

# Vibrational-Rotational Spectra of HCl and DCl

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This experiment is concerned with the rotational fine structure of the infrared vibrational spectrum of a linear molecule such as HCl. By interpreting the details of this spectrum, it is possible to obtain the moment of inertia of the molecule and thus the internuclear separation. In addition, the pure vibrational frequency determines the force constant that is a measure of the bond strength. By also investigating DCl, the isotope effect can be observed.<sup>1</sup>

<sup>1</sup> Transcribed (with corrections) from Nibler et al. [1].

## Introduction

The infrared region of the spectrum extends from the long-wavelength end of the visible region near  $1\text{ }\mu\text{m}$  out to the microwave region at approximately  $1000\text{ }\mu\text{m}$ . It is common practice to specify infrared frequencies in wavenumber units of “inverse centimeters”,  $\text{cm}^{-1}$ . The wavenumber of a photon,  $\tilde{\nu}$ , is calculated as:

$$\tilde{\nu} = 1/\lambda = \nu/c, \quad (1)$$

where  $c$  is the speed of light in  $\text{cm/s}$ ,  $\nu$  is the frequency in  $\text{Hz}$ , and  $\lambda$  is the wavelength in  $\text{cm}$ . Thus, this region extends from  $10\,000\text{ cm}^{-1}$  down to  $10\text{ cm}^{-1}$ . Although considerable work is now being done in the far-infrared region (below  $400\text{ cm}^{-1}$ ), the spectral range from  $4000\text{ cm}^{-1}$  to  $400\text{ cm}^{-1}$  has received the greatest attention because the vibrational frequencies of most molecules lie in this region.

## Theory

Almost all infrared work makes use of absorption techniques in which radiation from a source emitting broadband infrared radiation is passed through a sample of the material to be studied. When the frequency of this radiation is the same as the vibrational frequency of the molecule, the molecule may be vibrationally excited. This results in loss of energy from the radiation at that frequency and gives rise to an absorption band. The spectrum of a polyatomic molecule generally consists of several such bands arising from different vibrational motions of the molecule. This experiment involves diatomic molecules, which have only one vibrational mode.

The simplest model of a *vibrating* diatomic molecule is a harmonic oscillator, for which the potential energy depends quadratically on the change in internuclear distance. The allowed energy levels of a harmonic

oscillator, as calculated from quantum mechanics [2], are

$$E(v) = h\nu \left( v + \frac{1}{2} \right), \quad (2)$$

where  $v$  is the vibrational quantum number (having integer values 0, 1, 2, ...),  $\nu$  is the vibrational frequency in Hz, and  $h$  is the Planck constant,  $6.626 \times 10^{-34}$  J·s.

The simplest model of a *rotating* diatomic molecule is a rigid rotor, or “dumbbell”, model in which the two atoms of mass  $m_1$  and  $m_2$  are considered to be joined by a rigid, weightless rod. The allowed energy levels for a rigid rotor may be shown by quantum mechanics [2] to be

$$E(J) = \frac{h^2}{8\pi^2 I} J(J+1), \quad (3)$$

where the rotational quantum number,  $J$ , again takes integer values 0, 1, 2, .... The quantity  $I$  is the moment of inertia, which is related to the internuclear distance,  $r$ , and the reduced mass,  $\mu$ ,<sup>2</sup> by

$$I = \mu r^2. \quad (4)$$

<sup>2</sup> Recall that  $\mu$  is defined by the equation  $\mu u^{-1} = m_1^{-1} + m_2^{-1} = (m_1 m_2) / (m_1 + m_2)$ .

Since a real molecule is undergoing both rotation and vibration simultaneously, a first approximation to its energy levels,  $E(v, J)$  would be the sum of eqs. (2) and (3). A more complete expression for the energy levels of a diatomic molecule is given below [3, 4], with the levels expressed as *term values*,  $T$ , in frequency units of  $\text{cm}^{-1}$  rather than as energy values,  $E$ , in joules (J):

$$\begin{aligned} T(v, J) = \frac{E(v, J)}{hc} = & \tilde{\nu}_e \left( v + \frac{1}{2} \right) - \tilde{\nu}_e \chi_e \left( v + \frac{1}{2} \right)^2 + B_e J(J+1) \\ & - D_J J^2(J+1)^2 - \alpha_e \left( v + \frac{1}{2} \right) J(J+1), \end{aligned} \quad (5)$$

where  $c$  is the speed of light in cm/s,  $\tilde{\nu}_e$  is the frequency in  $\text{cm}^{-1}$  for the molecule vibrating about its equilibrium internuclear separation,  $r_e$ , in Å, and the equilibrium rotation frequency is defined as

