Math Mafia Manual

About Math Mafia

Description

- Math Mafia is a monopoly-based game that utilizes math problems for players to advance in the game.

Goals In Math Mafia

- Goal #1 Win

How Math is Implemented In The Game

- Buying, owning, and selling Real Estate
- Railroads
- Taxes
- Random Events (chance cards and community chest)

Background and Lore

- Former criminals
- They all were in the same prison
- Massive Prison Break
- Need to make lots of money, but want to fly under the radar, so they want to set up legitimate businesses

Game Mechanics

The Basics

1. Each player puts their chosen token piece in the “Go” space.
2. Roll 2d6 to determine how many spaces you move.
3. Go forward the number of spaces equal to what you rolled.
4. According to the space that your token lands on, you are entitled to complete a calculus question based on the calculus concept if it is a property card, pay rent, pay taxes, draw a chance card, etc.
5. If your roll landed on double(two 6’s, two 1’s, etc) then you get to go again. If you roll doubles up to three times, you go to jail.
6. If you go over the “Go” space whether by reaching through a card or by the roll, collect $200.

Obtaining Property

1. Whenever you land on an unobtained property, you may attempt to complete the math problem associated with it to obtain it.
2. If you succeed at solving the math problem, you now “legally” own that property. If you fail, you have to pay a fine equal to the mortgage cost.

   If you wish to not obtain the property, the banker can sell it at an auction by giving each buyer a problem about that property’s math subject. Whoever completes the problem the fastest and gets it correct wins. If the first buyer fails to answer it correctly, the second buyer to complete the problem next may attempt to obtain the property. If they fail to obtain it, then it is up to the next buyer down the line until a buyer has managed to either get the problem right or no player manages to get the problem right, in which case no one gets the property.

Paying Rent

1. When a player lands on a property owned by another player, that player has to either complete a math problem about that property’s subject or pay a bribe of at least the value of the rent. If the owner decides to not take the bribe, the player has to complete the problem regardless. If they fail to answer the problem

successfully, they have to pay the owner twice the amount of the rent.

If the property is mortgaged, no rent can be collected. When a property is mortgaged, its Title Deed card is placed face down in front of the owner. It is an advantage to hold all the Title Deed cards in a color group (e.g., Boardwalk and Park Place; or Connecticut, Vermont, and Oriental Avenues) because the owner may then charge double rent for unimproved properties in that color group. This rule applies to unmortgaged properties even if another property in that color group is mortgaged. It is even more advantageous to have houses or hotels on properties because rents are much higher than for unimproved properties. The owner may not collect the rent if he/she fails to ask for it before the second player throws the dice.

Chance and Community Chest

1. When you land on either of these spaces, take the top card from the deck indicated, follow the instructions and return the card face down to the bottom of the deck. The “Get Out of Jail Free” card is held until used and then returned to the bottom of the deck. If the player who draws it does not wish to use it, he/she may sell it, at any time, to another player at a price agreeable to both.

- **Types of Chance Cards**

1. Get out of jail free
2. **Break out of jail successfully**
3. Bank pays you a dividend of $150
4. **Acquire $150 “legally”**
5. Take a trip to Reading Railroad(if you pass Go, collect $200)
6. **Take a trip to Trigonometric Railroad(if you pass Go, collect $200)**
7. Go to jail. Go directly to jail, do not pass Go, do not collect $200
8. **Go to jail. Go directly to jail, do not pass Go, do not collect $200**

9. “Advance to” Cards
    a. Illinois Ave.(if you pass Go, collect $200)
    b. St. Charles Place(if you pass Go, collect $200)
    c. Boardwalk(if you pass Go, collect $200)
    d. Go(collect $200)
    e. **Go(collect $200)**
    f. Nearest Railroads x2(If unowned, you may buy it from the Bank.
    g. If owned, pay the owner twice the rental to which they are entitled)

h. **Nearest Railroads x2(If unowned, you may buy it from the Bank. If owned, pay the owner twice the rental to which they are entitled)**
i. Nearest Utility(If unowned, you may buy it from the Bank. If owned, throw dice and pay the owner a total of ten times the amount thrown.
j. **Nearest Utility(If unowned, you may buy it from the Bank. If owned, throw dice and pay the owner a total of ten times the amount thrown**.

