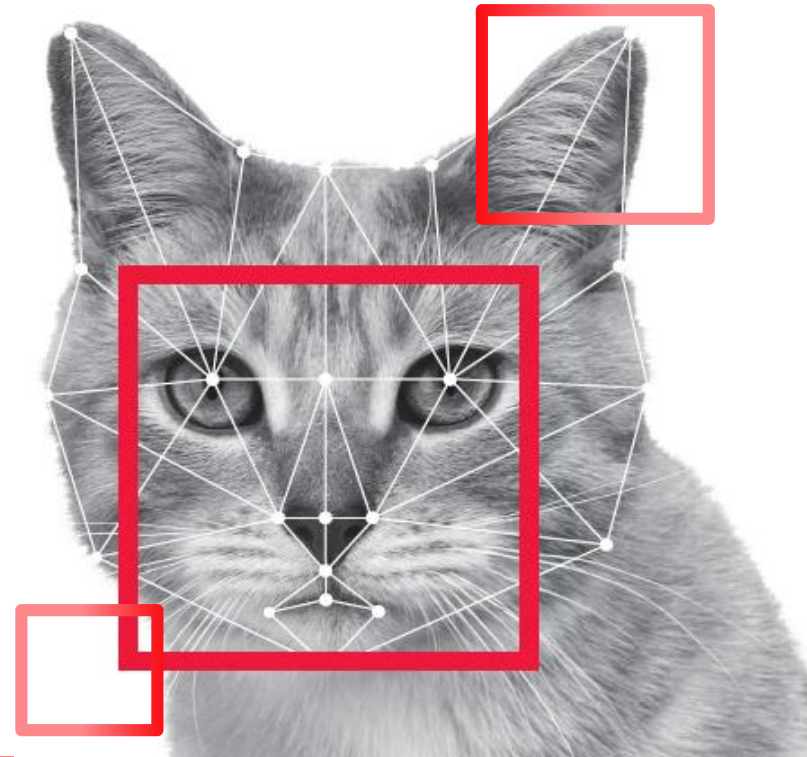


COMPUTER VISION

컴퓨터 비전

기본 개념부터 최신 모바일 응용 예까지



3장. 에지 검출

각 절에서 다루는 내용

1. 선분 검출

3.5 선분 검출

■ 에지 맵 → 에지 토막 → 선분

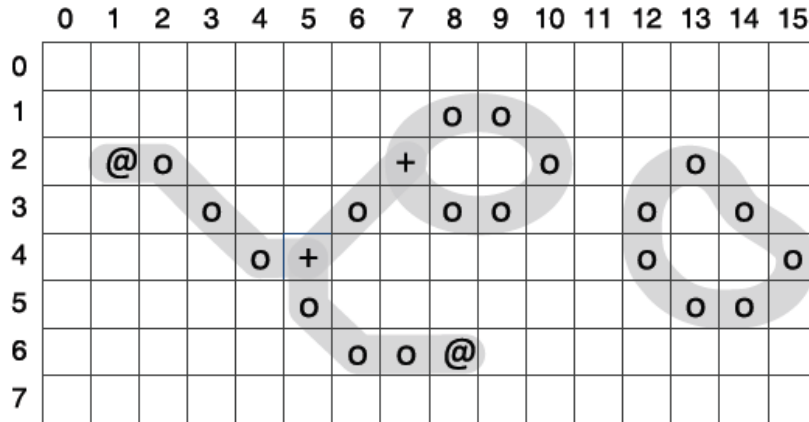
3.5.1 에지 연결과 선분 근사

3.5.2 허프 변환

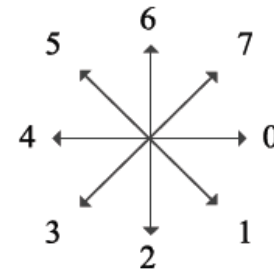
3.5.3 RANSAC

3.5.1 에지 연결과 선분 근사

■ 에지 연결과 표현



(a) 에지 영상



(b) 체인 코드 표시 기준

에지 토막	에지 열	체인 코드
1	(2,1)(2,2)(3,3)(4,4)(4,5)	(2,1)0110
2	(4,5)(5,5)(6,6)(6,7)(6,8)	(4,5)2100
3	(4,5)(3,6)(2,7)	(4,5)77
4	(2,7)(1,8)(1,9)(2,10)(3,9)(3,8)	(2,7)701345
5	(2,13)(3,14)(4,15)(5,14)(5,13)(4,12)(3,12)	(2,13)113567

그림 3-22 에지 토막의 에지 열과 체인 코드 표현

3.5.1 에지 연결과 선분 근사

■ 선분 근사

- 두 끝점을 잇는 직선으로부터 가장 먼 점까지의 거리 h 가 임계값 이내가 될 때까지 선분 분할을 재귀적으로 반복

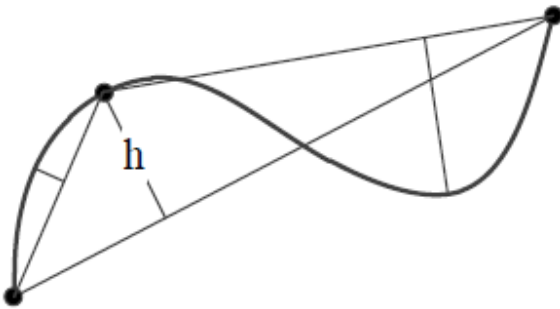


그림 3-27 선분 근사화 알고리즘

3.5.2 허프 변환

■ 허프 변환

- 에지 연결 과정 없이 선분 검출 (전역 연산을 이용한 지각 군집화)
- 영상 공간 y - x 를 기울기 절편 공간 b - a 로 매핑

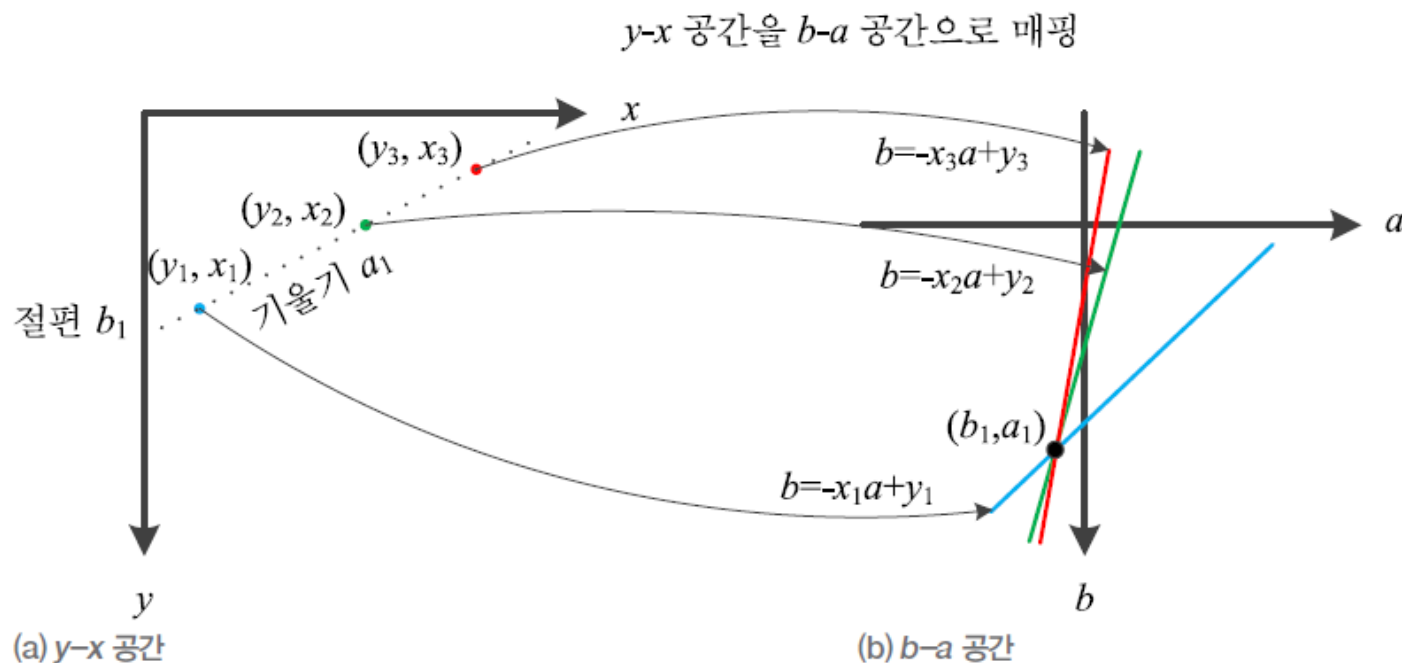


그림 3-28 허프 변환의 원리

3.5.2 허프 변환

■ 수직선의 기울기가 ∞ 인 문제

- 극좌표계 사용하여 해결

$$y \cos \theta + x \sin \theta = \rho \quad (3.16)$$

y - x 공간을 ρ - θ 공간으로 매핑

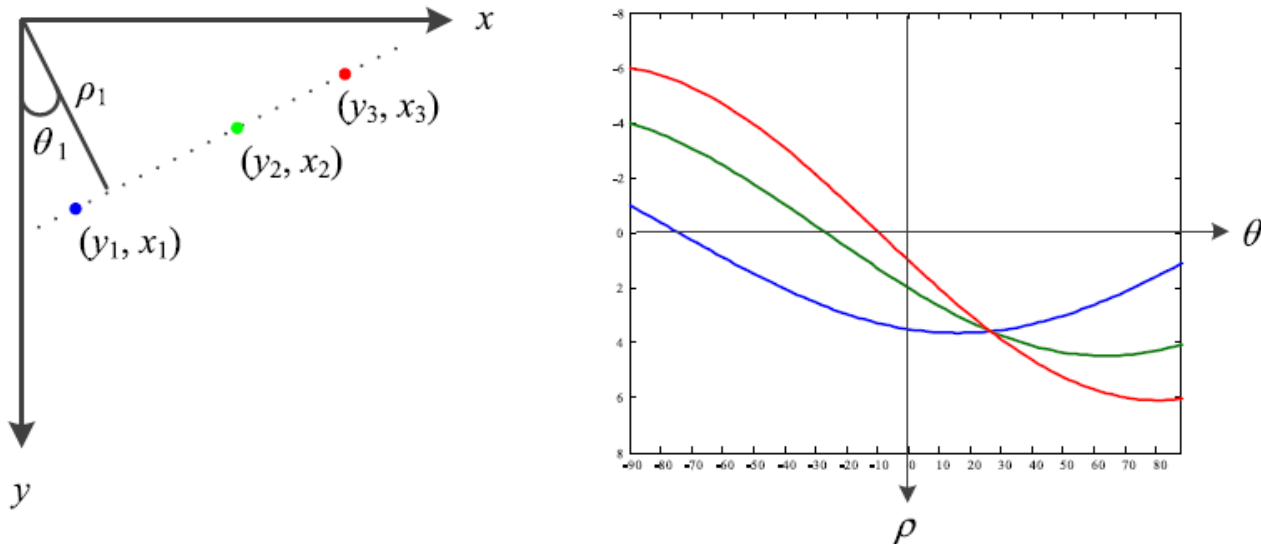


그림 3-29 ρ - θ 공간에서 허프 변환

3.5.2 허프 변환

■ 밀집된 곳 찾기

- 양자화된 누적 배열 이용하여 해결

알고리즘 3-7 직선 검출을 위한 허프 변환

입력 : 에지 영상 $e(j, i)$, $0 \leq j \leq M-1$, $0 \leq i \leq N-1$, 임계값 T // 에지는 1, 비에지는 0인 이진 영상

출력 : (ρ_k, θ_k) , $1 \leq k \leq n$ (n 개의 직선)

- 1 2차원 누적 배열 A 를 0으로 초기화한다.
- 2 for(에지 영상 e 에 있는 에지 화소 (y_i, x_i) 각각에 대해)
- 3 $y_i \cos \theta + x_i \sin \theta = \rho$ 가 지나는 A 의 모든 칸을 1만큼 증가시킨다.
- 4 A 에서 T 를 넘는 지역 최대점 (ρ_k, θ_k) 를 모두 찾아 직선으로 취한다.

■ 원 검출

- 3차원 누적 배열 사용

$$(y - b)^2 + (x - a)^2 = r^2 \quad (3.17)$$

3.5.2 허프 변환

예제 3-3 허프 변환

[그림 3-30]은 [그림 3-29]를 이산 공간에 다시 그린 것이다. 왼쪽 그림에서 세 점은 $(y_1, x_1)=(4,1)$, $(y_2, x_2)=(2,4)$, $(y_3, x_3)=(1,6)$ 이다. $(y_1, x_1)=(3.5,1)$ 이면 세 점이 정확히 일직선 상에 있지만, 디지털 영상의 특성상 약간의 위치 오차가 발생했다고 간주하자.

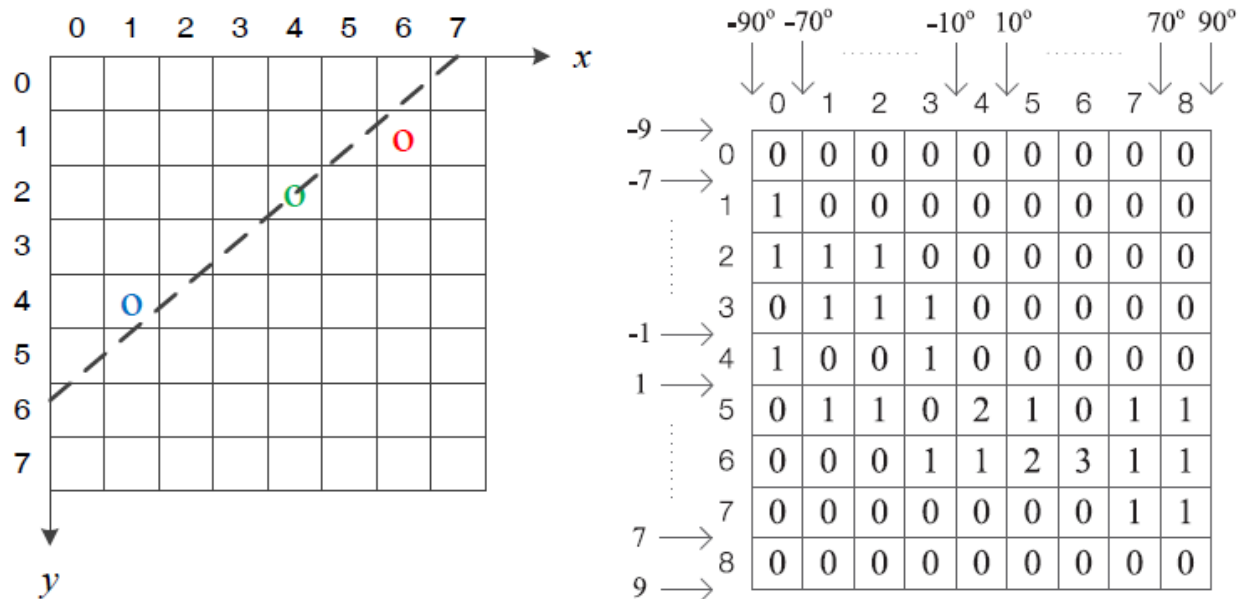
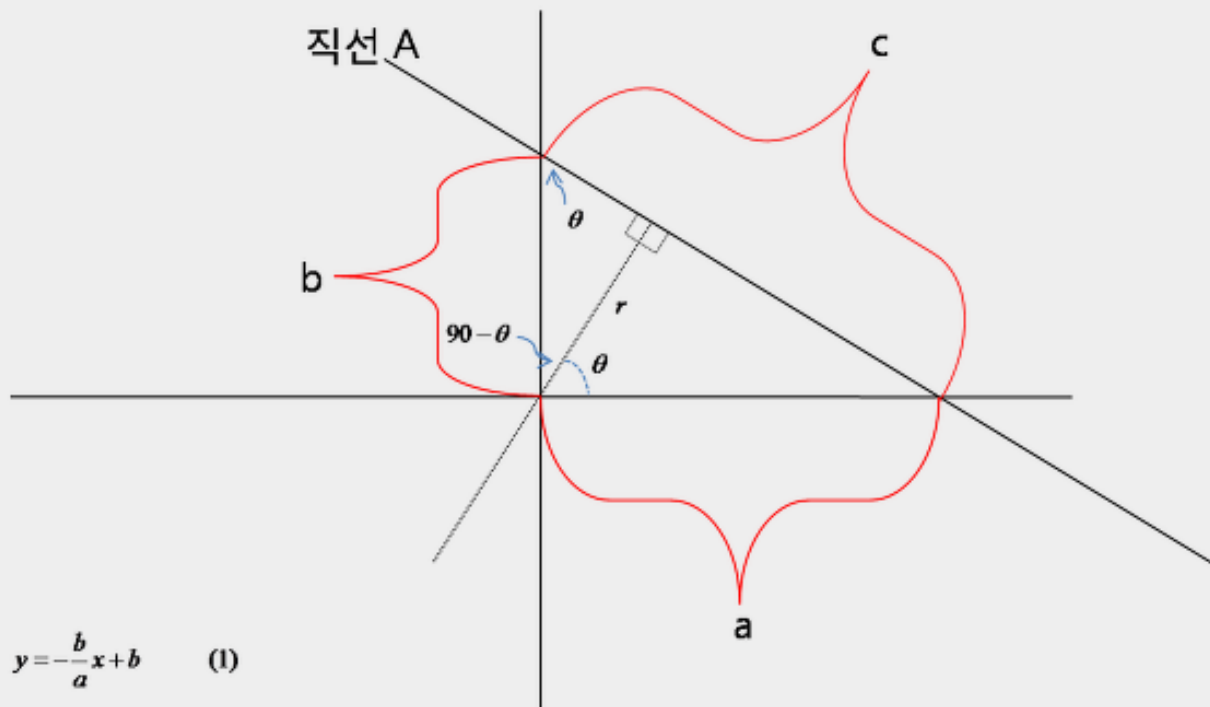


그림 3-30 이산 공간에서 허프 변환

θ 축은 20° 간격으로 양자화하여 총 아홉 개의 구간을 가지도록 하였다. ρ 축은 범위 $[-9, 9]$ 를 2 크기의 구간으로 나누어 총 아홉 개의 구간을 가지도록 양자화하였다. 따라서 누적 배열 A 는 9×9 이다. [알고리즘 3-7]에 따라 A 를 0으로 초기화한 후, 2~3행을 수행하여 세 점의 자취를 누적시키면 오른쪽 그림과 같은 배열이 된다. 이 배열에서 지역 최대점은 3을 갖는 $(6,6)$ 으로, $(\rho, \theta)=(4, 40^\circ)$ 에 해당한다. $y \cos 40^\circ + x \sin 40^\circ = 4$ 라는 직선을 검출한 셈이다. 왼쪽 그림에 있는 점선이 검출한 직선이다.

