A Study of the Sensitivity of Biomechanical Models of the Spine for Scoliosis Brace Design

Supplementary Material

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1. Error Analysis

Figs. 1 to 4 show the relative error of our sensitivity-based approximation of spine deformations. We apply changes of 1% and 10% to the parameter ratios, and we compare deformations computed using the sensitivity-based approximation vs. a full solve of static equilibrium. For each vertebra, we compute its feature vector $\mathbf{z}_i$ using both methods, and we evaluate the RMS error. The figures show the relative error normalized by the largest RMS feature vector of all vertebrae.

In the validation, we test independently the following parameter ratios:

- $k_{v,x}$ Lateral translation stiffness, intervert. joints.
- $k_{v,y}$ Vertical translation stiffness, intervert. joints.
- $k_{v,z}$ Sagittal translation stiffness, intervert. joints.
- $k_{v,\alpha}$ Flexion/extension stiffness, intervertebral joints.
- $k_{v,\beta}$ Axial rotation stiffness, intervertebral joints.
- $k_{v,\gamma}$ Lateral bending stiffness, intervertebral joints.
- $k_{r,x}$ Separation stiffness, rib-vertebrae joints.
- $k_{r,y}$ Alignment stiffness, rib-vertebrae joints.
- $k_{r,z}$ Alignment stiffness, rib-vertebrae joints.
- $k_{r,\alpha}$ Twist stiffness, rib-vertebrae joints.
- $k_{r,\beta}$ Bending stiffness, rib-vertebrae joints.
- $k_{r,\gamma}$ Bending stiffness, rib-vertebrae joints.
- $k_{s,x}$ Separation stiffness, rib-sternum joints.
- $k_{s,y}$ Alignment stiffness, rib-sternum joints.
- $k_{s,z}$ Alignment stiffness, rib-sternum joints.
- $k_{s,\alpha}$ Twist stiffness, rib-sternum joints.
- $k_{s,\beta}$ Bending stiffness, rib-sternum joints.
- $k_{s,\gamma}$ Bending stiffness, rib-sternum joints.
- μ Lamé constant (shear modulus).
- λ Lamé constant.

The different figures summarize results as follows:

- Fig. 1: 1% change, healthy subject.
- Fig. 2: 10% change, healthy subject.
- Fig. 3: 1% change, scoliosis subject.
- Fig. 4: 10% change, scoliosis subject.

2. Parameter Sensitivity

Figs. 5 and 6 show the (linearized) sensitivity of spine deformations with respect to a change of 1 in the parameter ratios, which is equivalent to a change in 100% in the actual parameters. In the analysis, we test independently the same parameters as in the error validation. The displayed values correspond to columns of the sensitivity matrix. We color each vertebra according to the sensitivity of its feature vector, after computing the RMS of the eight corners of the bounding box.

The different figures summarize results as follows:

- Fig. 5: healthy subject.
- Fig. 6: scoliosis subject.

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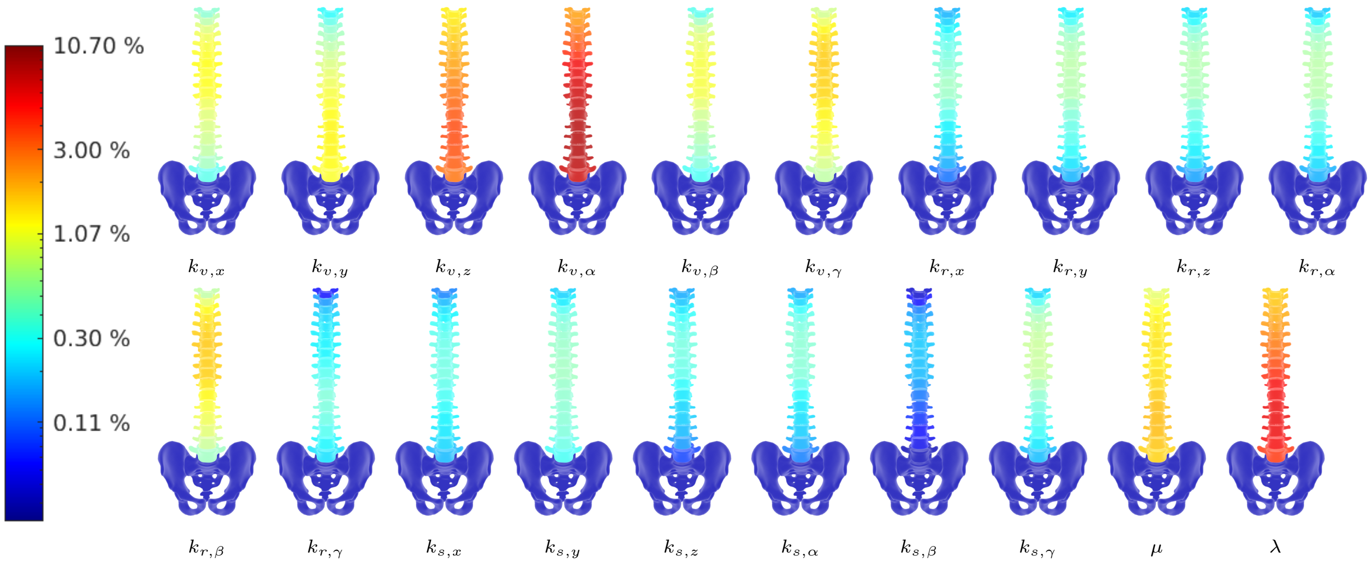