10. Your building and loan mature(collect $150)
11. **Your real estate investments increase(collect $150)**
12. Go back 3 spaces
13. **Go back 3 spaces**
14. Make general repairs on all your properties(pay $25 for each house, $100 for each hotel)
15. **Storage facility upkeep(pay $25 for each house, $100 for each hotel)**
16. You have been elected chairman of the board(pay each player $50)
17. **You have been blackmailed. Pay each player $50**
18. Speeding fine(pay $15)
19. **Bribe the police investigator(pay $15)**

- **Types of Community Chest Cards**

1. From the sale of stock(you get $45)
2. **The successful sale of "legal" goods(you get $45)**
3. Grand opera opening(collect $50 from each player)
4. **Blackmail the other gangs(collect $50 from each player)**
5. Go to jail. Go directly to jail, do not pass Go, do not collect $200
6. **Go to jail. Go directly to jail, do not pass Go, do not collect $200**
7. Income tax refund(collect $20)
8. **Commit tax evasion(collect $20)**
9. Pay hospital $100
10. **Bribe the jury(pay $100)**
11. You inherit $100
12. **You won a lawsuit(collect $100)**
13. Advance to Go(collect $200)
14. **Advance to Go(collect $200)**
15. Receive for your services $25
16. **Public donations(collect $25)**
17. Life insurance matures(collect $100)
18. **Successful Ponzi Scheme(collect $100)**
19. Pay a school tax of $150
20. **Bribe the politician to look the other way($150)**
21. Holiday fund matures(collect $150)
22. **Successful Money Laundering scheme(collect $150)**
23. You are assessed for street repairs($40 per house, $115 per hotel)

24. **Keep your facility security up to date**($40 per house, $115 per hotel)
25. Bank error in your favor(collect $200)
26. **Successful bank heist(collect $200)**
27. You have won second prize in a beauty contest(collect $10)
28. **Successful pump and dump scheme(collect $10)**
29. Doctor's fee(pay $50)
30. **Bribe the sheriff to not get caught(pay $50)**
31. It is your birthday(collect $10 from each player)
32. **Successful Coercion(collect $10 from each player)**

Income Tax

If you land here you have two options: You may estimate your tax at $200 and pay the Bank, or you may pay 10% of your total worth to the Bank. Your total worth is all your cash on hand, printed prices of mortgaged and unmortgaged properties, and the cost price of all buildings you own. You must decide which option you will take before you add up your total worth.

Jail

You land in Jail when… (1) your token lands on the space marked "Go to Jail"; (2) you draw a card marked "Go to Jail"; or (3) you throw doubles three times in succession. When you are sent to Jail you cannot collect your $200 salary in that move since, regardless of where your token is on the board, you must move it directly into Jail. Your turn ends when you are sent to Jail. If you are not "sent" to Jail but in the ordinary course of playland on that space, you are "Just Visiting," you incur no penalty, and you move ahead in the usual manner on your next turn. You get out of Jail by… (1) throwing doubles on any of your next three turns; if you succeed in doing this you immediately move forward the number of spaces shown by your doubles throw; even though you had thrown doubles, you do not take another turn; (2) using the "Get Out of Jail Free" card if you have it; (3) purchasing the "Get Out of Jail Free" card from another player and playing it; (4) paying a fine of $50 before you roll the dice on either of your next two turns. If you do not throw doubles by your third turn, you must pay the $50 fine. You then get out of Jail and immediately move forward the number of spaces shown by your throw. Even though you are in Jail, you may buy and sell property, buy and sell houses and hotels and collect rent.

Free Parking

A player landing on this place does not receive any money, property, or reward of any kind. This is just a “free” resting place.

Houses

When you own all the properties in a color group you may buy houses from the Bank and erect them on those properties. If you buy one house, you may put it on any one of those properties. The next house you buy must be erected on one of the unimproved properties of this or any other complete color group you may own. The price you must pay the Bank for each house is shown on your Title Deed card for the property on which you erect the house. The owner still collects double rent from an opponent who lands on the unimproved properties of his/her complete color group. Following the above rules, you may buy and erect at any time as many houses as your judgment and financial standing will allow. But you must build evenly, i.e., you cannot erect more than one house on any one property of any color group until you have built one house on every property of that group. You may then begin on the second row of houses, and so on, up to a limit of four houses to a property. For example, you cannot build three houses on one property if you have only one house on another property of that group. As you build evenly, you must also break down evenly if you sell houses back to the Bank (see SELLING PROPERTY).

Hotels

When a player has four houses on each property of a complete color group, he/she may buy a hotel from the Bank and erect it on any property of the color group. He/she returns the four houses from that property to the Bank and pays the price for the hotel as shown on the Title Deed card. Only one hotel may be erected on any one property.