$$B_e = \frac{h}{8\pi^2 I_e c}. \quad (6)$$

The first and third terms on the right-hand side of eq. (5) are the harmonic-oscillator and rigid-rotor terms with  $r = r_e$ . The second term (containing the *anharmonicity constant*,  $\chi_e$ ) takes into account the effect of anharmonicity on the vibrational levels. Since the real potential,  $U(r)$ , for a molecule differs from a harmonic potential,  $U_{\text{HO}}$ , the real vibrational levels are not quite those given by eq. (2) and a correction term is required.<sup>3</sup> The fourth term (involving the *centrifugal distortion constant*,  $D_J$ ) accounts for the effects of centrifugal stretching on the rotational levels. Since a chemical bond is not truly rigid but more like a stiff spring,

<sup>3</sup> See fig. 1

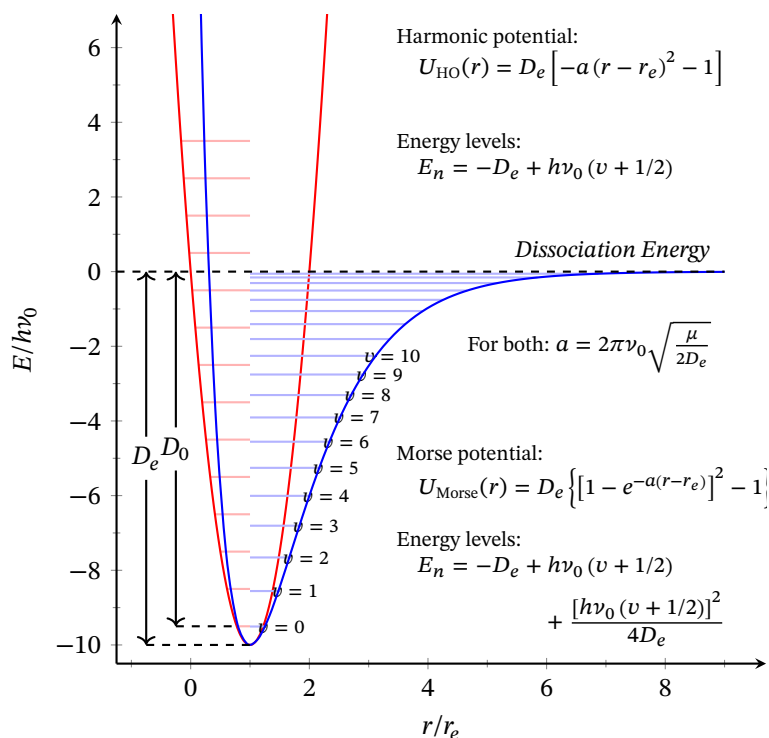


Figure 1: Schematic diagram showing potential energy curves as a function of internuclear separation,  $r$ , for a diatomic molecule. The curve for a harmonic oscillator is depicted in red while the Morse potential is drawn in blue. Vibrational levels are shown for each. Note that 0 on the y-axis corresponds to the dissociated system, not any physical representation of “zero energy.” This is important when considering your computational results later.

it stretches somewhat when the molecule rotates. Such an effect is only important for high  $J$  values, since the constant  $D_J$  is usually very small. The last term in eq. (5), involving  $\alpha_e$ , accounts for interactions between vibration and rotation. During a vibration, the internuclear distance  $r$  changes; this changes the moment of inertia and affects the rotation of the molecule. The constant  $\alpha_e$  is also quite small, but this term should not be neglected.

### Selection Rules

The harmonic-oscillator, rigid-rotor selection rules are  $\Delta v = \pm 1$  and  $\Delta J = \pm 1$ ; that is, infrared emission or absorption can occur only when these “allowed” transitions take place [4]. For an *anharmonic* diatomic molecule, the  $\Delta J = \pm 1$  selection rule is still valid, but weak transitions corresponding to  $\Delta v = \pm 2, \pm 3$ , etc. (overtones) can now be observed [4]. Since we are interested in the most intense absorption band (the “fundamental”), we are concerned with transitions from various  $J''$  levels of the vibrational ground state ( $v'' = 0$ ) to  $J'$  levels in the first excited vibrational state ( $v' = 1$ ). From the still-valid selection rule, we know that the transition must be from  $J''$  to  $J' = J'' \pm 1$ . Since  $\Delta E = h\nu = hc\tilde{\nu}$ , the frequency  $\tilde{\nu}$  (in wavenumbers) for this transition will be  $[T(v', J') - T(v'', J'')]$ . Apply-

