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AQ1

# Construction of analysis of variance-based metamodels for probabilistic seismic analysis and fragility assessment

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Finite element models used in industrial studies for seismic structural reliability analysis are in general very complex and computationally intensive. This is due to the important number of degrees of freedom as well as due to advanced damage models and failure modes that have to be simulated. In consequence, reliability studies are feasible only by means of simpler surrogate models able to represent essential physics. Then the choice of an accurate surrogate or metamodel is crucial for uncertainty propagation and sensitivity analysis. The construction of a pertinent metamodel is however a not simple task when random uncertainties, not explained by the model parameters, have to be accounted for. This is the case for transient seismic analysis where the ground motion, an intrinsically random phenomena, is modelled by a stochastic process that cannot be entirely described by a set of parameters. In this paper, we construct a versatile metamodel based on analysis of variance (ANOVA) decomposition. The ANOVA decomposition provides a convenient framework allowing both for parametric uncertainties and for stochastic variability introduced by seismic load. We then compute fragility curves and perform sensitivity analysis.

**Keywords:** metamodel; fragility curve; regression; uncertainty; sensitivity analysis

## 1. Introduction

Finite element models used in industrial studies for seismic structural reliability analysis are in general very complex and computationally intensive. This is, on the one hand, due to the important number of degrees of freedom, and on the other hand, due to the complex damage models and failure modes that have to be considered for these studies. A metamodel (also referred to as response surface, surrogate model or emulator) may then be used in order to replace the original physical model by a simpler, often analytical, relation between model input and output. This is very useful especially when repeated analyses are required, such as it is the case for probabilistic analyses or optimisation. This paper addresses the construction of metamodels when the model input is a random phenomenon that is represented by a stochastic process or field. In this case, we have to deal with variability that is not ‘explained’ or simply covered by model parameters. In consequence, the construction a pertinent metamodel is not straightforward.

We are here more precisely interested in transient seismic analyses where seismic load, an intrinsically random phenomenon, is modelled by a non-stationary random process that cannot be simply described by a small set of parameters. Each occurrence of an earthquake, given by a ground-motion time history, is characterised by one

or more ground-motion intensity measures that have to be added to the model parameters. Moreover, the ground-motion variability cannot be simply treated by adding an independent random noise to the model input–output relation. Indeed, the variability or the dispersion depends on ground-motion intensity and thus on the model parameters. This is also known as heteroscedastic behaviour. This problem has been tackled by only a few authors in the engineering community so far. In what follows, we will distinguish ‘parameter uncertainty’, related to the model parameters (including the ground-motion intensity parameter) from the ‘stochastic uncertainty’ or ‘variability’, that is, the model output variability that remains when all the model parameters (including the ground-motion intensity parameter) are fixed. This terminology will be used throughout this paper for the sake of simplicity and clarity. The distinction and definition of different kinds of uncertainty, in particular in terms of epistemic (reducible) and aleatory (intrinsic) uncertainty, are a subject discussed in literature (Celik and Ellingwood 2010; Der Kiureghian and Ditlevsen 2009) and will not be addressed in this paper.

Schotanus et al. (2004) and, more recently, Buratti, Ferracuti, and Savoia (2010) constructed metamodels for seismic fragility analysis by means of second-order polynomials. In these works, explicit variables, corresponding to model parameters, are distinguished from

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implicit variables, corresponding to random uncertainty not explained by the model parameters. The latter is due to seismic ground-motion variability. The response surface is constructed for structural capacity under the assumption that the effects of the implicit variables are additive and do not interact with the (explicit) model parameters by using ‘block design’ as introduced by Box and Draper (2007). The different blocks correspond to different realisations of the implicit random variables, that is, to different ground-motion time histories. Samples of structural capacity are obtained by scaling each of the chosen time histories until failure is observed. Besides, a lognormal transformation is applied so that capacity is modelled by a lognormal random variable. FORM or standard Monte Carlo simulation is then used for determining fragility curves. This approach is however based on a number of simplifying assumptions, and the ‘analysis by blocks’ strategy is not straightforward and computationally intensive. It occasionally requires the scaling of the ground motion over wide ranges, according to Schotanus et al. (2004), which might lead to bias in the analysis.

Conceptually, simpler approaches are known as lognormal seismic demand models where the model output is related to the parameter measuring ground-motion intensity by linear regression techniques (Ellingwood and Kinali 2009; Celik and Ellingwood 2010; Hamburger, Foutch, and Cornell 2003; Bai, Gardoni, and Hueste 2010; Tang and Zhang 2011). Heteroscedastic behaviour is accounted for through the properties of the lognormal distribution. These models have proven to fit well to earthquake data, but dependence on parameters other than the ground-motion parameter is not introduced. Behraman and Behnamfar (2009) constructed a seismic demand model by adding a bias term, the ‘explanatory basis functions’, to the deterministic seismic demand. This bias function depends on the mechanical model parameters. It is basically a first-order polynomial expression whose functional form is derived from simple relations used in design practice. The parameters of the bias term are evaluated using Bayesian updating.

In this paper, we present a straightforward manner to construct a metamodel for seismic fragility analysis where stochastic variability, not explained by the model parameters, has to be accounted for. The approach, based on Analysis of Variance-High-Dimensional Model Representation (ANOVA-HDMR), is conceptually simple and very versatile. It allows the performing of seismic fragility analysis without having to resort to ‘scaling’ of the ground-motion time histories and provides analytical formulas for global sensitivity indices. Non-parametric regression techniques based on recursive filtering and smoothing, as proposed by Ratto, Pagano, and Young

(2007), are used here for evaluating the other decomposition terms.

This paper is organised as follows. In Section 2, some general elements on HDMR decomposition are given. Section 2 describes how the ANOVA-HDMR metamodel can be used for seismic analysis where stochastic uncertainty due to seismic excitation has to be accounted for. Section 3 provides an illustration of the method through a simple study case. Eventually, fragility curves are evaluated for a reinforced concrete building studied during the international benchmark SMART. Section 6 finishes with some conclusions and perspectives.

## 2. Some elements on HDMR

ANOVA (or HDMR) is a generic tool for modelling high-dimensional input-output system behaviour. It consists of decomposing a function into terms of increasing dimensionality that can be used for principally two purposes:

- Construction of a computational model from field data or creation of metamodels.
- Identification of key model parameters and sensitivity analysis.

One of the main advantages of HDMR is orthogonality, implying that its terms can be evaluated independently one after the other. This property is used in this work for introducing ‘stochastic variability’ ~~due to seismic excitation by making use of seismic demand models available in literature~~. This kind of functional decomposition has been initially introduced by Hoeffding (1948) and further developed by Sobol’ (1993) for the purpose of sensitivity analysis. The decomposition has been further investigated by the latter author in the reference (Sobol’ 2003). Different approaches for evaluating the HDMR terms have been suggested in literature since these are principally cut-HDMR and ANOVA-HDMR. A comprehensive review and theoretical analysis of the different contributions and approaches can be found in Rahman (2011a), where it is pointed out that the two methods have been developed independently and from a completely different perspective by several researchers. The ANOVA framework has however several advantages with respect to the cut-HDMR, as will be detailed in what follows, and is used here. Let us define the general model:

$$Y = f(\mathbf{X}) \in \mathbb{R}, \quad (1)$$

where  $f$  is an analytical expression or a ‘black box’ code,  $Y$  is the model output, and  $\mathbf{X} = (X_1, X_2, \dots, X_p)$  are the uncertain parameters that are modelled by independent random variables. HDMR consists of decomposing a function  $f(\mathbf{X})$  in terms of increasing dimensionality

yielding an expression of the form:

$$f(\mathbf{X}) = f_0 + \sum_i f_i + \sum_{1 \leq i \leq j \leq p} f_{ij} + \dots + f_{12\dots p}, \quad (2)$$

where the total number of terms is  $2^p$ . The use of HDMR for metamodeling is based on the observation that in most physical models, higher-order interaction terms are negligible. This means that the series (2) could be truncated to a reduced number of low-order terms (in general first- and second-order terms are sufficient) without considerable loss of precision.