Building Storages

When the Bank has no houses to sell, players wishing to build must wait for some player to return or sell his/her houses to the Bank before building. If there are a limited number of houses and hotels available and two or more players wish to buy more than the Bank has, the houses or hotels must be sold at auction to the highest bidder

Selling Properties

Unimproved properties, railroads, and utilities (but not buildings) may be sold to any player as a private transaction for any amount the owner can get; however, no property can be sold to another player if buildings are standing on any properties of that color group. Any buildings so located must be sold back to the Bank before the owner can sell any property of that color group. Houses and hotels may be sold back to the Bank at any time for one-half the price paid for them. All houses in one color group must be sold one by one, evenly, in reverse of how they were erected. All hotels in one color group may be sold at once, or they may be sold one house at a time (one hotel equals five houses), evenly, in reverse of how they were erected.

Mortgages

Unimproved properties can be mortgaged through the Bank at any time. Before an improved property can be mortgaged, all the buildings on all the properties of its color group must be sold back to the Bank at half price. The mortgage value is printed on each Title Deed card. No rent can be collected on mortgaged properties or utilities, but rent can be collected on unmortgaged properties in the same group. To lift the mortgage, the owner must pay the Bank the amount of the mortgage plus 10% interest. When all the properties of a color group are no longer mortgaged, the owner may begin to buy back houses at full price. The player who mortgages property retains possession of it and no other player may secure it by lifting the mortgage from the Bank. However, the owner may sell this mortgaged property to another player at any agreed price. If you are the new owner, you may lift the mortgage at once if you wish by paying off the mortgage plus 10% interest to the Bank. If the mortgage is not lifted at once, you must pay the Bank 10% interest when you buy the property and if you lift the mortgage later you must pay the Bank an additional 10% interest as well as the amount of the mortgage.

Bankruptcy

You are declared bankrupt if you owe more than you can pay either to another player or to the Bank. If your debt is to another player, you must turn over to that player all that you have of value and retire from the game. In making this settlement, if you own houses or hotels, you must return these to the Bank in exchange for money to the extent of one-half the amount paid for them; this cash is given to the creditor. If you have mortgaged property you also turn this property

over to your creditor but the new owner must at once pay the Bank the amount of interest on the loan, which is 10% of the value of the property. The new owner who does this may then, at his/her option, pay the principal or hold the property until some later turn, then lift the mortgage. If he/she holds property in this way until a later turn, he/she must pay the interest again upon lifting the mortgage. Should you owe the Bank, instead of another player, more than you can pay (because of taxes or penalties) even by selling off buildings and mortgaging property, you must turn over all assets to the Bank. In this case, the Bank immediately sells by auction all property so taken, except buildings. A bankrupt player must immediately retire from the game. The last player left in the game wins.

Miscellaneous

Money can be loaned to a player only by the Bank and then only by mortgaging property. No player may borrow from or lend money to another player

Map

- ## Railroads
    - Rational Railroad
    - Related Rates Railroad
    - Trigonometric Transcontinental
    - Chain Rule Railroad

- ## Real Estate
    - **Note:** Premium Real Estate is just more expensive, so players need harder questions

    - ### Regular Real Estate
        - (two per region, 16 total)

        - Optimization Consulting
        - Simplification Security

        - Implicit Inspections
        - Inverse Investments

        - Logarithmic's LLC
        - Fundamental Theorem Foundation

        - Factored Financial
        - The First Derivative Fund

        - Limit's Limited
        - Derivative Research and Development Group

        - The Concavity Company
        - Chain Rule Railroad

    - Higher Order Derivatives HVAC
    - Mean Value Theorem Management Consulting

    - The Continuity Company
    - L'Hopital's LLC

- **Premium Real Estate**
    - (one per region, 8 total)

    - Integrated Trigonometry Incorporated
    - Antiderivative Apartments
    - Integral Investments
    - Rolle's Royce Estates
    - Extrema Electricity
    - Newton's News LLC
    - Combination Combatives (math problems with multiple subjects)

Penalty Areas

- **Recession**
    - Pay fees for each piece of real estate
    - if a player lands on it, it affects all players

- **Inflation**
    - only cash can be penalized, not real estate
    - if a player lands on it, it affects all players

- **Unexpected Business Expense**
    - The player is given a card and holds onto it until they pass GO
    - The player is charged 50% of the total income they made from when they got the card to when they pass GO
    - Once the player passes go, they can then return the card to the pile
    - If they miss their payment or forget about it, then when they are caught, they have to pay 20% of their entire net worth (so you better pay attention LOL)

Math Problems

- **(3-5) Per Check Item**

Railroads

- **Normal Trigonometric Problems**

If $f(x) = 2\sin x + 2\cos x$, then

$f'(x) =$ $2\cos(x) - 2\sin(x)$ ✓

If $f(x) = 2x(\sin x + \cos x)$, find

$f'(x) =$ $(2)(\sin(x) + \cos(x)) + (2x)(\cos(x) - \sin(x))$ ✓

Find $\frac{dy}{dx}$ for $y = \frac{\sin(x)}{x^5}$.

