AERO4450 Major Propulsion Assignment: Progress Report (5%)

Introduction

The Indian Defense Research and Development Laboratory and their partner organizations have designed and flight tested a hydrocarbon fuelled Mach 6.5 scramjet aboard the Hypersonic Technology Demonstrator Vehicle (HSTDV) shown in figure 1. The engine uses readily available fuel, materials and manufacturing techniques. The likely application for this engine is a Mach 6.5 cruise missile. Some details of the engine design can be publicly sourced and are provided here. Your task is to ascertain the performance of such an engine.

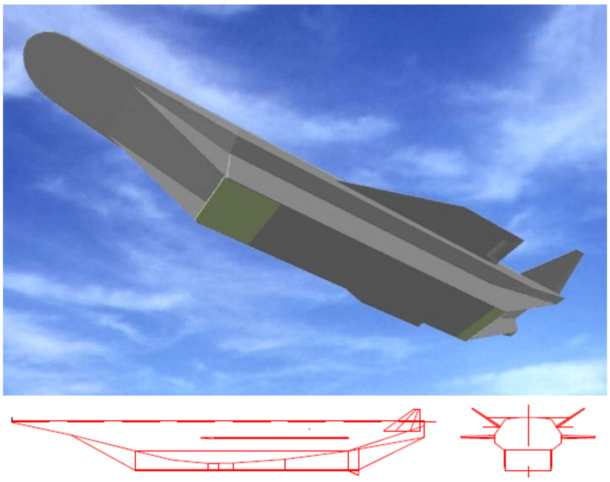


Figure 2: Hypersonic Technology Demonstrator Vehicle (HSTDV), J. Geetha, *ISHARA*, Bangalore, 2012

The engine geometry to be analysed is shown schematically in figure 2. The width of the engine is 0.46 m. Your final goal is to determine whether the thrust of the engine, when stoichiometrically fuelled, exceeds the 3180 N drag the vehicle experiences at a 6° angle of attack. The performance analysis can be most easily completed utilizing Matlab or python scripts as several plots are required to be submitted. Please e-submit copies of any scripts you use.