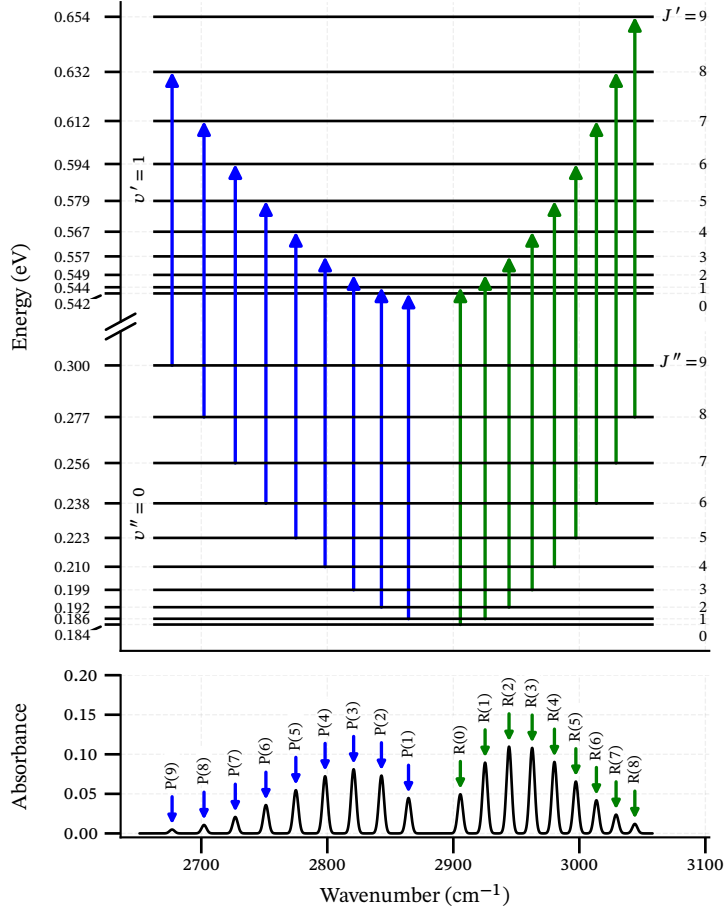


Figure 2: Rotational energy levels for the ground vibrational state ( $v'' = 0$ ) and the first excited vibrational state ( $v' = 1$ ) in a diatomic molecule. The vertical arrows indicated allowed transitions in the  $R$  and  $P$  branches; the numbers above the arrows index the value  $J''$  of the lower state. Transitions in the  $Q$  branch ( $\Delta J = 0$ ) are not shown, since they are not infrared-active.

ing the selection rule, we find from eq. (5) that

$$\tilde{\nu}_R = \tilde{\nu}_0 + (2B_e - 3\alpha_e) + (2B_e - 4\alpha_e)J'' - \alpha_e J''^2 \quad J'' = 0, 1, 2, \dots \quad (7)$$

$$\tilde{\nu}_P = \tilde{\nu}_0 - (2B_e - 2\alpha_e)J'' - \alpha_e J''^2 \quad J'' = 0, 1, 2, \dots \quad (8)$$

for  $\Delta J = +1$  and  $\Delta J = -1$ , respectively, where the  $D_J$  term has been omitted and  $\tilde{\nu}_0$ , the frequency of the *forbidden* transition from  $(v'', J'') = (0, 0)$  to  $v' = 1, J' = 0$  is defined as

$$\tilde{\nu}_0 = \tilde{\nu}_e - 2\tilde{\nu}_e\chi_e. \quad (9)$$

The two series of lines given in eqs. (7) and (8) are called  $R$  and  $P$  branches, respectively. These allowed transitions are indicated on the energy-level diagram given in fig. 2. If  $\alpha_e$  were negligible, eqs. (7) and (8) would predict a series of equally spaced lines separated by  $2B_e$  except for a missing line at  $\tilde{\nu}_0$ . The effect of interaction between rotation and vibration (non-zero  $\alpha_e$ ) is to squeeze lines in the  $R$  branch closer together and spread the lines in the  $P$  branch farther apart, as shown for a typical spectrum in fig. 2.

