
           if(!vis[i])
69
                dfs1(i);
70
71
72
73 zero();
```

```
74     int scc_n = 1;
 75
        while(!scc_stack.empty()){
           int a = scc_stack.top();
 76
            scc_stack.pop();
 77
            if(scc[a] == 0){
 78
                dfs2(a, scc_n);
 79
 80
                 scc_n++;
 81
       }
 82
        for(int i = 2; i <= m*2; i+=2){</pre>
 84
            if(scc[i] == scc[i+1]){
 85
                cout << "IMPOSSIBLE" << endl;</pre>
                 return 0;
 87
            }
 88
 89
        // 1 - false; 2 - true;
for(int i = 2; i <= 2*m; i+=2){
 90
 91
            int a = scc[i];
 92
            int b = scc[i+1];
 93
            if(scc_val[a] != 0){
                scc_val[b] = (scc_val[a] == 1 ? 2 : 1);
 95
            }
 96
 97
            if(scc_val[b] != 0){
                 scc_val[a] = (scc_val[b] == 1 ? 2 : 1);
98
99
100
            if(a < b){
                scc_val[a] = 1;
101
                 scc_val[b] = 2;
102
            }
103
            else{
104
                 scc_val[a] = 2;
105
                 scc_val[b] = 1;
106
107
108
109
       for(int i = 2; i <= m*2; i+=2)</pre>
110
            cout << (scc_val[scc[i]] == 2 ? "+" : "-") << " ";
112
113
        return 0;
114 }
```

4 Range Queries

4.1 Segment trees

BASE size table:

4.1.1 Point-Range Trees

```
int sal[2*BASE];
3 void add(int p, int val){
      p += BASE;
      sal[p] += val;
      while(p > 0){
6
          p >>= 1;
           sal[p] = sal[2*p] + sal[2*p+1];
9
10 }
11
int query(int p, int q){
      int ret =0 ;
13
      p += BASE -1;
14
     q += BASE +1;
15
    while (p>>1 != q>>1) {
    if (not (p&1)) {
16
17
```

```
18          ret += sal[p+1];
19          }
20          if(q&1){
21          ret += sal[q-1];
22          }
23          p >>= 1;
24          q >>= 1;
25     }
26     return ret;
27 }
```

4.1.2 Range-Range Trees

1. range ADD insert, range MAX value query

```
void max_add(int a, int b, int val, int k=1, int x=0, int y=BASE-1){
      if(a <= x and y <= b){</pre>
          max_tree[k][1] += val;
3
           return;
      }
5
      max_tree[2*k][1] += max_tree[k][1];
6
      max_tree[2*k+1][1] += max_tree[k][1];
      max_tree[k][0] += max_tree[k][1];
8
9
      max\_tree[k][1] = 0;
10
11
      int d = (x+y)/2;
       if(a <= d){
12
          max_add(a, b, val, 2*k, x, d);
13
14
           }
15
       if(b > d){
          max_add(a, b, val, 2*k+1, d+1, y);
16
17
      \max_{\text{tree}}[k][0] = \max(\max_{\text{tree}}[2*k+1][0] + \max_{\text{tree}}[2*k+1][1], \max_{\text{tree}}[2*k][0]
18
       + max_tree[2*k][1]);
20 }
1 ll max_query(int a, int b, int k=1, int x=0, int y=BASE-1){
     if(a \le x and y \le b){
           return max_tree[k][0] + max_tree[k][1];
3
4
5
      max_tree[2*k][1] += max_tree[k][1];
      max_tree[2*k+1][1] += max_tree[k][1];
6
       max_tree[k][0] += max_tree[k][1];
      max_tree[k][1] = 0;
       int d = (x+y)/2;
9
       11 ret = -INF;
10
      if(a <= d){</pre>
11
```

2. range ADD insert, range SUM querry

 $if(b > d){$

return ret:

12 13

14 15

16

17

ret = max(ret, max_query(a, b, 2*k, x, d));

ret = max(ret, max_query(a, b, 2*k+1, d+1, y));

```
void sum_add(int a, int b, int val, int k=1, int x=0, int y=BASE-1){
      if(a \le x and y \le b){
2
          sum_tree[k][1] += val;
3
          return;
4
5
      sum_tree[2*k][1] += sum_tree[k][1];
6
      sum_tree[2*k+1][1] += sum_tree[k][1];
7
      sum\_tree[k][0] += sum\_tree[k][1]*(y-x+1);
8
      sum_tree[k][1] = 0;
9
      int d = (x+y)/2;
10
      if(a <= d){
11
          int w = (d \le b ? d : b)+1;
12
          w -= (x \le a ? a : x);
13
          sum_tree[k][0] += val*(w);
          sum_add(a, b, val, 2*k, x, d);
15
16
```