### 2.1. ANOVA-HDMR

The ANOVA-HDMR has been first introduced by Sobol' (1993, 2003) for real-valued functions  $f$  defined on the  $p$ -dimensional unit hypercube  $K_p = [0,1]^p$ . More recent findings (Rahman 2008) reveal that the ANOVA decomposition also applies to an arbitrary probability measure, provided that all integrals involved exist and are finite. Moreover, Rahman (2008, 2009) shows that the polynomial dimensional decomposition (PDD) method represents a polynomial version of ANOVA-HDMR.

**ANOVA-HDMR:** There is a unique decomposition of Form (2) and the ANOVA component functions  $f_{i_1, \dots, i_s}(x_{i_1}, \dots, x_{i_s})$ ,  $\forall \{i_1, \dots, i_s\} \subseteq \{1, \dots, p\}$  have zero mean:

$$\int_{\mathbb{R}^p} f_{i_1, \dots, i_s}(x_{i_1}, \dots, x_{i_s}) p_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} = 0, \quad (3)$$

where  $f_0$  is a constant and  $\prod_{k=1}^p p_k(x_k)$ . Each term of Expression (2) is a function only of the variables in the index, that is,  $f_i = f(X_i)$ ,  $f_{ij} = f(X_i, X_j)$ , etc. Under these conditions, the terms of the decomposition are orthogonal:

$$\int_{\mathbb{R}^p} f_{i_1, \dots, i_s}(x_{i_1}, \dots, x_{i_s}) f_{j_1, \dots, j_s}(x_{j_1}, \dots, x_{j_s}) p_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} = 0, \quad (4)$$

if at least one index is not repeated:  $\{i_1, \dots, i_s\} \neq \{j_1, \dots, j_s\}$ . This means that the ANOVA-HDMR terms of equality (2) can be expressed as:

$$\begin{aligned} f_0 &= \int_{\mathbb{R}^p} f(\mathbf{x}) p_{\mathbf{X}}(\mathbf{x}) d\mathbf{x} = \mathbb{E}(Y), \\ f_i &= \int_{\mathbb{R}_{p-1}} f(\mathbf{x}) \prod_{k \neq i} p_k(x_k) dx_k - f_0 = \mathbb{E}(Y|X_i) - f_0, \\ f_{ij} &= \int_{\mathbb{R}_{p-2}} f(\mathbf{x}) \prod_{k \neq i, j} p_k(x_k) dx_k - f_0 - f_i - f_j \\ &= \mathbb{E}(Y|X_i, X_j) - f_0 - f_i - f_j, \dots \end{aligned} \quad (5)$$

Moreover, the ANOVA-HDMR is directly linked to the definition of Sobol' sensitivity indices. Indeed, if  $f$  is a

square integrable, then the variance of  $Y = f(\mathbf{X})$  can be decomposed as:

$$\text{Var}(Y) = \sum_i V_i + \sum_{1 \leq i \leq j \leq p} V_{ij} + \dots + V_{12\dots p}, \quad (6)$$

where  $V_i = \text{Var}(E(Y|X_i))$  and  $V_{ij} = \text{Var}(E(Y|X_i, X_j)) - V_i - V_j, \dots$ , leading to the classical definition of sensitivity indices (Sobol' 1993; Saltelli et al. 2008):

$$S_i = \frac{V_i}{\text{Var}(Y)}, S_{ij} = \frac{V_{ji}}{\text{Var}(Y)}, \dots \quad (7)$$

### 2.2. Cut-HDMR

In the last years, the cut-HDMR has become quite popular in the engineering community, (see, for instance, the references Chowdhury and Rao 2009; Rao et al. 2010; Adhikari, Chowdhury, and Friswell 2010; Xu and Rahman 2005; Unnikrishnan, Prasad, and Rao 2012; Ziehn and Tomlin 2009; Alibrandi 2014). Cut-HDMR is based on the same type of decomposition (2) as the ANOVA-HDMR (Rabitz et al. 1999; Xu and Rahman 2004, 2005). The difference between the two approaches relies on the choice of the weighting functions. In the cut-HDMR, the integrals of Equation (4) are replaced by Dirac delta functions  $\delta_{\theta}$  such that:

$$f_{i_1, \dots, i_s}(x_{i_1}, \dots, x_{i_s})|_{x_{i_s} = \theta_{i_s}} = 0, \quad (8)$$

$$\forall k = 1, \dots, s, \forall \{i_1, \dots, i_s\} \subseteq \{1, \dots, p\}$$

where  $\theta = (\theta_1, \theta_2, \dots, \theta_p)$  is a reference point.

Under these conditions, the component functions of equality (2) are given by the following expressions:

$$\begin{aligned} f_0 &= f(\theta), \\ f_i &= f(\theta_1, \dots, \theta_{i-1}, x_i, \theta_{i+1}, \dots, \theta_p) - f_0, \\ f_{ij} &= f(\theta_1, \dots, \theta_{i-1}, x_i, \theta_{i+1}, \dots, \theta_p) - f_0 - f_i - f_j, \end{aligned} \quad (9)$$

the following terms are defined accordingly. Cut-HDMR expansion gives exact results along the lines, planes, volumes, etc.

As opposed to ANOVA-HDMR decomposition or PDD, the cut-HDMR does not require high-dimensional integration. Therefore, it is computationally less expensive which explains its attractiveness. It is also noteworthy that the dimension reduction (Xu and Rahman 2004) and decomposition (Xu and Rahman 2005) methods, developed independently from the perspective of Taylor series expansion for statistical moments and reliability analysis, are the same as the cut-HDMR, as shown in Rahman (2011b). The cut-HDMR suffers however from a number of drawbacks. First, when the HDMR terms are truncated, considering only low-order terms, then the accurateness of the approximation can depend significantly on the choice of the reference point as pointed out by Sobol' (2003). Second, Rahman (2011b) shows that the error of cut-HDMR is on average



much higher than with ANOVA-HDMR. Besides, in some cases, the higher-order cut-HDMR approximation may commit larger expected error than a lower-order approximation. Third, since we have to sample along lines and planes, etc., the number of necessary model runs increases rapidly with the number of parameters making the approach inefficient. Eventually, the cut-HDMR cannot straightforwardly be used for global sensitivity analysis using Sobol' indices, cf. Equation (7). Also, it is not very useful when stochastic uncertainties, not fully described by the model parameters, have to be considered. In fact, in this case, the output is not deterministic but remains random even when all of the model parameters are fixed to their reference value. Likely, when fixing all but one of the model parameters, the output does not reduce to a curve as assumed by cut-HDMR but is given by a scatterplot (cf. Figure 2).

### 2.3. Considering stochastic input and truncation

In many real-life applications, the output variability cannot be entirely explained by the set of chosen input parameters. There may be some remaining variability called here 'stochastic uncertainty'. In order to account for this, we will consider the following truncated HDMR:

$$f(\mathbf{X}) = f_0 + \sum_i f_i + \sum_{1 \leq i < j \leq p} f_{ij} + \epsilon. \quad (10)$$

In this expression, we have put  $\epsilon = \epsilon_c + \epsilon_r$ , where  $\epsilon_c$  models the contribution of neglected higher-order terms of the expansion, while  $\epsilon_r$  represents the contribution due to effects not explained by the model parameters. The truncated terms can be modelled by a Gaussian random variable, considering them as a sum of a large number of independent centred random variables (see Ratto, Pagano, and Young 2007). The unexplained effects, due to the stochastic input, are accounted for by the second noise term. When noise is not dependent on the parameter values (homoscedastic behaviour), then it is easy to evaluate its characteristics from data and to introduce it into the model. This is unfortunately not the case for seismic analysis where the variance of the noise depends on the parameter values and, in particular, on the ground-motion intensity parameter (heteroscedastic behaviour). The following sections give a description of how the parameter-dependent noise is handled here.

