

$$1) \quad a) \quad (x_1, y_1) = (0.0 \pm 0.2, 0.0 \pm 0.3) \\ (x_2, y_2) = (3.0 \pm 0.3, 4.0 \pm 0.2)$$

$$\Delta x = 3.0 - 0.0 = 3.0 \\ \delta x = \sqrt{0.2^2 + 0.3^2} = 0.4$$

$$\Delta y = 4.0 - 0.0 = 4.0 \\ \delta y = \delta x = 0.4$$

$$d = \sqrt{x^2 + y^2}$$

$$\theta = \arctan(y/x)$$

$$\begin{aligned} \delta d &= \sqrt{\left[\frac{1}{2}(x^2 + y^2)^{-1/2}(2x\delta x)\right]^2 + \left[\frac{1}{2}(x^2 + y^2)^{-1/2}(2y\delta y)\right]^2} \\ &= \sqrt{\frac{1}{4}(3^2 + 4^2)^{-1}(6 \cdot 0.4)^2 + \frac{1}{4}(3^2 + 4^2)^{-1}(8 \cdot 0.4)^2} \\ &= \sqrt{0.0576 + 0.1024} = \sqrt{0.16} = 0.4 \end{aligned}$$

$$d = 5 \pm 0.4$$

$$\begin{aligned} \delta \theta &= \sqrt{\left[\frac{-y}{x^2}\left(\frac{y^2}{x^2} + 1\right)^{-1}\delta x\right]^2 + \left[\frac{1}{x}\left(\frac{y^2}{x^2} + 1\right)^{-1}\delta y\right]^2} \\ &= \sqrt{0.004096 + 0.000256} = 0.07 \text{ rad} \end{aligned}$$

$$\theta = 0.93 \pm 0.07 \text{ rad}$$

$$= 53 \pm 4^\circ$$

$$1) \quad b) \quad N = N_0 e^{-x/\lambda}$$

$$N = (10000)_{E6} e^{-1/0.25} = 1000 e^{-4} = 18.32 E6$$

$$\delta N^2 = \left(\frac{\partial N}{\partial N_0} \delta N_0 \right)^2 + \left(\frac{\partial N}{\partial x} \delta x \right)^2 + \left(\frac{\partial N}{\partial \lambda} \delta \lambda \right)^2$$

$$= \left(e^{-x/\lambda} \cdot \delta N_0 \right)^2 + \left(-\frac{N_0}{\lambda} e^{-x/\lambda} \delta x \right)^2 + \left(N_0 \frac{x}{\lambda^2} e^{-x/\lambda} \delta \lambda \right)^2$$

$$= e^{-2x/\lambda} \left(\delta N_0^2 + \left(\frac{N_0 \delta x}{\lambda} \right)^2 + \left(\frac{N_0 x \delta \lambda}{\lambda^2} \right)^2 \right)$$

$$\delta N = e^{-x/\lambda} \left(25 E12 + 1.6 E15 + 9.2 E17 \right)^{1/2}$$

$$0.0183 \cdot 9.6 E8$$

$$\delta N = 1.7 E7$$

$$N = 18 \pm 17 \times 10^6 \text{ particles}$$

1) c)

$$y = a + bx$$

$$y = 2.5 + 4 \cdot 0.05$$

$$= 2.7$$

$$a = 2.5 \pm 0.3 \text{ m}$$

$$b = (5.0 \pm 0.1) \text{ E}^{-2}$$

$$x = 4 \text{ m}, 4.0 \pm 0.1 \text{ m}$$

$$u = bx$$

$$\delta u = |x| \delta b \quad \text{or} \quad |bx| \sqrt{\left(\frac{\delta b}{b}\right)^2 + \left(\frac{\delta x}{x}\right)^2}$$

$$= 0.004 \quad = 0.2 \sqrt{0.004 + 0.00625} = 0.02$$

$$\delta y = \sqrt{\delta a^2 + \delta b^2}$$

$$= \sqrt{0.09041}$$

$$= 0.3$$

$$y = 2.7 \pm 0.3 \text{ m}$$

For both cases, δa is the major source of uncertainty, so δy is unchanged.