Figure 1: Relative error of sensitivity-based spine deformations. Healthy subject, 1% parameter change.

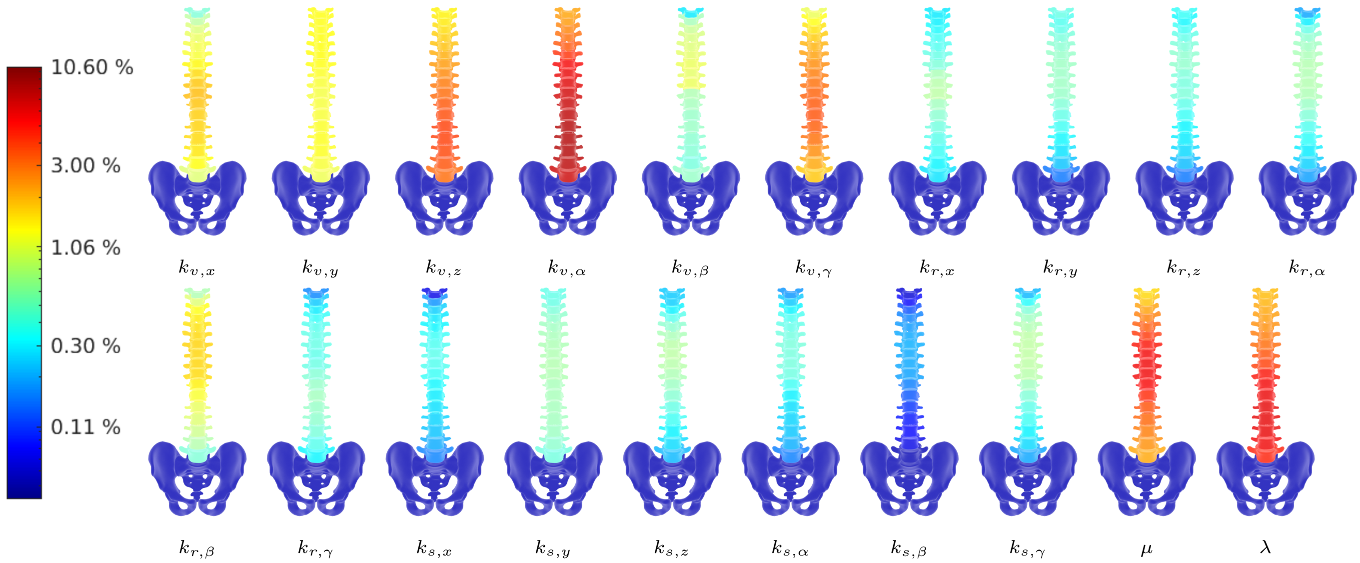

Figure 2: Relative error of sensitivity-based spine deformations. Healthy subject, 10% parameter change.

