Optimization and algorithms

Course overview

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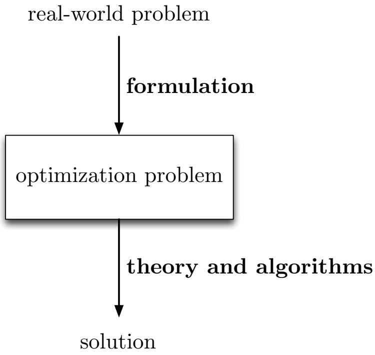
Outline

- What is the goal of optimization?
- The most important fact about optimization
- Your first optimization problem: placing a fire station
- Grading

What is the goal of optimization?

To save the world, one optimization at a time...



Why study optimization?

Optimization is applied in numerous fields:

- communications
- control
- power systems
- computer vision
- machine learning
- finance
- networks
- data science
- ...

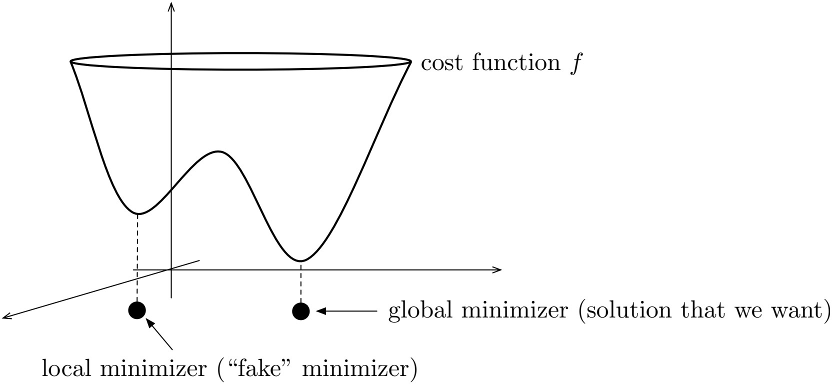
An optimization problem is a mathematical object of the following form:

$$
\begin{array}{lll}
\underset{x}{\text{minimize}} & f(x) & \\
\text{subject to} & h_1(x) & = 0 \\
 & & \vdots \\
 & h_p(x) & = 0 \\
 & g_1(x) & \leq 0 \\
 & & \vdots \\
 & g_m(x) & \leq 0
\end{array}
$$

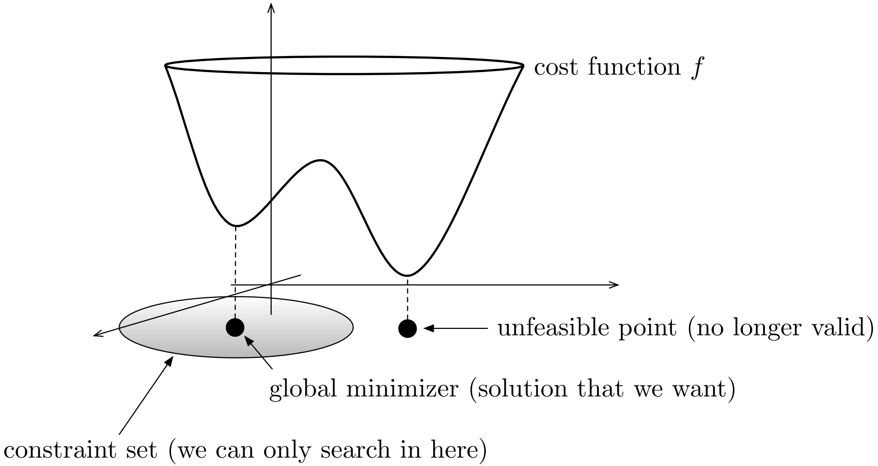
- $x \in \mathbf{R}^n$ is the optimization variable
- $f : \mathbf{R}^n \to \mathbf{R}$ is the objective or cost function that we want to minimize
- $h_1, \ldots, h_p, g_1, \ldots, g_m : \mathbf{R}^n \to \mathbf{R}$ are constraint functions

Solving an optimization problem means finding a global minimizer

An unconstrained optimization problem:

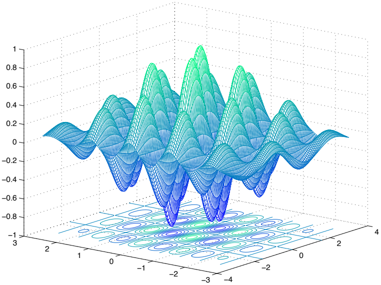


A constrained optimization problem:

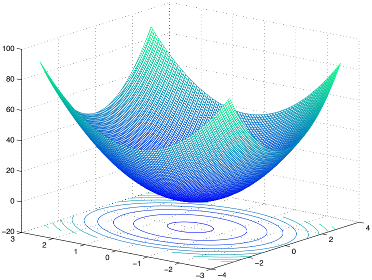


The most important fact about optimization

There are two classes of optimization problems:



Nonconvex

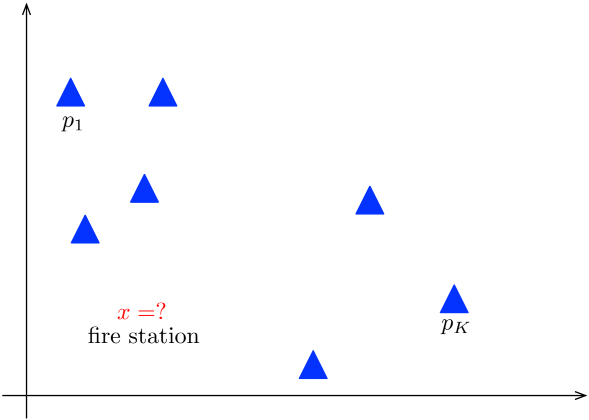


Convex

- Algortihms that solve typical nonconvex problems are slow
- Algorithms that solve typical convex problems are **very fast**

Your first optimization problem: placing a fire station

- A fire station is going to serve K villages
- Villages are located at given positions $p_1, p_2, \ldots, p_K \in \mathbf{R}^2$
- Where should you place the fire station?



- A possible problem formulation is as follows:

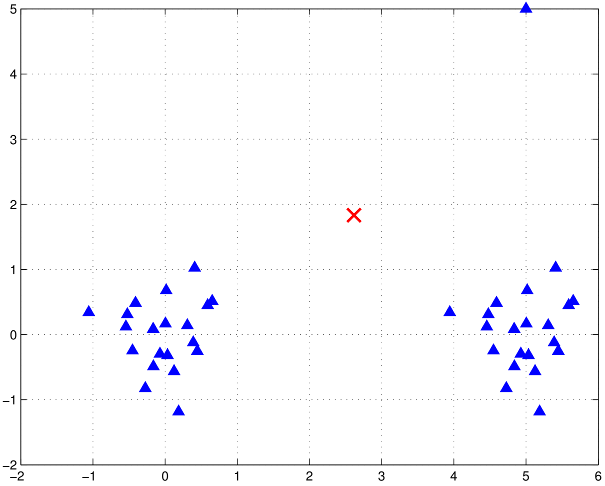
$$\underset{x}{\text{minimize}} \quad \underbrace{\max\left\{\|x-p_1\|, \|x-p_2\|, \ldots, \|x-p_K\|\right\}}_{f(x)}$$

with optimization variable $x \in \mathbf{R}^2$ (x represents the hospital location)

- Is this a convex or a nonconvex problem?

```
% firestation.m; uses package CVX from http://cvxr.com/cvx
KK = 20; % choose K = number of villages
p1 = 0.5*randn(2,KK); % generate random positions
p2 = p1+[5 ; 0]*ones(1,KK);
p = [ p1 , p2 , [ 5 ; 5 ] ]; K = size(p,2);

% plot the villages
figure(1); clf;
plot(p(1,:),p(2,:),'^','MarkerSize',8,'MarkerFaceColor','b');
grid on;

% solve the optimization problem
cvx_begin quiet
    variable x(2,1);

    % build cost function
    f = norm(x - p(:,1));
    for i = 2:K
        f = max(f,norm(x-p(:,i)));
    end;

    minimize(f);
cvx_end;

%plot solution
hold on; plot(x(1),x(2),'rx','MarkerSize',15,'LineWidth',2);
```

- Here is a typical output:



The course consists of three parts:

- Part 1: the art of formulating optimization problems
- Part 2: unconstrained optimization
  - how to tell if an unconstrained optimization problem is convex
  - numerical algorithms for convex and nonconvex problems
- Part 3: constrained optimization
  - how to tell if a constrained optimization problem is convex
  - numerical algorithms for convex and nonconvex problems

Grading

- Your grade is computed as follows:

$$\text{grade} = 50\% \text{ project} + 50\% \text{ max}\{\text{exam 1, exam 2}\}$$

- The minimum grade for either the project or the exam is 10 points

- Project is done by groups of four students

Project timeline

- Sep. 23: I post the link for registering the groups
- Sep. 23–Sep.28: you register your group
- Sep. 28: I remove the link for registering the groups
- Oct. 1: I post the list of groups and Part 1 of the project
- Nov. 1: I post part 2 of the project
- Dec. 1: I post part 3 of the project
- Dec. 14: you submit by email your project report (written in LaTeX)
- Dec. 16–Dec. 20: you present orally your project (20 min)