d) $c_M = aT + bT^3$

$$= u + v$$

$$\delta u = \sqrt{\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta T}{T}\right)^2}$$

$$\delta v = \sqrt{\left(\frac{\delta b}{b}\right)^2 + \left(\frac{3\delta T}{T}\right)^2}$$

$$\delta c_M = \sqrt{\left(u \frac{\delta u}{u}\right)^2 + \left(v \frac{\delta v}{v}\right)^2}$$

$$= \sqrt{(0.74)^2 + (0.797)^2} = 1.1$$

$$a = 1.35 \pm 0.05 \text{ mJ/mol} \cdot \text{K}^2$$

$$b = 0.021 \pm 0.001 \text{ mJ/mol} \cdot \text{K}^4$$

$$T = 5.0 \pm 0.5 \text{ K}$$

$$\frac{\delta u}{u} = 0.11 \quad \frac{\delta v}{v} = 0.30$$

$$c_M = 9.38 \pm 1.1$$

$$c_M = 9 \pm 1$$

$$2. a) \left. \begin{array}{r} 5.6 \pm 0.7 \\ + 3.70 \pm 0.03 \end{array} \right\} \text{ - Error } \ll 1/3, \text{ can be ignored}$$

$$\boxed{9.3 \pm 0.7} \quad \text{ - direct addition}$$

$$((0.7)^2 + (0.03)^2)^{1/2} = \sqrt{0.4909}$$

$$\boxed{\pm 0.7} \quad \text{ - quadrature}$$

$$b) \left. \begin{array}{r} 5.6 \pm 0.7 \\ + 2.3 \pm 0.1 \end{array} \right\} \text{ - Error } < 1/3, \text{ can be ignored}$$

$$\boxed{7.9 \pm 0.8} \quad \text{ - direct}$$

$$((0.7)^2 + (0.1)^2)^{1/2} = \sqrt{0.50}$$

$$\boxed{\pm 0.7} \quad \text{ - quadrature}$$

$$c) \left. \begin{array}{r} 5.6 \pm 0.7 \\ + 4.1 \pm 0.2 \end{array} \right\} \text{ - Error } < 1/3, \text{ can be ignored}$$

$$\boxed{9.7 \pm 0.9} \quad \text{ - direct} \quad \text{ - Error negligible } (< 1/3)$$

$$((0.7)^2 + (0.2)^2)^{1/2} = \sqrt{0.53}$$

$$\boxed{\pm 0.7} \quad \text{ - quadrature}$$

$$d) \left. \begin{array}{r} 5.6 \pm 0.7 \\ + 1.9 \pm 0.3 \end{array} \right\}$$

$$\boxed{7.5 \pm 1} \quad \text{ - direct}$$

$$((0.7)^2 + (0.3)^2)^{1/2} = \sqrt{0.58}$$

$$\boxed{\pm 0.8} \quad \text{ - quadrature}$$

3 a)

$$i. \delta q = \delta a + \delta b + \delta c = \boxed{4 \text{ cm}}$$

$$\frac{\delta q}{|q|} = \frac{\delta a + \delta b + \delta c}{|a + b + c|} = \boxed{10\%}$$

$$ii. \delta q = \delta a + \delta b + \delta c = \boxed{4 \text{ cm}}$$

$$\frac{\delta q}{|q|} = \frac{\delta a + \delta b + \delta c}{a + b - c} = \boxed{40\%}$$

$$iii. \delta q = |q| \left(\frac{\delta c}{|c|} + \frac{\delta t}{|t|} \right) = \boxed{8 \text{ cm} \cdot \text{s}}$$

$$\frac{\delta q}{|q|} = \frac{\delta c}{|c|} + \frac{\delta t}{|t|} = \boxed{30\%}$$

