Time Value of Money

The time value of money is an important concept in business and finance. Leases, mortgages, bonds, retirement contributions, and all investments apply the time value of money. If recognizes that a dollar today is worth more than a dollar tomorrow or one year from now because of the influence of three factors: interest, risk, and inflation. To calculate such values, financial formulas are used accordingly, as explained below.

$$FV_{SA} = PV_{SA} \times (1+r)^n$$

$$PV_{SA} = \frac{FV_{SA}}{(1+r)^n}$$

$$FV_{OA} = PMT\left[\frac{(1+r)^n - 1}{r}\right]$$

$$FV_{AD} = PMT\left[\frac{(1+r)^n - 1}{r}\right](1+r)$$

$$PV_{OA} = PMT\left[\frac{1-(1+r)^{-n}}{r}\right]$$

$$PV_{AD} = PMT\left[\frac{1-(1+r)^{-n}}{r}\right](1+r)$$

$$PV_{perpetuity} = \frac{PMT}{r}$$

$$Effective\ Rate\ for\ Payment\ Period * = \left(1+\frac{r_{nom}}{m}\right)^{\frac{m}{p}} - 1$$

$$Effective\ Rate\ for\ Payment\ Period\ when\ rate\ is\ stated\ as\ an\ APR = \frac{APR}{p}$$

FV = **Future** value

PV = **Present** value

SA = **Single** amount

OA = Ordinary annuity – **Payment or receipts occur at the end of a pay period**

AD = Annuity due – **Payments or receipts being at the start of a pay period**

r = interest rate or effective rate as you have calculated it

n = number of periods over which investment is being calculated

PMT = payment amount

r_{nom} = the nominal rate given (used to calculate effective rate)

m = number of compounding periods per year for the nominal rate

p = payment periods during the year

** The effective rate for payment period is used when the number of compound periods per year differs from the payment periods during the year*

How to properly present Time Value of Money solutions:

1. Variables: Firstly, state the known variable values (in relation to the equations stated above) based on the information given in the question – including the unknown variable
2. Equation(s): Secondly, state the equation(s) of which will be used to calculate the unknown variable (without variables substituted within the equation)
3. Numbers: Thirdly, insert the known variables within the equation and simply the formula to isolate the unknown variable and calculate its value (remember: include 4-6 decimals after each calculation step – although include only two decimals in the final value)
4. Conclusion: Lastly, state "Therefore, the [unknown variable] is [calculated value]."

Application of Appropriate Formulae to Problems

To indicate the appropriate formula based on the word problem, recognize words that align with the associated equations, examples include:

GIVEN INFORMATION	DESIRED VALUE	NECESSARY EQUATION
Single payment investment	**Future** amount of investment	$FV_{SA} = PV_{SA} \times (1+r)^n$
Single payment investment	**Present** amount of investment	$PV_{SA} = \frac{FV_{SA}}{(1+r)^n}$
Multiple payments investment beginning at the **end** of the payment period	**Future** amount of investment	$FV_{OA} = PMT\left[\frac{(1+r)^n - 1}{r}\right]$
Multiple payments investment beginning at the **start** of the payment period	**Future** amount of investment	$FV_{AD} = PMT\left[\frac{(1+r)^n - 1}{r}\right](1+r)$
Multiple payments investment beginning at the **end** of the payment period	**Present** amount of investment	$PV_{OA} = PMT\left[\frac{1-(1+r)^{-n}}{r}\right]$
Multiple payments investment beginning at the **start** of the payment period	**Present** amount of investment	$PV_{AD} = P$)

- Annually: n = 1
- Semi-annually: n = 2
- Quarterly: n = 4
- Monthly: n = 12
- Weekly: n = 52

** Remember to be aware of different types of payment periods and multiply the amount of years based on the type.*

To calculate n:

Variables: State the variables given

1. Equation: Choose the proper formula based on the situation and variables given within the question
2. Numbers: Insert the variable values and simplify the equation

ln(x) both sides of the equation and isolate for the n value, as shown below:

- $y = x^n$
- $\ln(y) = n\ (\ln(x))$
- $n = \ln(y) / \ln(x)$

If n contains a decimal value, round up to the next whole number.

