1) a) $(x_1, y_1) = (0.0 \pm 0.2,\ 0.0 \pm 0.3)$

$(x_2, y_2) = (3.0 \pm 0.3,\ 4.0 \pm 0.2)$

$\Delta x = 3.0 - 0.0 = 3.0$ $\qquad \Delta y = 4.0 - 0.0 = 4.0$

$\delta x = \sqrt{0.2^2 + 0.3^2} = 0.4$ $\qquad \delta y = \delta x = 0.4$

$d = \sqrt{x^2 + y^2}$

$\theta = \arctan(y/x)$

$$\delta d = \sqrt{\left[\tfrac{1}{2}(x^2+y^2)^{-1/2}(2x\delta x)\right]^2 + \left[\tfrac{1}{2}(x^2+y^2)^{-1/2}(2y\delta y)\right]^2}$$

$$= \sqrt{\tfrac{1}{4}(3^2+4^2)^{-1}(6\cdot 0.4)^2 + \tfrac{1}{4}(3^2+4^2)^{-1}(8\cdot 0.4)^2}$$

$$= \sqrt{0.0576 + 0.1024} = \sqrt{.16} = 0.4$$

$$\boxed{d = 5 \pm 0.4}$$

$$\delta\theta = \sqrt{\left[\tfrac{-y}{x^2}\left(y^2/x^2 + 1\right)^{-1}\delta x\right]^2 + \left[\tfrac{1}{x}\left(y^2/x^2 + 1\right)^{-1}\delta y\right]^2}$$

$$\sqrt{0.004096 + 0.000256} = 0.07_{\text{rad}}$$

$$\boxed{\begin{aligned}\theta &= 0.93 \pm 0.07 \text{ rad} \\ &= 53 \pm 4^\circ\end{aligned}}$$

1) b) $N = N_0 e^{-x/\lambda}$

$$N = (1000\text{E}6)\, e^{-1/0.25} = 1000 e^{-4} = 18.32\text{E}6$$

$$\delta N^2 = \left(\frac{\partial N}{\partial N_0}\delta N_0\right)^2 + \left(\frac{\partial N}{\partial x}\delta x\right)^2 + \left(\frac{\partial N}{\partial \lambda}\delta\lambda\right)^2$$

$$= \left(e^{-x/\lambda}\cdot\delta N_0\right)^2 + \left(\frac{-N_0}{\lambda} e^{-x/\lambda}\delta x\right)^2 + \left(N_0 x/\lambda^2\, e^{-x/\lambda}\delta\lambda\right)^2$$

$$= e^{-2x/\lambda}\left(\delta N_0^2 + \left(\frac{N_0\delta x}{\lambda}\right)^2 + \left(\frac{N_0 x\,\delta\lambda}{\lambda^2}\right)^2\right)$$

$$\delta N = \underset{0.0183}{e^{-x/\lambda}} \cdot \underset{9.6\text{E}8}{\left(25\text{E}12 + 1.6\text{E}15 + 9.2\text{E}17\right)^{1/2}}$$

$$\delta N = 1.7\text{E}7$$

$$\boxed{N = 18 \pm 17 \times 10^6 \text{ particles}}$$

1) c)

$$y = a + bx$$

$a = 2.5 \pm 0.3$ m
$b = (5.0 \pm 0.1)E^{-2}$
$x = 4$m, 4.0 ± 0.1 m

$$y = 2.5 + 4 \cdot 0.05$$
$$= 2.7$$

$$u = bx$$
$$\delta u = |x|\,\delta b \quad \text{or} \quad |bx|\sqrt{\left(\frac{\delta b}{b}\right)^2 + \left(\frac{\delta x}{x}\right)^2}$$
$$= 0.004 \qquad = 0.2\sqrt{0.004 + 0.00625} = 0.02$$

$$\delta y = \sqrt{\delta a^2 + \delta b^2}$$
$$= \sqrt{0.09041}$$
$$= 0.3$$

$$\boxed{y = 2.7 \pm 0.3 \text{ m}}$$

For both cases, δa is the major source of uncertainty, so δy is unchanged.

d)

$$c_M = aT + bT^3$$
$$= u + v$$

$a = 1.35 \pm 0.05$ mJ/mol·K²
$b = 0.021 \pm 0.001$ mJ/mol·K⁴
$T = 5.0 \pm 0.5$ K

$$\delta u = \sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta T}{T}\right)^2}$$

$$\delta v = \sqrt{\left(\frac{\delta b}{b}\right)^2 + \left(\frac{3\,\delta T}{T}\right)^2}$$

$$\frac{\delta u}{u} = 0.11 \qquad \frac{\delta v}{v} = 0.30$$

$$\delta c_M = \sqrt{\left(u\frac{\delta u}{u}\right)^2 + \left(v\frac{\delta v}{v}\right)^2}$$

$$c_M = 9.38 \pm 1.1$$

$$= \sqrt{(0.74)^2 + (0.797)^2} = 1.1$$

$$\boxed{c_M = 9 \pm 1}$$

2. a) $\begin{array}{r} 5.6 \pm 0.7 \\ +\ 3.70 \pm 0.03 \end{array}\Big\}$ - Error ≪ 1/3, can be ignored

