Supplemental Material for "Complexity-stability relationships disordered dynamical systems"

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Cut-off.— In this section we consider a case amenable to analytical treatment to exemplify the argument behind the cut-off Λ introduced in the main text to deal with diverging moments distributions.

Consider the case of a power law distribution

$$P(x) = \frac{x^{-\beta}}{\mathcal{Z}}, \tag{12}$$

defined from 1 to ∞ and with

$$\mathcal{Z} = \int_1^\infty dx x^{-\beta}. \tag{13}$$

Let us choose $\beta = 3/2$. Notice that this power law describes the behavior of the distribution we consider in the main text for our example with $\alpha = \gamma = 1$ and $\beta = 3/2$. The distribution is normalized with $\mathcal{Z} = 2$, but the mean diverges. However, we would like to be able to describe the behavior of the sample mean

$$\bar{x}_N \equiv \frac{1}{N}\sum_{i=1}^N x_i, \tag{14}$$

where the x_i are extracted from $P(x)$. For this purpose, we can define the quantity

$$\langle x\rangle_\Lambda \equiv \int_1^\Lambda dx P(x) x, \tag{15}$$

with the cut-off Λ defined such that $\int_\Lambda^\infty dx P(x) = 1/N$, i.e., such that there is statistically less than 1 variable with value above Λ out of N extracted variables. For the case $\beta = 3/2$ we have

$$\frac{1}{2}\int_\Lambda^\infty dx x^{-3/2} = \Lambda^{-1/2}, \tag{16}$$

and therefore $\Lambda = N^2$. We have for the mean

$$\langle x\rangle_\Lambda = \frac{1}{2}\int_1^{N^2} dx x^{-1/2} = N - 1. \tag{17}$$

The result is plotted in Fig. 1, alongside the sample mean for extractions of $N = 10$, 10^2, 10^3, 10^4, 10^5, 10^6.

Heterogeneous exponents.— In this section we explore the robustness of our findings in the case in which the exponents characterizing the dynamics of each degree of freedom x_i are not identical for all i.

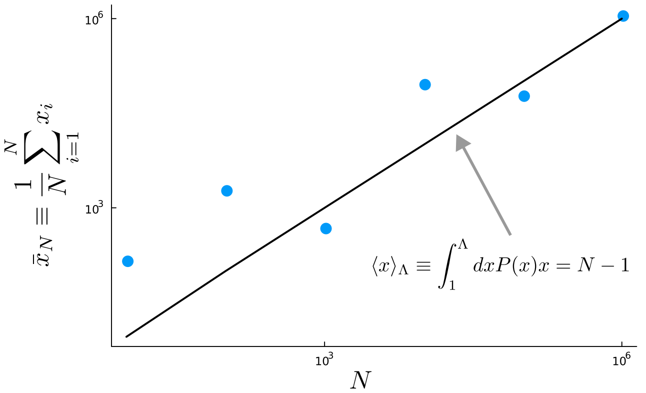

FIG. 1: Probability of stability vs. σ_e for a system with $S = 100$, $\mu = \sigma = 0.01$, $\alpha = \gamma = 1$ and $\beta = 3/2$.

Consider the system

$$\dot{x}_i = x_i^{\alpha_i} - \sum_j A_{ij} x_i^{\beta_i} x_j^{\gamma_i} \tag{18}$$

where the exponents α_i, β_i, γ_i, are extracted from Gaussian distributions: $\alpha_i \sim \mathcal{N}(\alpha, \sigma_e)$, $\beta_i \sim \mathcal{N}(\beta, \sigma_e)$ and $\gamma_i \sim \mathcal{N}(\gamma, \sigma_e)$, respectively with mean α, β and γ, and with the same standard deviation σ_e. The interaction coefficients A_{ij} are drawn independently from a distribution with mean μ and standard deviation σ. Our results are robust when σ_e is small enough and we observe a loss of stability when $\min_i \beta_i > \max_i \alpha_i$. As an example, we show in the plot in Fig. 2 the probability of stability vs. σ_e for a system with $S = 100$, $\mu = \sigma = 0.01$, $\alpha = \gamma = 1$ and $\beta = 3/2$. The probability of stability is obtained as the fraction of stable systems out of 100 realizations for each value of σ_e.

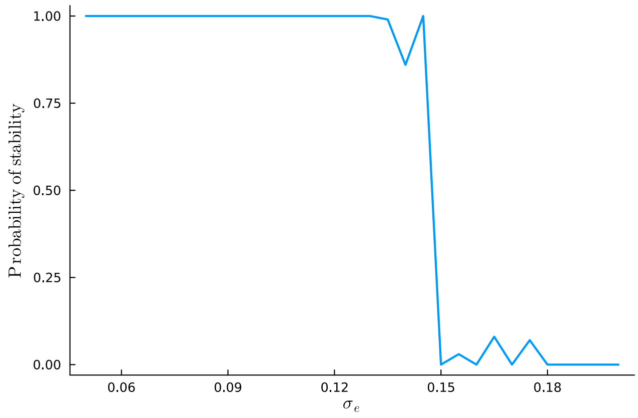

FIG. 2: Probability of stability vs. σ_e for a system with $S = 100$, $\mu = \sigma = 0.01$, $\alpha = \gamma = 1$ and $\beta = 3/2$.