4. Conclusion: "Therefore, the amount of time to reach the desired investment amount is [n]."

** Remember to be aware of different types of payment periods and multiply the amount of years based on the type.*

To calculate the mortgage payment per period: (applicable to mortgages and loans)

1. Variables:

PV_{OA} = Borrowed amount = (value of house) – (down payment)

PMT = UNKNOWN

r_{nom} = the nominal rate given (used to calculate effective rate)

n = number of periods = (remaining years) x (payment per year)

p = payment periods during the year

m = number of compounding periods per year for the nominal rate

2. Equations:

Calculate effective rate: $Effective\ Rate\ for\ Payment\ Period = \left(1 + \frac{r_{nom}}{m}\right)^{\frac{m}{p}} - 1$

Calculate payment amount (PMT): $PV_{OA} = PMT\left[\frac{1-(1+r)^{-n}}{r}\right]$

3. Numbers: Insert variable values and isolate for PMT

4. Conclusion: "Therefore, the mortgage payment per period is [PMT]."

To calculate the outstanding balance after a certain period: (applicable to mortgages and loans)

1. Variables:

PV_{OA} = UNKNOWN

PMT = payment amount

r_{nom} = the nominal rate given (used to calculate effective rate)

n = number of periods = (remaining years) x (payment per year)

p = payment periods during the year

m = number of compounding periods per year for the nominal rate

2. Equations:

Calculate effective rate: $Effective\ Rate\ for\ Payment\ Period = \left(1 + \frac{r_{nom}}{m}\right)^{\frac{m}{p}} - 1$

Calculate present value: $PV_{OA} = PMT\left[\frac{1-(1+r)^{-n}}{r}\right]$

3. Numbers: Insert variable values and calculate for balance (PV_{OA})
4. Conclusion: "Therefore, the outstanding balance is [PV_{OA}]."

To calculate the interest and principal repayment:

Principal paid = (borrowed amount) – (outstanding balance)

Interest paid = (PMT) x (amount of completed payment periods) – (principal paid)

To calculate the payments per period for a car: (processes also apply to leases)

1. Variables:

PMT = UNKNOWN

Cost of car = (price of car) + (freight and PDI) – (down payment)

Residual amount = (price of car) x (residual percentage)

n = number of periods over which investment is being calculated

r = Effective rate for payment period = APR / p

2. Equations:

Calculate present value (SA): $PV_{SA} = \frac{Residual\ amount}{(1+r)^n}$

Calculate present value (AD): **PV_{AD}** = Cost of car – PV_{SA}

Calculate payment amount (PMT): $PV_{AD} = PMT\left[\frac{1-(1+r)^{-n}}{r}\right](1+r)$

3. Numbers: Insert variable values and isolate for PMT
4. Conclusion: "Therefore, the lease payment per period is [PMT]."

To calculate payments per period for retirement contributions:

a) Determine the investment amount needed at time of retirement

** This amount is sometimes given as a single entity; therefore, no calculations are needed for section a. If the amount during retirement needed is given as PMT, calculations are necessary.*

1. Variables:

PV_{AD} = UNKNOWN

n = number of periods over which investment is being calculated

PMT = payment amount

r = interest rate or effective rate as you have calculated it **(effective monthly rate equation if needed)**

2. Equations:

Calculate investment amount needed (PV_{AD}): $PV_{AD} = PMT\left[\frac{1-(1+r)^{-n}}{r}\right](1+r)$

3. Numbers: Insert variable values and calculate PV_{AD}

b) Payments per period needed before retirement to achieve retirement amount

1. Variables:

PMT = UNKNOWN

n = number of periods over which investment is being calculated

FV_{OA} = PV_{AD} = investment amount needed at time of retirement (calculated previously)

r = interest rate or effective rate as you have calculated it **(effective monthly rate equation if needed)**

2. Equations:

Calculate payment amount (PMT): $FV_{OA} = PMT\left[\frac{(1+r)^n-1}{r}\right]$

3. Numbers: Insert variable values and isolate for PMT
4. Conclusion: "Therefore, the payment per period before retirement must be [PMT]."

To calculate the maximum price you would be willing to pay for a preferred share:

1. Variables:

PV = UNKNOWN

PMT = (percent preferred share) x (par value)

r = interest rate

2. Equations:

Calculate present value: $PV_{perpetuity}$ = PMT/r

3. Numbers: Insert variable values and calculate $PV_{perpetuity}$
4. Conclusion" Therefore, the amount you are willing to pay for this investment is [$PV_{perpetuity}$]."

To calculate the maximum price you would be willing to pay for a bond:

1. Variables:

Price of Bond = UNKNOWN

r = (interest rate) / (payments periods per year)

n = number of periods over which investment is being calculated

PMT = ((face value) x (coupon rate)) / (payment periods per year)

2. Equations:

Calculate present value (OA): $PV_{OA} = PMT\left[\frac{1-(1+r)^{-n}}{r}\right]$

Calculate present value (SA): $PV_{SA} = \frac{FV_{SA}}{(1+r)^n}$

Calculate Price of Bond: Price of Bond = $PV_{OA} + PV_{SA}$

3. Numbers: Insert variable values and calculate the Price of Bond
4. Conclusion: "Therefore, the amount you are willing to pay for this investment is [Price of Bond]."

Remember: Although these calculation instructions give specific equations, the AD and OA may vary based on question. Recognize key words that determine which equation is appropriate.