One of the advantages of the ANOVA-HDMR is the linear independence of the decomposition terms (uncorrelated terms). This allows for a step-by-step evaluation of the functional terms using only input-output samples. In particular, it allows here to treat parametric and stochastic uncertainties separately.

The ANOVA-HDMR terms corresponding to uncertain model parameters are evaluated by non-parametric regression as described in Section 2.4, while terms

related to stochastic input  $\epsilon_r$  are accounted for by a residual noise model. This is illustrated in Section 3.

### 2.4. Evaluation of the ANOVA-HDMR terms by non-parametric regression techniques

The simplest approach to obtain the ANOVA-HDMR terms, defined as conditional means, is to evaluate the terms of Expression (5) by integration. This is however too costly or impossible when complex or black box functions are considered. In the latter cases, smoothing and regression techniques have to be used in order to approximate the HDMR terms. In this paper, we evaluate the HDMR terms by means of state-dependent regression (SDR) as proposed by Ratto, Pagano, and Young (2007), and Borgonovo, Castaings, and Tarantola (2012). The SDR algorithm is based on Kalman filtering combined with fixed interval smoothing. It can be used to deal with non-smooth or even discontinuous patterns, making it very versatile for complex industrial problems. The evolution of each state-dependent parameter is accomplished by an integrated random walk (IRW) process. In the applications of this paper, the MATLAB software provided by the authors (Ratto, Pagano, and Young 2007) upon request is used. This software requests uniform random variables as input. This is why, in the applications of the Section 3, the random variables have been transformed to uniform before using the SDR programme.

The properties and advantages of different recursive smoothing algorithms are also discussed in Ratto and Pagano (2004). Other smoothing spline or regression techniques have been proposed in literature (see, for example, the references Storlie et al. 2009; Spall 2003; Storlie and Helton 2008; Storlie et al. 2011; Rahman 2011a). Such methods could be used in combination with the proposed approach. In particular, Rahman (2008, 2009) has successfully estimated the expansion coefficients of measure-consistent orthogonal polynomials by means of dimension reduction integration (Xu and Rahman 2004), and in Rahman (2011a), the PDD is used to calculate variance-based global sensitivity indices. Rahman, furthermore, compared the performance of different approaches for evaluating HDMR models, among them the SDR. The author concludes that the PDD method provides more accurate estimates of global sensitivity indices than the SDR method.

## 3. Application to seismic analysis

### 3.1. Construction of an ANOVA-HDMR metamodel by means of seismic transient response analysis

When performing transient analysis, the seismic motion is modelled by a non-stationary stochastic process (it is non-stationary both in amplitude and in frequency



content). Each realisation of the stochastic process models a seismic event and is characterised by a so-called ‘ground-motion indicator’. ~~Without loss of generality, absolute maximal acceleration is used in this paper to characterise the seismic ground-motion intensity.~~ Let  $\mathcal{T}$  be the considered time interval, a compact subset of  $\mathbb{R}$  and  $\mathcal{A}(t), t \in \mathcal{T}$  be the second-order, zero mean stochastic process defined on a probability space  $(D, \mathcal{D}, P)$  with values in  $\mathbb{R}$ . Let  $a(t, d)$  be a realisation of this stochastic process and  $\alpha(d) = \max_{t \in \mathcal{T}} |a(t, d)|$ . The parameter  $\alpha$  is generally called Peak Ground Acceleration (PGA). It is a common parameter for characterising seismic ground-motion intensity. However, it is clear that this description is not exhaustive. Ground-motion time histories having same maximum  $\alpha$  can be very different, leading to a great deal of variability in the model response. This ~~variability, due to seismic excitation,~~ cannot be explained by the model parameters and is called here ‘stochastic’ uncertainty.

The terms  $f_i, f_{ij}$  of Expression (10) are evaluated by means of non-parametric regression (cf. Section 2.4). The ‘stochastic’ part is accounted for by a term introduced ad hoc and reflects the physical knowledge of the problem, in particular, heteroscedasticity. As we will see, the ad hoc term is closely related to common seismic demand models used for seismic fragility analysis (Hamburger, Foutch, and Cornell 2003; Ellingwood and Kinali 2009; Behraman and Behnamfar 2009; Bai, Gardoni, and Hueste 2010). As in the latter models, we make use of lognormal noise terms in order to represent the stochastic variability ~~due to seismic load and this cannot be explained by the model parameters.~~ The use of lognormal models for seismic demand has proven to provide good approximations in many applications (e.g., Ellingwood and Kinali 2009; Behraman and Behnamfar 2009; Bai, Gardoni, and Hueste 2010; Zentner et al. 2011; Schotanus et al. 2004). Moreover, lognormal expressions have been retained in regulatory recommendations and guidelines (ATC-58 2009; FEMA-445 2006; Reed and Kennedy 1994).

The steps for the construction of the HDMR metamodel for seismic analysis are the following:

- (1) Consider solely stochastic uncertainties (seismic excitation) and evaluate the ad hoc (seismic-demand type) model based on HDMR input–output relation (this is illustrated in Section 4.1).
- (2) Consider stochastic and parametric uncertainty and compute the ANOVA-HDMR metamodel, neglecting higher-order terms and unexplained effects (this is illustrated in Section 4.2).
- (3) Introduce the unexplained variability in the model obtained at Step (2) by using the terms identified at Step (1).

In summary, the limitations of this methodology are the following:

- Only uncorrelated input variables can be considered: this assumption is true for the ground-motion parameter and the mechanical model parameters (seismic hazard does not depend on the properties of the building structure), but there might be correlations between the mechanical model parameters.
- It is assumed that the seismic demand, neglecting mechanical model parameter uncertainty, follows a lognormal distribution (lognormal noise term).
- The model parameters have been transformed to uniform random variables for the construction of the ANOVA-HDMR metamodel: this is not a theoretical limitation of the approach but due to the algorithm used here (and can thus be overcome easily by adopting other implementations, cf. also Section 2.4).

The whole procedure for the construction of the ANOVA-HDMR metamodel is illustrated in Section 4. Since it is our aim to determine seismic fragility curves, we give a brief definition in what follows.

### 3.2. Definition of fragility curves

Seismic fragility curves give the failure probability of a structure as a function of seismic ground-motion intensity. Each point of the curve is a conditional probability, that is, the probability that the structural response exceeds the admissible value ~~conditioned on~~ ground-motion intensity  $\alpha$ . Let  $Y$  be the variable of interest that is used in order to characterise structural damage. Then, the fragility curve  $P_f(\alpha)$  can be expressed as:

$$P_f(\alpha) = P(Y > Y_{\max} | \alpha), \quad (11)$$

that is, the probability of failure for given ground-motion intensities is  $\alpha$ . In the common *capacity-demand* framework, this can be expressed as:

$$P_f(\alpha) = P(D > C | \alpha) = P(C - D < 0 | \alpha), \quad (12)$$

where ~~the capacity  $C$~~  and the demand  $D$  depends on the intensity of seismic load and mechanical model parameters. Uncertainty on capacity can be accounted for by modelling it as a random variable.

## 4. Illustration of the method for a simple study case

We consider here a simple single degree of freedom oscillator subjected to seismic excitation whose equation of motion reads:

$$\ddot{x} + 2\zeta\omega_0\dot{x} + \omega_0^2x = -a(t).$$

Table 1. Characteristics of random variables.