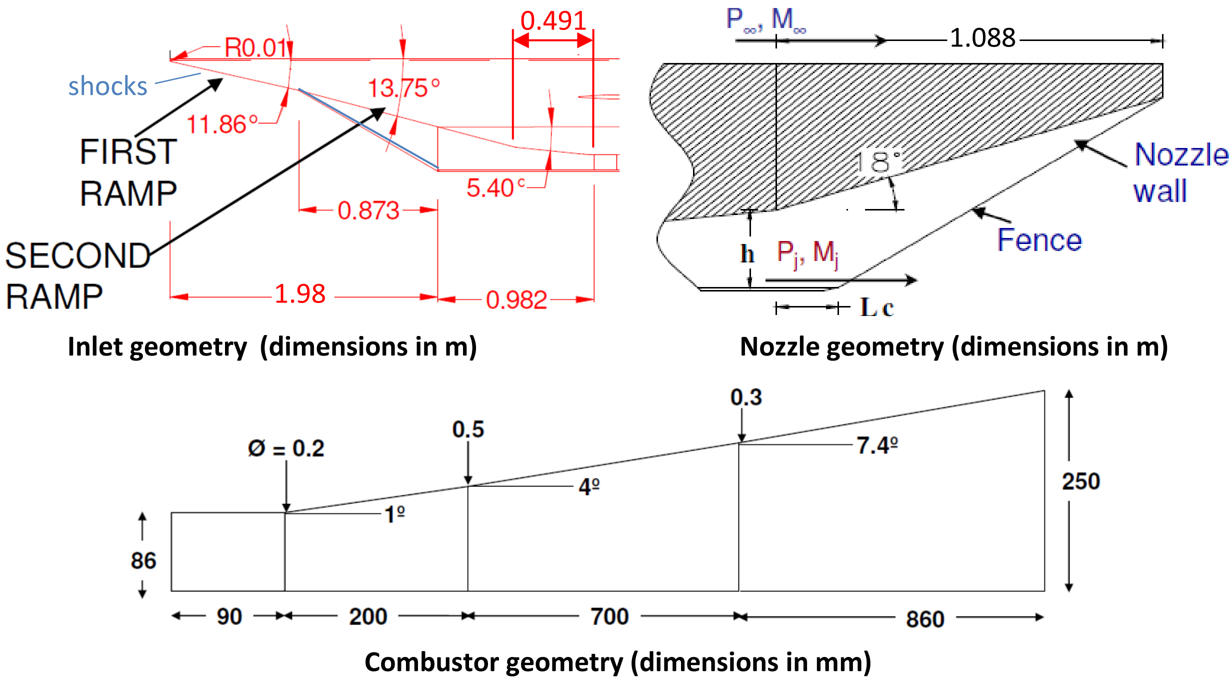


Figure 1: HSTDV engine geometry, S. Panneerselvam, *ISHARA*, Bangalore, 2012 (Note: forebody length increased).

In the Progress Report, you will set-up the combustion modelling based on AERO4450 lectures.

Combustion part for the Progress Report: 5 Marks

Fuel F2 is burned with air as specified by the following stoichiometric reaction

(1) F2+2(O2+bN2) -> 2FO2 + 2bN2

The composition of air is 23% O2 and 77%N2 on mass basis. The molecular mass of the fuel is M_{F2}=44 g/mol, while M_{O2}=32 g/mol and M_{N2}=28 g/mol. The reaction rate is determined by the equation

(2) $$\frac{dY_{\mathrm{FO2}}}{dt} = W = Y_{\mathrm{F2}} Y_{\mathrm{O2}} A \exp\left(-\frac{T_{act}}{T}\right)$$

where Y_S are the mass fractions of species "S", T_{act}=40000K and A=5x10^{11} 1/s. The thermodynamic properties of the species are given in the table (assume constant heat capacities)

Species	Enthalpy of formation (MJ/kg)	Heat capacity Cp (J/kgK)
F2	0	2
FO2	-12	2.2
O2	0	1.09
N2	0	1.17

Fuel and air are supplied to combustion chamber at the rates of 30g/s and 900g/s correspondingly. Combustion is complete. The inlet temperature is 800K.

I. Preliminaries (2 marks)

1. Determine the stoichiometric air/fuel mass ratio and the stoichiometric value of the mixture fraction Z_{st} and the average (i.e. average in the combustion chamber) values of the air/fuel mass ratio and the mixture fraction Z_{av}
2. Determine $Y_{pc}(Z)$ the mass fraction Y_{FO2} of the product FO2 as a function of the mixture fraction Z for conditions when combustion is complete. Plot $Y_{pc}(Z)$ and indicate $Y_{max}=Y_{pc}(Z_{st})$.
3. Determine total enthalpy at Z=0, Zst and 1, as well as the maximal adiabatic temperature T_{max}
4. Consider mass balance and determine constants a_o and a_f in the equation

(3) $Y_{O2} = 0.23 \times (1 - Z) - a_0\, Y_{FO2}$ and $Y_{\mathrm{F2}} = Z - a_f Y_{\mathrm{FO2}}$

Note that Z and Y_{FO2} should stay within the following ranges: $0 \le Z \le 1$ and $0 \le Y_{\mathrm{FO2}} \le Y_{pc}(Z)$ to remain physical. Program a function that returns Y_{O2}, Y_{F2} and Y_{N2} for a given input of Z and Y_{FO2}. Check that the mass fractions of the fuel, oxygen, nitrogen, and the combustion product sum up to unity for any inputs Z and Y_{FO2}. Implement checks that all mass fractions are physical (non-negative). Note that Y_{N2} is a conservative scalar.