$\boxed{9.3 \pm 0.7}$ - direct addition

$\left((0.7)^2+(0.03)^2\right)^{1/2} = \sqrt{0.4909}$

$\boxed{\pm\ 0.7}$ - quadrature

b) $\begin{array}{r} 5.6 \pm 0.7 \\ +\ 2.3 \pm 0.1 \end{array}\Big\}$ - Error < 1/3, can be ignored

$\boxed{7.9 \pm 0.8}$ - direct

$\left((0.7)^2+(0.1)^2\right)^{1/2} = \sqrt{0.50}$

$\boxed{\pm 0.7}$ - quadrature

c) $\begin{array}{r} 5.6 \pm 0.7 \\ +\ 4.1 \pm 0.2 \end{array}\Big\}$ - Error < 1/3, can be ignored

$\boxed{9.7 \pm 0.9}$ - direct - Error negligible (< 1/3)

$\left((0.7)^2+(0.2)^2\right)^{1/2} = \sqrt{0.53}$

$\boxed{\pm 0.7}$ - quadrature

d) $\begin{array}{r} 5.6 \pm 0.7 \\ +\ 1.9 \pm 0.3 \end{array}$

$\boxed{7.5 \pm 1}$ - direct

$\left((0.7)^2+(0.3)^2\right)^{1/2} = \sqrt{0.58}$

$\boxed{\pm 0.8}$ - quadrature

3 a)

i. $\delta q = \delta a + \delta b + \delta c = \boxed{4\,cm}$

$$\frac{\delta q}{|q|} = \frac{\delta a + \delta b + \delta c}{|a+b+c|} = \boxed{10\%}$$

ii. $\delta q = \delta a + \delta b + \delta c = \boxed{4\,cm}$

$$\frac{\delta q}{|q|} = \frac{\delta a + \delta b + \delta c}{a+b-c} = \boxed{40\%}$$

iii. $\delta q = |q| \left(\frac{\delta c}{|c|} + \frac{\delta t}{|t|} \right) = \boxed{8\,cm \cdot t}$

$$\frac{\delta q}{|q|} = \frac{\delta c}{|c|} + \frac{\delta t}{|t|} = \boxed{30\%}$$

iv. $\delta q = \frac{\delta q}{|q|} \cdot |q| = \boxed{70 \ \frac{cm \cdot g}{s}}$

$$\frac{\delta q}{|q|} = \frac{\delta m}{|m|} + \frac{\delta b}{|b|} + \frac{\delta t}{|t|}$$
$$= \boxed{40\%}$$

b)

i. $\delta q = \sqrt{(\delta a)^2 + (\delta b)^2 + (\delta c)^2}$
$= \boxed{2\,cm}$

$$\frac{\delta q}{|q|} = \sqrt{\frac{(\delta a)^2 + (\delta b)^2 + (\delta c)^2}{(a+b+c)^2}}$$
$$= \boxed{7\%}$$

ii. $\delta q = \sqrt{(\delta a)^2 + (\delta b)^2 + (\delta c)^2}$
$= \boxed{2\,cm}$

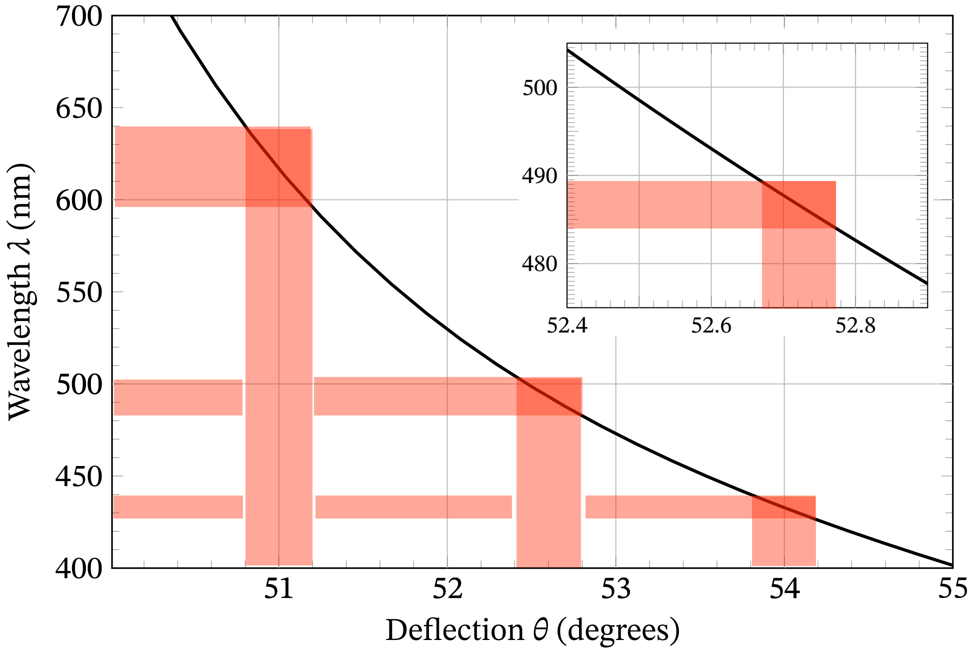
$$\frac{\delta q}{|q|} = \sqrt{\frac{(\delta a)^2 + (\delta b)^2 + (\delta c)^2}{(a+b-c)^2}}$$
$$= \boxed{20\%}$$