$\frac{dy}{dx} =$ $\frac{(x^5)(\cos(x)) - (\sin(x))(5x^4)}{x^{10}}$ ✓

- **Chain Rule**

a. Use the chain rule to find the derivative of $8\sqrt{5x^9 + 10x^5}$

Type your answer without fractional or negative exponents. Use sqrt(x) for $\sqrt{x}$.

$\frac{4(45x^8 + 50x^4)}{\sqrt{(5x^9 + 10x^5)}}$ ✓

b. Use the chain rule to find the derivative of $2e^{-7x^{10}+3x^9}$

Use e^x for e^x.

$2\left(e^{-7x^{10}+3x^9}\right)\left(-70x^9+27x^8\right)$ ✓

c. Find the derivative of: $-5\sin^2(-2x^9)$.

Hint: $\sin^2(x)=[\sin(x)]^2$...so use the **chain rule** (twice!).

$180x^8\left(\sin\left(-2x^9\right)\right)\left(\cos\left(-2x^9\right)\right)$ ✓

- **Problems Involving Rational**

- **Related Rates**

Gravel is being dumped from a conveyor belt at a rate of 10 cubic feet per minute. It forms a pile in the shape of a right circular cone whose base diameter and height are always equal. How fast is the height of the pile increasing when the pile is 23 feet high?

The height of the pile is increasing by 0.0241 ✓ $\frac{\text{ft}}{\text{min}}$ (round your answer to 3 decimal places)

A spherical snowball is melting in such a way that its radius is decreasing at rate of 0.4 cm/min. At what rate is the volume of the snowball decreasing when the radius is 16 cm. (Note the answer is a positive number).

The volume is decreasing by 1286.796 ✓ $\frac{\text{cm}^3}{\text{min}}$ (enter your answer rounded to 3 decimal places)

A circle's radius is increasing at 0.1 feet/minute. How rapidly is its area increasing when the radius is 2 feet?

The circle's area is increasing at 1.257 ✓ $\frac{\text{feet}^2}{\text{minute}}$.

Region 1

- **First Derivative Test**

Use the first derivative test to find any local min/ max of the function:

$f(x) = x^4 + x^3$

Write DNE if there is no min or max.

The local max of $f(x)$ occurs at $x =$ DNE ✓ .

The local min of $f(x)$ occurs at $x =$ $-\frac{3}{4}$ ✓ .

The below function has one local max and one local min. Use the first derivative test to find where they occur.

$f(x) = 2x^2e^x - 5$

The local max of $f(x)$ occurs at $x =$ -2 ✓ .

The local min of $f(x)$ occurs at $x =$ 0 ✓ .

The below function has one local max and one local min. Use the first derivative test to find where they occur.

$f(x) = -2x^3 - 3x^2 + 12x - 8$

The local max of $f(x)$ occurs at $x =$ 1 ✓ .

The local min of $f(x)$ occurs at $x =$ -2 ✓ .

- **Concavity**

Determine the open intervals on which the graph is concave up or concave down. If there is more than one answer, separate them with a comma.

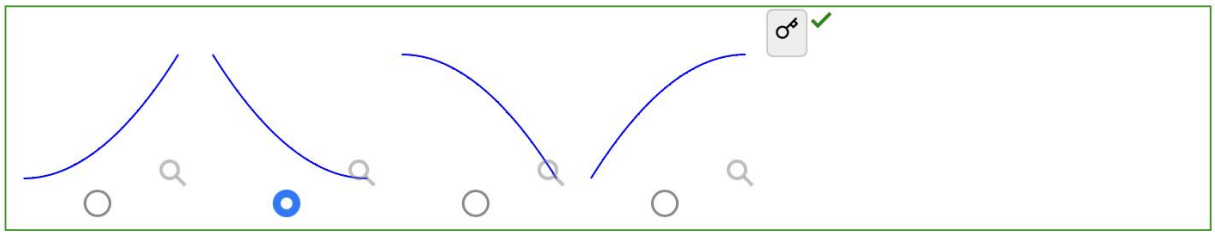
$f(x) = -x^3 + 6x^2 - 9x - 1$

a) $f''(x) =$ $-6x + 12$ ✓

b) $f''(x) = 0$ when $x =$ 2 ✓

c) $f(x)$ is concave up on $(-\infty,2)$ ✓ and concave down on $(2,\infty)$ ✓ .