	Distribution	Parameters	Values
$\omega_0$	lognormal	Mean, std, min	4.0, 0.04, 1.0
$\xi$	Uniform	Min, max	0.01, 0.06

This model is characterised by two parameters: its eigenfrequency  $\omega_0$  and damping ratio  $\xi$ . The distributions chosen for these parameters are resumed in Table 1. We have at our disposal a database containing 50 ground-motion time histories (generally called accelerograms) that are supposed to be entirely representative of seismic events that might occur at the considered site. The response spectra relative to these accelerograms are shown on Figure 1 (grey lines) together with their median and confidence intervals (black lines). The variable of interest for this study is spectral acceleration  $S_a$ , that is, the peak acceleration of the structure subjected to seismic load. PGA has been chosen as a ground-motion indicator. The uncertain structural model parameters are damping and eigenfrequency. Thus, the model output depends on the parameters  $\alpha$  (the PGA),  $\omega_0$ ,  $\xi$  as well as the not modelled effects represented by variable  $\mathcal{U}$ :  $S_a = f(\alpha, \omega_0, \xi, \mathcal{U})$ . Thus, for the HDMR metamodel, we consider:

$$X_1 = \alpha, X_2 = \omega_0, X_3 = \xi, X_4 = \mathcal{U}.$$

The terms  $f_0, f_1, f_2, f_3$  and  $f_{12}, f_{13}, f_{23}$  are estimated by means of non-parametric regression as described in Section 2.4, while  $f_4 = f_{\mathcal{U}}$  and  $f_{14} = f_{\alpha\mathcal{U}}$  are the ad hoc terms. The latter are introduced in Section 4.1.

#### 4.1. Step 1: seismic demand considering only seismic excitation

At the first time, we consider the simplified case where only seismic excitation is modelled and no parameter uncertainty is accounted for.

We introduce the following ad hoc model expressing the relation between ground-motion parameter and model output:

$$S_a = \frac{1}{\exp(0.5\beta^2)} f_{SH} \exp(\beta\mathcal{U}), \mathcal{U} \sim \mathcal{N}(0, 1). \quad (13)$$

where  $f_{SH} = \mathbb{E}(S_a | \alpha) \simeq f_0 + f_\alpha$  is the fitted regression curve with respect to the ground-motion parameter PGA corresponding to the first two ANOVA-HDMR terms. The remaining variability, not explained by the model parameter (PGA), is characterised by parameter  $\mathcal{U}$ , while  $\beta$  is the lognormal standard deviation.

With these notations, we can determine  $f_{\mathcal{U}}$  and  $f_{\alpha\mathcal{U}}$  as:

$$\begin{aligned} f_{\mathcal{U}} &= f_0(\exp(\beta\mathcal{U} - 0.5\beta^2) - 1), \\ f_{\alpha\mathcal{U}} &= f_\alpha(\exp(\beta\mathcal{U} - 0.5\beta^2) - 1). \end{aligned} \quad (14)$$

In Equation (13), for given ground-motion intensity  $\alpha$ , maximal spectral acceleration is modelled by a lognormal random variable with mean  $\mathbb{E}_{\mathcal{U}}(S_a) = f_{SH}$  and variance  $\text{Var}_{\mathcal{U}}(S_a) = f_{SH}^2(\exp(\beta^2) - 1)$ . The model parameter  $\beta$  is estimated by means of maximum likelihood method and using the property:

$$\log(S_a) \sim \mathcal{N}\left(\log\left(\frac{f_{SH}}{\exp(0.5\beta^2)}\right), \beta\right). \quad (15)$$

This leads to  $\beta = 0.48$ . In Figure 2, the scatterplot of the data (results of 50 structural analysis) is compared to the scatterplot of a sample of predicted outputs obtained with the HDMR metamodel. The blue line on this figure

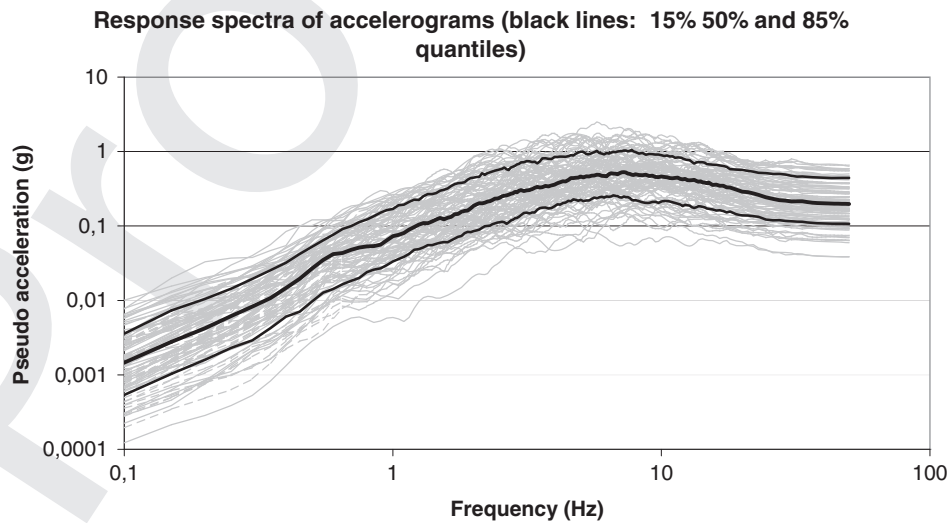


Figure 1. Response spectra of synthetic accelerograms (grey) and respective quantiles (black).

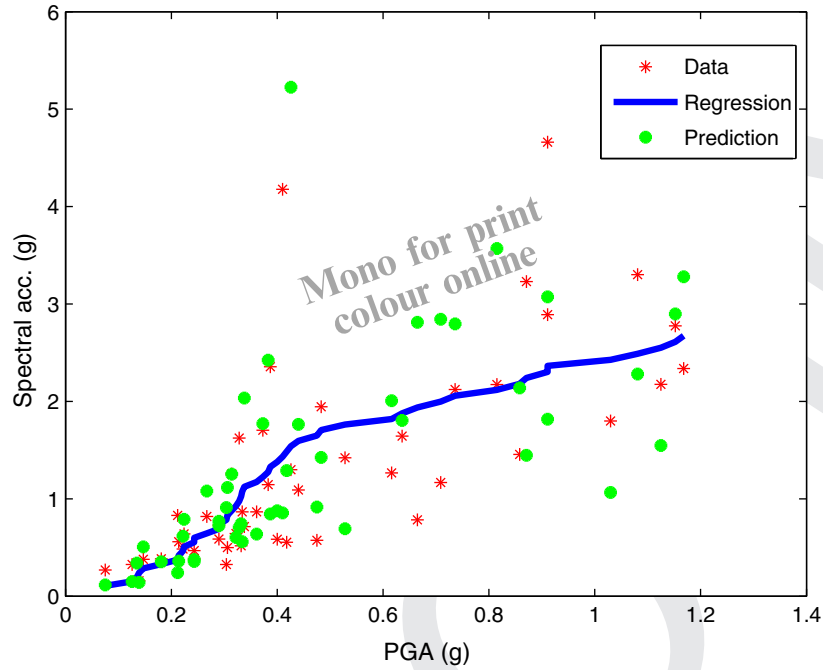


Figure 2. Scatter plots for data (red), prediction (green) and regression  $f_{SH}$  (blue line).

is the regression curve  $f_{SH}$ . The cumulative distribution functions (cdf) of the initial data and the predicted values are plotted for comparison in Figure 3. It can be seen that the metamodel represents the distribution of the model output (spectral acceleration) very accurately.