For convenience, let us introduce a new quantity  $m$ , where  $m = J'' + 1$  for the  $R$  branch and  $m = -J''$  for the  $P$  branch, as shown in fig. 2. It is now possible to replace eqs. (7) and (8) with a single equation:

$$\tilde{\nu}(m) = \tilde{\nu}_0 + (2B_e - 2\alpha_e)m - \alpha_e m^2, \quad (10)$$

where  $m$  takes all integral values and  $m = 0$  yields the frequency  $\tilde{\nu}_0$  of the forbidden “purely vibrational” transition. If one retains the  $D_J$  term of eq. (5) (which assumes  $D'' = D' = D_J$ ), eq. (10) takes the form

$$\tilde{\nu}(m) = \tilde{\nu}_0 + (2B_e - 2\alpha_e)m - \alpha_e m^2 - 4D_J m^3. \quad (11)$$

Thus, a multiple linear regression can be performed to determine  $\tilde{\nu}_0$ ,  $B_e$ ,  $\alpha_e$ , and  $D_J$ .

### Isotope Effect

When an isotopic substitution is made in a diatomic molecule, the equilibrium bond length,  $r_e$ , and the force constant,  $k$ , are unchanged, since they depend only on the behavior of the bonding electrons. However, the reduced mass,  $\mu$ , does change, and this will affect the rotation and vibration of the molecule. In the case of rotation, the isotope effect can be easily stated. From the definitions of  $B_e$  and  $I$ , we see that

$$\frac{B_e^*}{B_e} = \frac{\mu}{\mu^*}, \quad (12)$$

where an asterisk is used to distinguish one isotopologue<sup>4</sup> from another.

For a harmonic oscillator model, the frequency  $\tilde{\nu}_e$  in wavenumbers is given by

$$\tilde{\nu}_e = \frac{1}{2\pi c} \sqrt{\frac{k}{\mu}}, \quad (13)$$

which leads to the relation

$$\frac{\tilde{\nu}_e^*}{\tilde{\nu}_e} = \sqrt{\frac{\mu}{\mu^*}}. \quad (14)$$

The ratio  $\tilde{\nu}_e^*/\tilde{\nu}_e$  differs slightly from this harmonic ratio due to deviation of the true potential function from a quadratic form, as depicted in fig. 1. A closer approximation to the solid curve can be obtained by adding cubic and higher anharmonic terms to the interatomic potential  $U(r)$ , like so:

$$U(r) = \frac{1}{2}k(r - r_e)^2 + c_1(r - r_e)^3 + c_2(r - r_e)^4 + \dots \quad (15)$$

Although somewhat complicated, it can be shown [4, 5] that the  $c_1$  and  $c_2$  terms yield, as the first correction to the energy levels, precisely the  $-\tilde{\nu}_e \chi_e (\nu + 1/2)^2$  term given in eq. (5). A similar conclusion is reached if  $U(r)$  is taken to have the Morse potential form given by<sup>5</sup>

<sup>4</sup> two molecules which differ only by the nuclear isotopes which make up the molecule

<sup>5</sup> The parameter  $a$  is determined by equating  $k_e$  to the curvature of the Morse potential at  $r = r_e$ , yielding

$$a = \sqrt{\frac{k_e}{2D_e}}.$$

This three-parameter function has the desired values of  $U(r = r_e) = -D_e$  and  $U(r = \infty) = 0$ . It provides a very good approximation to the real potential energy curve at all distances except  $r \ll r_e$ , a region of no practical significance.

$$U(r) = D_e \{ [1 - \exp\{-a(r - r_e)\}]^2 - 1 \}. \quad (16)$$

In both cases, the mass dependence of  $\tilde{\nu}_e \chi_e$  is found to be greater than for  $\tilde{\nu}_e$  and is

$$\frac{\tilde{\nu}_e^* \chi_e^*}{\tilde{\nu}_e \chi_e} = \frac{\mu}{\mu^*}. \quad (17)$$

Equations (14) and (17) are useful in obtaining the  $\tilde{\nu}_0^*$  counterpart of eq. (9),

$$\tilde{\nu}_0^* = \tilde{\nu}_e^* - 2\tilde{\nu}_e^* \chi_e^* = \tilde{\nu}_e \sqrt{\frac{\mu}{\mu^*}} - 2\tilde{\nu}_e \chi_e \frac{\mu}{\mu^*}, \quad (18)$$

and it is seen that a measurement of  $\tilde{\nu}_0$  for HCl and DCl suffices for a determination of  $\tilde{\nu}_e$  and  $\tilde{\nu}_e \chi_e$ . Alternatively or course, the latter constants can be determined from overtone vibrations ( $\Delta v > 0$ ) of a single isotopic form.<sup>6</sup> However, such overtones generally have low intensity and the transitions may fall outside the range of many infrared instruments, so the isotopic shift method is used in the present experiment.