Voting schemes

- Let each feature vote for all the models that are compatible with it
- Hopefully the noise features will not vote consistently for any single model
- Missing data doesn't matter as long as there are enough features remaining to agree on a good model



기울기

$$\sin \theta = \frac{a}{c} \quad (2)$$

$$\cos \theta = \frac{b}{c} \quad (3)$$

$$-\frac{b}{a} = -\frac{\cos \theta \cdot c}{\sin \theta \cdot c} = -\frac{\cos \theta}{\sin \theta} \quad (4)$$

Y절편

$$\sin \theta = \frac{r}{b} \quad (5)$$

$$b = \frac{r}{\sin \theta} \quad (6)$$

$$y = -\frac{\cos \theta}{\sin \theta}x + \frac{r}{\sin \theta} \quad (7)$$

$$\cos \theta \cdot x + \sin \theta \cdot y = r \quad (8)$$

Hough transform

- An early type of voting scheme
- General outline:
 - Discretize parameter space into bins
 - For each feature point in the image, put a vote in every bin in the parameter space that could have generated this point
 - Find bins that have the most votes

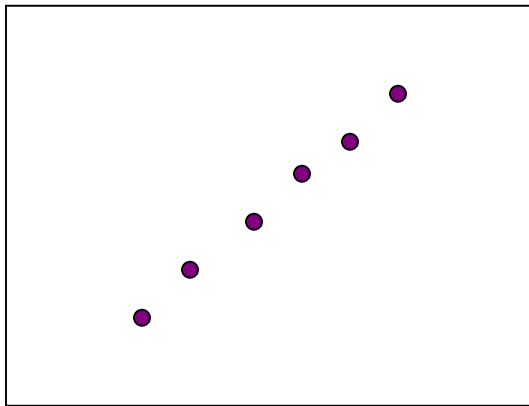
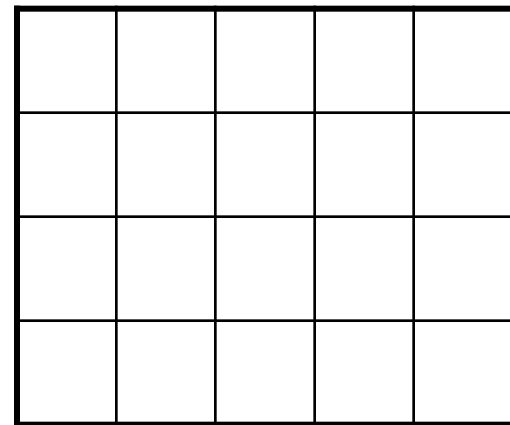
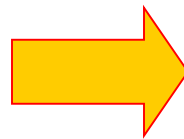


Image space

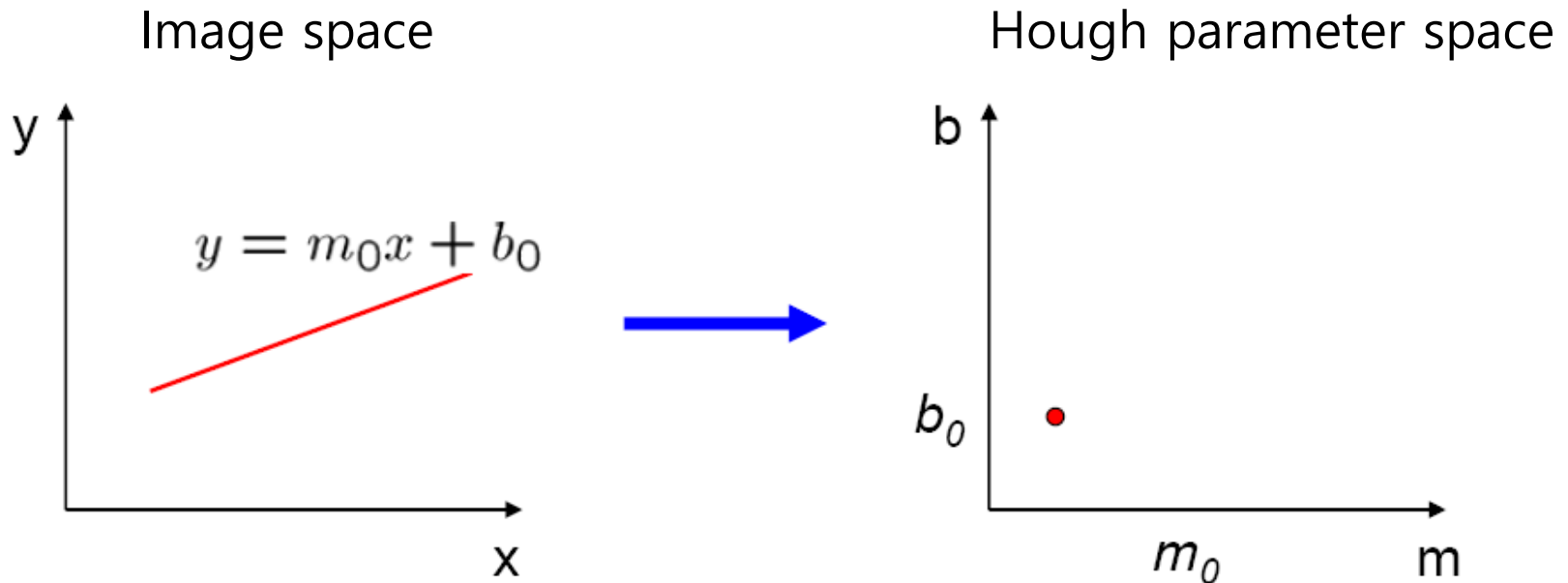


Hough parameter space

P.V.C. Hough, *Machine Analysis of Bubble Chamber Pictures*, Proc. Int. Conf . High Energy Accelerators and Instrumentation, 1959

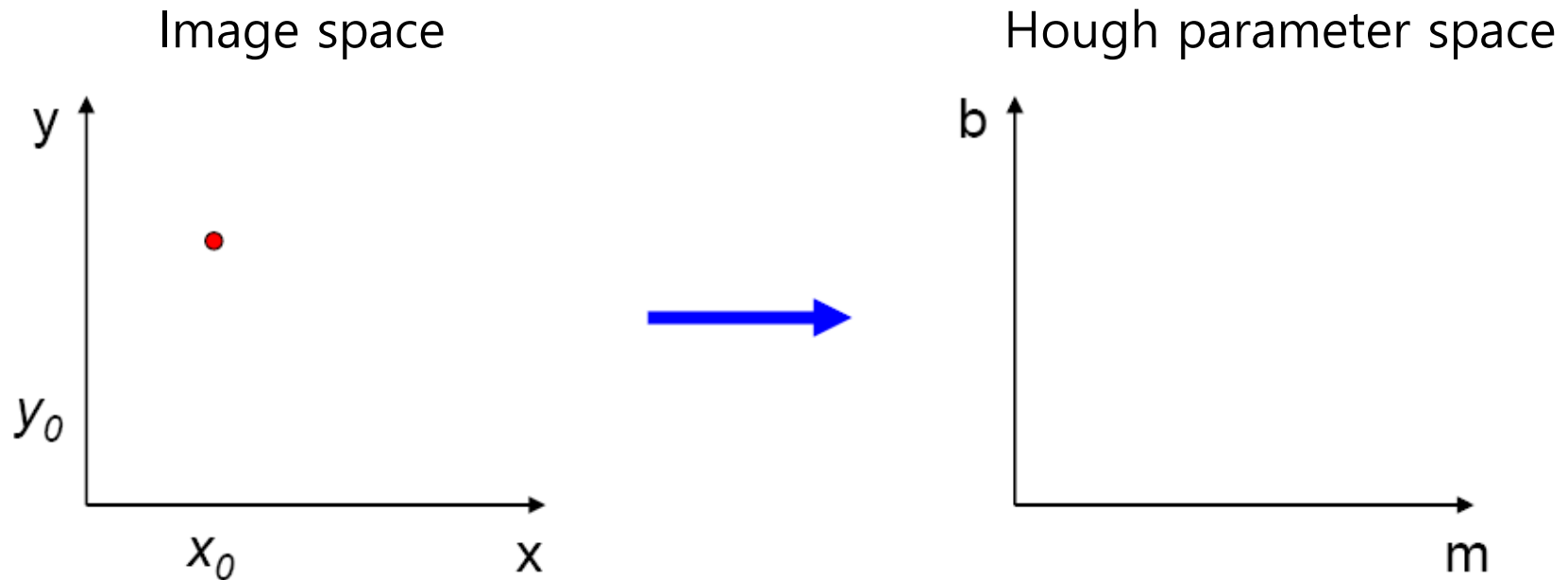
Parameter space representation

- A line in the image corresponds to a point in Hough space



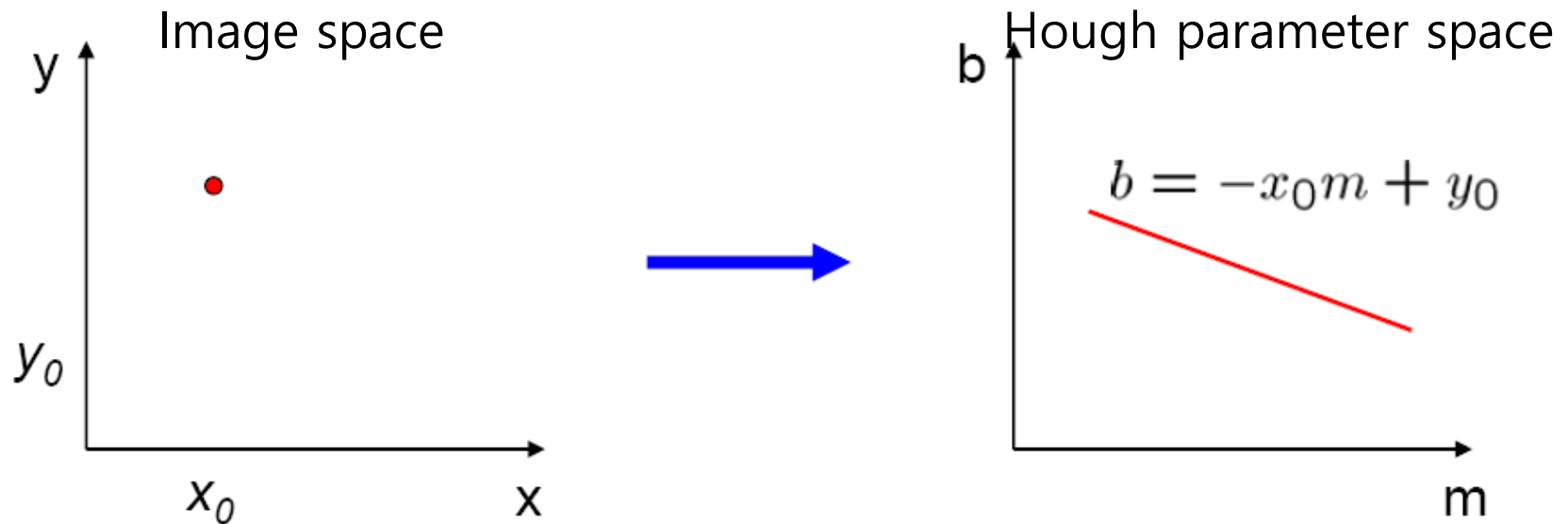
Parameter space representation

- What does a point (x_0, y_0) in the image space map to in the Hough space?



Parameter space representation

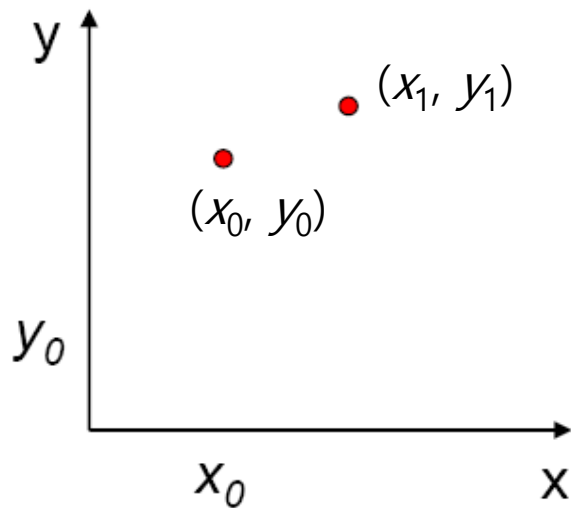
- What does a point (x_0, y_0) in the image space map to in the Hough space?
 - Answer: the solutions of $b = -x_0m + y_0$
 - This is a line in Hough space



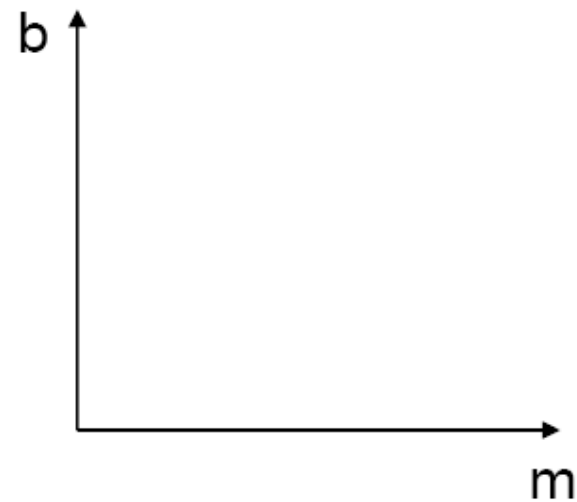
Parameter space representation

- Where is the line that contains both (x_0, y_0) and (x_1, y_1) ?

