PROCESY MARKOWA

LISTA 2 ZADANIE 7

Rozpatrzmy rzuty symetryczną kostką i umówmy się, że $X_n = j$, jeżeli j jest największą liczbą wyrzuconą w pierwszych n rzutach. Czy $\{X_n\}$ jest jednorodnym łańcuchem Markowa? Jeżeli tak, to znaleźć jego macierz prawdopodobieństw przejść $\mathbb{P}$ oraz $\mathbb{P}^n$

$$\mathbb{P} = \begin{pmatrix} 1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 0 & 2/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & 3/6 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & 4/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & 0 & 5/6 & 1/6 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{vmatrix} 1/6-\lambda & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 0 & 2/6-\lambda & 1/6 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & 3/6-\lambda & 1/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & 4/6-\lambda & 1/6 & 1/6 \\ 0 & 0 & 0 & 0 & 5/6-\lambda & 1/6 \\ 0 & 0 & 0 & 0 & 0 & 1-\lambda \end{vmatrix}$$

$$= \left(\frac{1}{6}-\lambda\right)\left(\frac{2}{6}-\lambda\right)\left(\frac{3}{6}-\lambda\right)\left(\frac{4}{6}-\lambda\right)\left(\frac{5}{6}-\lambda\right)(1-\lambda) = 0$$

1° $\lambda = \frac{1}{6}$

$$\begin{pmatrix} 0 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 0 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & 2/6 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & 3/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & 0 & 4/6 & 1/6 \\ 0 & 0 & 0 & 0 & 0 & 5/6 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow b = c = d = e = f = 0$$

$W_1 = (1, 0, 0, 0, 0, 0)^T$

2° $\lambda = \frac{2}{6}$

$$\begin{pmatrix} -1/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & 1/6 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & 2/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & 0 & 3/6 & 1/6 \\ 0 & 0 & 0 & 0 & 0 & 4/6 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow c = d = e = f = 0, \quad a = b$$

$W_1 = (1, 1, 0, 0, 0, 0)^T$

$3^\circ \quad \lambda = \frac{3}{6}$

$$\begin{pmatrix} -2/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 0 & -1/6 & 0 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & 0 & 2/6 & 1/6 \\ 0 & 0 & 0 & 0 & 0 & 3/6 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow d = e = f = 0, \quad a = b = c$$

$W_1 = (1, 1, 1, 0, 0, 0)^T$

$4^\circ \quad \lambda = \frac{4}{6}$

$$\begin{pmatrix} -3/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 0 & -2/6 & 0 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & -1/6 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & 0 & 1/6 & 1/6 \\ 0 & 0 & 0 & 0 & 1/6 & 1/6 \\ 0 & 0 & 0 & 0 & 0 & 2/6 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow e = f = 0, \quad a = b = c = d$$

$W_1 = (1, 1, 1, 1, 0, 0)^T$

$5^\circ \quad \lambda = \frac{5}{6}$

$$\begin{pmatrix} -4/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 0 & -3/6 & 0 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & -2/6 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & -1/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & 0 & 0 & 1/6 \\ 0 & 0 & 0 & 0 & 0 & 1/6 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow f = 0, \quad a = b = c = d = e$$

$W_1 = (1, 1, 1, 1, 1, 0)^T$

$6^\circ \quad \lambda = 1$

$$\begin{pmatrix} -5/6 & 1/6 & 1/6 & 1/6 & 1/6 & 1/6 \\ 0 & -4/6 & 0 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & -3/6 & 1/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & -2/6 & 1/6 & 1/6 \\ 0 & 0 & 0 & 0 & -1/6 & 1/6 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \\ e \\ f \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow a = b = c = d = e = f$$

$W_1 = (1, 1, 1, 1, 1, 1)^T$

$$\mathbb{P}^n = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1/6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2/6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3/6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4/6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5/6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}^n \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} (1/6)^n & (2/6)^n & (3/6)^n & (4/6)^n & (5/6)^n & 1 \\ 0 & (2/6)^n & (3/6)^n & (4/6)^n & (5/6)^n & 1 \\ 0 & 0 & (3/6)^n & (4/6)^n & (5/6)^n & 1 \\ 0 & 0 & 0 & (4/6)^n & (5/6)^n & 1 \\ 0 & 0 & 0 & 0 & (5/6)^n & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} (1/6)^n & (2/6)^n - (1/6)^n & (3/6)^n - (2/6)^n & (4/6)^n - (3/6)^n & (5/6)^n - (4/6)^n & 1 - (5/6)^n \\ 0 & (2/6)^n & (3/6)^n - (2/6)^n & (4/6)^n - (3/6)^n & (5/6)^n - (4/6)^n & 1 - (5/6)^n \\ 0 & 0 & (3/6)^n & (4/6)^n - (3/6)^n & (5/6)^n - (4/6)^n & 1 - (5/6)^n \\ 0 & 0 & 0 & (4/6)^n & (5/6)^n - (4/6)^n & 1 - (5/6)^n \\ 0 & 0 & 0 & 0 & (5/6)^n & 1 - (5/6)^n \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$