```
if(b > d){
           int w = (y \le b ? y : b)+1;
           w -= (d+1 \le a ? a : d+1);
19
           sum_tree[k][0] += val*(w);
20
21
           sum_add(a, b, val, 2*k+1, d+1, y);
22
23 }
1 ll sum_query(int a, int b, int k=1, int x=0, int y=BASE-1){
      if(a <= x and y <= b){</pre>
           return sum_tree[k][0] + sum_tree[k][1]*(y-x+1);
3
4
       sum_tree[2*k][1] += sum_tree[k][1];
5
       sum_tree[2*k+1][1] += sum_tree[k][1];
6
       sum_tree[k][0] += sum_tree[k][1]*(y-x+1);
sum_tree[k][1] = 0;
       int d = (x+y)/2;
9
      ll ret = 0;
10
      if(a <= d){</pre>
11
          ret += sum_query(a, b, 2*k, x, d);
12
13
      if(b > d){
14
          ret += sum_query(a, b, 2*k+1, d+1, y);
15
17
       return ret;
18 }
int sal[2*BASE];
```

4.2 Max Subarray Sum

```
1 11 ssum [2*BASE];
2 11 prfx[2*BASE];
3 11 sufx[2*BASE];
4 11 tsum [2*BASE];
6 void insert(int val, int p){
      p += BASE;
       ssum[p] = val;
prfx[p] = val;
9
       sufx[p] = val;
10
       tsum[p] = val;
11
       int 1, r;
12
       while(p){
13
14
            p >>= 1;
15
            1 = 2*p;
            r = 2*p+1;
16
17
            tsum[p] = tsum[1] + tsum[r];
            prfx[p] = max(prfx[1], tsum[1] + prfx[r]);
sufx[p] = max(sufx[1] + tsum[r], sufx[r]);
18
19
            ssum[p] = max(sufx[1] + prfx[r], max(ssum[1], ssum[r]));
21
22 }
```

5 Tree Algorithms

5.1 Binary Lifitng

```
int main(){
   int n, q;
   cin >> n >> q;
   boss[1][0] = 0;
   for(int i = 2; i <= n; i++){
      cin >> boss[i][0];
}

for(int i = 1; i < LOG_MX; i++){
   for(int emp = 1; emp <= MX; emp++){
      boss[emp][i] = boss[boss[emp][i-1]][i-1];
}

}</pre>
```

5.2 Tree Traversal

```
vector <int> adj[MX];
1 int subtree[MX];
3 int flat[MX];
4 int vals[MX];
6 int idx = 1;
7 int dfs(int a, int p){
      flat[a] = idx;
      idx++;
9
     for(int b : adj[a]){
10
          if(b == p)
11
             continue;
12
          subtree[a] += dfs(b, a)+1;
13
14
      return subtree[a];
15
```

5.3 LCA

LCA can be found using binary lifting or by building a segment tree build on pre- and postorder.

6 String Algorithms

6.1 Trie

```
int trie[TMX][30];
bool stop[TMX];
3 int next_node = 1;
4 int dp[MX];
6 void insert(string s){
     int idx = 0;
      for(char c : s){
          if(trie[idx][c-'a'] == 0){
9
              trie[idx][c-'a'] = next_node;
              next_node ++;
11
          }
12
13
          idx = trie[idx][c-'a'];
14
15
      stop[idx] = true;
16 }
17
18 int main(){
     string text;
19
20
      int n;
21
      cin >> text >> n;
      for(int i = 0; i < n; i++){</pre>
22
          string s;
          cin >> s;
24
          insert(s);
25
```

6.2 Pattern Finding

6.2.1 KMP

```
pi[i] = j;

int res = 0;

for(int i = 0; i < s.size(); i++){
    if(pi[i] == pattern.size())
        res++;

cout << res;
return 0;</pre>
```

6.2.2 Hashing

```
const int MOD = 1e9+9;
2 const int P = 9973;
3 const int MX = 1e6+7;
5 11 ppow[MX];
6 ll pfx_hash[MX];
8 int res = 0;
10 int main(){
      string s, pattern;
11
      cin >> s >> pattern;
12
      ppow[0] = 1;
13
      for (int i = 1 ; i < MX; i++) {</pre>
14
       ppow[i] = (ppow[i-1]*P)%MOD;
}
15
17
     11 pattern_hash = 0;
for(int i = 0; i < pattern.size(); i++){</pre>
18
19
           pattern_hash = (pattern_hash + (pattern[i]-'a'+1)*ppow[i])%MOD;
20
21
22
      for(int i = 0; i < s.size(); i++){</pre>
23
           pfx_hash[i+1] = (pfx_hash[i] + (s[i]-'a'+1)*ppow[i])%MOD;
24
25
26
27
      for(int i = 0; i+pattern.size()-1 < s.size(); i++){</pre>
           int a = i, b = i+pattern.size();
ll sub_hash = pfx_hash[b] - pfx_hash[a]+MOD;
28
29
           sub_hash %= MOD;
30
           if(sub_hash == (pattern_hash*ppow[a])%MOD){
31
               res++;
33
     }
34
     cout << res << endl;</pre>
35
    return 0;
36
```

6.3 Palindormes

Fiding longest palindromic substring.