Image space



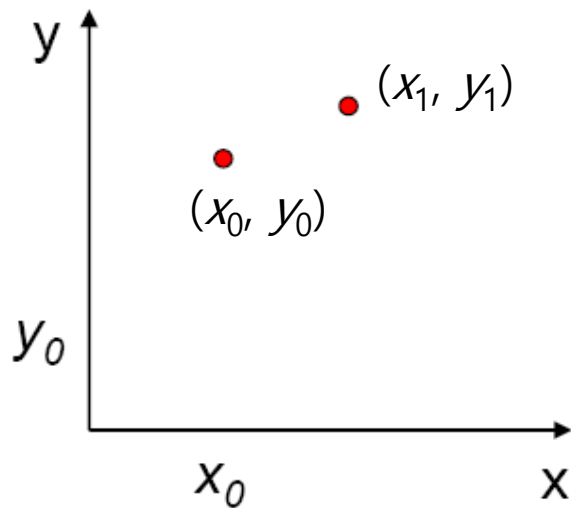
Hough parameter space



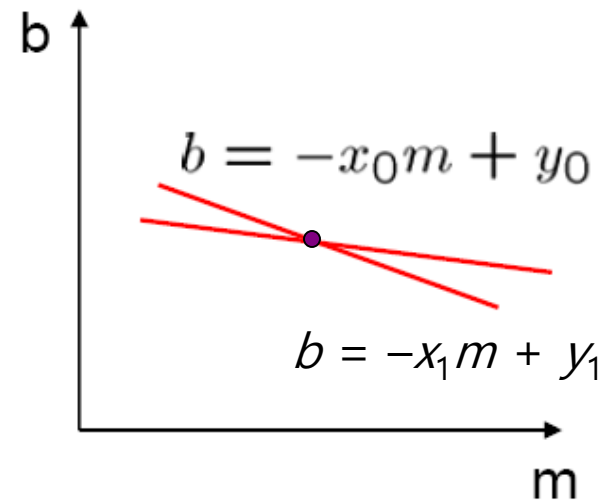
Parameter space representation

- Where is the line that contains both (x_0, y_0) and (x_1, y_1) ?
 - It is the intersection of the lines $b = -x_0m + y_0$ and $b = -x_1m + y_1$

Image space



Hough parameter space

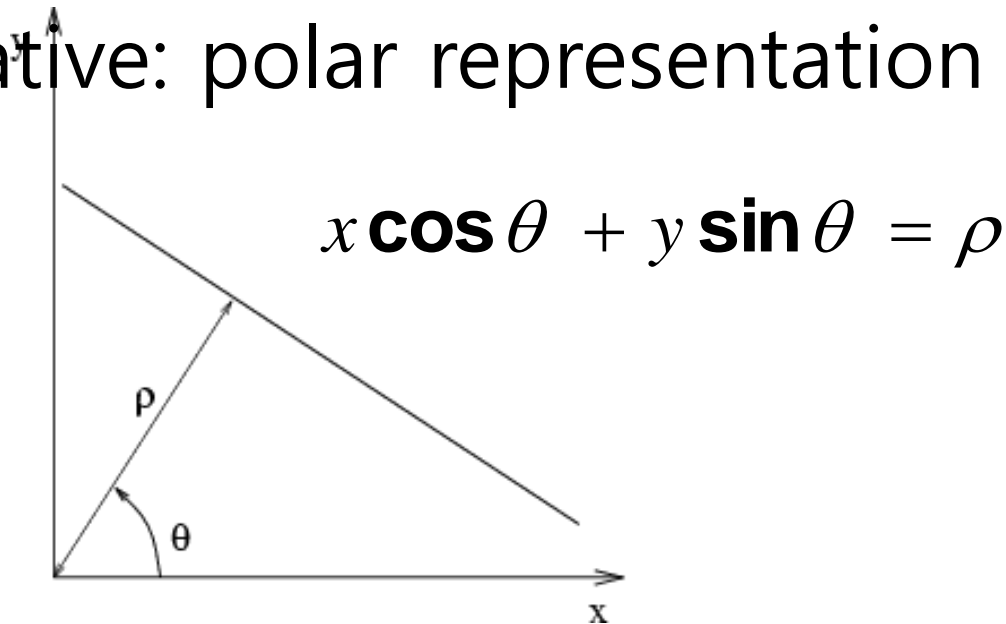


Parameter space representation

- Problems with the (m,b) space:
 - Unbounded parameter domains
 - Vertical lines require infinite m

Parameter space representation

- Problems with the (m,b) space:
 - Unbounded parameter domains
 - Vertical lines require infinite m
- Alternative: polar representation



Each point will add a sinusoid in the (θ, ρ) parameter space

Algorithm outline

- Initialize accumulator H to all zeros
- For each edge point (x,y) in the image

For $\theta = 0$ to 180

$$\rho = x \cos \theta + y \sin \theta$$

$$H(\theta, \rho) = H(\theta, \rho) + 1$$

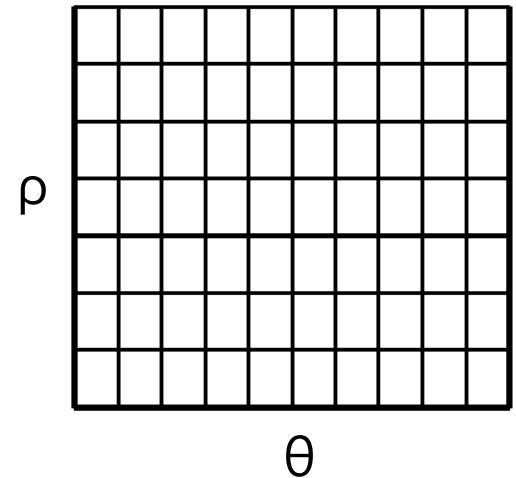
end

end

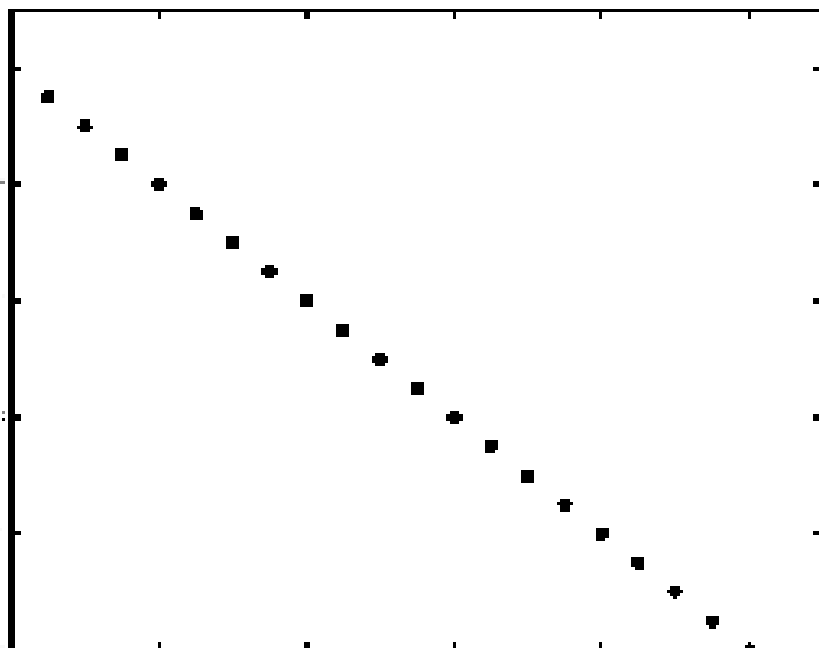
- Find the value(s) of (θ, ρ) where $H(\theta, \rho)$ is a local maximum
 - The detected line in the image is given by

$$\rho = x \cos \theta + y \sin \theta$$

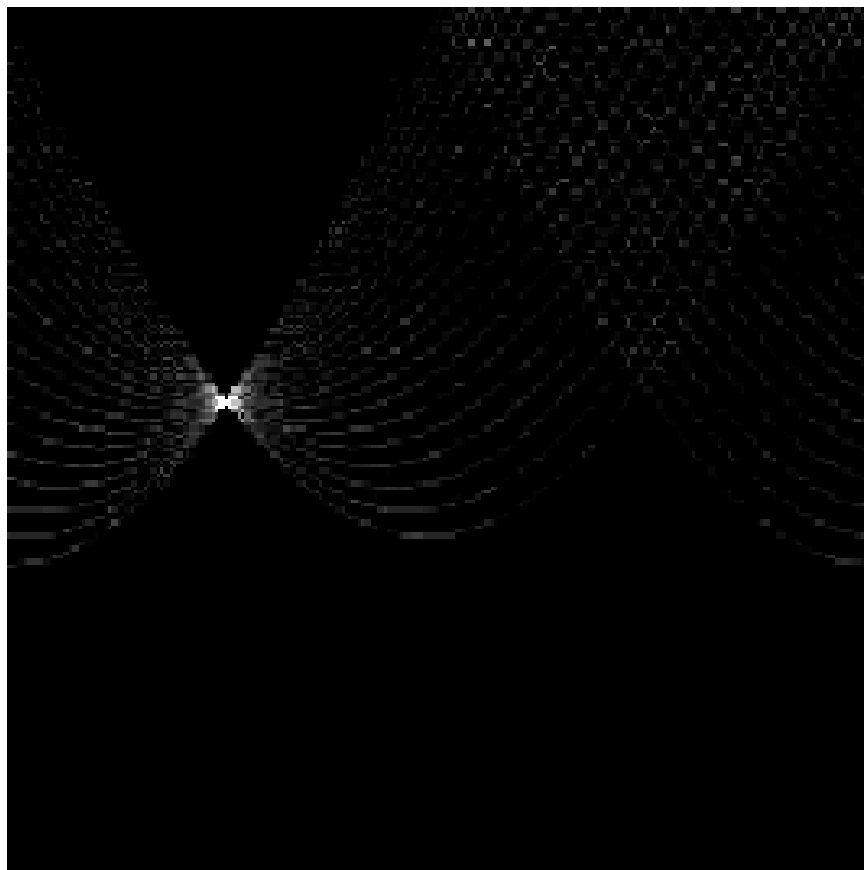
H: accumulator array (votes)



Basic illustration



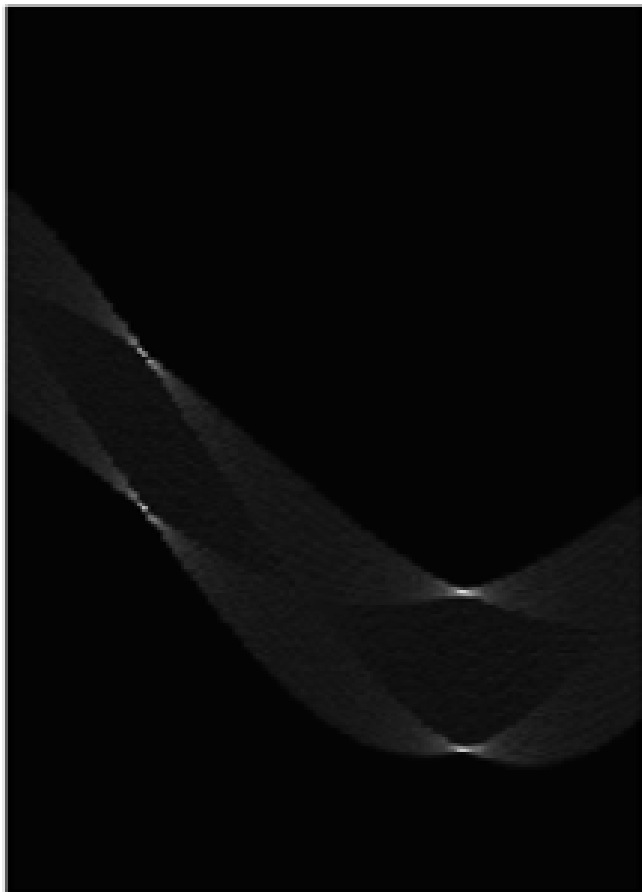
features



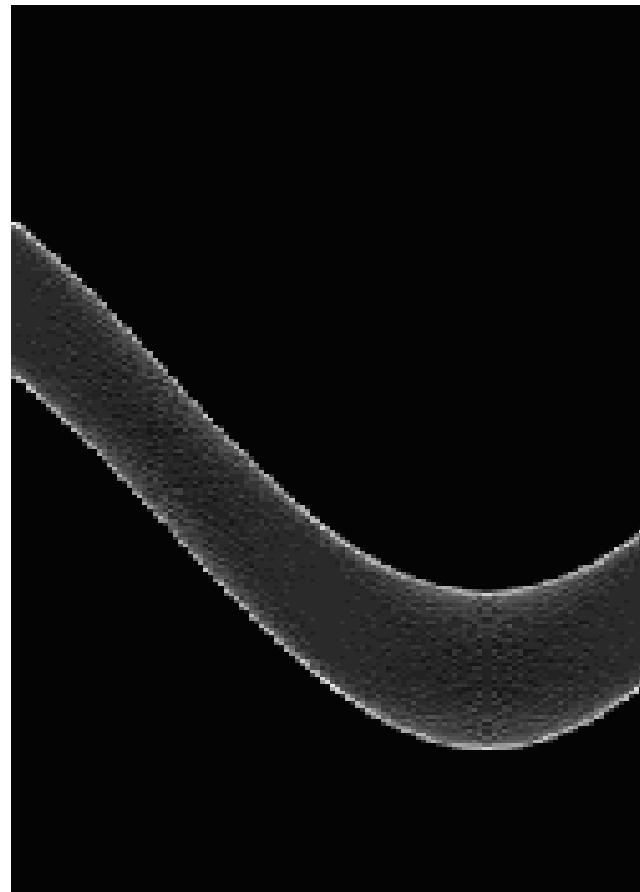
votes

Other shapes

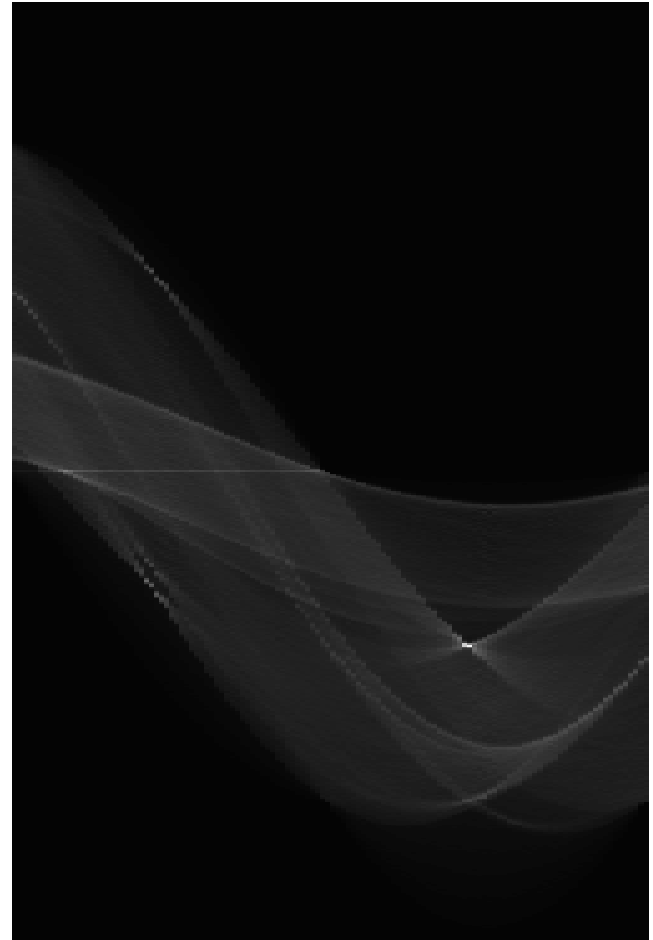
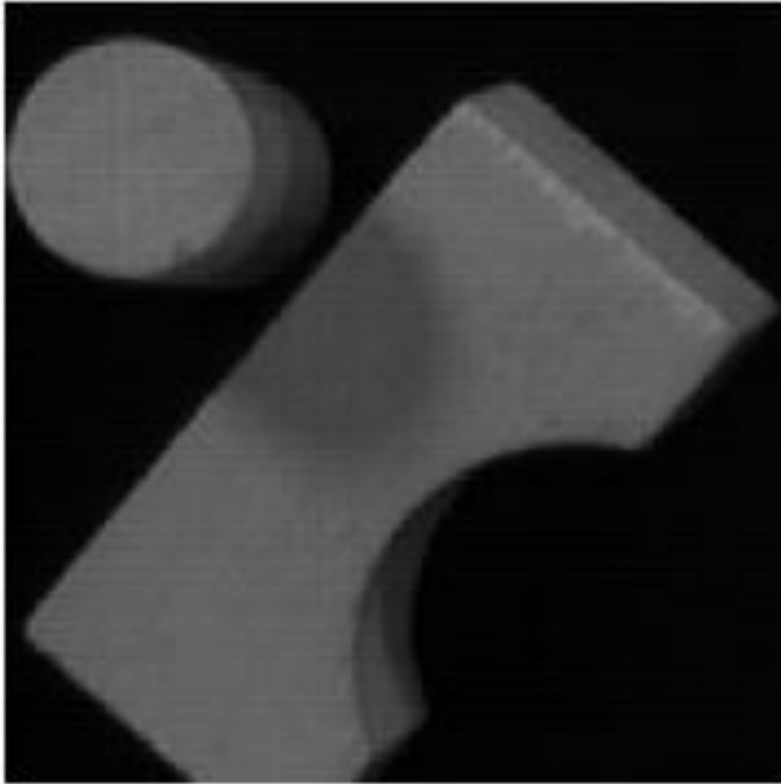
Square