$$iv. \delta q = \frac{\delta q}{|q|} \cdot |q| = \boxed{70 \frac{\text{cm} \cdot \text{g}}{\text{s}}}$$

$$\frac{\delta q}{|q|} = \frac{\delta m}{|m|} + \frac{\delta b}{|b|} + \frac{\delta t}{|t|} = \boxed{40\%}$$

b)

$$i. \delta q = \sqrt{(\delta a)^2 + (\delta b)^2 + (\delta c)^2} = \boxed{2 \text{ cm}}$$

$$\frac{\delta q}{|q|} = \sqrt{\frac{(\delta a)^2 + (\delta b)^2 + (\delta c)^2}{(a + b + c)^2}} = \boxed{7\%}$$

$$ii. \delta q = \sqrt{(\delta a)^2 + (\delta b)^2 + (\delta c)^2} = \boxed{2 \text{ cm}}$$

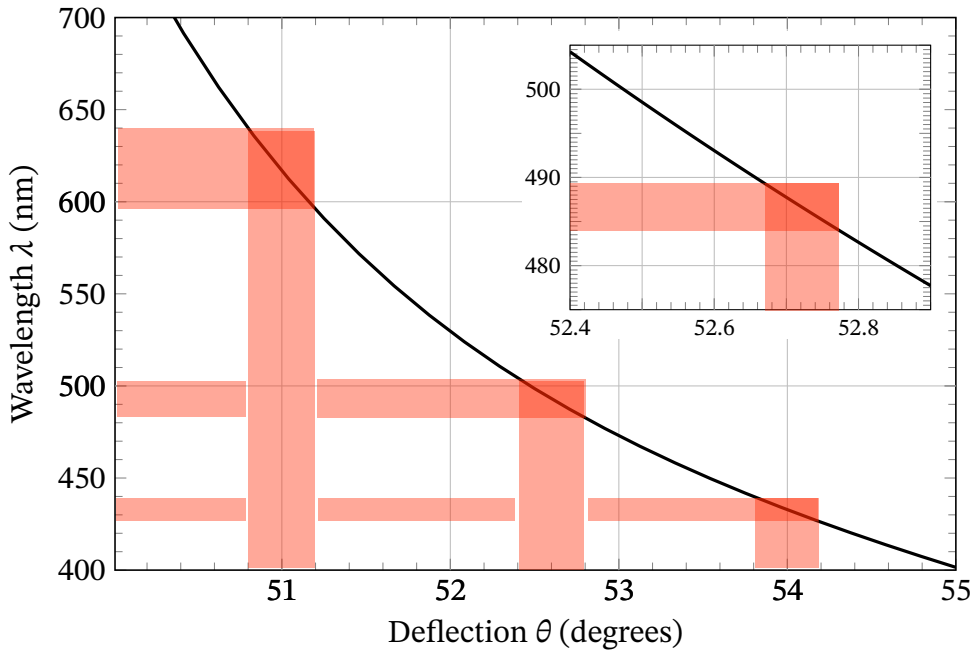
$$\frac{\delta q}{|q|} = \sqrt{\frac{(\delta a)^2 + (\delta b)^2 + (\delta c)^2}{(a + b - c)^2}} = \boxed{20\%}$$

$$iii. \delta q = \frac{\delta q}{|q|} \cdot |q| = \boxed{6 \text{ cm} \cdot \text{s}}$$

$$\frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta c}{|c|} \right)^2 + \left(\frac{\delta t}{|t|} \right)^2} = \boxed{30\%}$$

$$iv. \delta q = \frac{\delta q}{|q|} \cdot |q| = \boxed{50 \frac{\text{cm} \cdot \text{g}}{\text{s}}}$$

$$\frac{\delta q}{|q|} = \sqrt{\left(\frac{\delta m}{|m|} \right)^2 + \left(\frac{\delta b}{|b|} \right)^2 + \left(\frac{\delta t}{|t|} \right)^2} = \boxed{30\%}$$



