Optimization and Algorithms
Project report

Group 00
Matyas Skalicky 194904, Carolina Guariglia 123456,
Manel Freitas 123456 and Carina Fernandes 123456

1 Task 1

2 Task 2

3 Task 3

4 Task 4

5 Task 5

Task 5. Using simple geometric arguments, give a closed-form expression for $d\,(p, D(c, r))$.

Based on whether the robot passes within the disc or not, the following cases might occur

$$d\,(p, D(c, r)) = \begin{cases} 0 & \text{if } d\,(p(\tau_k), c_k) \leq r_k \\ d\,(p(\tau_k), c_k) - r_k & \text{if } d\,(p(\tau_k), c_k) > r_k \end{cases} \tag{1}$$

The whole equation can be then expressed in closed form as

$$max\,(0, d\,(p(\tau_k), c_k) - r_k) \tag{2}$$

6 Task 6

7 Task 7

8 Task 8

The function ϕ outputs a zero if its argument is zero and outputs a one, otherwise. That is, we have $\phi : \mathbf{R}^2 \to \mathbf{R}$, with

$$\phi(x) = \begin{cases} 0, & \text{if } x = 0, \\ 1, & \text{if } x \neq 0. \end{cases}$$

Task 8. Show that the function ϕ is nonconvex.

Function is called convex if

$$\forall x_1, x_2 \in X, \forall t \in [0,1] : \qquad f(tx_1 + (1-t)x_2) \leq tf(x_1) + (1-t)f(x_2) \tag{3}$$

A function is convex if for any two points x_1, x_2 ant $t \in [0,1]$ the above inequality is true. We select $x_1 = 0$, $x_2 = 1$ and $t = 0.5$. The inequality therefore looks like

$$\begin{aligned} f(t \cdot x_1 + (1-t) \cdot x_2) &\leq t \cdot f(x_1) + (1-t) \cdot f(x_2) \\ f(0.5 \cdot 0 + 0.5 \cdot 1) &\leq 0.5 \cdot f(0) + 0.5 \cdot f(1) \\ f(0.5) &\leq 0.5 \cdot f(0) + 0.5 \cdot f(1) \\ 1 &\leq 0.5 \end{aligned} \tag{4}$$

which is in case of ϕ a contradiction for selected x_1, x_2 and t. Since ϕ does not fulfill the requirements for a convex function, we conclude that it is non convex.