Suppose that over an interval $f'(x)$ is negative and $f''(x)$ is positive. Which of the following could be a sketch of the graph over that interval?



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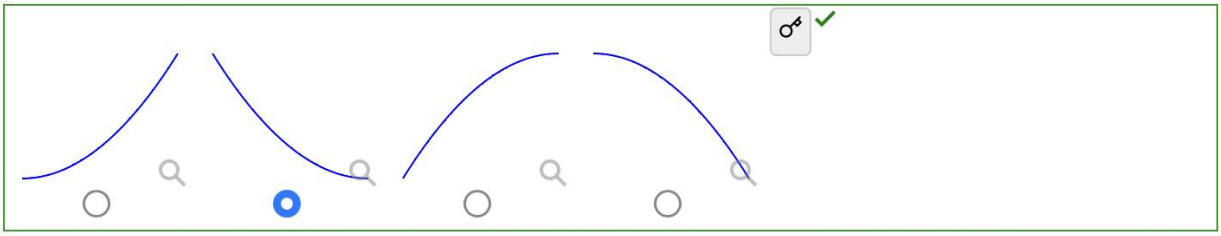


- **Integrated Trigonometry**

Find the standard answer for

$$\int x^1 + \tan(x)\, dx = \frac{x^2}{2} + \ln\left(\left|\sec(x)\right|\right) + C$$

Find the standard answer for

$$\int 5^x + \sin(x)\, dx = \frac{5^x}{\ln(5)} - \cos(x) + C$$

Evaluate the definite integral

$$\int_0^{\pi} 8\sin(x)dx$$

16

Region 2

- **Higher Order Derivatives**

Let $f(x) = \dfrac{1-3x}{1+3x}$.

Then $f'(1)$ is -0.375 ✓

and $f''(1)$ is 0.5625 ✓

$f(x) = x^5 + 9x^4 + 2x^3 + 8x^2 - 6x - 1$

$f'(x) =$ $5x^4 + 36x^3 + 6x^2 + 16x - 6$ ✓

$f''(x) =$ $20x^3 + 108x^2 + 12x + 16$ ✓

Let $f(x) = \dfrac{1-4x}{1+4x}$.

Then $f'(3)$ is -0.0473372781 ✓

and $f''(3)$ is 0.029130632680929 ✓

- **Mean Value Theorem**

Consider the function $f(x) = 5x^3 - 6x$ on the interval $[-5, 5]$. Find the average or mean slope of the function on this interval. 119

By the Mean Value Theorem, we know there exists at least one c in the open interval $(-5, 5)$ such that $f'(c)$ is equal to this mean slope.
For this problem, there are two values of c that work.

The smaller one is -2.88675

and the larger one is 2.88675

Consider the function $f(x) = 1 - 5x^2$ on the interval $[-4, 4]$. Find the average or mean slope of the function on this interval, i.e.

$$\frac{f(4) - f(-4)}{4 - (-4)} = 0$$

By the Mean Value Theorem, we know there exists a c in the open interval $(-4, 4)$ such that $f'(c)$ is equal to this mean slope. For this problem, there is only one c that works. Find it.

0

Consider the function $f(x) = \frac{1}{x}$ on the interval $[2, 11]$. Find the average or mean slope of the function on this interval.

$-\frac{1}{22}$

By the Mean Value Theorem, we know there exists a c in the open interval $(2, 11)$ such that $f'(c)$ is equal to this mean slope. For this problem, there is only one c that works. Find it.

4.690

- **Antiderivatives**

Find the standard answer for

$\int \cos(x) + e^x \, dx =$ $\sin(x) + e^x$ ✓ $+C$

Find $\int \left(\frac{2}{x^5} + 3x + 5 \right) dx$

$-\frac{1}{2x^4} + \frac{3x^2}{2} + 5x$ ✓ + C

Consider the function $f(x) = 5x^{10} + 8x^6 - 2x^4 - 9$.

Enter an antiderivative of $f(x)$. Do not enter $+c$ as part of your answer.