4. a) $\theta_1 = 51.0 \pm 0.2^\circ$
 $\theta_2 = 52.6 \pm 0.2^\circ$
 $\theta_3 = 54.0 \pm 0.2^\circ$

$$\lambda_1 = 615 \pm 20 \text{ nm}$$

$$\lambda_2 = 495 \pm 10 \text{ nm}$$

$$\lambda_3 = 430 \pm 5 \text{ nm}$$

b) λ_1 : 656 nm not in range
 λ_2 : 586 nm in range
 λ_3 : 434 nm in range

c) Assuming plot is approx. straight in the region, scale error by $1/4$ for each

$$\lambda_1 = 615 \pm 5 \text{ nm}$$

$$\lambda_2 = 595 \pm 3 \text{ nm}$$

$$\lambda_3 = 430 \pm 1 \text{ nm}$$

d) $\lambda_2 = 487 \pm 3 \text{ nm}$

Prediction was reasonable.

5)

$$x = 6.0 \pm 0.1$$

$$y = 3.0 \pm 0.1$$

$$q = xy + x^2/y$$

$$(\delta q)^2 = \frac{\partial q}{\partial x} \delta x + \frac{\partial q}{\partial y} \delta y$$

$$= \left[\left(y + 2x/y \right) \delta x \right]^2 + \left[\left(x - x^2/y^2 \right) \delta y \right]^2$$

$$= \left[\left(3 + 2 \cdot \frac{6}{3} \right) \cdot 0.1 \right]^2 + \left[\left(6 - \left(\frac{6}{3} \right)^2 \right) \cdot 0.1 \right]^2$$

$$\delta q^2 = (0.7)^2 + (0.2)^2$$

$$\delta q = 0.7$$

$$q = 30.0 \pm 0.7$$

$$6) \quad \frac{1}{f} = \frac{1}{p} + \frac{1}{q} \quad f = \frac{pq}{p+q}$$

$$a) \quad \delta f^2 = \left(\frac{\partial f}{\partial p} \delta p \right)^2 + \left(\frac{\partial f}{\partial q} \delta q \right)^2$$

$$\delta f^2 = \left[\left(\frac{(p+q) \cdot q - pq}{(p+q)^2} \right) \delta p \right]^2 + \left[\left(\frac{(p+q) \cdot p - pq}{(p+q)^2} \right) \delta q \right]^2$$

$$= \frac{1}{(p+q)^4} \left[(pq + q^2 - pq) \delta p \right]^2 + \left[(p^2 + pq - pq) \delta q \right]^2$$

$$\delta f = \frac{1}{(p+q)^2} \sqrt{q^4 \delta p^2 + p^4 \delta q^2}$$

$$b) \quad f = \left(\frac{1}{p} + \frac{1}{q} \right)^{-1} \quad \text{Sub } u = \frac{1}{p}, v = \frac{1}{q}$$

$$f = (u+v)^{-1}$$

$$\delta u = \left| \frac{1}{p^2} \right| \delta p \quad \delta v = \left| \frac{1}{q^2} \right| \delta q$$

$$w = u+v \Rightarrow f = w^{-1}$$

$$\delta f = \left(\frac{df}{dw} \right) \delta w$$

$$= w^{-2} \delta w$$

$$\delta w = \sqrt{\delta u^2 + \delta v^2}$$

$$= \sqrt{\frac{1}{p^4} \delta p^2 + \frac{1}{q^4} \delta q^2}$$

$$f^2 \delta w = \sqrt{\left(\frac{1}{p^4} \delta p^2 + \frac{1}{q^4} \delta q^2 \right)} \cdot \sqrt{\frac{(pq)^4}{(p+q)^4}} = \frac{1}{(p+q)^2} \sqrt{q^4 \delta p^2 + p^4 \delta q^2}$$