If the analyst is interested only in global structural response, then the simple surrogate described above

might be sufficient. Parametric uncertainty can be propagated through the model and be accounted for through the parameter  $\mathcal{U}$  representing the unexplained effects (variability) including parameter uncertainty. This is feasible because variability introduced by parameter uncertainty is much less important than unexplained variability due to seismic excitation. Moreover, fragility

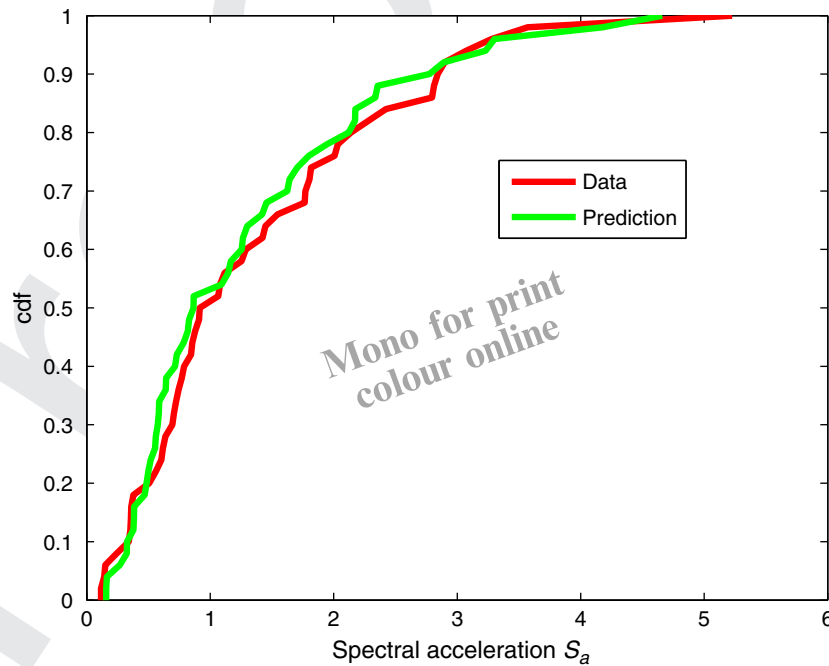


Figure 3. Estimated cdf of spectral acceleration: data (red) and sample of predictions (green).

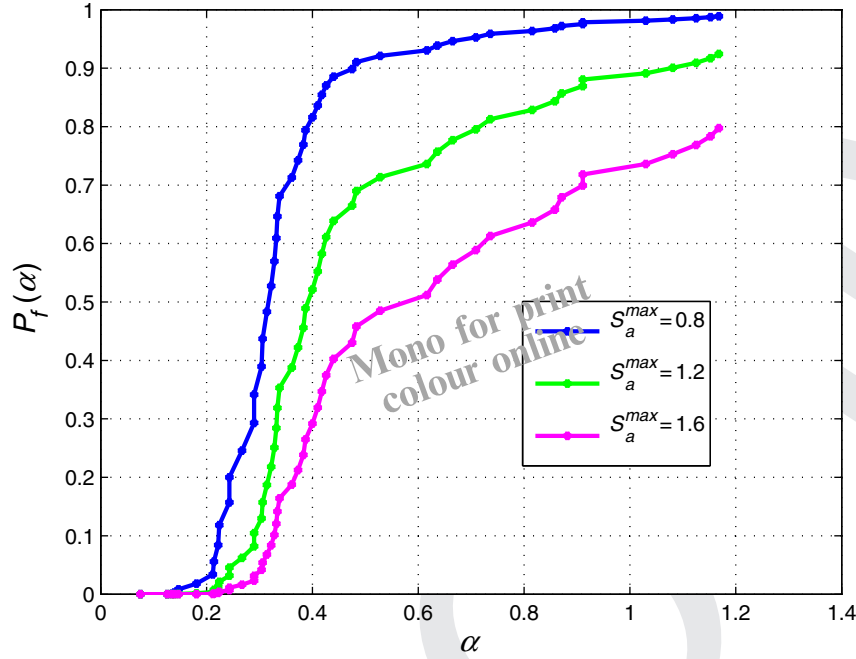


Figure 4. Fragility curves for different  $S_a^{\max}$ .

curves can be evaluated analytically using Expressions (13), (15) and (11). Indeed, the failure probability for a given ground-motion parameter value  $\alpha$ , reads:

$$\begin{aligned} P_f(\alpha) &= 1 - P(S_a \leq S_a^{\max} | \alpha), \\ &= 1 - \Phi\left(\frac{\log(S_a^{\max}/f_{SH}(\alpha)) + 0.5\beta^2}{\beta}\right), \quad (16) \\ &= \Phi\left(\frac{\log((f_0 + f_x(\alpha))/S_a^{\max}) - 0.5\beta^2}{\beta}\right), \end{aligned}$$

where  $\Phi$  denotes the standard normal cdf. The fragility curves determined for this case study are shown in Figure 4.

**Remark 1:** Uncertainty related to the admissible threshold value (here maximal admissible spectral acceleration  $S_a^{\max}$ ) can be introduced without further effort if the latter is supposed to be lognormally distributed. Let  $S_a^{\max}$  be a lognormal random variable such that  $\log(S_a^{\max}) \sim \mathcal{N}(\mu, \sigma)$  and define the margin as  $F = \frac{S_a^{\max}}{S_a}$ . Under these conditions,  $F$  is a lognormal random variable verifying:

$$\log(F) \sim \mathcal{N}\left(\log(\mu) - \log\left(\frac{f_{SH}}{\exp(0.5\beta^2)}\right), \sqrt{\beta^2 + \sigma^2}\right),$$

and the fragility curve can be easily obtained by calculating:

$$P_f(\alpha) = P(F \leq 1 | \alpha). \quad (17)$$

However, if the analyst wants to know about the particular effect of parametric uncertainties on the output variability and perform sensitivity analysis, then the uncertain parameters have to be accounted for in the construction of the metamodel. This is discussed in the following section.

#### 4.2. Step 2: construct full ANOVA-HDMR model considering seismic excitation and parameter uncertainty

We now consider not only seismic load, but we also account for model parameter uncertainty. The characteristics of the distributions of the two model parameters, eigenfrequency and damping, are given in Table 1. We recall that we consider the model parameters  $X_1 = \alpha$ ,  $X_2 = \omega_0$ ,  $X_3 = \xi$ ,  $X_4 = \mathcal{U}$ . Then  $f_0$  is simply the mean value of the model output, while the terms  $f_1, f_2, f_3, f_{12}, f_{13}, f_{23}$  are estimated by means of non-parametric regression. The terms  $f_4 = f_{\mathcal{U}}$  and  $f_{14} = f_{\alpha\mathcal{U}}$ , given by Expression (14), are evaluated by using the previously defined  $f_0$  and  $f_{\alpha}$  and by introducing the dispersion  $\beta$  determined in Section 4.1.

Using the same data basis as in Section 4.1 containing 50 ground-motion time histories we have computed an output sample of size  $N_s = 200$  by performing four replications of the initial Latin Hypercube sample (Helton and Davis 2003), each of size  $N_{SH} = 50$ .

For this simple case study, there was no interaction between parameters such that  $f_{12} = f_{13} = f_{23} = 0$ . The scatterplot, the regression curves and a sample of predicted outputs plotted against PGA values are shown

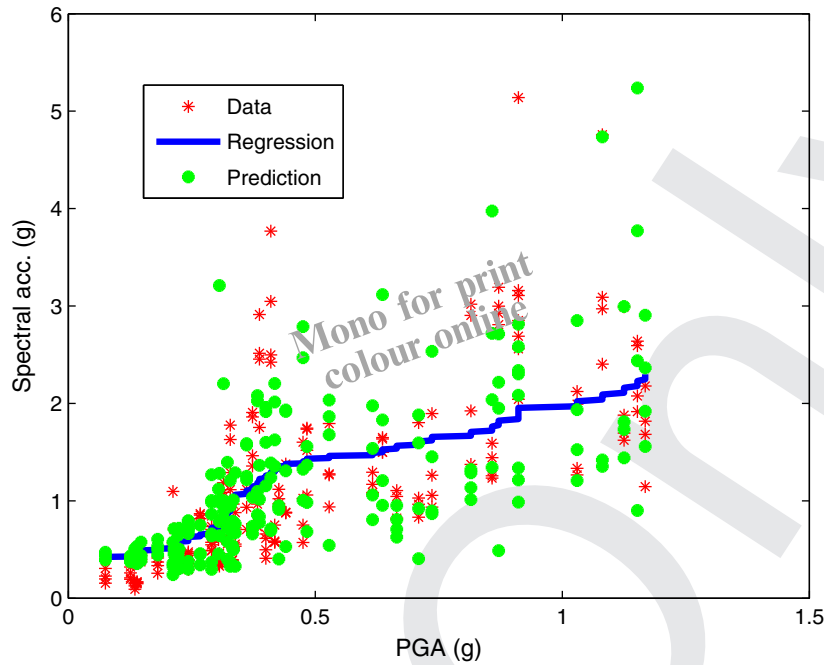


Figure 5. Regression curve (blue) and scatterplots of the initial data (red) and a predicted sample (green).

in Figure 5. The scatterplots and fitted curves for the other two parameters are shown in Figure 6. The cumulative distribution functions of the initial data and the predicted values are compared in Figure 7. It can be seen again that the metamodel represents the distribution of the model output (spectral acceleration) very accurately.