Answer: $\frac{5x^{11}}{11} + \frac{8x^7}{7} - \frac{2x^5}{5} - 9x$ ✓ $+c$

Region 3

- **Optimization**

- **Problems Where You Need to Simplify Things**

a. Let $f(x) = \sqrt{5x+3}$. Expand and simplify:

$\frac{f(x+h) - f(x)}{h} = \frac{5}{\sqrt{5x+5h+3} + \sqrt{5x+3}}$ ✓

For the function $f(x) = \dfrac{5}{x}$, apply the definition of the derivative.

Evaluate each of the following and enter your answers in simplest form:

Step 1 $f(x+h) =$ $\dfrac{5}{x+h}$ ✓

Step 2 $f(x+h) - f(x) =$ $\dfrac{-5h}{x(x+h)}$ ✓

Step 3 $\dfrac{f(x+h) - f(x)}{h} =$ $-\dfrac{5}{x(x+h)}$ ✓

Step 4 $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} =$ $-\dfrac{5}{x^2}$ ✓

So, $f'(x) =$ $-\dfrac{5}{x^2}$ ✓

b.

Let $f(x) = \sqrt{4x - 8}$. Expand and simplify:

$\dfrac{f(x+h) - f(x)}{h} =$ $\dfrac{4}{\sqrt{4x+4h-8} + \sqrt{4x-8}}$ ✓

c.

- **Chain Rule**

Use the chain rule to find the derivative of

$$4\sqrt{5x^5+9x^7}$$

Type your answer without fractional or negative exponents. Use **sqrt(x)** for $\sqrt{x}$.

$$\frac{2\left(25x^4+63x^{(6)}\right)}{\sqrt{5x^5+9x^7}}$$

Use the chain rule to find the derivative of

$$2\left(8x^3+5x^4\right)^{13}$$

You do not need to expand out your answer.

$$26\cdot\left(8x^3+5x^4\right)^{12}\cdot\left(24x^2+20x^3\right)$$

Use the chain rule to find the derivative of

$$5e^{-3x^8-7x^3}$$

Use **e^x** for e^x.

$$5e^{-3x^8-7x^3}\cdot\left(-24x^7-21x^2\right)$$

Region 4

- **Implicit Differentiation**

Use implicit differentiation to find $\frac{dy}{dx}$.

$y^2 + 3x^3 = 8y - 9x^2$

$\frac{dy}{dx} =$ $\frac{9x^2 + 18x}{8 - 2y}$ ✓

Use implicit differentiation to find $\frac{dy}{dx}$ given the equation $\cos(xy) = y^5$.

$\frac{dy}{dx} =$ $\frac{-\sin(xy) \cdot y}{5y^4 - x(-\sin(xy))}$ ✓

Use implicit differentiation to determine $\frac{dy}{dx}$ given the equation $x^3 + y^4 = 10$.

$\frac{dy}{dx} =$ $\frac{3x^2}{-4y^3}$ ✓

- **Inverse Functions**

Differentiate $f(x) = \arctan(6x)$. Use exact values.

$f'(x) =$ $\frac{6}{1 + 36x^2}$

Differentiate $f(x) = \cos^{-1}(4x)$. Use exact values.

$f'(x) =$ $-\dfrac{4}{\sqrt{1-16x^2}}$

Differentiate $f(x) = 5\sin^{-1}\left(\dfrac{x}{5}\right)$. Use exact values. You should simplify the complex fraction.

$f'(x) =$ $\dfrac{5}{\sqrt{25-x^2}}$ ✓

- **Newton's Method**

Use Newton's method to approximate a root of the equation $4x^7 + 6x^4 + 3 = 0$ as follows.
Let $x_1 = 2$ be the initial approximation.

The second approximation x_2 is 1.69203629 ✓

and the third approximation x_3 is 1.419182738 ✓

Use Newton's method to approximate a root of the equation $\ln(5x) = \arctan(x - 0.1)$ as follows.
Let $x_1 = 0.1$ be the initial approximation.

The fourth approximation x_4 is 0.22677 ✓

and the fifth approximation x_5 is 0.2269 ✓

Use Newton's method to approximate a root of the equation $\cos(x^2 + 4) = x^3$ as follows.
Let $x_1 = 2$ be the initial approximation.

The second approximation x_2 is 1.4895 ✓

Region 5

- **Logarithmic-centered Problems**

Find $\frac{d}{dx}(7\ln(x))$

$\frac{7}{x}$ ✓

Let $f(x) = \ln(x^2 - 16x + 66)$

$f'(x)$ =

$$\frac{2x - 16}{x^2 - 16x + 66}$$

Find the first derivative of $f(x) = 7x\ln(4x + 15)$.