```
int p[MX];
3 int main(){
        string txt1, txt = "#";
        cin >> txt1;
        for(char c : txt1){
6
             string s{c};
             txt += s + "#";
8
9
        txt = "^" + txt + "$";
10
        int l = 1, r = 1;
for(int i = 1; i < txt.size(); i++){
   p[i] = max(0, min(r-i, p[l+r-i]));</pre>
11
12
13
             while(txt[i-p[i]] == txt[i+p[i]])
14
15
                  p[i]++;
             if(i+p[i] > r){
16
                  1 = i-p[i];
r = i+p[i];
17
18
19
        }
20
21
        int maxi = 0;
        int imaxi;
for(int i = 0; i < txt.size(); i++){</pre>
22
23
             if(p[i] > maxi){
24
                  maxi = p[i];
imaxi = i;
25
26
27
        }
28
        string res = txt.substr(imaxi-maxi+1, maxi*2-1);
        for(auto c : res){
   if(c != '#')
30
31
                  cout << c;
32
```

7 Mathematics

7.1 Fermat's Theorem

Modular inverse:

If m is prime, then: $x^{-1} \mod m = x^{m-2} \mod m$

$$x^{\varphi(m)} \, mod \, m = x^{k \, mod \, (m-1)} \, mod \, m$$

```
int exp(int a, int b, int MOD){
    if(b == 0)
        return 1;
    if(b & 1){
        return ((ll)a * exp(a, b-1, MOD))%MOD;
    }
    ll tmp = exp(a, b/2, MOD);
    return (tmp*tmp)%MOD;
}
```

7.2 Fast Fibonacci

```
void mul(11 a[][2], 11 b[][2]){
    ll res[2][2];
    res[0][0] = a[0][0]*b[0][0] + a[0][1]*b[1][0];
    res[0][1] = a[0][0]*b[0][1] + a[0][1]*b[1][1];
    res[1][0] = a[1][0]*b[0][0] + a[1][1]*b[1][0];
    res[1][1] = a[1][0]*b[0][1] + a[1][1]*b[1][1];
```

```
a[0][0] = res[0][0] % MOD;
8
       a[0][1] = res[0][1] % MOD;
9
       a[1][0] = res[1][0] % MOD;
a[1][1] = res[1][1] % MOD;
10
11
12 }
13
14 int main(){
       11 n;
       cin >> n;
16
       11 m_pow[2][2] =
17
                 {{0, 1},
18
19
                  {1, 1}};
       11 m[2][2] =
20
                 {{1, 0},
21
                  {0, 1}};
22
23
       auto pow = bitset <64> (n);
24
25
       for(int i = 0 ; i < 64; i++){</pre>
26
27
            if(pow[i]){
                mul(m,m_pow);
28
            }
29
            mul(m_pow, m_pow);
30
31
32
33
       cout << m[0][1];
34
       return 0;
36 }
```

8 Geometry

8.1 Nie wiem co to robi

```
#include <iostream>
3 using namespace std;
5 struct P {
      int x, y;
      void read() {
          cin >> x >> y;
9
      P operator-(const P &b) const {
10
11
          return P{x - b.x, y - b.y};
12
      void operator -= (const P &b) {
13
       x = b.x;
14
          y -= b.y;
15
16
      long long operator *(const P& b) const {
17
          return (long long)x * b.y - (long long) y * b.x;
18
19
20
21
      bool operator <(const P& b) const;</pre>
22 };
23
P most_left;
25
26 bool P::operator < (const P& b) const {</pre>
27
     long long res = (*this - most_left) * (b - most_left);
      if (res == 0) {
28
          return x < b.x;</pre>
29
31
      return res > 0;
32 }
33 int main() {
34
     int t;
      cin >> t;
35
      while (t--) {
36
   P a, b, c;
```

```
a.read();
b.read();
c.read();

long long res = (b - a) * (c - a);
if (res > 0)
cout << "LEFT\n";
else if (res < 0) {
    cout << "RIGHT\n";
} else {
    cout << "TOUCH\n";
}
</pre>

return 0;
```

9 Other Algorithms