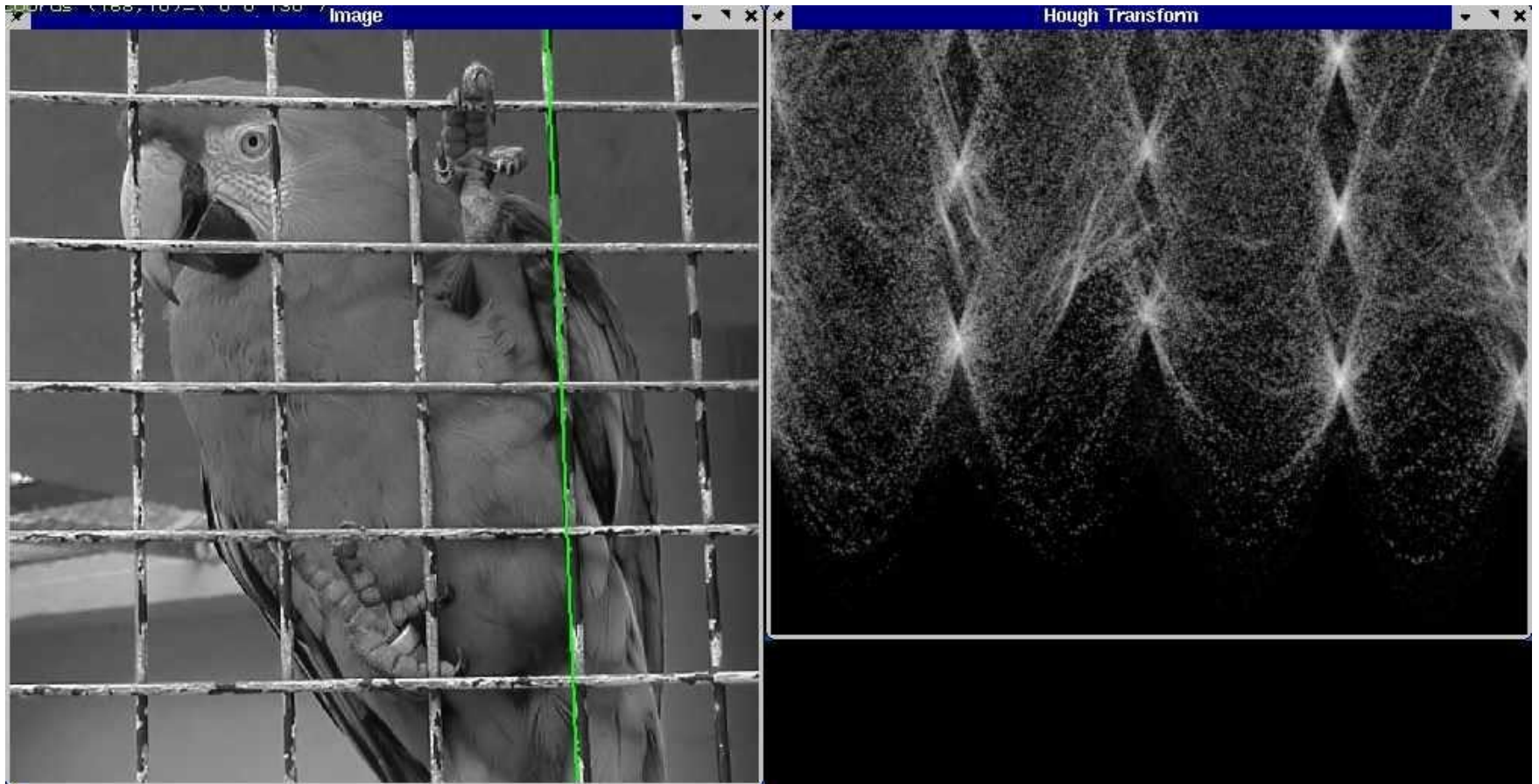
Circle



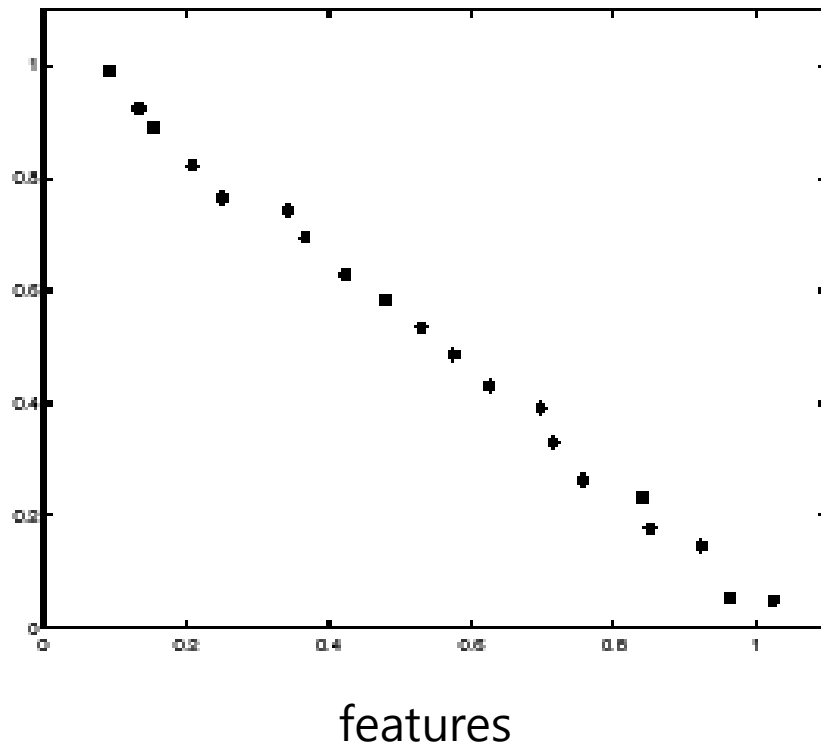
Several lines



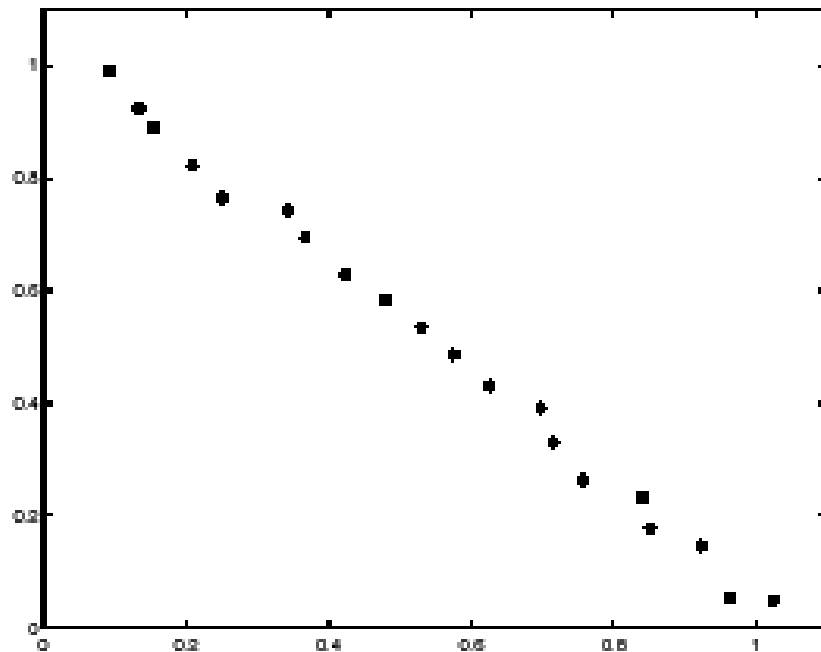
A more complicated image



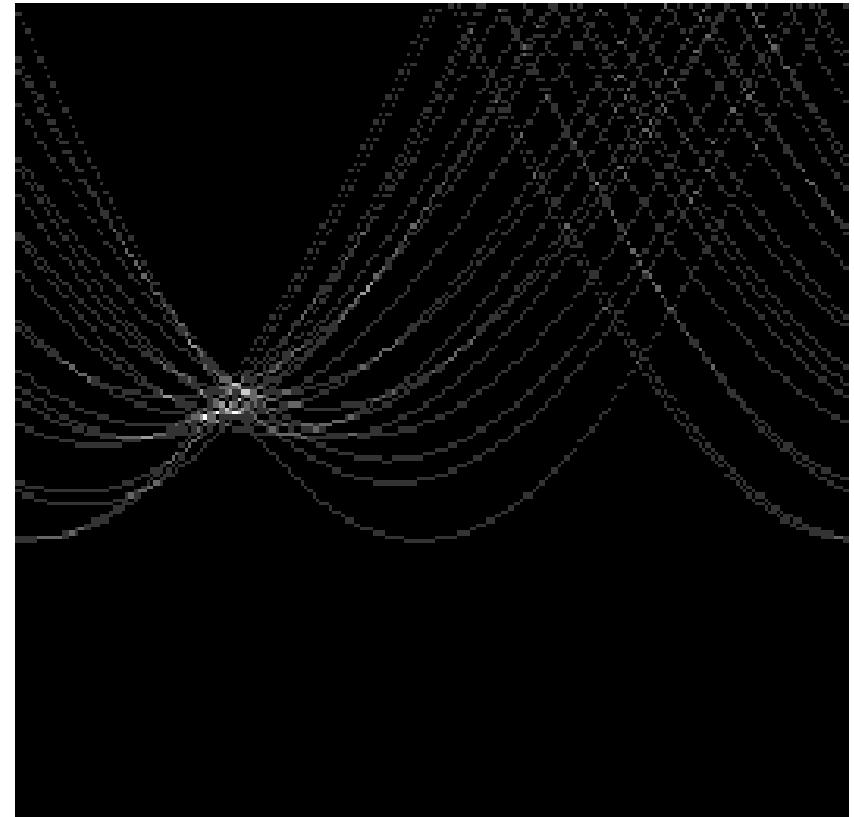
Effect of noise



Effect of noise



features

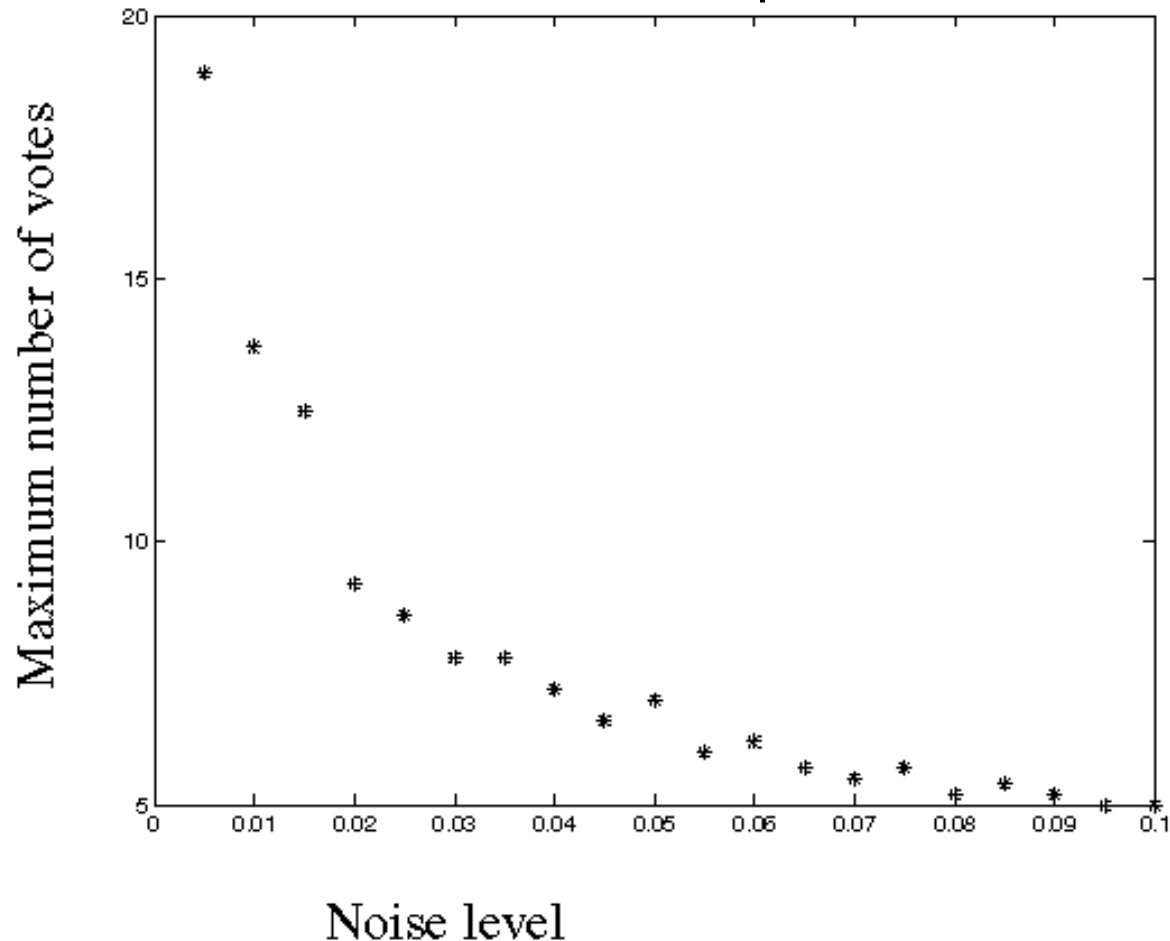


votes

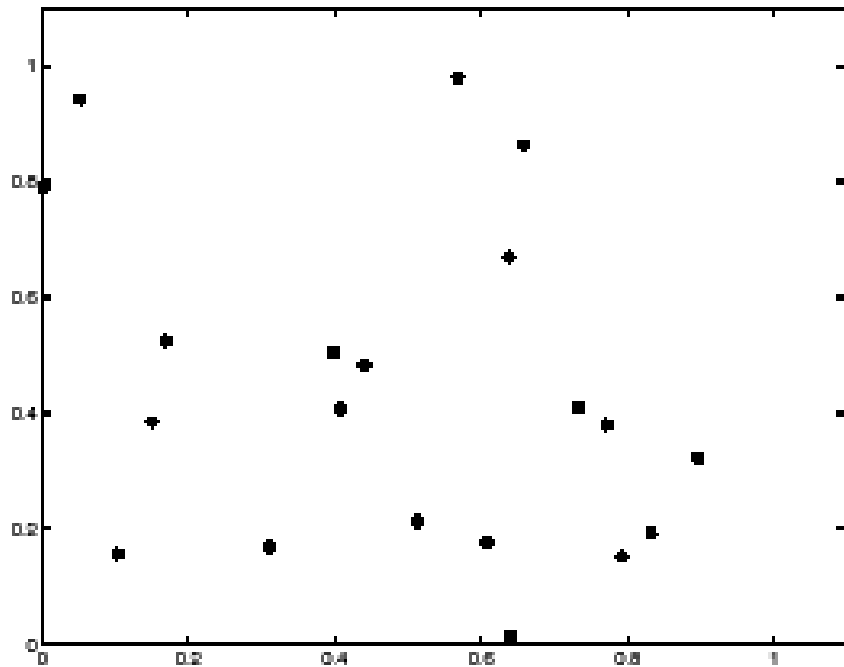
- Peak gets fuzzy and hard to locate

Effect of noise

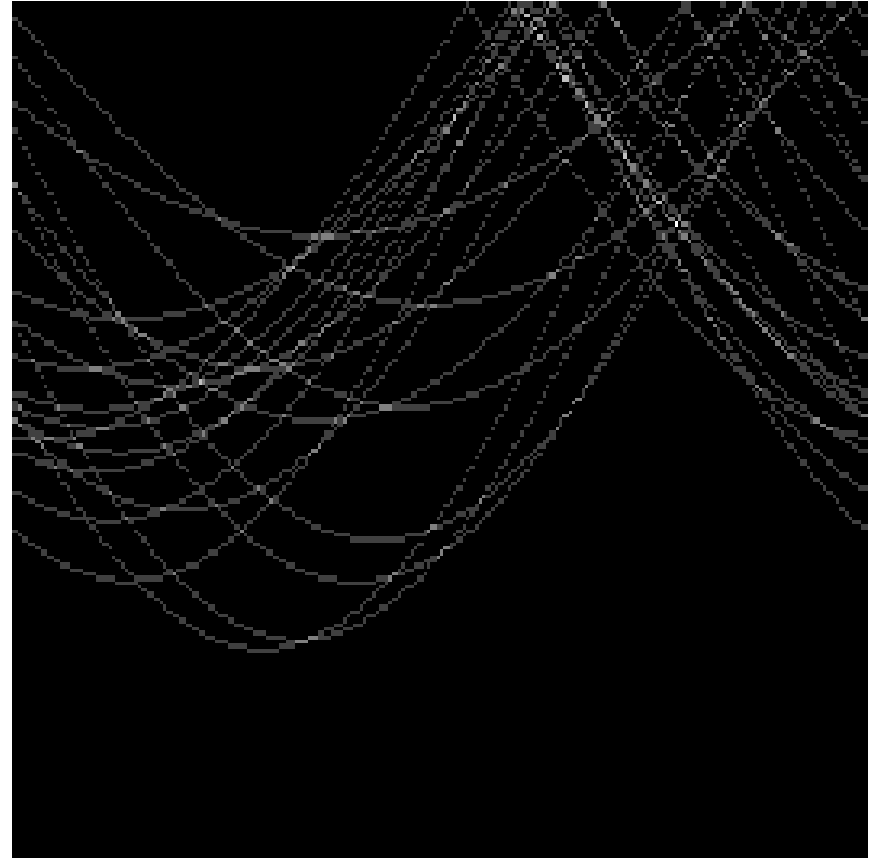
- Number of votes for a line of 20 points with increasing noise:



Random points



features

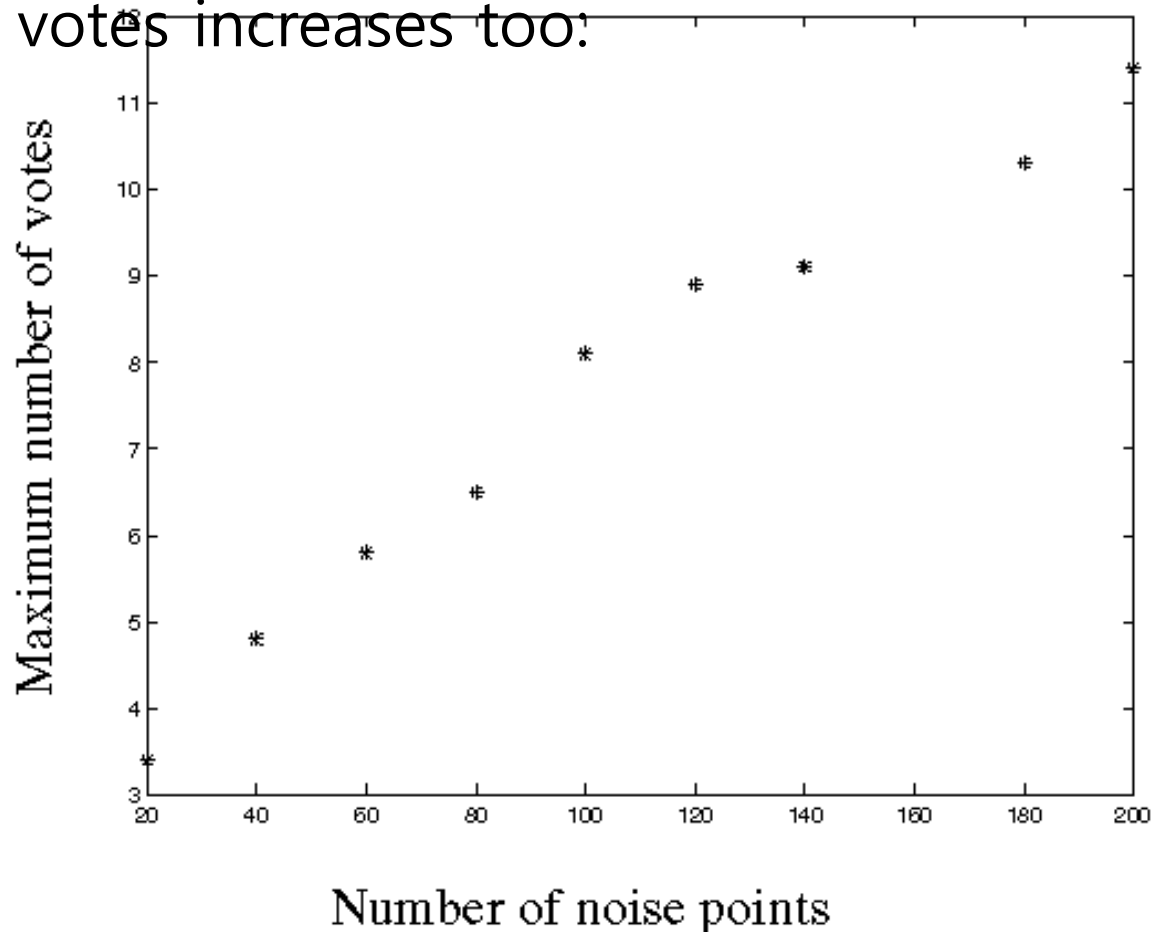


votes

■ Uniform noise can lead to spurious peaks in the array

Random points

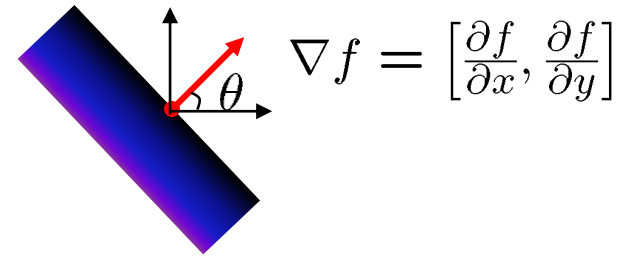
- As the level of uniform noise increases, the maximum number of votes increases too:



Dealing with noise

- Choose a good grid / discretization
 - **Too coarse:** large votes obtained when too many different lines correspond to a single bucket
 - **Too fine:** miss lines because some points that are not exactly collinear cast votes for different buckets
- Increment neighboring bins (smoothing in accumulator array)
- Try to get rid of irrelevant features
 - Take only edge points with significant gradient magnitude

Incorporating image gradients



- Recall: when we detect an edge point, we also know its gradient direction
- But this means that the line is uniquely determined!
- Modified Hough transform:

$$\theta = \tan^{-1} \left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x} \right)$$

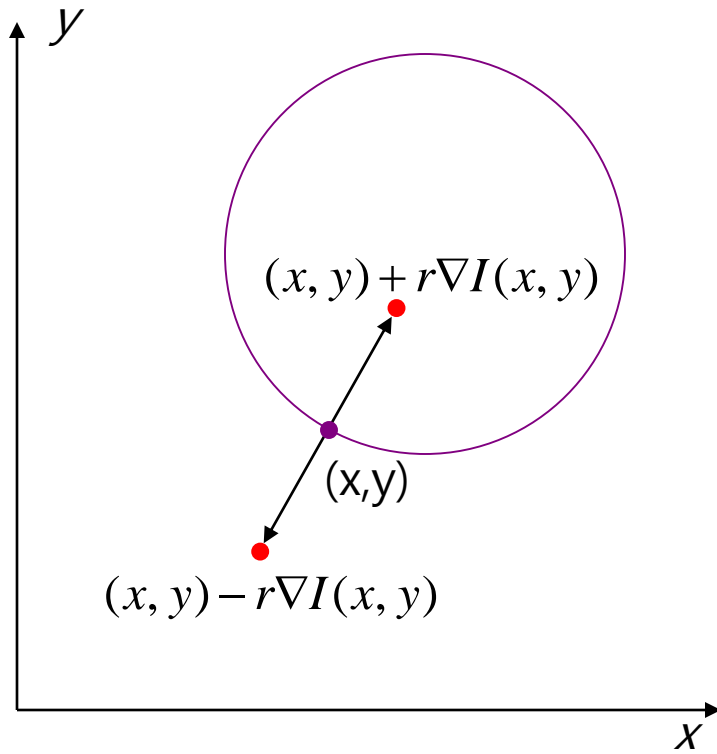
- For each edge point (x,y)
 - θ = gradient orientation at (x,y)
 - $\rho = x \cos \theta + y \sin \theta$
 - $H(\theta, \rho) = H(\theta, \rho) + 1$
- end

Hough transform for circles

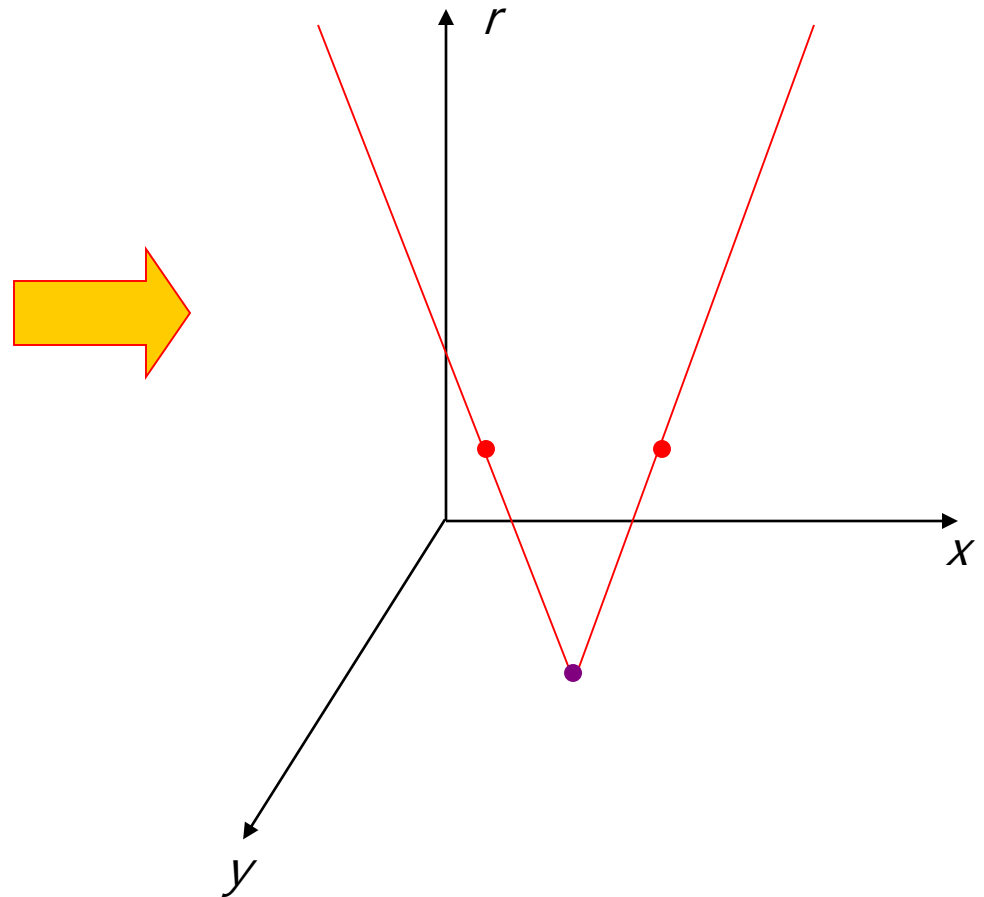
- How many dimensions will the parameter space have?
- Given an oriented edge point, what are all possible bins that it can vote for?

Hough transform for circles

image space

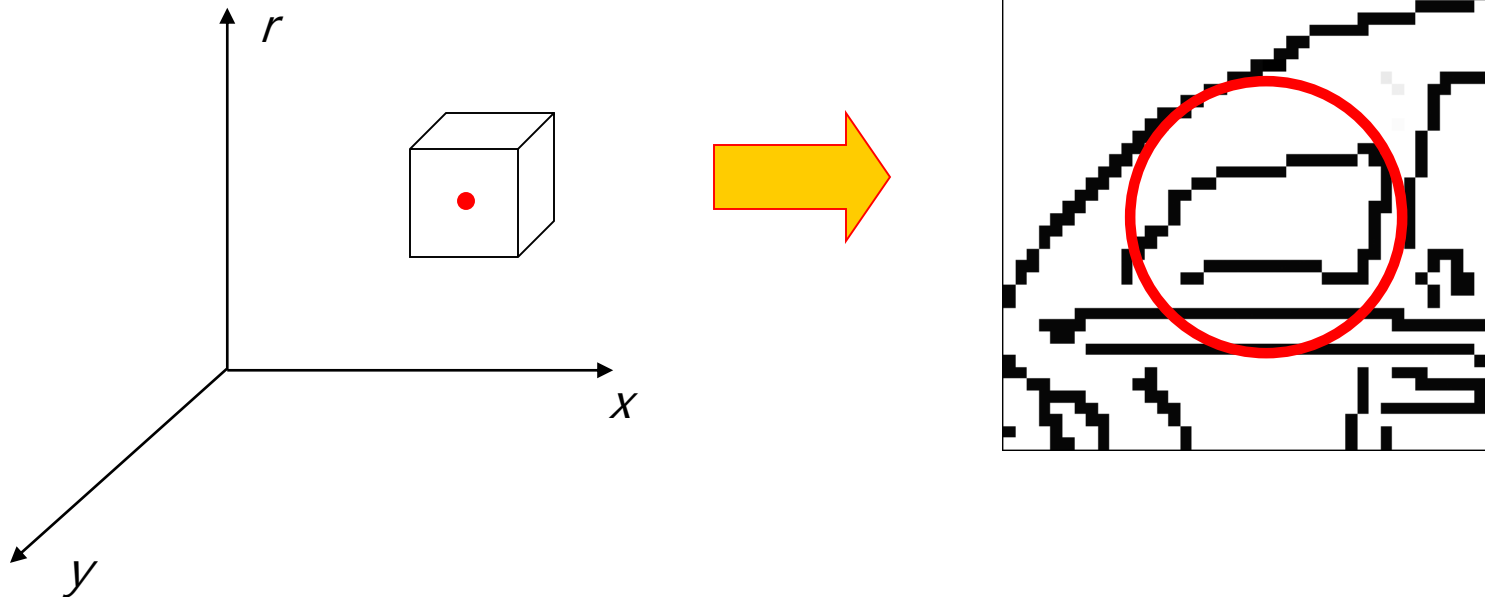


Hough parameter space



Hough transform for circles

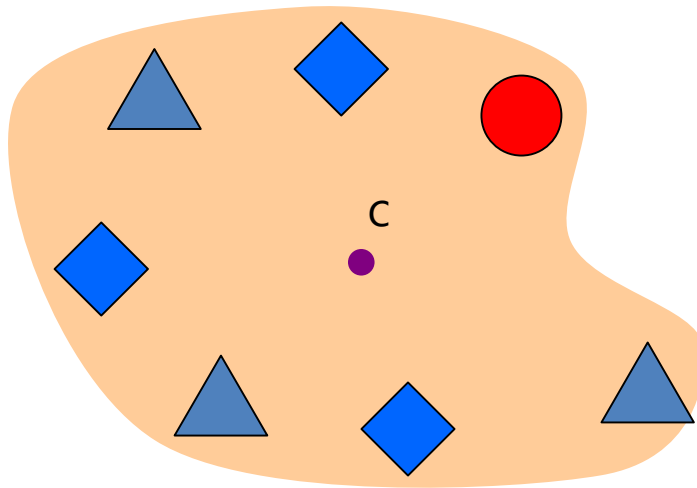
- Conceptually equivalent procedure: for each (x,y,r) , draw the corresponding circle in the image and compute its "support"



Is this more or less efficient than voting with features?

- # Generalized Hough transform
- We want to find a template defined by its reference point (center) and several distinct types of landmark points in stable spatial configuration

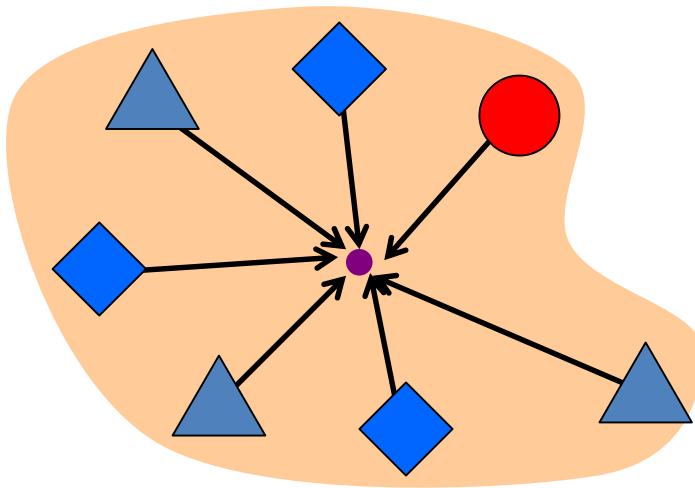
Template



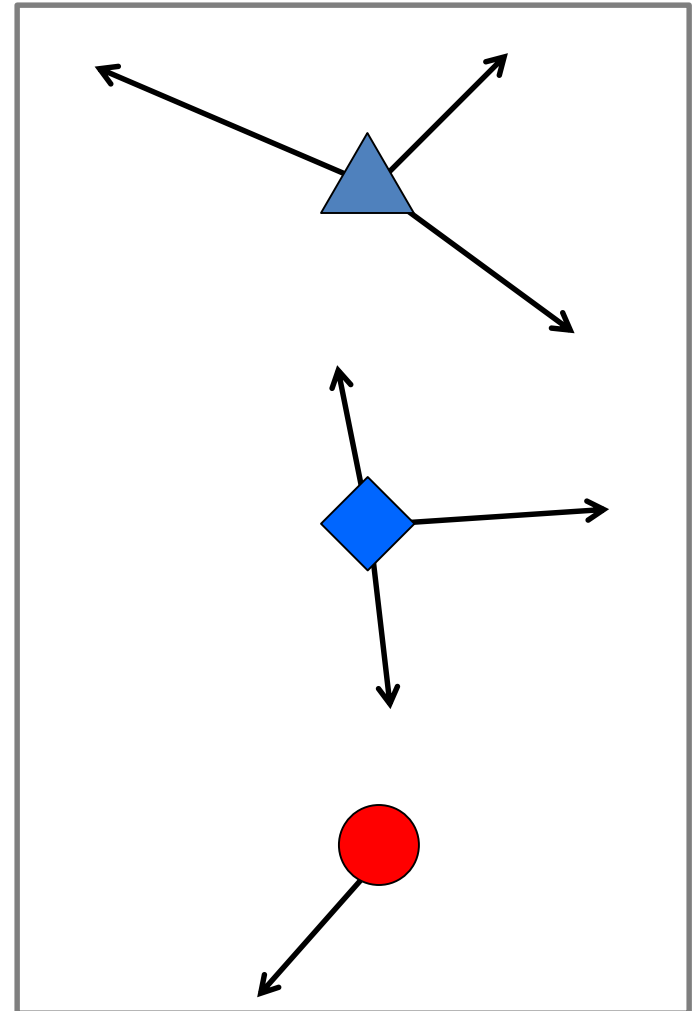
Generalized Hough transform

- Template representation: for each type of landmark point, store all possible displacement vectors towards the center