$f'(x) =$ $7\left(\ln(4x + 15) + \frac{4x}{4x + 15}\right)$ ✓

- **Indefinite Integrals**

$\int \cos(x) + e^x \, dx =$ $\sin(x) + e^x$ ✓ $+C$

Find $\int \left(\frac{2}{x^5} + 3x + 5 \right) dx$

$-\frac{1}{2x^4} + \frac{3x^2}{2} + 5x$ ✓ + C

Find $\int (x + 7)(x - 5) dx$

$\frac{x^3}{3} + x^2 - 35x$ ✓ + C

- **Rolle's Theorem**

If we try to apply Rolle's Thorem to the function $f(x) = 2x^2 - 20x - 4$ on the interval $[-3, 7]$, which of the following conditions is not met?

- ☐ differentiability on $(-3, 7)$
- ☐ continuty on $[-3, 7]$
- ☑ $f(a) \neq f(b)$

✓

Consider the function $f(x) = \cos(2\pi x)$ on the interval $\left[-\frac{3}{2}, -\frac{1}{2}\right]$. Evaluate this function at the endpoints of the interval.

$f\left(-\frac{3}{2}\right) =$ -1 ✓

$f\left(-\frac{1}{2}\right) =$ -1 ✓

Does Rolle's Theorem apply to f on this interval?

- ○ No
- ◉ Yes

✓

If Rolle's Theorem applies, find c in $\left(-\frac{3}{2}, -\frac{1}{2}\right)$ such that $f'(c) = 0$. If Rolle's Theorem does not apply, enter "DNE".

$c =$ -1 ✓

Consider the function $f(x) = 4x^2 + 24x + 4$ on the interval $[-6, 0]$. Evaluate this function at the endpoints of the interval.

$f(-6) =$ 4 ✓

$f(0) =$ 4 ✓

Does Rolle's Theorem apply to f on this interval?

- ◉ Yes
- ○ No

✓

If Rolle's Theorem applies, find c in $(-6, 0)$ such that $f'(c) = 0$. If Rolle's Theorem does not apply, enter "DNE".

$c =$ -3 ✓

Region 6

- **Problems that Require Factoring**

Evaluate the limit:

$$\lim_{x \to 7} \frac{4x - 28}{x^2 - 10x + 21} = \boxed{1}$$

- **First Derivative Test**

Use the first derivative test to find any local min/ max of the function:

$f(x) = 9x + e^{-5x}$

Write DNE if there is no min or max.

You may use a calculator to evaluate $f'(x)$ at various points, but give an exact answer (no decimal approx).

The local max of $f(x)$ occurs at $x =$ DNE .

The local min of $f(x)$ occurs at $x =$ $-\frac{1}{5}\ln\left(\frac{9}{5}\right)$.

The below function has one local max and one local min. Use the first derivative test to find where they occur. *Give exact answers, no decimals.*

$f(x) = 15x^3 - 9x - 4$

The local max of $f(x)$ occurs at $x =$ $-\sqrt{\frac{9}{45}}$.

The local min of $f(x)$ occurs at $x =$ $\sqrt{\frac{9}{45}}$.

Use the first derivative test to find any local min/ max of the function:

$f(x) = 3\ln(x) - 5x$

Write DNE if there is no min or max.

The local max of $f(x)$ occurs at $x =$ $\frac{3}{5}$.

The local min of $f(x)$ occurs at $x =$ DNE .

Note that the domain of $f(x)$ is $(0, \infty)$; any "test points" must lie in this interval.

- **Extrema On an Interval**

Find the absolute min and max of the function

$f(x) = 2x^2 - 20x + 5$

on the interval $[3, 9]$.

The absolute max of $f(x)$ (on this interval) is -13 ✓ ,

occurring at $x =$ 9 ✓ .

The absolute min of $f(x)$ (on this interval) is -45 ✓ ,

occurring at $x =$ 5 ✓ .