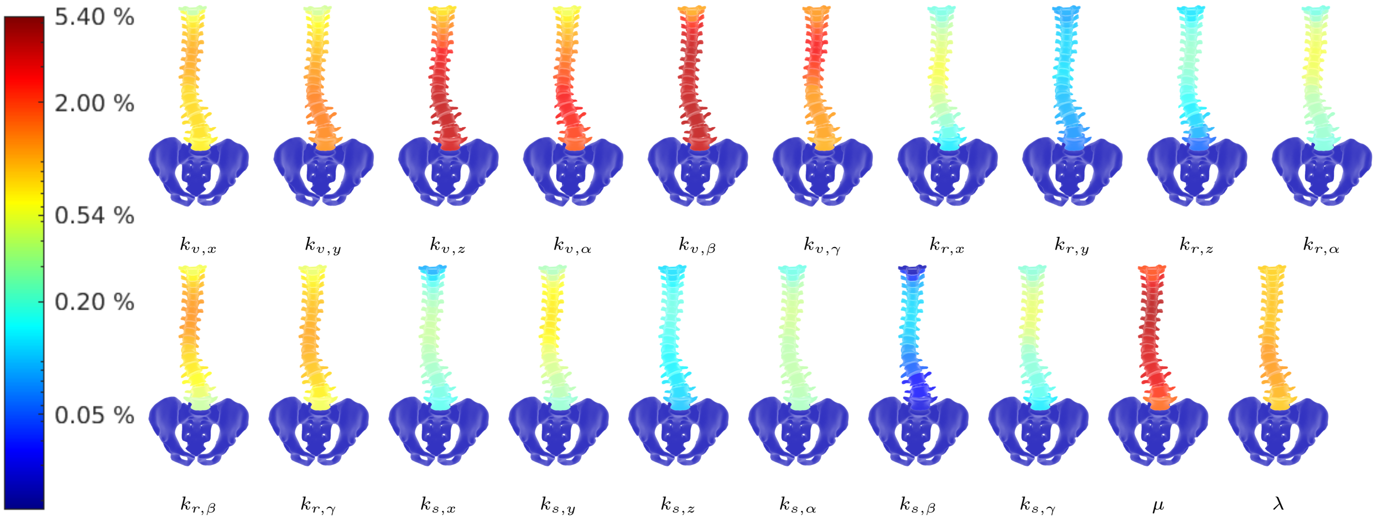

Figure 3: Relative error of sensitivity-based spine deformations. Scoliosis subject, 1% parameter change.

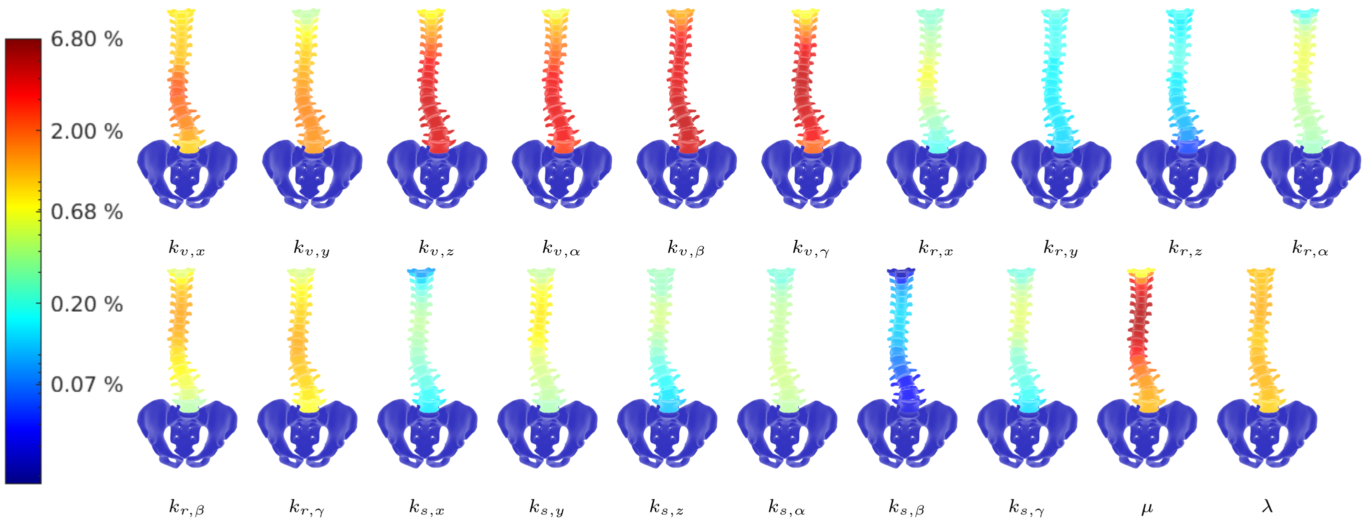

Figure 4: Relative error of sensitivity-based spine deformations. Scoliosis subject, 10% parameter change.