Template



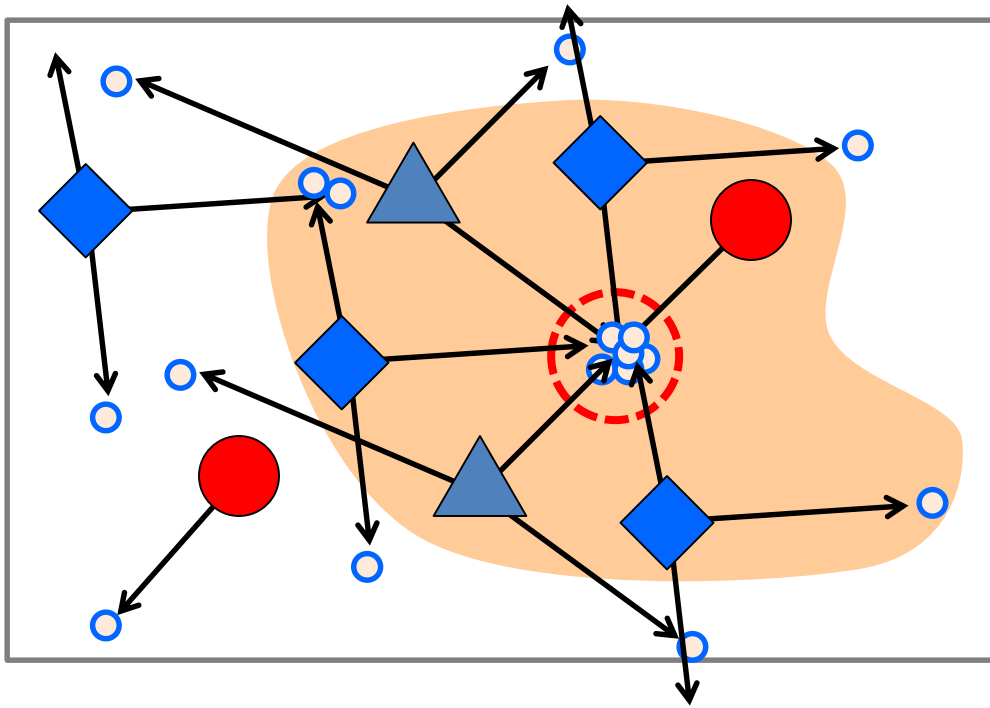
Model



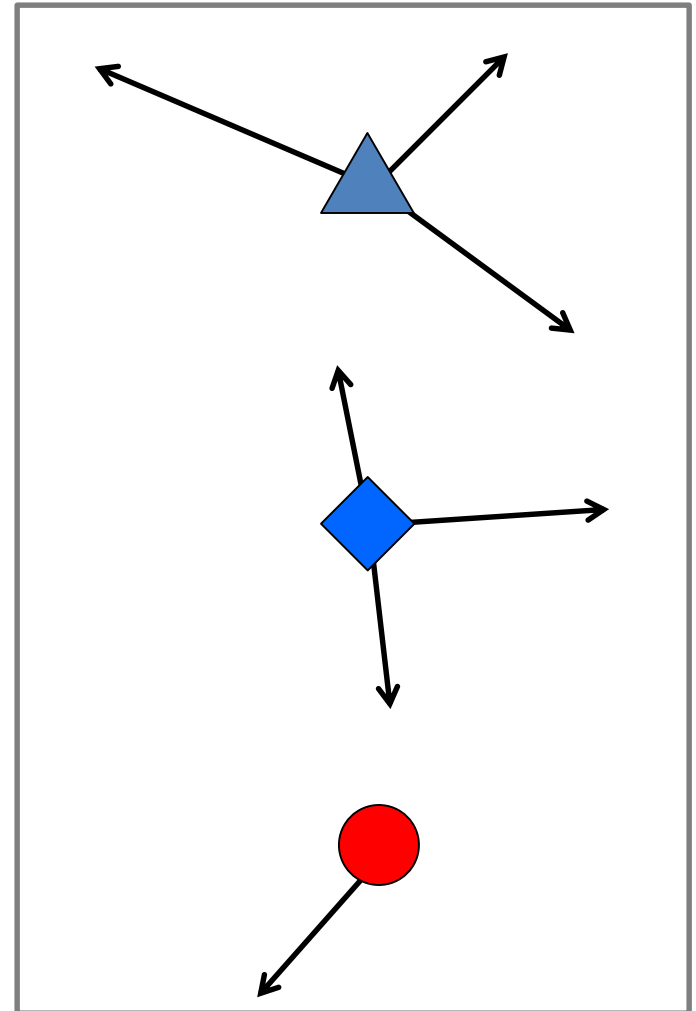
Generalized Hough transform

- Detecting the template:
 - For each feature in a new image, look up that feature type in the model and vote for the possible center locations associated with that type in the model

Test image

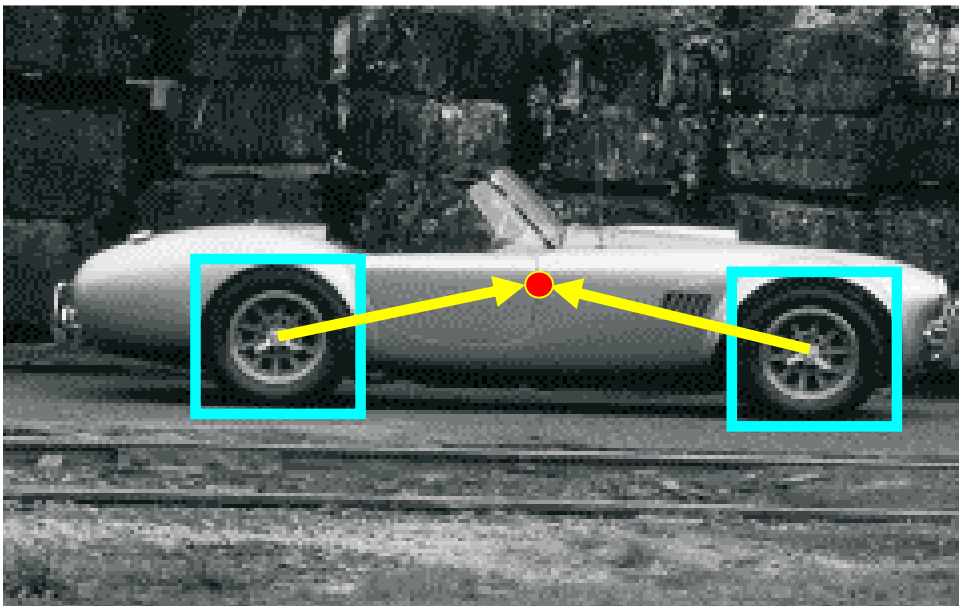


Model

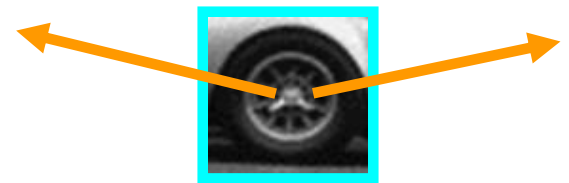


Application in recognition

- Index displacements by “visual codeword”



training image



visual codeword with
displacement vectors

B. Leibe, A. Leonardis, and B. Schiele, [Combined Object Categorization and Segmentation with an Implicit Shape Model](#), ECCV Workshop on Statistical Learning in Computer Vision 2004

Application in recognition

- Index displacements by “visual codeword”

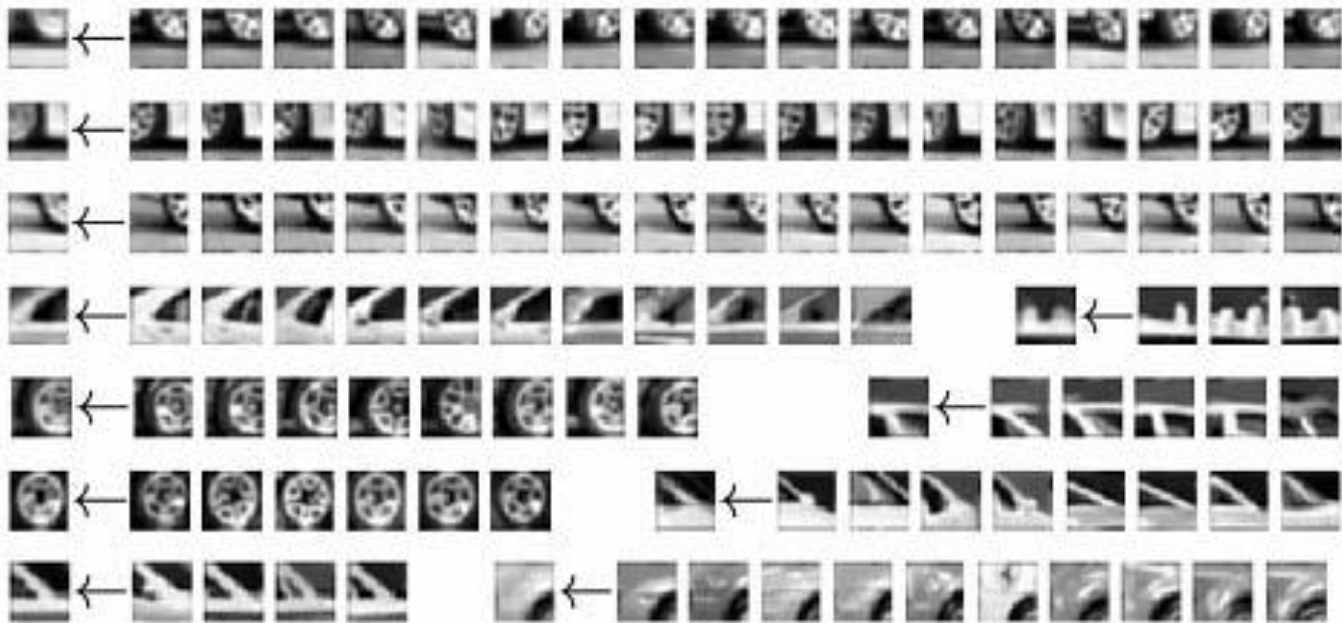


test image

B. Leibe, A. Leonardis, and B. Schiele, [Combined Object Categorization and Segmentation with an Implicit Shape Model](#), ECCV Workshop on Statistical Learning in Computer Vision 2004

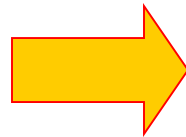
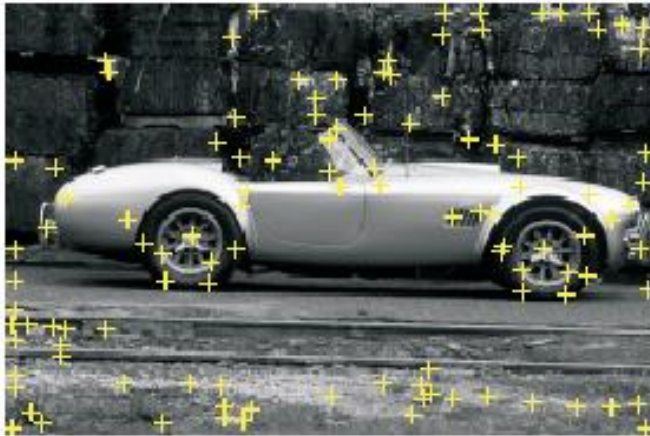
Implicit shape models: Training

1. Build codebook of patches around extracted interest points using clustering (more on this later in the course)



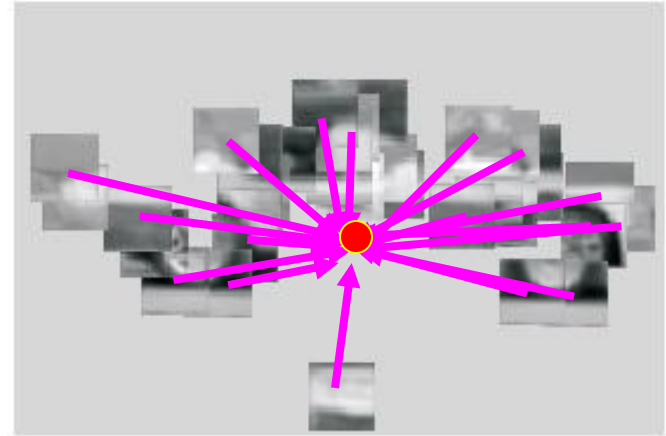
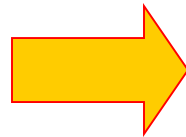
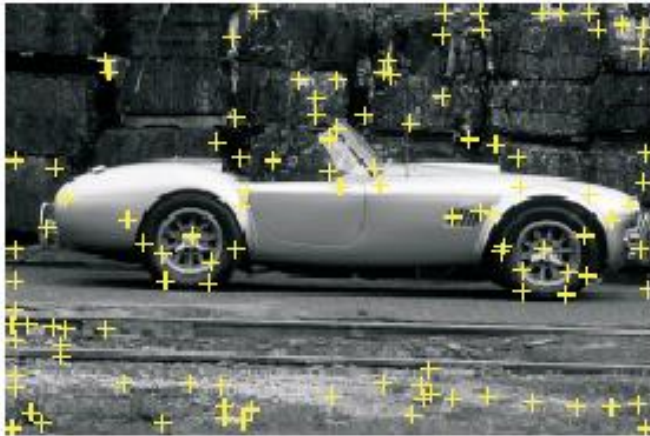
Implicit shape models: Training

1. Build codebook of patches around extracted interest points using clustering
2. Map the patch around each interest point to closest codebook entry



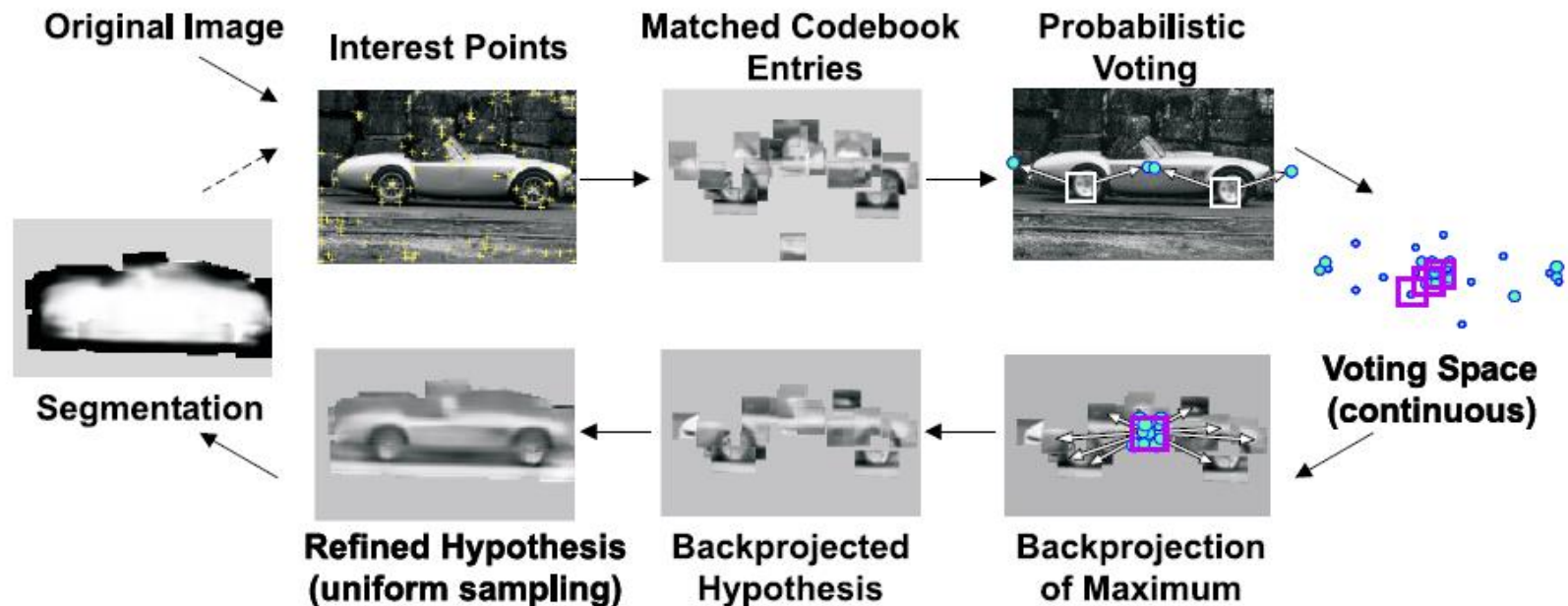
Implicit shape models: Training

1. Build codebook of patches around extracted interest points using clustering
2. Map the patch around each interest point to closest codebook entry
3. For each codebook entry, store all positions it was found, relative to object center