Find the absolute min and max of the function

$f(x) = 30\sqrt{x} - 5x - 2$

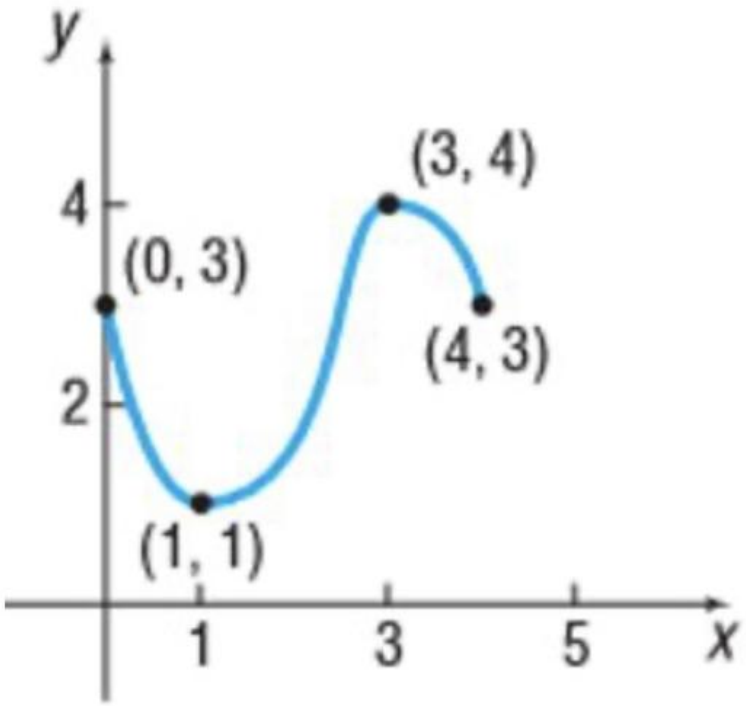
on the interval $[0, 25]$.

The absolute max of $f(x)$ (on this interval) is 43 ✓ ,

occurring at $x =$ 9 ✓ .

The absolute min of $f(x)$ (on this interval) is -2 ✓ ,

occurring at $x =$ 0 ✓ .



relative min: 1, 3

absolute min: 1

relative max: 3, 4

absolute max: 4

Region 7

- **Limits**

a. Estimate the limit numerically or state that the limit does not exist:

$$\lim_{x \to 0} \frac{8\sin(5x)}{\sin(3x)} = 13.333$$

b. Estimate the limit numerically or state that the limit does not exist:

$$\lim_{x \to 0} \frac{6^x - 1}{x} = 1.792$$

c. Let $f(x) = \begin{cases} 2 - x - x^2 & \text{if } x \leq 5 \\ 2x - 38 & \text{if } x > 5 \end{cases}$

Calculate the following limits. Enter "DNE" if the limit does not exist.

$$\lim_{x \to 5^-} f(x) = -28$$

$$\lim_{x \to 5^+} f(x) = -28$$

$$\lim_{x \to 5} f(x) = -28$$

- **Derivatives**

a. Given $f(x) = \frac{3+x}{9-x}$, find $f'(x)$ using the limit definition of the derivative.

$$f'(x) = \frac{12}{(9-x)^2}$$

b. If $h(x) = \frac{7x^2 + 7x - 3}{2 - 5x}$, find:

$$h'(x) = \frac{\left((14x+7)(2-5x) - \left(7x^2+7x-3\right)(-5)\right)}{(2-5x)^2}$$

c. Let $f(x) = 3x^5 - 3x^3 - 10$. Find the equation of the line tangent to the graph of $y = f(x)$ at the point $(1, -10)$.

The equation of the tangent line is $y =$ $6x - 16$ ✓

- **Definite Integrals**

Evaluate the definite integral

$$\int_3^9 \frac{4x^2 + 5}{\sqrt{x}} dx$$

376.538 ✓

Find $\int_{-3}^{4} (6x^7 + 5x^6) dx$

57495.25 ✓

Evaluate the definite integral

$$\int_0^{\pi} 10 \sin(x) dx$$

20 ✓

Region 8

- **Continuity On an Interval**

- **L'Hopital's Rule**

Evaluate the limit $\lim_{x \to \infty} 15xe^{-5x}$ using L'Hôpital's rule.

After applying L'Hôpital's rule, the limit becomes (fill in a function):

$\lim_{x \to \infty}$ $\frac{15}{5e^{5x}}$ ✓

Then the answer is (fill in a number):

$\lim_{x \to \infty} 15xe^{-5x} =$ 0 ✓

Evaluate the limit using L'Hospital's rule

$$\lim_{x \to 0} \frac{e^x - 1}{\sin(2x)}$$

$\frac{1}{2}$ ✓

Use L'Hospital to determine the following limit. Use exact values.

$\lim_{x \to 0^+} (1 + \sin 8x)^{\frac{1}{x}} =$ e^8 ✓

- **Combination (math problems with multiple subjects)**