The variance-based sensitivity indices can be straightforwardly obtained from the HDMR metamodel. They are given in Table 2. It can be seen that, as expected, parameters related to seismic excitation ( $\alpha$  and  $\mathcal{U}$ ) are the most influential ones. The importance of structural model parameters is very small; this might be due to the simplicity of the model and the introduced

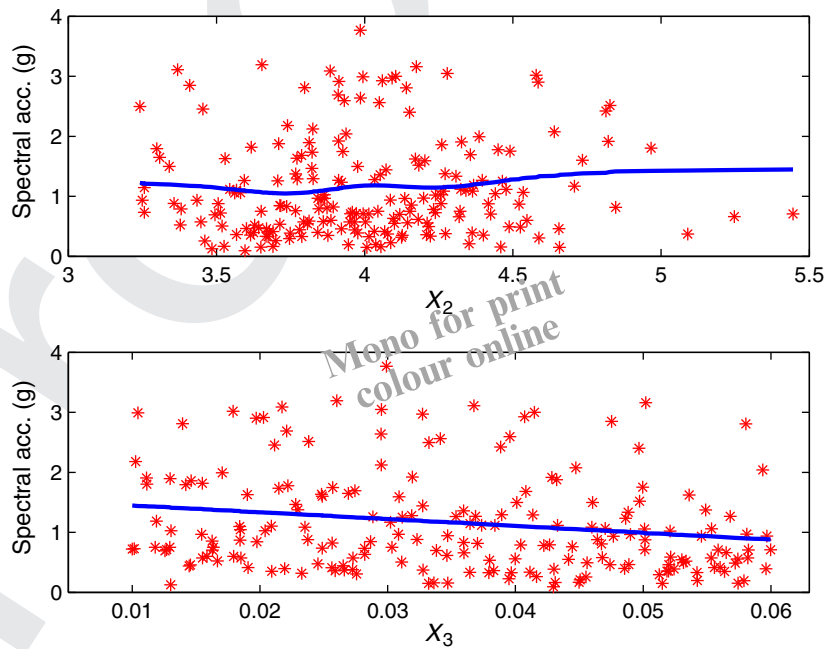


Figure 6. Scatter plots and regression curves for parameters  $\omega_0$  and  $\xi$ .

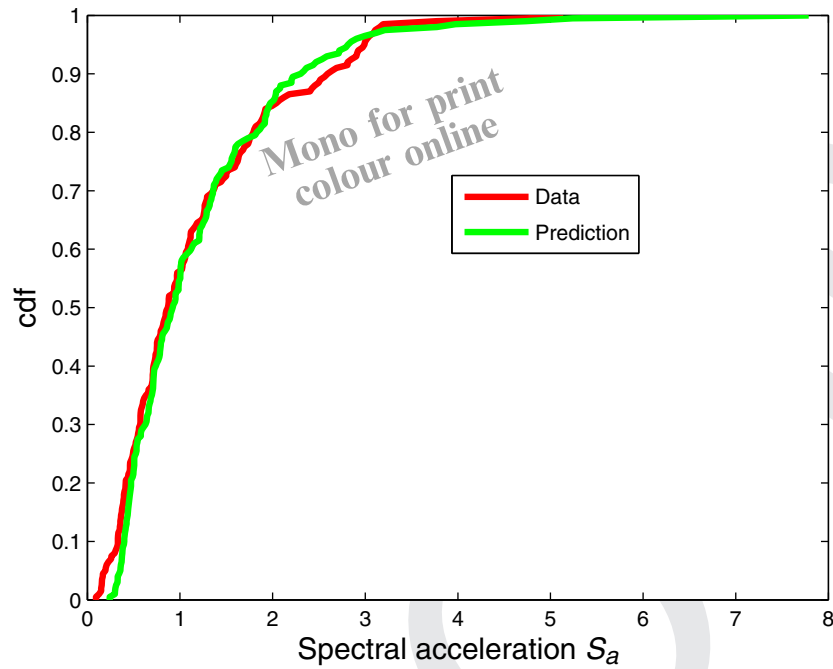


Figure 7. Estimated cdf: data (red) and model (green).

Table 2. Sobol' sensitivity indices for the oscillator.

$S_1$	$S_2$	$S_3$	$S_4$	$S_{14}$
0.515	0.005	0.010	0.400	0.10

uncertainty. In particular, the model output is not very sensitive to the eigenfrequency, which means that this parameter could be fixed to its best-estimate value for further analysis.

**Remark 2:** The sum of the sensitivity indices in Table 2 exceeds unity. This is due to numerical errors and approximations. The ANOVA-HDMR metamodel used

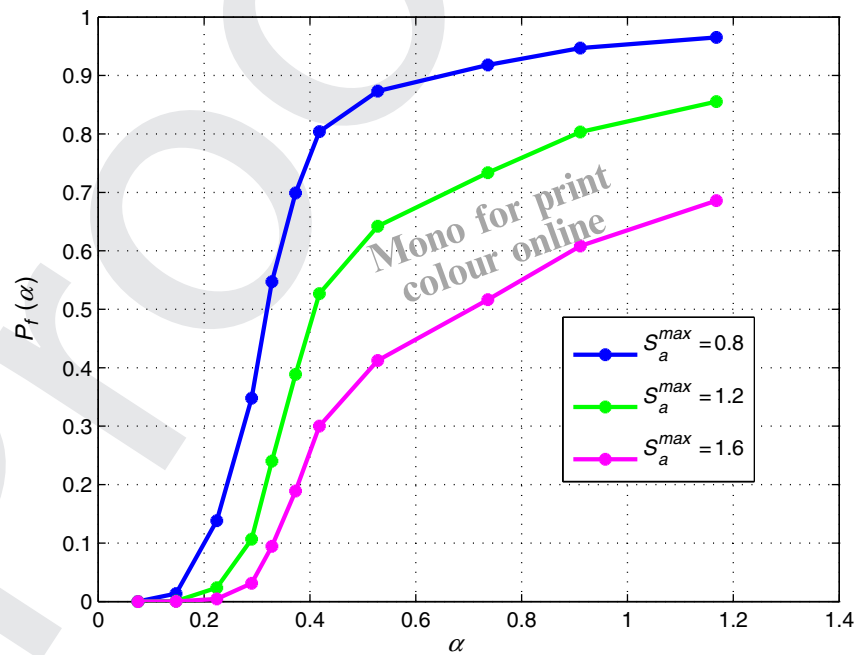
Figure 8. Fragility curves for different  $S_a^{\max}$ .



Table 3. Characteristics of random variables for SMART benchmark.