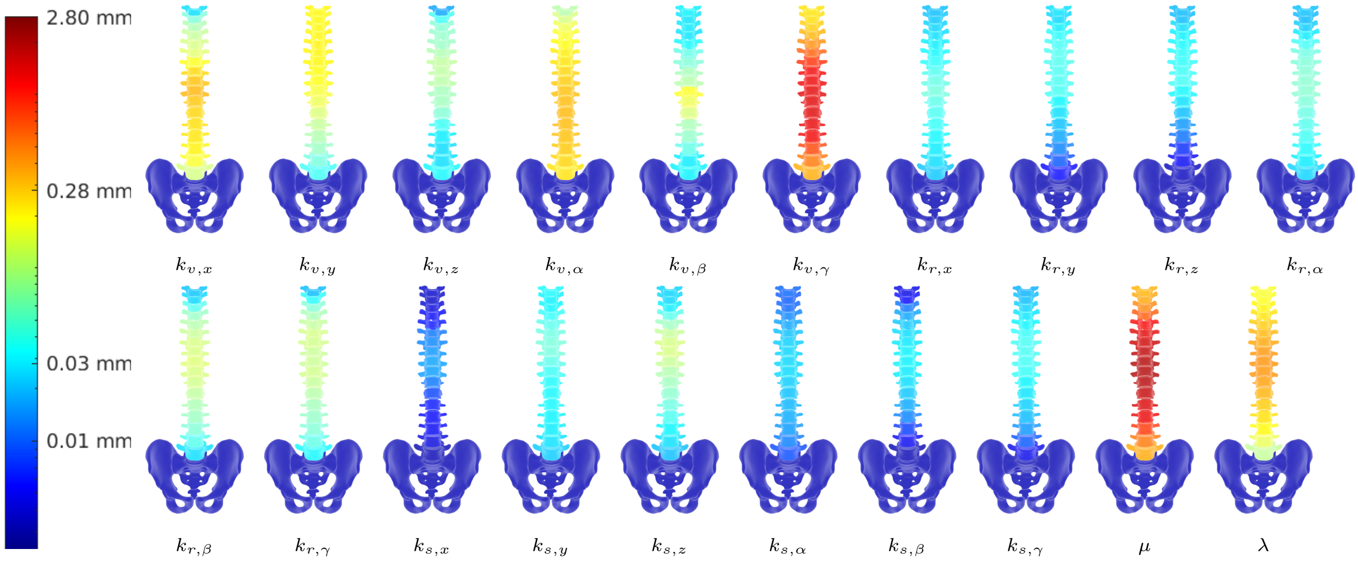

Figure 5: Parameter sensitivity in spine deformations. Healthy subject.

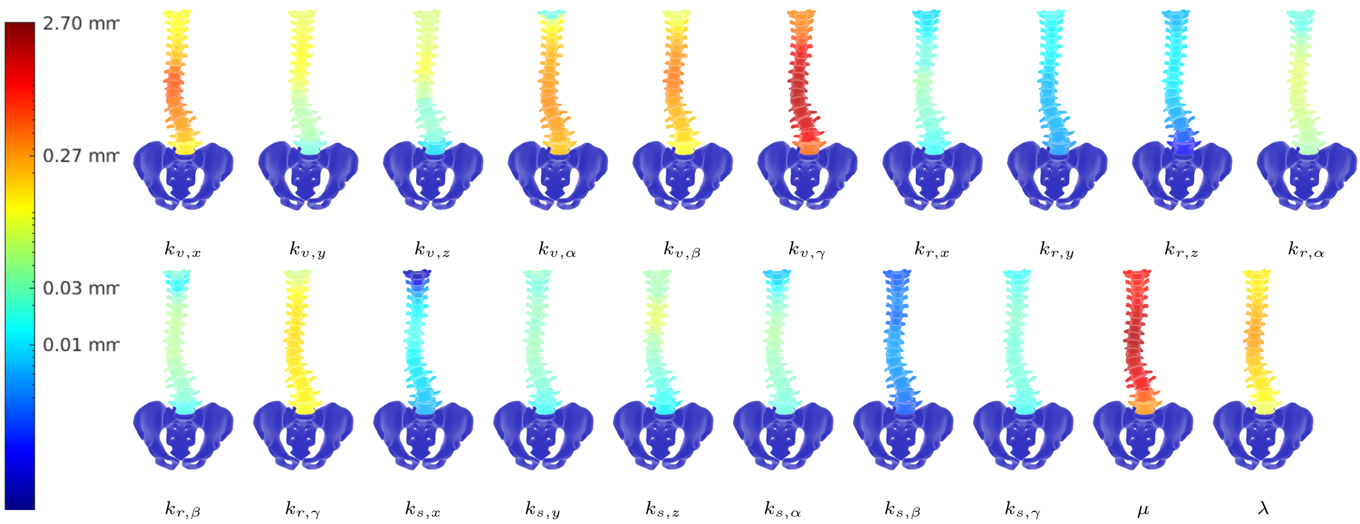

Figure 6: Parameter sensitivity in spine deformations. Scoliosis subject.