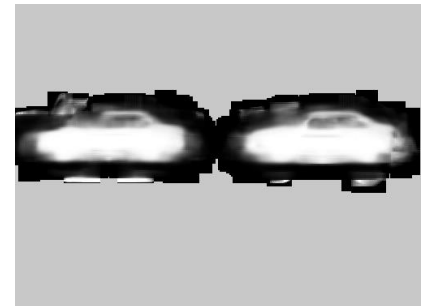
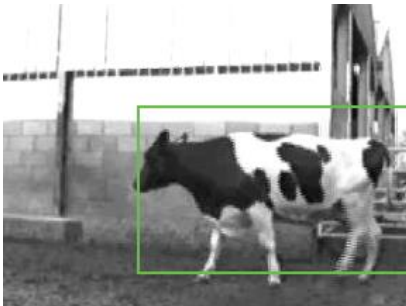
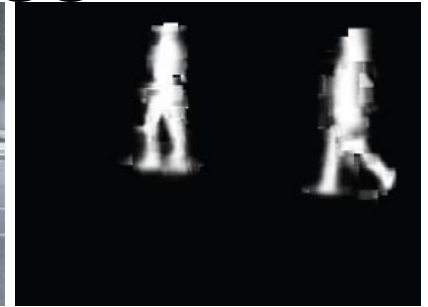
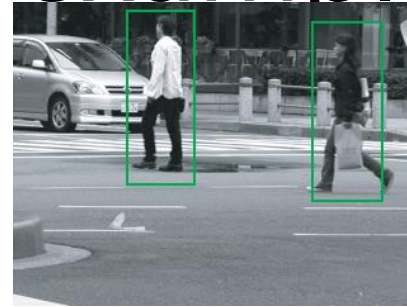
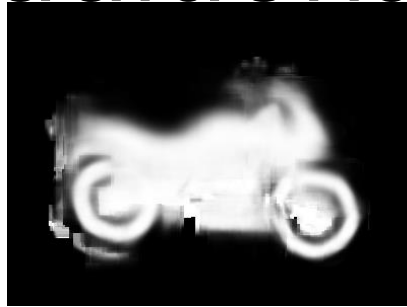


Implicit shape models: Testing

1. Given test image, extract patches, match to codebook entry
2. Cast votes for possible positions of object center
3. Search for maxima in voting space
4. Extract weighted segmentation mask based on stored masks for the codebook occurrences



Additional examples



B. Leibe, A. Leonardis, and B. Schiele, [Robust Object Detection with Interleaved Categorization and Segmentation](#), IJCV 77 (1-3), pp. 259-289, 2008.

Implicit shape models: Details

- Supervised training
 - Need reference location and segmentation mask for each training car
- Voting space is continuous, not discrete
 - Clustering algorithm needed to find maxima
- How about dealing with scale changes?
 - Option 1: search a range of scales, as in Hough transform for circles
 - Option 2: use interest points with characteristic scale
- Verification stage is very important
 - Once we have a location hypothesis, we can overlay a more detailed template over the image and compare pixel-by-pixel, transfer segmentation masks, etc.

Hough transform: Discussion

- Pros
 - Can deal with non-locality and occlusion
 - Can detect multiple instances of a model
 - Some robustness to noise: noise points unlikely to contribute consistently to any single bin
- Cons
 - Complexity of search time increases exponentially with the number of model parameters
 - Non-target shapes can produce spurious peaks in parameter space
 - It's hard to pick a good grid size

3.5.3 RANSAC

■ RANSAC

- 1981년 Fischler&Bolles이 제안 [Fischler81]
- 인라이어를 찾아 어떤 모델을 적합시키는 기법
- 난수 생성하여 인라이어 군집을 찾기 때문에 임의성 지님

■ 선분 검출에 적용

- 모델은 직선의 방정식 $y=ax+b$

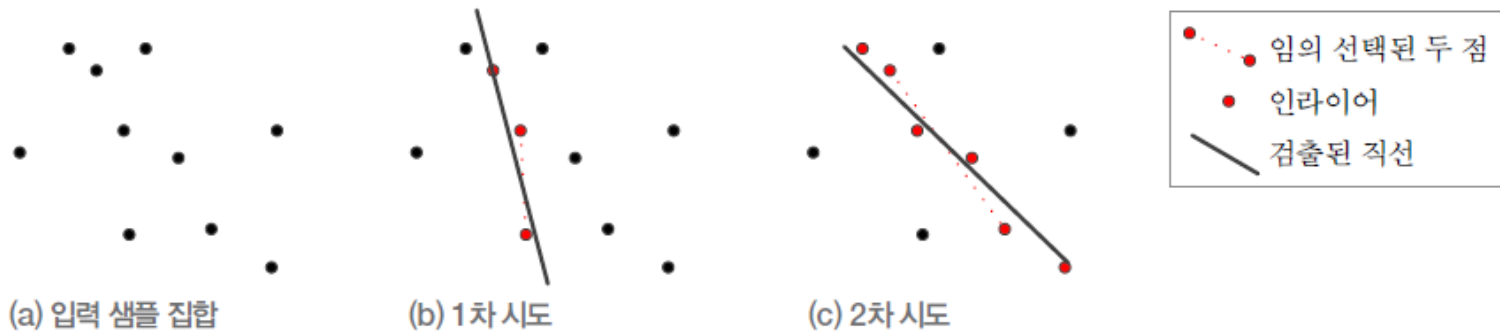
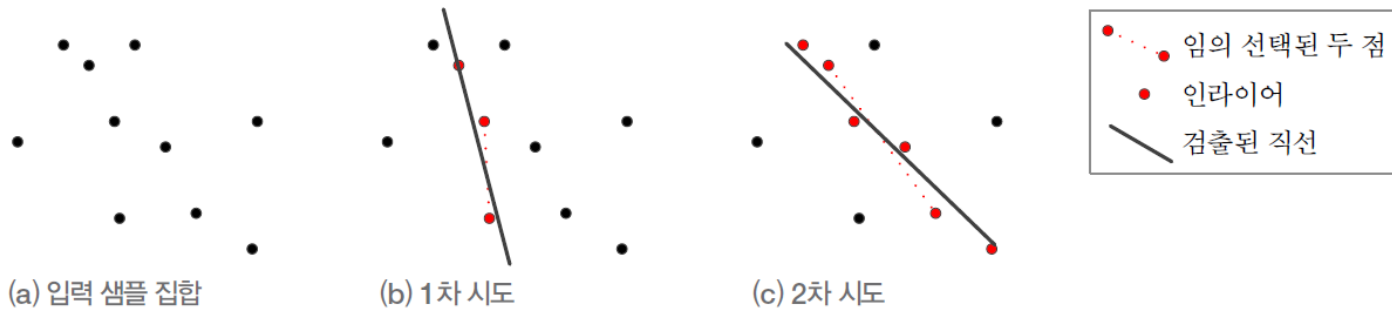