<sup>6</sup> See Exp. 39 in 1.

Since HCl gas is a mixture of  $\text{H}^{35}\text{Cl}$  and  $\text{H}^{37}\text{Cl}$  molecules, a chlorine isotope effect will also be present. Since the ratio of the reduced masses is only 1.0015, high resolution is required to detect this effect. HCl is predominantly  $\text{H}^{35}\text{Cl}$ . If a single peak is seen in the HCl manifold, it can be assumed that the bands are due to  $\text{H}^{37}\text{Cl}$ . At a resolution of  $<1 \text{ cm}^{-1}$ , the band should consist of a series of doublets, with the larger peak due to  $\text{H}^{35}\text{Cl}$  and the smaller peak due to  $\text{H}^{37}\text{Cl}$ . If deuterium is substituted for hydrogen, the ratio of the reduced masses,  $\mu(\text{D}^{35}\text{Cl})/\mu(\text{H}^{35}\text{Cl})$ , is 1.946 and the isotope effect is quite large.

### Vibrational Partition Function [6–8]

The thermodynamic quantities for an ideal gas can usually be expressed as a sum of translational, rotational, and vibrational contributions. We shall consider here the heat capacity at constant volume. At room temperature and above, the translational and rotational contributions to  $C_v$  are constants that are independent of temperature. For HCl and DCl,<sup>7</sup> the molar quantities are

<sup>7</sup> diatomic, and thus linear, molecules

$$\begin{aligned} \tilde{C}_v(\text{trans}) &= \frac{3}{2}R \\ \tilde{C}_v(\text{rot}) &= R. \end{aligned} \quad (19)$$

The vibrational contribution to  $\tilde{C}_v$  varies with temperature and can be calculated from the vibrational partition function,  $q_{\text{vib}}$  using

$$\tilde{C}_v(\text{vib}) = R \frac{\partial}{\partial T} \left( T^2 \frac{\partial}{\partial \ln q_{\text{vib}} T} \right). \quad (20)$$

The partition function,  $q_{\text{vib}}$ , of HCl or DCl is well-approximated by the harmonic-oscillator partition function,  $q_{\text{HO}}$ . Since the energy levels of a

harmonic oscillator are given by  $\left(v + \frac{1}{2}\right)h\nu$ , one obtains [8]

$$q_{\text{HO}} = \sum_{v=0}^{\infty} \exp \left[ \frac{-\left(v + \frac{1}{2}\right)h\nu}{kT} \right] = \frac{e^{-h\nu/(2kT)}}{1 - e^{-h\nu/(kT)}}. \quad (21)$$

Combining eqs. (20) and (21), we find

$$\tilde{C}_v(\text{vib}) = R \frac{u^2 e^{-u/2}}{(1 - e^{-u})^2}, \quad (22)$$

where  $u = h\nu/(kT) = hc\tilde{\nu}/(kT) = 143.88\tilde{\nu}/T$ .

### *Safety Precautions*

- HCl is a strong acid should be handled cautiously.
- All chemical work should be done in the fume hood and gloves should be worn at all times.
- Always wear safety glasses in the laboratory.
- The cell used to obtain the infrared spectra is valuable and non-replaceable. Handle it with extreme care. The windows are made from NaCl and are moisture sensitive. Do not touch the lenses with your fingers or expose them to moisture of any kind. If the cell filled with HCl breaks outside the fume hood, notify the instructor immediately and clear the area.

### *Procedure*

This experiment must be performed with an IR spectrometer capable of medium to high resolution ( $> 2\text{ cm}^{-1}$  resolution). A high-quality grating spectrometer and most FTIR instruments are generally capable of this, and FTIR instruments don't require calibration with external gas standards.

The infrared gas cell is constructed from a short (5 cm to 10 cm) length of large-diameter (4 cm to 5 cm) Pyrex tubing with a vacuum stopcock attached. Infrared-transparent windows are clamped against O-rings at the ends of the cell or are sealed on the ends with Glyptal resin. For studies concentrating on the region  $4000\text{ cm}^{-1}$  to  $500\text{ cm}^{-1}$ , NaCl windows will suffice. When the spectrum of interest extends down to  $\sim 400\text{ cm}^{-1}$ , KBr windows are necessary. Both types of salt windows become "foggy" on prolonged exposure to a moist atmosphere and should be protected<sup>8</sup> when not in use. In the present experiment, inert sapphire windows with transmission down to  $1600\text{ cm}^{-1}$  are particularly convenient and relatively inexpensive.