iii. $\delta q = \frac{\delta q}{|q|} \cdot |q| = \boxed{6\,cm \cdot t}$

$$\frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta c}{|c|}\right)^2 + \left(\frac{\delta t}{|t|}\right)^2}$$
$$= \boxed{30\%}$$

iv. $\delta q = \frac{\delta q}{|q|} \cdot |q| = \boxed{50 \frac{cm \cdot g}{s}}$

$$\frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta b}{b}\right)^2 + \left(\frac{\delta t}{t}\right)^2}$$
$$= \boxed{30\%}$$



700
650
600
550
500
450
400
Wavelength λ (nm)
51
52
53
54
55
Deflection θ (degrees)
500
490
480
52.4
52.6
52.8

4. a) $\theta_1 = 51.0 \pm 0.2°$

$\theta_2 = 52.6 \pm 0.2°$

$\theta_3 = 54.0 \pm 0.2°$

$\boxed{\begin{aligned} \lambda_1 &= 615 \pm 20 \text{ nm} \\ \lambda_2 &= 495 \pm 10 \text{ nm} \\ \lambda_3 &= 430 \pm 5 \text{ nm} \end{aligned}}$

b)

$\cancel{\lambda_1}$: 656 nm not in range

λ_2: 586 nm in range

λ_3: 434 nm in range

c) Assuming plot is approx. straight in the region, scale error by 1/4 for each

$\boxed{\begin{aligned} \lambda_1 &= 615 \pm 5 \text{ nm} \\ \lambda_2 &= 595 \pm 3 \text{ nm} \\ \lambda_3 &= 430 \pm 1 \text{ nm} \end{aligned}}$

d) $\lambda_2 = 487 \pm 3$ nm

Prediction was reasonable.

5)

$x = 6.0 \pm 0.1 \qquad y = 3.0 \pm 0.1$

$q = xy + x^2/y$

$$(\delta q)^2 = \frac{\partial q}{\partial x}\delta x + \frac{\partial q}{\partial y}\delta y$$

$$= \left[\left(y + 2\,x/y\right)\delta x\right]^2 + \left[\left(x - x^2/y^2\right)\delta y\right]^2$$

$$= \left[\left(3 + \underbrace{\cancel{2 \cdot 6/3}}_{4}\right)\cdot 0.1\right]^2 + \left[\left(6 - \underbrace{\cancel{(6/3)^2}}_{4}\right)\cdot 0.1\right]^2$$

$$\delta q^2 = (0.7)^2 + (0.2)^2$$

$$\delta q = 0.7$$

$$\boxed{q = 30.0 \pm 0.7}$$

6) $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$ $\qquad f = \frac{pq}{p+q}$

a) $\delta f^2 = \left(\frac{\partial f}{\partial p}\delta p\right)^2 + \left(\frac{\partial f}{\partial q}\delta q\right)^2$

$$\delta f^2 = \left[\left(\frac{(p+q)\cdot q - pq}{(p+q)^2}\right)\delta p\right]^2 + \left[\left(\frac{(p+q)\cdot p - pq}{(p+q)^2}\right)\delta q\right]^2$$

$$= \frac{1}{(p+q)^4}\left[(\cancel{pq} + q^2 - \cancel{pq})\delta p\right]^2 + \left[(p^2 + pq - pq)\delta q\right]^2$$

$$\boxed{\delta f = \frac{1}{(p+q)^2}\sqrt{q^4\delta p^2 + p^4\delta q^2}}$$

b) $f = \left(\frac{1}{p} + \frac{1}{q}\right)^{-1}$ $\qquad$ Sub $u = 1/p$, $v = 1/q$

$f = (u+v)^{-1}$ $\qquad \delta u = \left|-\frac{1}{p^2}\right|\delta p \quad \delta v = \left|-\frac{1}{q^2}\right|\delta q$

$w = u + v \implies f = w^{-1}$

$\delta f = \left(df/dw\right)\delta w \qquad \delta w = \sqrt{\delta u^2 + \delta v^2}$

$= w^{-2}\,\delta w \qquad = \sqrt{\frac{1}{p^4}\delta p^2 + \frac{1}{q^4}\delta q^2}$

$$f^2\delta w = \sqrt{\left(\frac{1}{\cancel{p^4}}\delta p^2 + \frac{1}{\cancel{q^4}}\delta q^2\right)}\cdot\sqrt{\frac{(\cancel{pq})^4}{(p+q)^4}} = \boxed{\frac{1}{(p+q)^2}\sqrt{q^4\delta p^2 + p^4\delta q^2}}$$