5 Implement a function that calculates overall thermodynamic quantities ϕ (per unit mass, where ϕ can be any of h, hf, Cp) in terms of the corresponding properties of the mixture components for given values of the of the mass fraction Y_{F2}, Y_{O2}, Y_{FO2} and Y_{N2} using

(4) $$\phi = \sum_i \phi_i Y_i$$

6 Note that the overall enthalpy of the mixture is a conserved scalar, that is

(5) $$h = h_{Z=0} + Z\left(h_{Z=1} - h_{Z=0}\right)$$

And implement an algorithm evaluating the temperature according to

(6) $$T = T^0 + \frac{h(Z, Y_{FO2}) - h_f(Z, Y_{FO2})}{C_p(Z, Y_{FO2})}$$

and the reaction rate W according to equation (2).

7 Draw a figure showing the temperature versus Z for the reaction progress (the extent of the reaction) $c=Y_{FO2}/Y_{pc}$ taking the values 0, 1/3, 2/3, 1. Draw a figure showing the reaction rate versus Z for c=1/3 and c=2/3. What is W when c=0 or c=1?

II. The flamelet model (3 marks)

Consider the unsteady version of the flamelet model

(7) $$\frac{\partial Y_{\mathrm{FO2}}}{\partial t} - N_{st}\frac{\partial^2 Y_{\mathrm{FO2}}}{\partial Z^2} = W$$

1. Implement a numerical scheme (explicit or implicit) based on finite-difference approximation with 101 grid points in the domain $0 \le Z \le 1$. Solve equation (7) for W=0 as the initial test.

Always use the following initial conditions

(8) $$Y_{\mathrm{FO2}}\big|_{t=0} = Y_{pc}(Z)$$

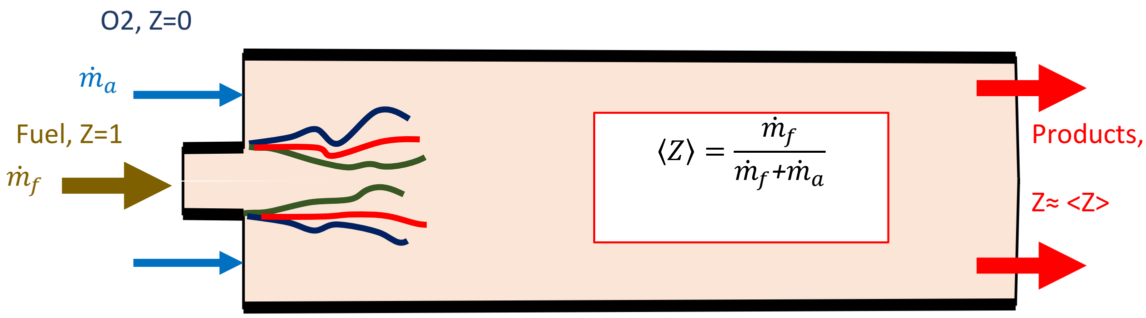
and control the time step to avoid numerical instabilities (without making the time step excessively small).

2. Find a stable solution of equation (7) for a subcritical value of N_{st} by starting from the initial conditions $Y_{\mathrm{FO2}}\big|_{t=0} = Y_{pc}(Z)$, using a finite-difference scheme with 100 grid points across 0<Z<1 and progressing forward in time with a good selection of the CFL number until a steady-state solution is reached.

3. Gradually increase N_{st} until extinction is observed. Report critical value N_{cr} of N_{st}, (i.e. the largest value of N_{st}, not causing extinction). Plot the solution $Y_{FO2}(Z)$ and $T(Z)$ for $N_{st} = N_{cr}$ and indicate adiabatic temperature on the figure.

4. Use critical value $N_{st} = N_{cr}$ to estimate the residence time in the combustion chamber shown required for combustion to be stable

$$\tau_{res} \approx \frac{\langle (Z')^2 \rangle}{2N_{cr}} \quad \text{where} \quad \langle (Z')^2 \rangle = \left(\langle Z^2 \rangle - \langle Z \rangle^2 \right)_{\text{inlet}}$$



Other Instructions: You are required to work in self-arranged groups of up to two students, for both parts of the Major Assignment and submit it as an engineering report via Turnitin on Blackboard. The report will consist of showing the work leading to the expressions required for the calculations performed in this assignment, resulting key results and any plots generated. Please list your assumptions in your write-up. You are also required to submit your scripts (in a scripts format) to the tutors via Turnitin/email.