그림 3-31 RANSAC의 원리

7.3.2 RANSAC

■ 원리

- 직선 검출하는 3장의 그림 3-31과 같은 원리



(a) 입력 샘플 집합

(b) 1차 시도

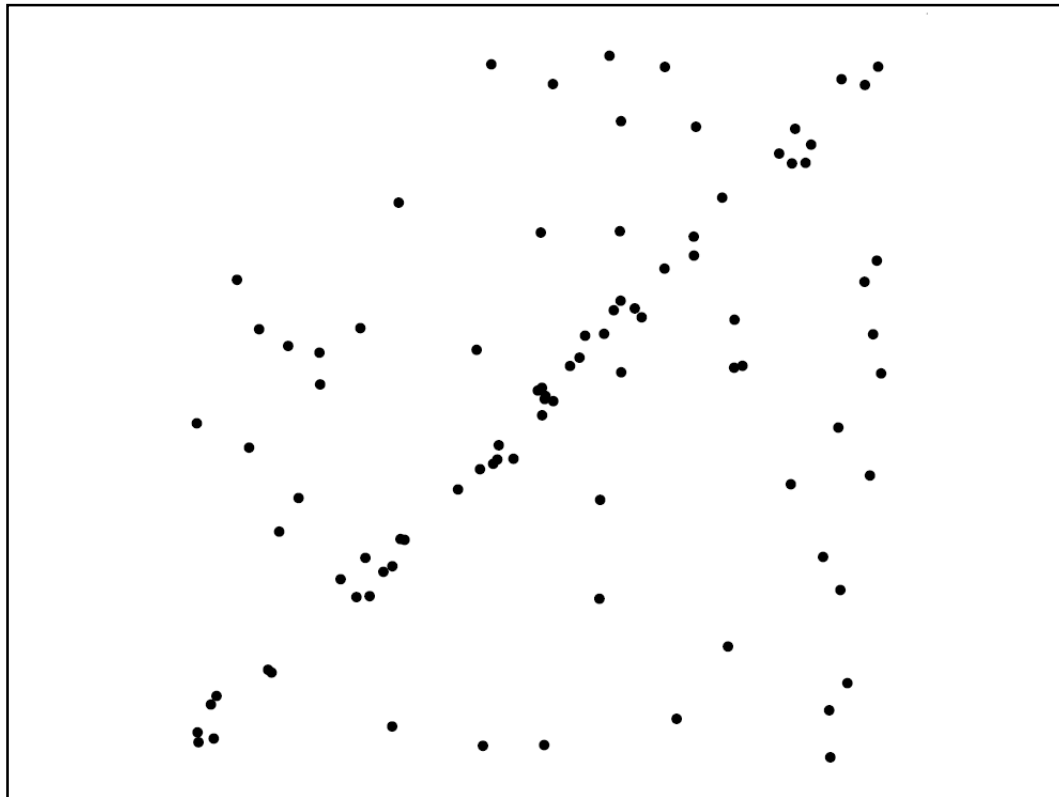
(c) 2차 시도

그림 3-31 RANSAC의 원리

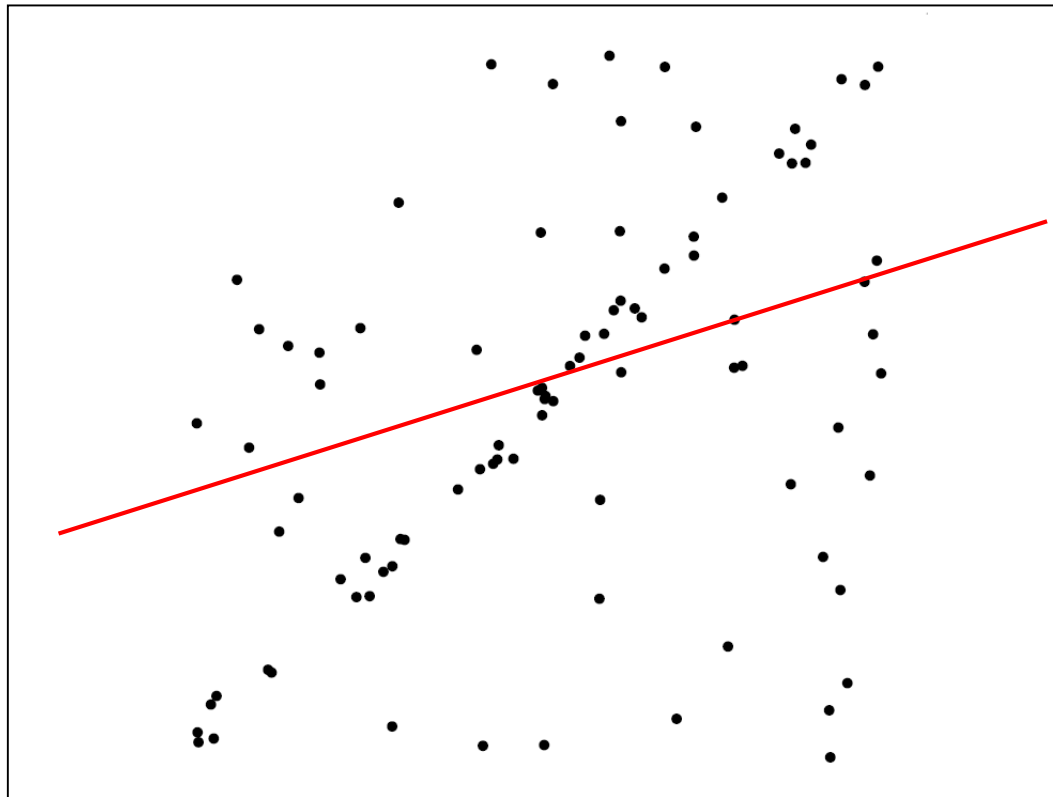
■ 여기서,

- 매칭 쌍 집합 $X=\{(a_1, b_1), (a_2, b_2), \dots, (a_n, b_n)\}$ 을 처리할 수 있게 확장

7.3.2 RANSAC

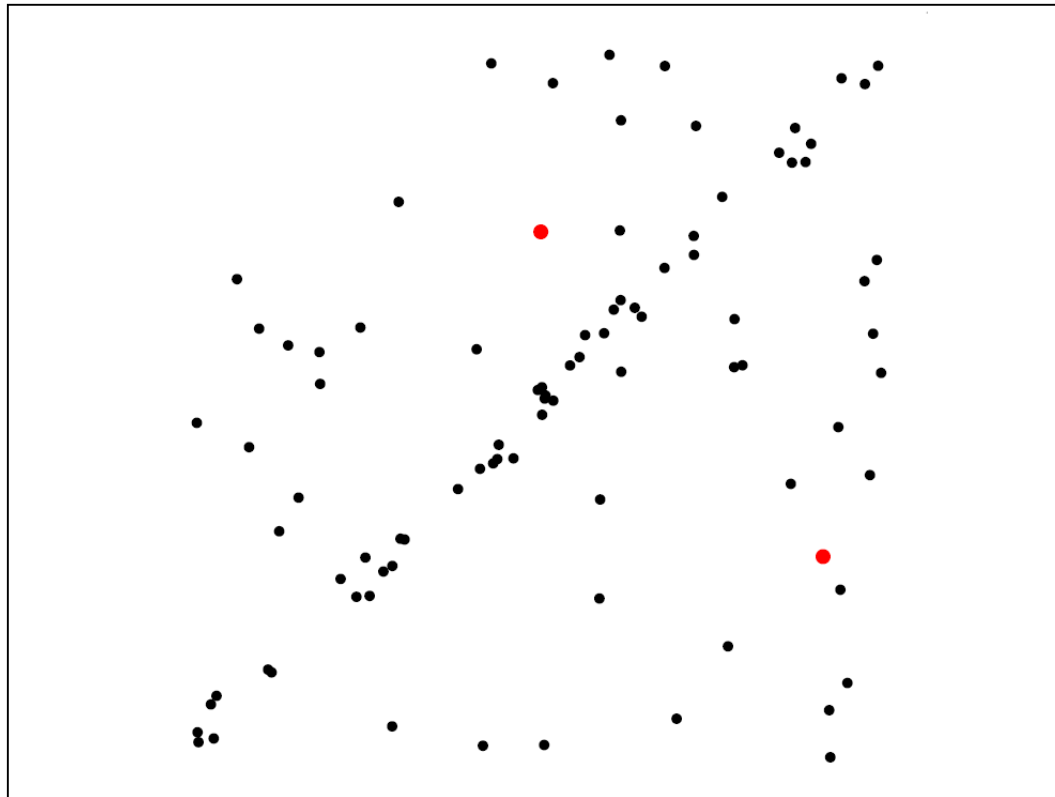


7.3.2 RANSAC



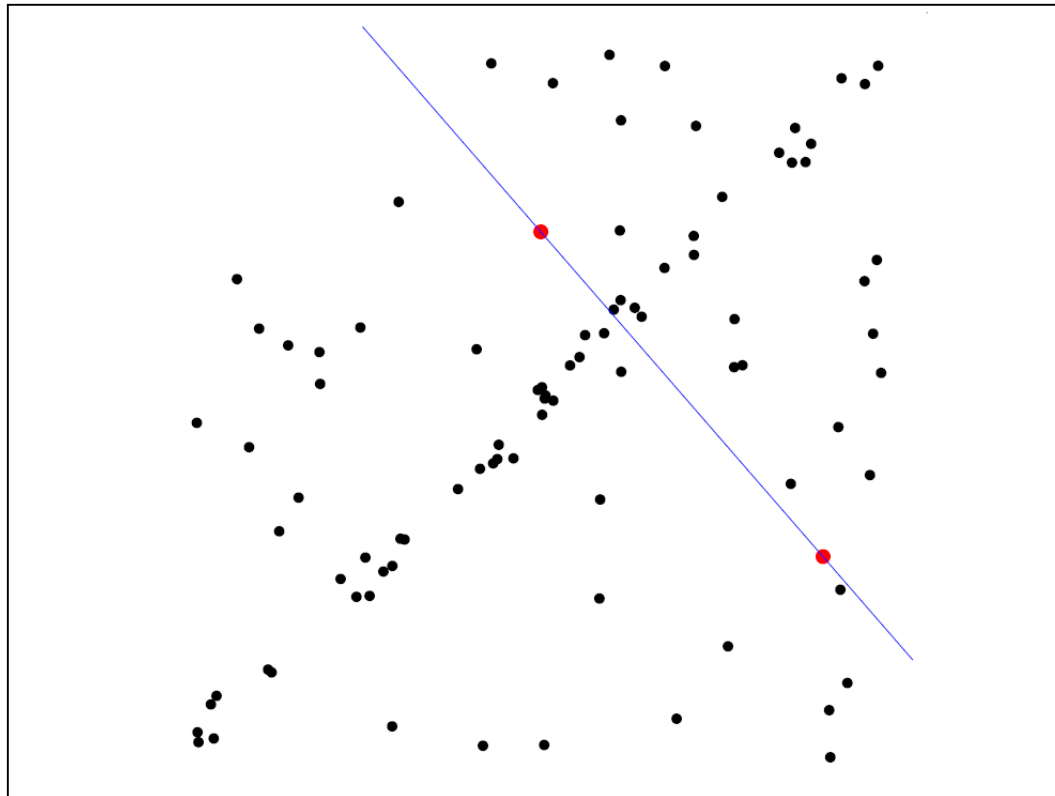
Least-squares fit

7.3.2 RANSAC



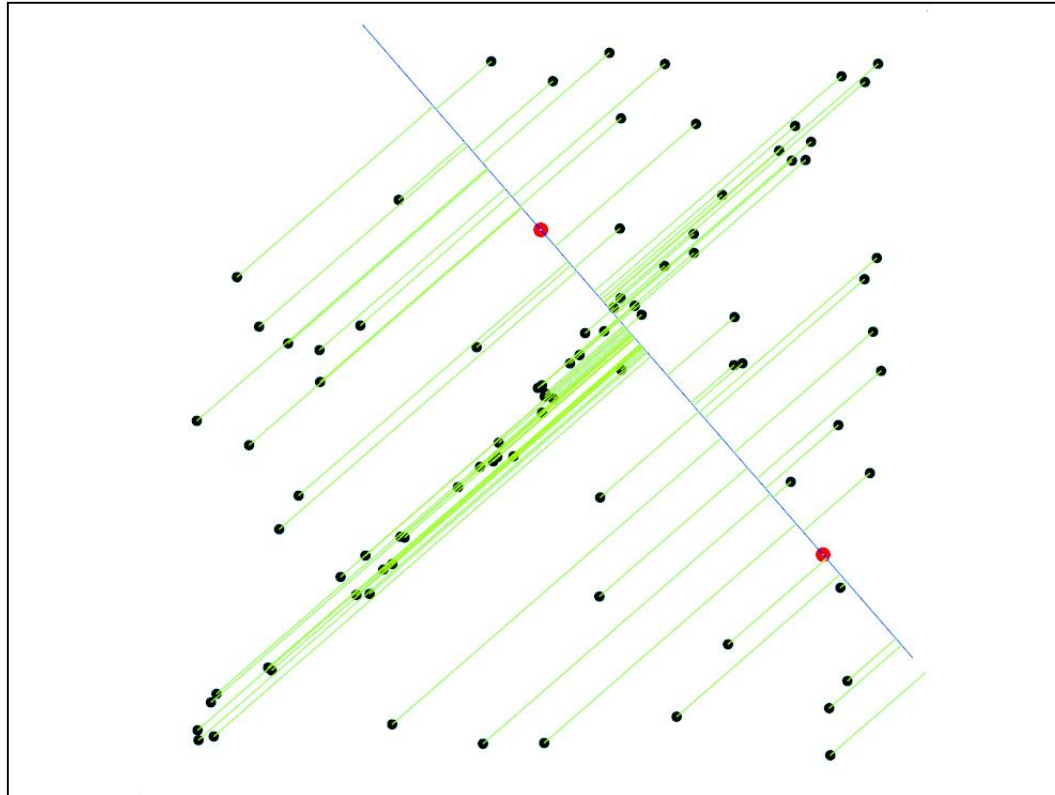
1. Randomly select minimal subset of points

7.3.2 RANSAC



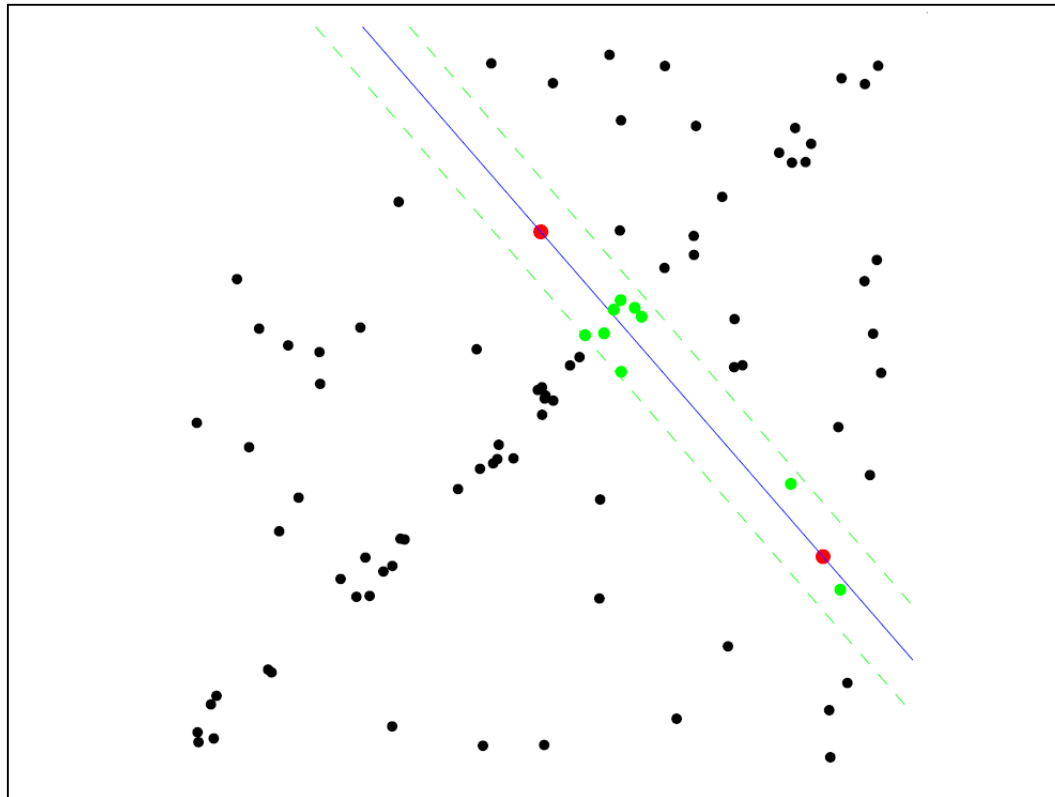
1. Randomly select minimal subset of points
2. Hypothesize a model

7.3.2 RANSAC



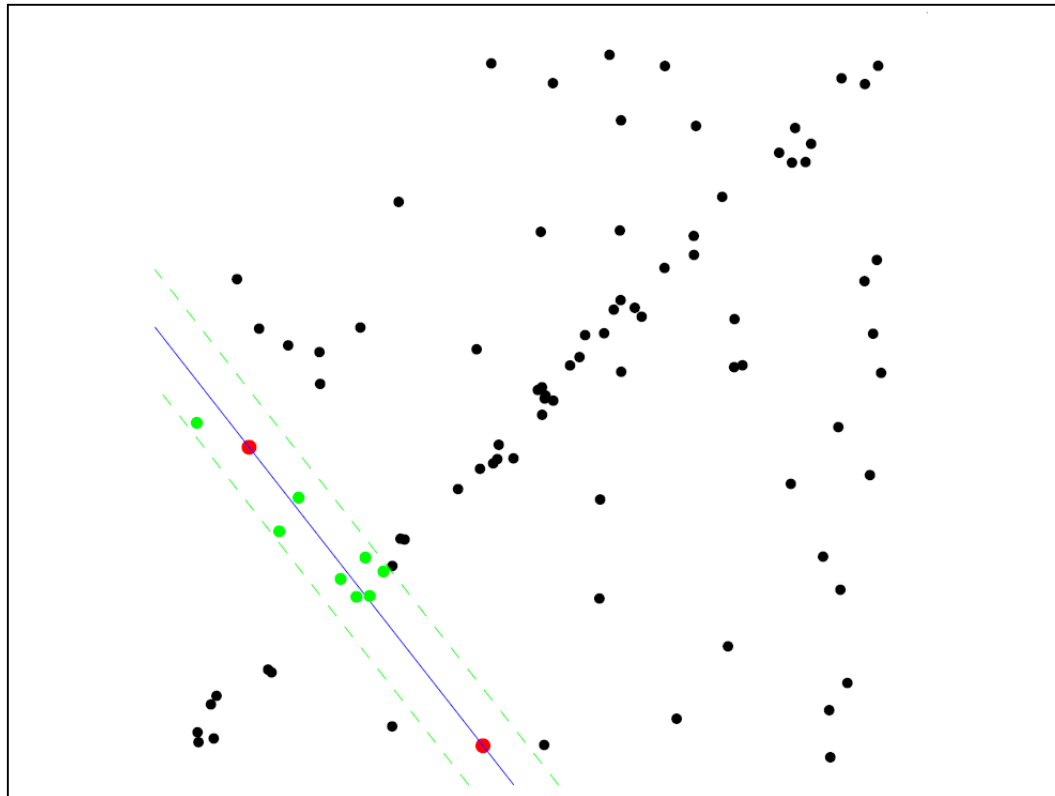
1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function

7.3.2 RANSAC



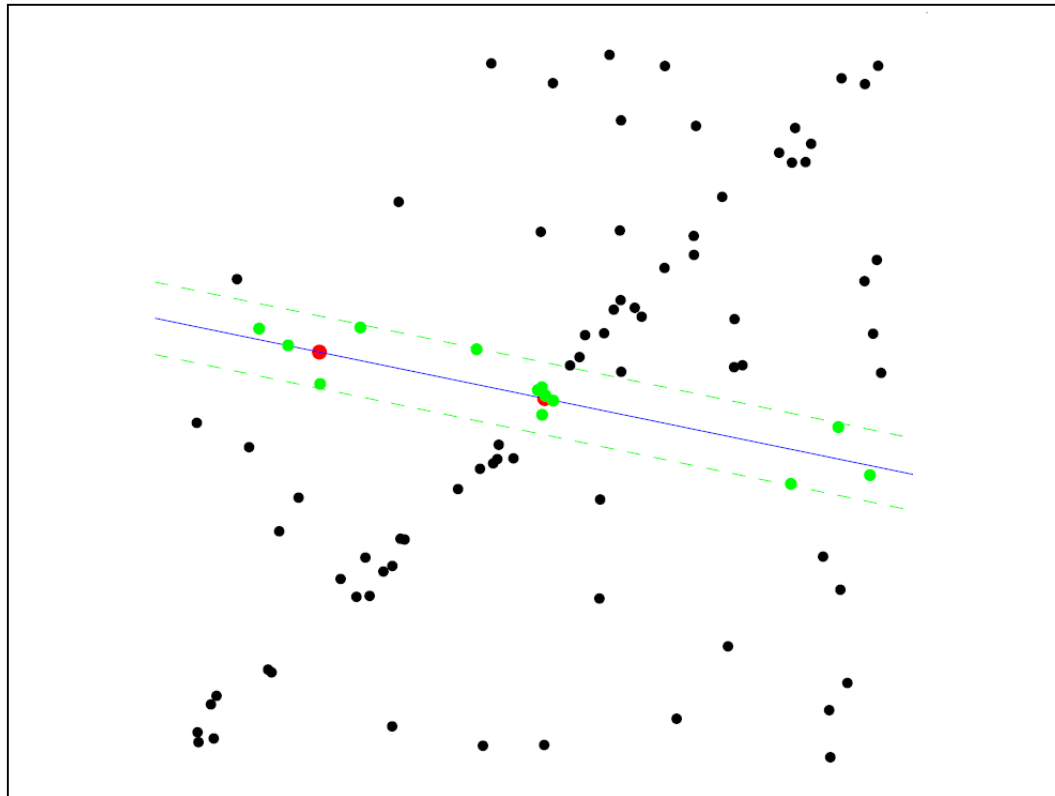
1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model

7.3.2 RANSAC



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. Repeat *hypothesize-and-verify* loop

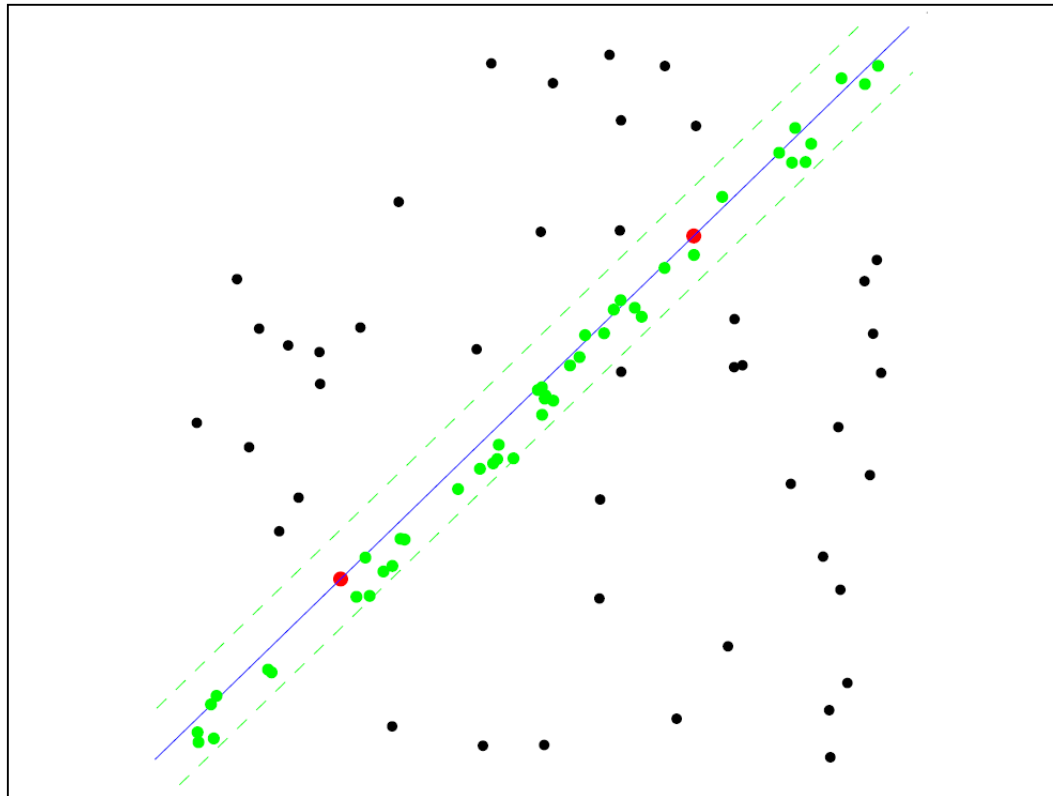
7.3.2 RANSAC



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. Repeat *hypothesize-and-verify* loop

7.3.2 RANSAC

Uncontaminated sample



1. Randomly select minimal subset of points
2. Hypothesize a model
3. Compute error function
4. Select points consistent with model
5. Repeat *hypothesize-and-verify* loop

7.3.2 RANSAC

알고리즘 7-9 기하 변환을 추정하기 위한 RANSAC

입력 : $X = \{(a_i, b_i), i=1, 2, \dots, n\}$ // 매칭 쌍 집합

반복 횟수 k , 인라이어 판단 t , 인라이어 집합의 크기 d , 적합 오차 e

출력 : 기하 변환 행렬 T

```
1  Q = ∅;
2  for(j=1 to k) {
3      X에서 세 개 대응점 쌍을 임의로 선택한다.
4      이들 세 쌍을 입력으로 식 (7.14)를 풀어  $T_j$ 를 추정한다.
5      이들 세 쌍으로 집합 inlier를 초기화한다.
6      for(이 세 쌍을 제외한 X의 요소  $p$  각각에 대해) {
7          if( $p$ 가 허용 오차  $t$  이내로  $T_j$ 에 적합)  $p$ 를 inlier에 넣는다.
8      }
9      if( $|inlier| \geq d$ ) // 집합 inlier가  $d$ 개 이상의 샘플을 가지면
10         inlier에 있는 모든 샘플을 가지고 새로운  $T_j$ 를 계산한다.
11         if( $T_j$ 의 적합 오류  $< e$ )  $T_j$ 를 집합  $Q$ 에 넣는다.
12     }
13     Q에 있는 변환 행렬 중 가장 좋은 것을  $T$ 로 취한다.
```