Parameter	Distribution	Mean	Standard deviation
$\rho$	$\rho$ lognormal	Nominal value	30% of nominal value
$E_b$	Lognormal	32,000 MPa	2200 MPa
$\delta$	$\delta$ lognormal	0.02	0.01

here provides analytical formulas for sensitivity indices and fragility curves. The complete expressions related to the unexplained variability have not been derived but are evaluated by performing Monte Carlo analysis in this work. This introduces supplementary statistical errors due to finite samples.

The fragility curves determined for the case when seismic excitation and parametric uncertainty are considered are shown in Figure 8.

The applications shown in this section highlight several advantages of the proposed metamodel.

First of all, 'scaling' of accelerograms to different ground-motion levels is not necessary due to the use of regression techniques (Grigoriou 2011; Watson-Lamprey and Abrahamson 2006). Then, the metamodel allows the performing of repeated seismic response analyses and the evaluation of fragility curves at a very low cost. The approach is in particular affordable when a limited number of ground-motion time histories, in agreement with the seismic scenario, are provided by seismologists for structural fragility analysis. Such accelerograms are

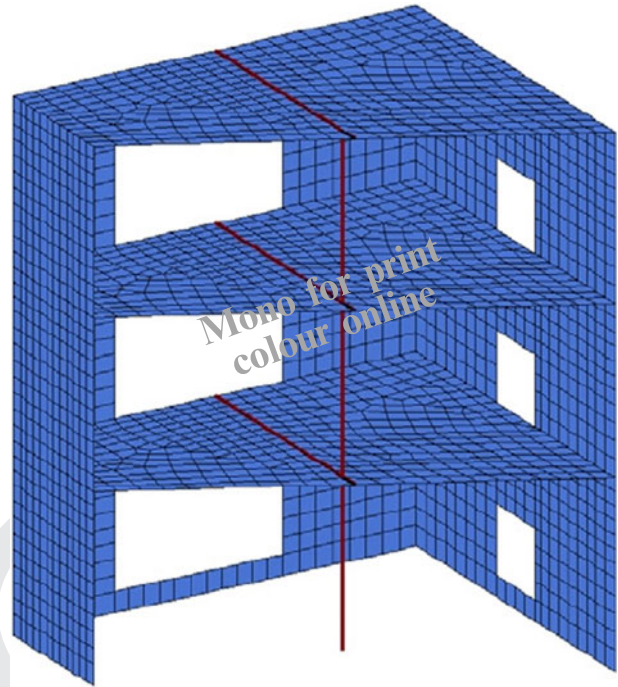


Figure 9. Finite element model used for SMART benchmark studies.

generally selected from ground-motion data basis (see e.g., Baker et al. 2011; Katsanos, Sextos, and Manolis 2010) and references herein. This is generally the case when seismic probabilistic risk assessment studies (Wakefield et al. 2003) are performed in nuclear industry or in the framework of performance-based earthquake

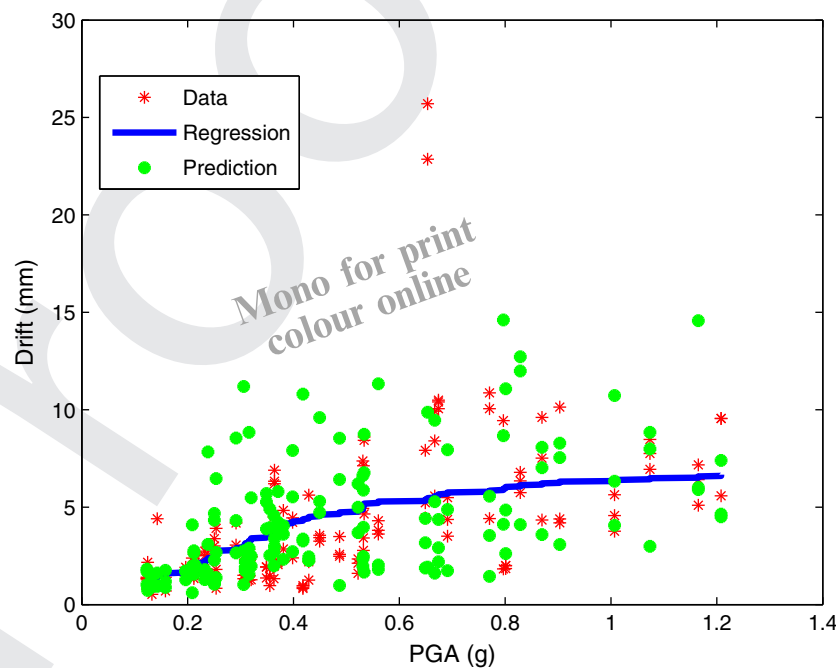


Figure 10. Regression curve (blue) and scatterplots of the initial data (red) and a predicted sample (green) for the SMART benchmark model.



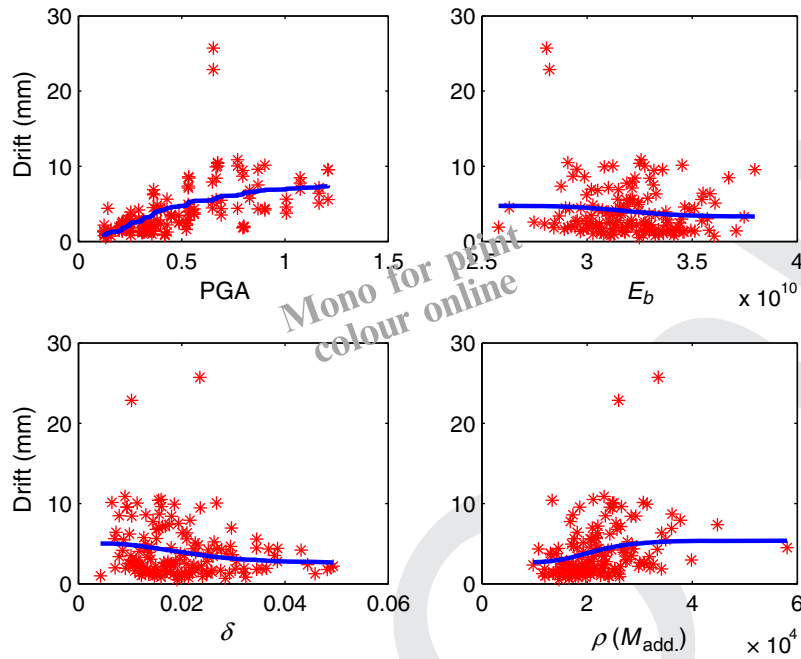


Figure 11. Scatter plots and regression curve (blue) for the four parameters: PGA, concrete Young's modulus  $E_b$ , damping  $\delta$  and additional mass  $\rho$ .

engineering (FEMA-445 2006; ATC-58 2009) for civil structures. Last but not least, the metamodel allows for analytical expressions of Sobol' indices. In consequence, global sensitivity analysis can be carried out at a very low cost without having to perform supplementary (mechanical) model runs.