<sup>8</sup> e.g., stored in a desiccator

### *Filling the Cell and Recording the Spectra*

Before beginning the synthesis, power on the FTIR. Prior to starting, purge the 5-cm gas cell (Thermo Fisher) with nitrogen gas to clean out the cell. This can be accomplished by attaching the tubing from the nitrogen tank regulator to one of the glass ports on the gas cell. Open both stopcocks on the gas cell and slowly open the needle valve on the nitrogen tank<sup>9</sup> until a steady stream of gas is exiting the open port of the gas cell. After allowing the gas to flow for 2 min, close the stopcock on the gas cell port releasing the gas and immediately close the stopcock on the port receiving the flowing gas, effectively sealing the gas cell. Immediately close

<sup>9</sup> Ask the instructor if you are unfamiliar with the use of a regulator.



the knob above the regulator nozzle, then close the needle valve controlling the tank's gas flow. Remove the tubing from the gas cell port and take the gas cell to the FTIR to obtain a background spectrum. Return the gas cell to the fume hood for the gas synthesis.

Prepare an indicator solution by dissolving 3 to 4 drops of bromocresol green in 300 mL of DI water in a 400 mL beaker. Attach Tygon tubing to the two stems of the gas cell, lubricating with glycerin to help the tubing slide on the glass. Take the end of one tube and submerge it in your indicator solution.<sup>10</sup>

To set up the rest of the apparatus, clamp a 50 mL round-bottom flask to a ring stand. Place ~5 mL of D<sub>2</sub>O with ~5 mL concentrated HCl in the round-bottom flask. Prepare an air condenser (10 cm) by placing a plug of glass wool at the bottom and filling the rest with CaCl<sub>2</sub> (approximately 12 mesh). Insert the stem of the condenser into a stopper, again using glycerin for lubrication, and insert the stopper into the neck of the round-bottom flask. Insert the end of the second tube (attached to the gas cell) into another stopper with a hole, then insert that stopper into the top of the air condenser.

Suspend the round-bottom flask in a water bath held around 80 °C. Gaseous HCl will slowly evolve from the solution and bubble through the indicator solution. When the indicator changes color from blue to green/yellow, close the stopcocks on the gas cell and remove the round-bottom flask from the water bath. This procedure should take ~20 min. Carefully remove the tubes from the gas cell and proceed to the spectrometer for analysis.

Make sure the experimental settings in the FTIR software are appropriate. The most important change to make is changing the resolution to the smallest possible value. This will allow you to distinguish the individual rotational transitions in the spectrum.

An arrangement for filling the cell is shown in fig. 3.

### *Theoretical Calculations*

The following work will be done during the second week of this lab. You will need to calculate the physical properties of HCl and DCl using Gaussian [9], including the potential energy, equilibrium bond length, and the predicted vibrational and rotational energies for the molecule.

Using Gaussian, calculate the potential well for the HCl molecule. Because of the methods used for calculation, the mass of the nuclei doesn't influence geometry, so results will be the same for the four isotopologues. To simplify this process, you can use the scan trick from the H<sub>3</sub>O<sup>+</sup> PES scan in the Introduction to Computational Chemistry lab. A number of files are available for your use in the hcl-dcl\_lab folder in Jupyter-Lab. You will need to modify the scan parameters in the hcl\_bond.com

<sup>10</sup> This will fill two roles: as a bubbler, it will prevent atmospheric gases from diffusing back into the cell; and as an indicator, it will show when sufficient HCl has passed through the cell. HCl is highly soluble in water, so the gaseous HCl will quickly acidify the water.

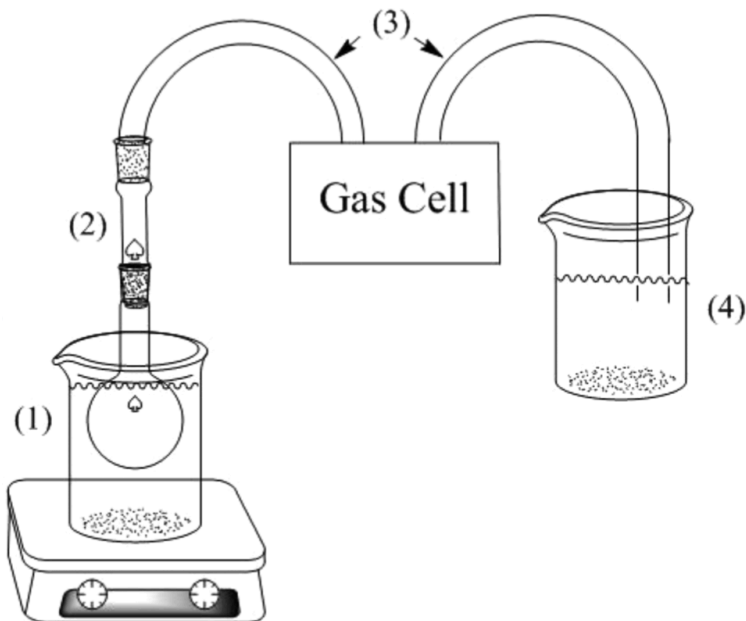


Figure 3: Experimental apparatus. (1) Water bath heating a round-bottom flask containing a mixture of  $\text{HCl}_{(\text{aq})}$  and  $\text{DCl}_{(\text{aq})}$ . (2) Condenser containing a drying agent. (3) Tygon tubing leading to and from the gas cell. (4) Beaker containing indicator solution. Figure reproduced from ??.

file (printed below). The selected basis set and level of theory make this a quick calculation, so you can set the scan parameters quite large. I recommend scanning from  $0.6 \text{ \AA}$  to  $4.5 \text{ \AA}$  in steps less than  $0.05 \text{ \AA}$ . With this resolution, you should obtain a plot that gives good fidelity around the minimum while showing realistic behavior at long bond lengths.

#### Contents of hcl\_bond.com

```

1  %nproc=2
2  %mem=800MB
   # PBE1PBE 6-311+g(d,p)
4  # scan scf=tight

6  Scan of HCl bond length

8  0 1
   H
10 Cl 1 dist

12 dist <start> <steps> <step-size>

```

Once you have calculated the potential energy surface for the molecule, perform an optimization and frequency analysis of the HCl molecule.<sup>11</sup> You need to do this for  $\text{H}^{35}\text{Cl}$ ,  $\text{H}^{37}\text{Cl}$ ,  $\text{D}^{35}\text{Cl}$ , and  $\text{D}^{37}\text{Cl}$ . Here is an example header for the input file for this calculation.

<sup>11</sup> As outlined for  $\text{H}_3\text{O}^+$  in Problem 3 in the Introduction to Computational Chemistry lab.

Template Gaussian input file vibrotational calculations on a diatomic molecule.

```

1 %nproc=2
2 %mem=800MB
  # MP2 def2qzvpp
4 # opt freq=(anharm,vibrot) scf=tight

6 <title information>

8 <charge> <multiplicity>
  <atom1>(Iso=<mass_num>) <x1> <y1> <z1>
10 <atom2>(Iso=<mass_num>) <x2> <y2> <z2>
  <empty line>

```

Line 3 sets the theory level and basis set to reasonable values for our computational abilities and time. Line 4 tells Gaussian to optimize the geometry, then do a frequency analysis including anharmonic and vibrational-rotational coupling terms. You will need to replace all items in angle brackets with appropriate data. Note the addition of the (Iso=<mass-num>) option after the element labels.<sup>12</sup> By inserting the relevant value (total number of protons and neutrons) for your isotope (2 for deuterium), you can tell Gaussian to calculate using a different atomic mass.

You will need to import the files into your Jupyter notebook the same way you imported the files for the  $\text{H}_3\text{O}^+$  molecule in the Introduction to Computational Chemistry lab. From there, you need to extract the values for the equilibrium bond length,  $r_0$ , and zero-point energy  $D_e$ .

You will need to search the output file for the relevant information on the energy of the ground state (relative to the zero-point energy),  $D_0$ , and the vibrational and rotational energies,  $\tilde{\nu}_e$  and  $B_e$  (cc1ib doesn't yet parse these values). The vibrational and rotational constants are listed in tables under the heading "Vibrational Energies at Anharmonic Level". You will need to find the *two* values for the first-mode vibrational energy: one calculated using a harmonic analysis, the second incorporating the effects of anharmonicity. In this same section will be equilibrium rotational constants for the two degenerate rotational modes (labeled Ba(y) and Ca(z)) and the energy (relative to the zero-point energy) of the ground state ( $D_0$ ).<sup>13</sup>

<sup>12</sup> i.e., <atom1>(Iso=<mass\_num>) should become He(Iso=3) for a  $^3\text{He}$  atom)

<sup>13</sup> All energy values are listed in  $\text{cm}^{-1}$ .

## Data Analysis

1. Select your best HCl and DCl spectra and index the lines with the appropriate  $m$  values, as seen in fig. 2. If the  $^{35}\text{Cl}/^{37}\text{Cl}$  splitting is seen, index both sets, but be sure to separate them appropriately. Make a table of these  $m$  values and the corresponding frequencies,  $\tilde{\nu}(m)$ .