### 5. Seismic fragility analysis of a reinforced concrete building

In this section, we apply our methodology to a more complex structural model. We determine fragility curves for a reinforced concrete building that has been studied in the framework of the international benchmark

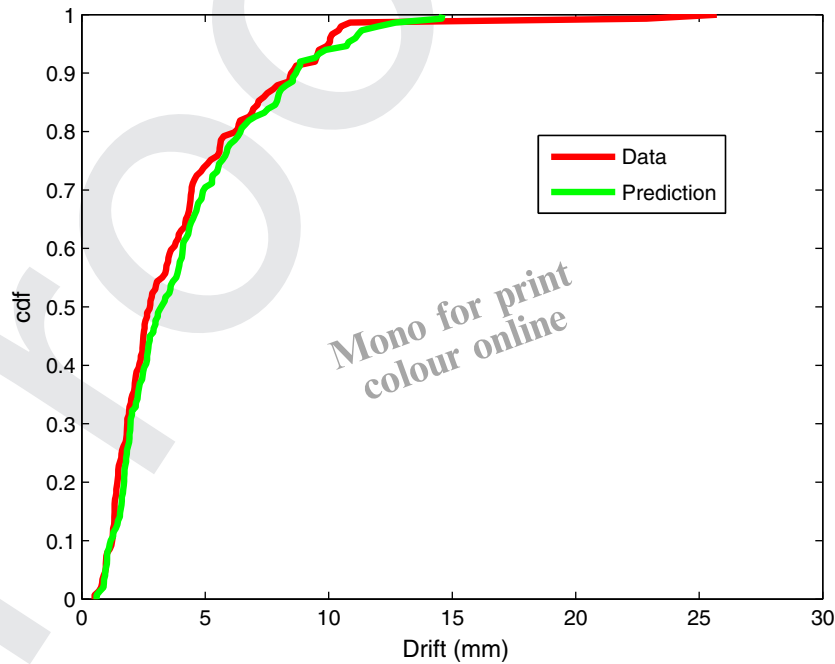


Figure 12. Estimated cdf for drift: data (red) and model (green).

Table 4. Sobol' sensitivity indices for SMART model.

$S_\alpha$	$S_{E_b}$	$S_\delta$	$S_\rho$	$S_{\mathcal{U}}$	$S_{\mathcal{U}\mathcal{U}}$
0.307	0.013	0.035	0.047	0.482	0.115

SMART 2008 (Lermitte et al. 2008). The finite element model is shown in Figure 9. Non-linear material behaviour of reinforced concrete is accounted for by a global damage model available with *Code\_Aster* finite element software (XXX). The non-linear homogenised material behaviour is represented by a cyclic three-slope model (Markovic et al. 2007). Peak interstory drift (differential displacement between two stories) is chosen as an indicator for the damage state of the structure. In what follows, we will simply write drift for this quantity of interest and denote  $Y$ . We consider two damage states defined as 'light damage' and 'controlled damage' in the framework of the benchmark (Lermitte et al. 2008).

Ground motion is given by the same 50 artificial time histories used in Section 4. As specified for the benchmark, we consider the uncertainties of the three model parameters. These are damping  $\delta$ , the concrete's Young's modulus  $E_b$  and the mass density of the dead load  $\rho$ . The latter models additional masses that are representing the equipment on the three concrete slabs of the building (cf. Figure 9).

One model run takes between 4 and 7 hours, depending on the parameter values and the seismic load. As explained in the previous section, we first carry out the 50 structural analysis (remember that 50 time histories are available) without parameter uncertainty. We then consider seismic load together with parameter uncertainty. We perform Latin Hypercube sampling, with three replications of the initial sample of size  $N_{SH} = 50$  leading to a sample of size  $N_s = 150$ . It has been checked that there is no significant refinement of the results when more than three replications are used. Interactions between the parameters, except interaction between the ground-motion parameter and unexplained effects, were again found to be negligible such that most of the second-order terms in Expansion (10) could be eliminated.

The scatterplots of the 150 model outputs and a sample of outputs using predictions from the metamodel, both plotted against PGA values, are shown in Figure 10. The regression curve  $\mathbb{E}(Y|\alpha)$  is given by the blue line on this same figure. The scatterplots and conditional expectations  $\mathbb{E}(Y|X_i) = f_i + f_0$  determined by non-parametric regression are shown in Figure 11 for the four model parameters  $X_1 = \alpha$  (PGA),  $X_2 = E_b$ ,  $X_3 = \delta$ , and  $X_4 = \rho$ . The cdf of the initial data and the predicted values are compared in Figure 12. Once again, the metamodel represents the distribution of the model output (drift) very accurately.

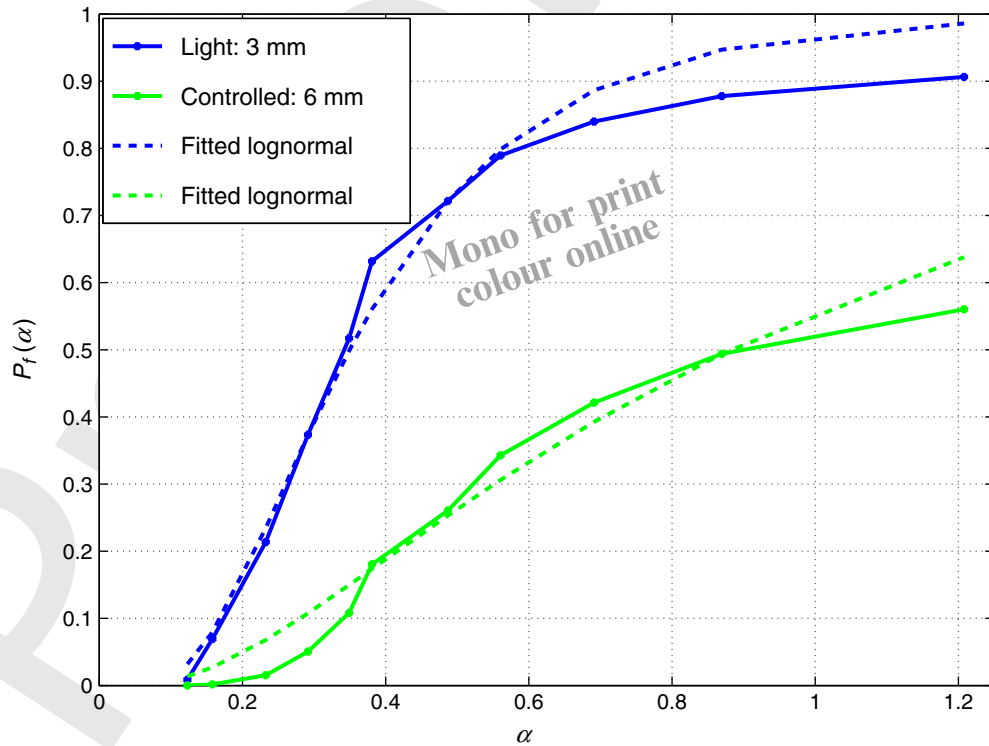


Figure 13. Fragility curves obtained with the metamodel and fitted lognormal curves (dotted lines).

Global sensitivity analysis results are reported in Table 4. The obtained sensitivity indices show that the influence of the mechanical model parameters is very small compared to seismic excitation (ground-motion level and unexplained variability). This is probably due to the fact that only a few sources of uncertainty have been introduced in the structural model and that damage and thus non-linear effects remained small. These findings are consistent with the previous results reported in literature (Celik and Ellingwood 2010) for moderate seismic zones. Uncertainty on non-linear material behaviour might however have more significant influence on predicted collapse performance for stronger shaking (Liel et al. 2009). The influence of uncertainty on other mechanical model parameters, more directly related to structural capacity, should be investigated in future studies.

## 6. Conclusions and perspectives

When complex engineering structures have to be studied, then the feasibility of uncertainty treatment and sensitivity analysis is closely linked to the availability of an accurate metamodel. The task is even more tricky when some of the model inputs are not parametric but of stochastic nature (stochastic processes or fields). This is the case for transient seismic response analysis where ground motion is modelled by a non-stationary stochastic process. In this paper, we have presented an ANOVA-based metamodel that can be used for seismic fragility analysis. The terms of the decomposition are evaluated by non-parametric regression techniques. The ANOVA decomposition provides a convenient framework to account for stochastic uncertainties. It has been shown that the cumulative densities of the model outputs of interest are represented very accurately by the metamodel. In consequence, seismic non-linear analysis and the assessment of structural fragility can be performed at a reasonably low cost. Especially, the ANOVA metamodel can be constructed with a limited number of given time histories without having to resort to scaling of the latter. This makes it very useful for seismic fragility assessment in the framework of performance-based earthquake engineering and seismic probabilistic risk assessment studies. Moreover, it allows to perform global sensitivity analysis without further evaluations of the mechanical model.

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