Express the frequencies in  $\text{cm}^{-1}$  to the tenth of a  $\text{cm}^{-1}$ . Then, list the differences between adjacent lines,  $\Delta\tilde{\nu}(m)$ , which will be roughly  $2B_e$ , but will vary with  $m$ .

2. Plot  $\Delta\tilde{\nu}(m)$  against  $m$  and plot the trendline for the series. If any values seem out of line, check the calculation for that cell.
3. Carry out a multiple linear least-squares fit to the data using eq. (10) to determine  $\tilde{\nu}_0$ ,  $B_e$ ,  $\alpha_e$ , and their standard errors. Repeat this fitting procedure using eq. (11), noting that high  $m$  transitions will be the most important ones in determining  $D_J$ , due to its  $m^3$  dependence.
4. Using your values of  $\tilde{\nu}_0$  for HCl and DCl, determine  $\tilde{\nu}_e$  and  $\tilde{\nu}_e\chi_e$  for HCl. From  $\tilde{\nu}_e$ , calculate  $k$  for HCl.
5. Calculate  $I_e$ , the moment of inertia, and  $r_e$ , the internuclear distance, for both HCl and DCl. Tabulate your results, along with your estimates of the experimental uncertainty. Compare your results with literature values found in Kerr, Huber, and Herzberg [3].
6. Using your value of  $\tilde{\nu}_0$  for HCl, calculate  $\tilde{C}_v$  (vib) at 298 K and at 1000 K from eq. (22). Compare the spectroscopic value  $\tilde{C}_v = 5/2R + \tilde{C}_v(\text{vib})$  with the experimental  $\tilde{C}_v$  value obtained from directly measured values [6, 10] of  $\tilde{C}_p$  and the expression  $\tilde{C}_v = \tilde{C}_p - R$ :  $\tilde{C}_v = 20.80 \text{ J}\cdot\text{K/mol}$  at 298 K and  $23.20 \text{ J}\cdot\text{K/mol}$  at 1000 K.

### Questions and Further Thoughts

1. Compute the ratio of  $B_e^*/B_e$  and compare with the rigid-rotor prediction of eq. (12). How constant is  $r_e$  for HCl and DCl?
2. Compute  $B_v = B_e - \alpha_e\left(v + \frac{1}{2}\right)$  for the  $v = 0, 1$ , and 2 levels of HCl and DCl and, from these, obtain average  $r_v$  values for these levels. Comment in your report on the changes in these distances.
3. Compare your  $\tilde{\nu}_0^*/\tilde{\nu}_0$  ratio with the ratio  $\sqrt{\mu/\mu^*}$  expected for a harmonic oscillator. How anharmonic is the HCl molecule: i.e., how large is  $\chi_e$ ?
4. Use your values of  $\tilde{\nu}_e$  and  $\tilde{\nu}_e\chi_e$  and eq. (5) to predict the frequencies of the first overtone transitions of HCl and DCl (ignore the rotational terms). Did you observe any evidence of these overtones in your spectra?
5. Do your spectra show any evidence of a  $^{35}\text{Cl}/^{37}\text{Cl}$  isotope effect? Use eq. (12) to calculate the splitting expected for this effect for several  $P$  and  $R$  branch transitions in HCl and DCl.

6. Compare your experimental results with the values deduced from your theoretical calculations. For each calculation, report the values for total energy ( $D_e$ ), vibration frequency ( $\tilde{\nu}_0$ ), bond length ( $r_e$ ), rotation constant  $B_e$ , and centrifugal distortion constant  $D_J$ . Additionally, you should plot the potential energy curve output by Gaussian against the theoretical Morse potential for the molecule.

### *Lab Report Guidelines*

As previously stated, your lab report should consist of the following parts:

#### *Title, Author and Date*

*Introduction and Objective* A paragraph describing what we hope to find in this experiment, and how.

*Experimental Procedure* This should be a very brief general outline of the procedure, written out as a paragraph or two. Give the make and model for any major instruments you used, as well as any important settings. For IR spectroscopy, this especially means the spectral resolution and number of scans.

*Results and Discussion* This should include answers to the questions in the section “Questions and Further Thoughts”. This should not be a separate section, but should instead be included organically in the discussion as a way of filling it out.

#### *Conclusion*

#### *References*

*Appendix* At the very end of your report, include examples of any calculations that you did by hand. Provide digital copies of the Excel (or other) files that you used to generate your graphs and copies of your Gaussian input files.

### *References*

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### *Further Reading*

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